

The Least Squares Estimation and Complementary Kalman Filtering Methods of Delays in Antenna Arraying for Deep Space Communications

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Abstract—The estimation accuracy of antenna delay is one of the most important parameters for arraying combining performance in deep space network. The least squares estimation method of delays is presented, considering the geometric relation of delays estimated by cross-correlation, and theoretical analysis of the method is also presented. The complementary Kalman filtering method of delays is also presented, