A Six-Dimensional Hyperchaotic System Selection and Its Application in DS-CDMA System

Yaqin Zhao, Shuying Li, Huan Lian, and Zhilu Wu
School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin, China
Email: {yaqinzhao, wuzhilu}@hit.edu.cn; hitlishuying@gmail.com

Abstract —In this paper, we analyze the hyperchaotic properties of cellular neural network (CNN) systems based on Lyapunov exponents. A four-step algorithm for selecting the hyperchaotic system is proposed. Firstly, calculate the maximum absolute value of time sequences, which is one of the properties of chaotic system, noted as boundedness. Secondly, check another property of chaotic system: sensitive dependence on the initial conditions. Several systems can be identified as non-chaotic systems through above two steps, which saves lots of time to calculate the Lyapunov exponents. Then, calculate the Lyapunov exponents. If the largest Lyapunov exponent is negative, the system is identified as non-chaotic system. Finally, determine the given system is chaotic or not by the strange attractors and the time sequences. The binary hyperchaotic spread spectrum sequences can be generated by a six-dimensional CNN system. Some of the binary hyperchaotic sequences can be used in a direct sequence code division multiple access (DS-CDMA) system after selecting through a special rule. Simulation results prove the effectiveness of the six-dimensional CNN hyperchaotic sequences, compared with the *m*-sequence and second-order Chebyshev polynomial function.

Index Terms—Cellular neural network, hyperchaotic system, lyapunov exponents, DS-CDMA, sensitive dependence on the initial conditions

I. INTRODUCTION

Chaos was defined by T. Li and J. York in 1975 [1]. It was widely studied since Pecora and Carroll achieved the synchronization in identical systems in 1990 [2], [3]. For a given system, there are several methods for identifying the presence of chaotic, at present the main methods are Lyapunov exponents, Kolmogorov entropy, attractor, Poincare section, time domain waveform, and bifurcation diagram [4], [5]. A system is determined to be chaotic system if there is at least one positive Lyapunov exponent when using Lyapunov exponent, or the locus is restricted in a limited phase space and the dynamic behavior looks like disorder when using attractor. Lyapunov exponent provides a quantitative measure of the sensitivity of the system to perturbations of initial conditions.

Rossler proposed the first hyperchaotic system and defined it as a system with two or more positive Lyapunov exponents [6]. Compared with chaotic attractor, the hyperchaotic has much richer dynamical behaviors; therefore, it is widely used in secure communication systems.

Cellular neural network (CNN) was first proposed by Chua and Yang in 1988 [7], [8]. CNN is a system of coupled n-dimension array (n = 1, 2,…) of large number of interconnected nonlinear oscillators [9]. The delay parameter or system parameters enable CNN to generate various kinds of dynamic behaviors, including chaos [10]. Many scholars have developed the chaotic system based on CNN. A strange attractor was produced by a delayed 2-cell CNN in [11]. The Hopf-like bifurcation in the 2-cell autonomous system and the chaos in a 3-cell autonomous CNN were analyzed in [12]. The authors in [13] analyzed the dynamic character of 3-order CNN system and proposed a secure communication system based on this 3-order CNN. The synchronization of 4-order CNN system with hyperchaotic character was studied in [14]. In [15] and [16], the authors proposed the 6-order CNN system. In [17], the authors proposed a model of CNN with transient chaos via adding negative self-feedbacks. In [18], a fractional-order 4-cell CNN was proposed. The authors analyzed the dynamical behaviors, such as periodic, chaotic and hyperchaotic motions.

In a direct sequence code division multiple access (DS-CDMA) system, different users' information signals are separated by different spread spectrum sequences, such as *m*-sequences, gold sequences, and chaotic sequences. However, the *m*-sequences or gold sequences have weak security and insufficient quantity [19]. While, chaotic or hyperchaotic sequences have strong security and can generate lots of spread spectrum sequences. Thus, the chaotic or hyperchaotic sequences are suitable for DS-CDMA system. In [20], the authors studied two different classes of chaotic systems for DS-CDMA system. In [21], the authors proposed a genetic algorithm-based method for the optimal generation of chaotic sequences. In order to improve the performance of chaotic sequences, the authors proposed an optimized selection algorithm based on the balance and correlation [22]. In [23], a new approach to compute the exact bit error rate expression for asynchronous multi-user chaos-based DS-CDMA system over multipath channel was proposed. In [26], the authors proposed a method for generating advanced hyperchaotic complex binary sequences and an optimal filter strategy in 2D-CDMA for best selection of complex spreading sequences.

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Corresponding author email: yaqinzhao@hit.edu.cn.
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In this work, we first propose a new scheme to determine whether the system exhibit chaotic or hyperchaotic phenomena for different parameters. And then generate several binary CNN hyperchaotic sequences as the spread spectrum sequences and apply them in DS-CDMA system. The remainder of this paper is organized as follows. In Section II, we give a brief description of the CNN model. In Section III, we propose a novel algorithm to select chaotic or hyperchaotic systems from numerous of given systems. In Section IV, simulations are given for the proposed hyperchaotic selection scheme. The binary CNN hyperchaotic sequences are generated from a six-dimensional CNN system and applied in a DS-CDMA system in Section V and followed by the concluding remarks in Section VI.

II. CNN Model

CNN was first proposed by Chua and Yang in 1988 [7-8]. The basic unit of CNN is cell. Each cell has a linear resistance, a linear capacitance, and several voltage-controlled current sources. The state equation of each cell is given by.

$$C\frac{dx_{ij}(t)}{dt} = I + \sum_{c_{kl}\in N_r(i,j)} A(i,j;k,l)y_{kl}(t) - \left(\frac{1}{R_x}\right)x_{ij}(t) + \sum_{c_{kl}\in N_r(i,j)} B(i,j;k,l)u_{kl}(t) \tag{1}$$

where R_x is the linear resistor, the output equation $y_{ij}(t) = (|x_{ij}(t)+1| - |x_{ij}(t)-1|)/2$, parameters $A(i, j; k, l)$ and $B(i, j; k, l)$ effect the output feedback and input control, respectively.

Define $E(t)$ as the Lyapunov function of a CNN by the following equation.

$$\begin{aligned} E(t) = &-\frac{1}{2}\sum_{(i,j)}\sum_{(k,l)} A(i,j;k,l)y_{ij}(t)y_{kl}(t) \\ &-\sum_{(i,j)}\sum_{(k,l)} B(i,j;k,l)y_{ij}(t)u_{kl}(t) \\ &+\frac{1}{2R_x}\sum_{(i,j)} y_{ij}(t)^2 - \sum_{(i,j)} I y_{ij}(t). \end{aligned} \tag{2}$$

According to [7], the scalar Lyapunov function is a monotone decreasing function, which is $\mathrm{d}E(t)/\mathrm{d}t \le 0$.

According to previous works on 3-order CNN, 4-order CNN and 6-order CNN systems [12, 15, 16, 24], we adopt the following 6-order CNN system.

$$\begin{aligned} \frac{dx_j}{dt} = &-x_j + a_j f(x_j) + \sum_{k=1,k\ne j}^{6} a_{jk} f(x_k) \\ &+\sum_{k=1}^{6} S_{jk}x_k + I_j \quad (j = 1,2,\cdots 6) \end{aligned} \tag{3}$$

where j is the label of each cell, x_j is the state variable, a_j is a constant, and $f(x_j)=(|x_j+1|-|x_j-1|)/2$ is the output of j-th cell.

Let some parameters are fixed and others are selected from given sets as follows

$$\begin{aligned} &a_{jk} = 0\ (j,k = 1,2,\cdots,6;\ j\ne k) \\ &I_j = 0\ (j = 1,2,\cdots,6) \\ &a_j = 0\ (j = 1,2,3,4,5)\ , \\ &S_{12} = S_{21} = S_{24} = S_{34} = S_{42} = S_{43} = S_{43} = \\ &S_{53} = S_{54} = S_{55} = S_{56} = S_{61} = S_{63} = S_{64} = 0 \end{aligned}$$

Therefore, equation (3) can be written as

$$\begin{cases} \dot{x}_1 = S_{13}x_3 + S_{14}x_4 \\ \dot{x}_2 = S_{22}x_2 + S_{23}x_3 \\ \dot{x}_3 = S_{31}x_1 + S_{32}x_2 \\ \dot{x}_4 = S_{41}x_1 + S_{44}x_4 + S_{45}x_5 + S_{46}x_6 + a_4 f(x_4) \\ \dot{x}_5 = S_{51}x_1 + S_{52}x_2 + S_{55}x_5 \\ \dot{x}_6 = S_{62}x_2 + S_{65}x_5 + S_{66}x_6 \end{cases} \tag{4}$$

where $f(x_4) = (|x_4+1| - |x_4-1|)/2$.

III. Hyperchaotic System Selection

In this section, a new algorithm is proposed for selecting the chaotic or hyperchaotic systems from systems (4) with different parameters. Motivated by "necessity and sufficiency condition in mathematical", we have the following assertion that "*Q* is necessary for *P*" is colloquially equivalent to "*P* cannot be true unless *Q* is true", or "if *Q* is false then *P* is false". For a given chaotic or hyperchaotic system, it is well known that the system has the following properties: the sensitive dependence on initial conditions, the attractors have limited value which is noted as boundedness in this work, at least one positive Lyapunov exponent for chaotic system and two positive Lyapunov exponents for hyperchaotic system. Similarly, we can say that "the sensitive dependence on initial conditions, boundedness, positive Lyapunov exponents" are necessary for "(4) is chaotic or hyperchaotic system", which is similar as necessity and sufficiency condition in mathematical.

A. Prepare for Proposed Algorithm

For a given 6-order CNN system, let $\mathbf{x_0}$ and $\mathbf{x_0}'$ respect for the initial values and the initial values with a very small difference, such as $\mathbf{x_0} = [0.1, 0.2, 0.2, 0.2, 0.2, 0.2]^T$, $\mathbf{x_0}' = [0.1, 0.2, 0.2, 0.2, 0.2, 0.2+10^{-5}]^T$, respectively. The system equations can be solved using 4-order Runge-Kutta method. Let $\mathbf{y}$ and $\mathbf{y}'$ be the solution for a system with initial values $\mathbf{x_0}$ and $\mathbf{x_0}'$. Calculate the maximum absolute value of $\mathbf{y}$ and $\mathbf{y}'$ and note as $y_{\max}$ and $y_{\max}'$, respectively.

$$\begin{aligned} y_{max} &= \max_{i=1}^{6}\max_{j=1}^{N} |y_{ij}| \\ y_{max}' &= \max_{i=1}^{6}\max_{j=1}^{M} |y'_{ij}| \end{aligned} \tag{5}$$

where N and M are the length of sequence $\mathbf{y}$ and $\mathbf{y}'$, respectively.

Let $\mathbf{e}_j = (\mathbf{y}_j - \mathbf{y}_j')^2$, $j \in \{1,2,\ldots,6\}$, and D_j respects for the sensitive dependence on initial conditions, which is given as follows

$$D_j = \log_{10} \sum_{i=1}^{L} e_{i,j} \tag{6}$$

where $L = \min\{M, N\}$ is the length of sequence $\mathbf{e}_j$.

B. Proposed Algorithm

The proposed algorithm is based on the properties of chaotic or hyperchaotic system, which are the sensitive dependence on initial conditions, boundedness, and positive Lyapunov exponents.

First of all, we use the boundedness property. From the point of theory, it is said that chaos is bounded, which means that its orbital motion is always confined in a certain area (chaotic attractor). In mathematical, for a 6-cell system (4), it is given as follows

$$y_{i,j} \le B_{up}, \forall i \in \{1,2,\cdots,6\},\ j \in \{1,2,\cdots,N\} \tag{7}$$

where B_{up} is an infinitely large number. To our best knowledge, B_{up} is smaller than 500, according to exist chaotic or hyperchaotic system and numerous of simulation results. For a given system, we assume that it is not a chaotic system if $y_{max} > B_{up}$.

Secondly, we adopt the sensitive dependence on initial conditions property. It means that there will be huge difference of the results for the same system with a very small difference, such as 10^{-5} [28]. We defined D_j which is given in Eq. (6), to measure this difference. D_j should be a huge number when the sequence length $N \rightarrow \infty$. For a given system, we assume that it is not a chaotic system if $D_j \le B_{low}$, where B_{low} is an infinitely small number.

Then, we use the positive Lyapunov exponents. For a given chaotic or hyperchaotic system, it has at least one positive Lyapunov exponent. Therefore, we select the systems that have positive Lyapunov exponent and delete the systems that don't have positive Lyapunov exponents.

We could not promise the selected systems are chaotic systems through the above three steps, however, we are sure that the deleted systems that don't satisfy the above three conditions are not chaotic systems. Finally, we analyze the attractor and the time sequence waves. If the attractor focus in a limited phase space and looks like disorder and the time sequence waves are not periodic, we determine this system as chaotic or hyperchaotic system. A new algorithm for chaotic or hyperchaotic system selection is proposed according to the above four steps, which is given as Algorithm 1.

Algorithm 1—the proposed algorithm

```
Initialize: x0 and x0' for a given system.
Solve (3) using 4-order Runge-Kutta method, and obtain the solution y and y'.
Find ymax = max(y) and ymax' = max(y').
If  ymax ≥ Bup or ymax' ≥ Bup
    Let LEindex = 0, and LE = -∞.
else
    Let ej = (yj - yj')^2, j ∈ {1,2,…,6}, and
      D = log10 Σ_{i=1}^{L} e_{i,j} , where L is the length of ej and e_{i,j}
      is the i-th element of ej.
    If D ≤ Blow
        Let LEindex = 0, and LE = -∞.
    else
        Calculate Lyapunov exponent λi
        If there exist positive Lyapunov exponents, let
          LE = [λ1, …, λ6]^T
            If attractor looks like disorder and the time
            sequence waves are not periodic,
            LEindex = 1.
            else
            LEindex = 0.
            end If.
        else
            Let LEindex = 0, and LE = -∞.
        end If.
    end If.
end If.
Output  LEindex, and LE.
```

In Algorithm 1, $LE_{index} = 1$ and $LE_{index} = 0$ respect for chaotic and non-chaotic system, respectively.

IV. Simulation Results for Hyperchaotic Selection Algorithm

A. Parameters Setup

In this section, lots of simulation results are given in order to proof the efficiency of the proposed algorithm. We use hp Z800 workstation (Inter Xeon CPU: X5660 @ 2.8GHz, RAM: 48GB) for all of the simulations.

First of all, the parameters are given as follows: $S_{23} = 1$, $S_{31} = 11$, $S_{32} = -12$, $S_{41} = 92$, $S_{44} = -95$, $S_{45} = 1$, $S_{46} = -1$, $S_{51} = 0$, $S_{52} = 0$, $S_{53} = 5$, $S_{55} = -1$, $S_{62} = 5$, $S_{65} = 0$, $S_{66} = -1$ and S_{13}, S_{14}, and a_4 are selected from some given sets. Thus, Eq. (4) is given as follows.

$$\begin{cases} \dot{x}_1 = S_{13}x_3 + S_{14}x_4 \\ \dot{x}_2 = S_{22}x_2 + x_3 \\ \dot{x}_3 = 11x_1 - 12x_2 \\ \dot{x}_4 = 92x_1 - 95x_4 + x_5 - x_6 + a_4 f(x_4) \\ \dot{x}_5 = 5x_3 - x_5 \\ \dot{x}_6 = 5x_2 - x_6 \end{cases} \tag{8}$$

where S_{13}, S_{14}, S_{22} and a_4 are selected from particular sets. Let the infinitely small number $B_{low} = 0.1$ and the infinitely large number $B_{up} = 10^3$.

In order to save time and reduce the calculation complexity, we use the two steps scheme: first of all, we select the hyperchaotic systems with a coarse selection; and then the fine selection as the second step. In the first step, the above four parameters are selected from coarse selection sets, which are given as follows: S_{13} and S_{14} are changing from -3 to 3 with step 1, S_{22} is changing from -3 to 6 with step 1, and a_4 is changing from 20 to 300 with step 20. There are total 11760 (7×7×16×15) systems. However, we find that a small difference with one parameter can also determine the system is chaotic or not with similar Lyapunov exponents by an unexpected experiment. Therefore, we do the following refine experiments with a smaller step. In the second step, we refine the range of each parameter at a narrow sets, which are given as follows: S_{13} is changing from -1.8 to –1 with step 0.1, S_{14} is changing from –1.4 to -0.6 with step 0.1, S_{22} is changing from 1.5 to 2.4 with step 0.1, and a_4 is changing from 180 to 282 with step 2. There are total 42120 (9×9×10×52) systems.

B. *Computation Time Compare*

Case 1: (For compare)

In this case, we just calculate Lyapunov exponents and then observe the attractors and the time sequences to determine whether a given system is chaotic or not. In our simulation system, it will take 20.863 seconds for calculating the Lyapunov exponents for each system. Therefore, it takes about 68 hours and 244 hours for the coarse and fine selection, respectively. It should take almost one minute to observe the time sequences (six figures) and the attractors (six figures). Therefore, it is at least 80 seconds for determine a system is chaotic or not (only Lyapunov exponents, attractors, and time sequences are considered). If all of the above works are done by one person, it will take about 261 and 936 hours for coarse and fine selection, respectively. In this case, it will takes about 119 days (working 10 hours each day). One important thing is that better computer could not help us too much because observing the attractors and time sequences are also time consuming.

Case 2: (Proposed algorithm)

In this case, we do the same simulation with Case 1. The difference with Case 1 is that we adopt the proposed algorithm.

For the coarse selection, first, we use the boundedness property, which is given on lines 4 and 5 in Algorithm 1, for these 11760 systems, and there are 9299 systems are deleted. Then, we deal with the left 2461 systems with the sensitive dependence of initial conditions property, which is given on lines 8 and 9 in Algorithm 1, and there are 2126 systems are deleted. Moreover, calculate the Lyapunov exponents for 335 systems and only 294 systems left for the next step, which is observing the attractors and time sequences. During the above three steps, we can run 12 Matlabs together and finished all of the simulations in one day. And use one more day to observe the attractors and time sequences for 294 systems.

C. *Fine Selection Analyze*

For the fine selection, we adopt similar process as coarse selection. There are 9007 (total 42120) systems are left for observing the attractors and time sequences. We deal with these 9007 systems in the subsection according to D values. After several days observing the attractors and the time sequences, we divided those systems into three ranges according to D values. Let R_1, R_2, and R_3 respect for rang of (0, 3.98), (3.98, 6.03), and (6.03, 50), respectively. There are 2226, 6726, and 55 systems in range R_1, R_2, and R_3, respectively.

We first analyze the systems in range R_1. There are only few chaotic or hyperchaotic systems (about 15%). Then we focus on the 55 systems in range R_3. The simulation results show that there are no chaotic or hyperchaotic systems in range R_3. Finally, we focus on the 6726 systems in range R_2. There are lots of chaotic or hyperchaotic systems according to the proposed algorithm. The projections of the attractors in three dimensional spaces and time sequences that can produce hyperchaotic phenomena are given as Fig. 1 and Fig. 2, respectively.

The system model for this example is given by

$$\begin{cases} \dot{x}_1 = -1.7x_3 - 1.4x_4 \\ \dot{x}_2 = 1.7x_2 + x_3 \\ \dot{x}_3 = 11x_1 - 12x_2 \\ \dot{x}_4 = 92x_1 - 95x_4 + x_5 - x_6 + 180 f(x_4) \\ \dot{x}_5 = 5x_3 - x_5 \\ \dot{x}_6 = 5x_2 - x_6. \end{cases} \tag{9}$$

In Eq. (9), the value of D is equal to 4.9230 and the Lyapunov exponents are: λ_1 = 0.5476, λ_2 = 0.3223, λ_3 = –0.2143, λ_4 = –1.7447, λ_5 = –2.0578, and λ_6 = –88.5710.

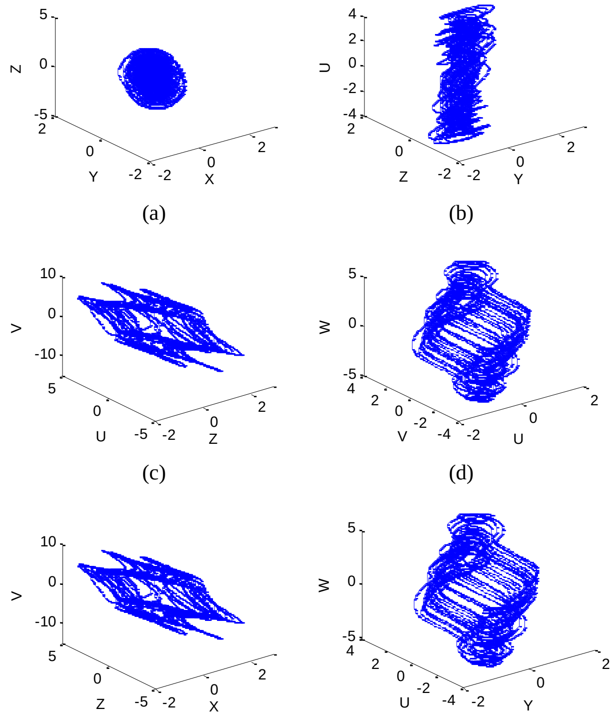

Fig. 1. The projections of the attractors in three dimensional spaces.

From Fig. 1, we can see that the attractors are disordered and does not like periodic signal, while Fig. 2 also imply the property of disorder, boundedness and the sensitive dependence on initial condition. The time sequences in Fig. 2 are changing without any order and sub-figure (f) shows the different values with a small difference 10^{-5}, which is the sensitive dependence on initial conditions property of hyperchaotic system. In Fig. 2(f), the red line and blue line respect for the time sequences of **y** and **y**', respectively. From Fig. 2(f), it can be seen that a small difference is introduced between the initial values, the two signals separate rapidly from each other after a short time period. This simulation results are matched with Fig. 1.1 in [28]. Recall that this system has two positive Lyapunov exponents, thus it is a hyperchaotic system.

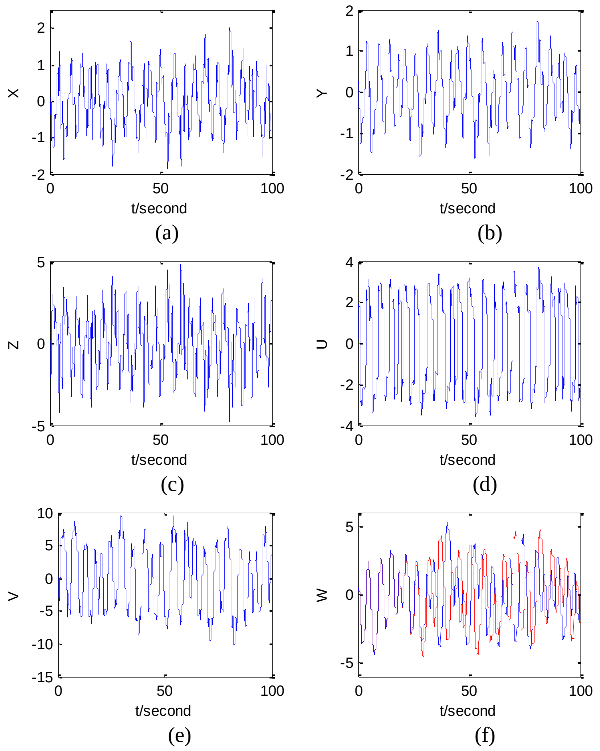

Fig. 2. The time sequences.

V. Application in DS-CDMA

In this section, a hyperchaotic-based *U*-user DS-CDMA system is considered. The spread spectrum sequences are generated by the 6-CNN hyperchaotic system in Eq. (9).

A. Binary Hyperchaotic Sequences Generater

According to Eq. (9), there are six sequences: $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, $x_5(t)$, $x_6(t)$. In this paper, we select $x_1(t)$ as the hyperchaotic sequences. In [27], the authors proposed a novel multilevel quantifying spread spectrum pseudo noise (PN) sequence based on the chaos of CNN. The binary hyperchaotic sequences are generated by the following two steps [4]: first of all, map the sequence $x_1(t)$ into interval [0, 1] as $x_1'(t)$; and then transform the decimal number $x_1'(t)$ into binary number $b_1(t)$, such as

$$x_1'(t) = b_1(t) = 0.b_{11}b_{12}\cdots b_{1T} \tag{10}$$

where $\{b_{1i}\}\in\{0,1\}$, i=1,2,…,T is the binary hyperchaotic sequences. Let N_s be the length of spread spectrum sequence. Thus, for the k-th user, (k = 1, 2, ... , U), a valid spread spectrum sequence SS_k can be given as follows.

$$SS_k = \{\mathrm{b}_{13}(t_0), \mathrm{b}_{13}(t_1), \dots, \mathrm{b}_{13}(t_{N_s})\} \tag{11}$$

B. Fine Selection of Spread Spectrum Sequences

However, sometimes, the spread spectrum sequences that generated by Eq. (11) are not suitable for spread spectrum sequences. Here, a fine rule, named as Algorithm 2, is given in order to generate some valid spread spectrum sequences.

Algorithm 2 — fine selection of spread sequences

```
Initialize: k = 0.
Repeat;
Calculate auto-correlation of SS_k, noted as R_x;
If R_x < η1
    calculate frequency P-value, noted as P1;
    If P1 < η2
        calculate runs P-value, noted as P2;
        If P2 < η3
            If k > 1
                calculate cross-correlation with the stored spread
                  spectrum sequences in matrix SS, and note the
                  maximum value as R_xy;
                If R_xy < η4
                    update k = k + 1,
                    and save it in matrix SS;
                end If;
            else
                update k = k + 1, and save it in matrix SS;
            end If;
        end If;
    end If;
end If;
Until k = U;
output spread spectrum sequences matrix SS.
```

In Algorithm 2, we aim to select the valid hyperchaotic spread spectrum sequences according to the auto-correlation, frequency P-value, runs P-value, and cross-correlation. The frequency P-value and runs P-value are calculated according to [25]. The threshold η_1, η_2, η_3, and η_4 are setting according to the simulation results of *m*-sequence.

C. Performance Analyze in DS-CDMA System

In this part, the selected binary hyperchaotic sequences are used in a multi-user DS-CDMA system. The simulation parameters are given as follows: the length of spread sequences N_s = {31, 63, 127}; the number of users U = {4, 6, 10}; the threshold η_1 = 0.2, η_2 = 0.8, η_3 = 0.8, η_4 = 0.2. In order to compare the BER performance, un-selected binary hyperchaotic sequences by Algorithm 2 and second-order Chebyshev polynomial function (CPF) are given in the simulation results. The differential equation of CPF is given by $x_{j+1} = 1 - 2x_j^2$.

The BER curves are given in Fig. 3 for different spread sequences with the number of users U = 4 and the length of spread sequences N_s = 31. It can be seen that the selected CNN hyperchaotic sequence has better performance than the other three sequences. The un-selected CNN hyperchaotic sequence has the worst performance, which proves the necessary of the fine selection of spread sequences in Algorithm 2. The selected CNN hyperchaotic sequence has better performance than CPF sequence, which satisfies the conclusion in [20].

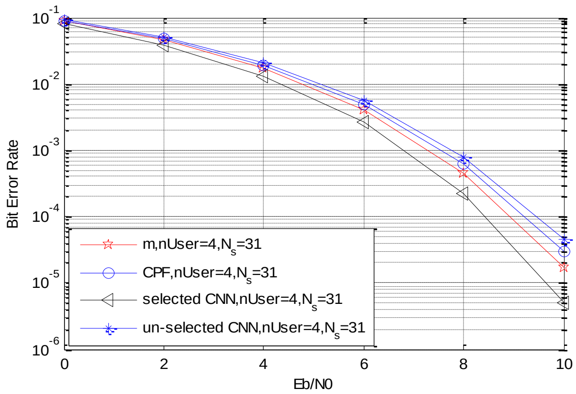

Fig. 3. The BER of DS-CDMA system with U = 4 and N_s = 31 for different spread sequences.

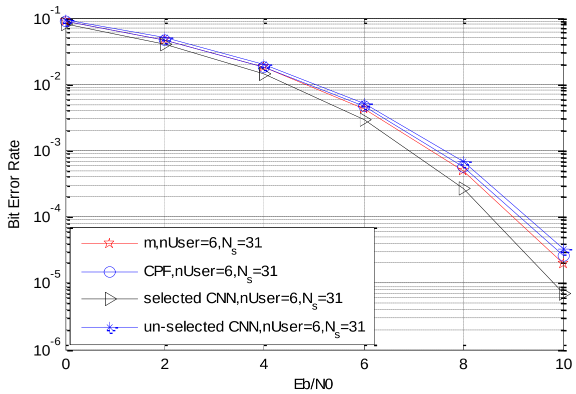

Fig. 4. The BER of DS-CDMA system with U = 6 and N_s = 31 for different spread sequences.

Fig. 4 shows the BER performance of different spread sequences for 6 users with N_s = 31. It can be seen that the selected CNN hyperchaotic sequence has better performance than the others. Note that, for spread spectrum length N_s = 31, the maximum user number for *m*-sequence is six. However, the CNN hyperchaotic sequence and CPF chaotic sequence can serve numerous of users. It means that there is no limitation of the quantity for the CNN spread spectrum sequences. It is very difficult to harvest the CNN spread spectrum sequences by exhaust method. Therefore, the selected CNN hyperchaotic sequences have strong security, which is suitable for security communication.

Fig. 5, Fig. 6 and Fig. 7 show the BER performance for four users, six users and ten users with spread spectrum sequences length N_s = 63 and N_s = 127, respectively. The selected CNN hyperchaotic sequence shows better performance. For six user with N_s = 63 case, the *m*-sequence has worst performance, while in other cases, the *m*-sequence has better performance than un-selected CNN hyperchaotic sequence and CPF chaotic sequence. In other words, the performances of CPF chaotic sequence or un-selected CNN hyperchaotic, maybe, fluctuate sometimes, which shows the importance of selecting the target spread spectrum sequences with Algorithm 2.

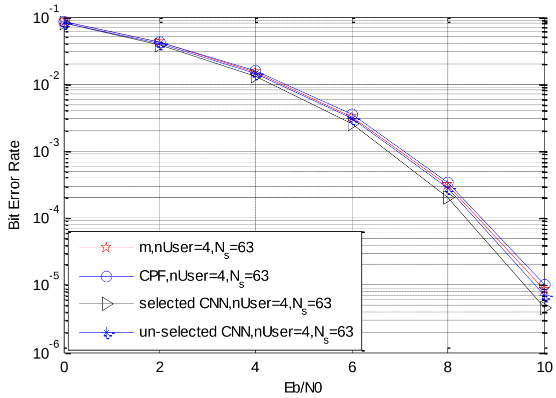

Fig. 5. The BER of DS-CDMA system with U = 4 and N_s = 63 for different spread sequences.

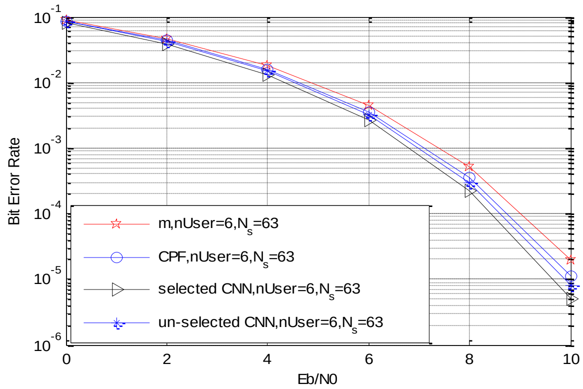

Fig. 6. The BER of DS-CDMA system with U = 6 and N_s = 63 for different spread sequences.

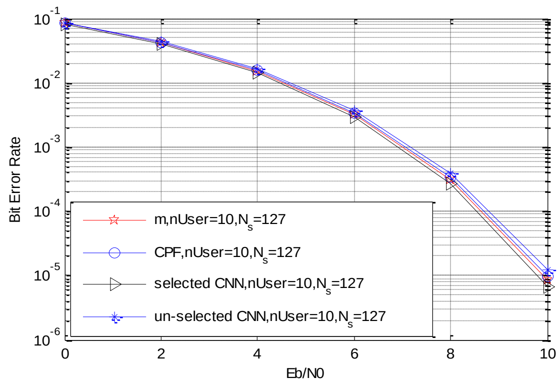

Fig. 7. The BER of DS-CDMA system with U = 10 and N_s = 127 for different spread sequences. s.

From Fig. 4- Fig. 7, we can see that the selected 6-CNN hyperchaotic spread sequences have better performance than m-sequences and CPF sequences. Moreover, from Fig. 2(f), we can see that the output of the sequences are different with a very small initial conditions. In other words, numerous of such 6-CNN hyperchaotic sequences can produced with different initial conditions and the receiver should know the exact values of the initial conditions. Therefore, the 6-CNN hyperchaotic sequences have stronger security than *m*-sequences. Compared with *m*-sequences, the selected 6-CNN hyperchaotic spread sequences i) have stronger security; ii) can serve more users; iii) have better BER performance.

VI. Conclusions

In this paper, we have proposed an effective algorithm to select hyperchaotic system from numerous of 6-CNN systems. And then, for a selected 6-CNN hyperchaotic system, some binary hyperchaotic sequences are generated as spread spectrum sequences in a DS-CDMA system. Simulation results show that the proposed hyperchaotic selection algorithm can save lots of time and effective for selecting hyperchaotic systems. From the simulation results of DS-CDMA system, it can be seen that the selected CNN spread spectrum sequence has better performance than *m*-sequence and CPF chaotic sequence. Compared with *m*-sequence, there is no limitation of the quantity for the selected CNN spread spectrum sequence. The selected CNN hyperchaotic sequences have strong security, which is suitable for security communication.

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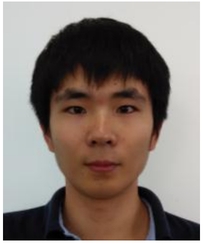
Yaqin Zhao was born in Heilongjiang Province, China, in 1978. She is a professor with the School of Electronics and Information Engineering at Harbin Institute of Technology, Harbin, China. She received the B.S. degree in electronics and information engineering in 1998, M.S. degree in signal and information processing in 2000, and Ph.D. degree in information and communication engineering in 2008, respectively, from the Harbin Institute of Technology (HIT). Her research interests include wireless communication, softer radio, CDMA, spectrum sensing in cognitive radio, wireless video communication, and chaotic theory.

Shuying Li was born in Hebei Province, China, in 1984. He received the B.S. degree in Electronic and Information Engineering (Underwater Acoustics) from Harbin Engineering University, Harbin, in 2008 and the M.S. degree in Information and communication engineering from HIT, in 2010. During Mar. 2011 and Mar. 2013, he did his research as a visiting student in Electrical Engineering Department, Columbia University, New York, US. He is currently pursuing the Ph.D. degree with School of Electronics and Information Engineering, HIT. His research interests include spectrum sensing in cognitive radio, wireless video communication, and chaotic theory.



Huan Lian was born in Hebei Province, China, in 1990. He received the B.S. degree in Electronic and Information Engineering from HIT in 2013. He is currently studying for the M.S. degree with School of Electronics and Information Engineering, HIT. His research interests include spread spectrum communication, cellular neural network and chaos communication.

Zhilu Wu was born in Heilongjiang Province, China, in 1961. He is a professor with the School of Electronics and Information Engineering and the dean of the Information Engineering Department at HIT, Harbin, China. He received the B.S. degree in electronic instrument and measurement technology in 1983, the M.S. degree in signal and information processing in 1989, and the Ph.D. in information and communication engineering in 2008 from HIT. His research interests are wireless communication, spectrum sensing in cognitive radio, cognitive radio engine design, software radio, and artificial neural networks.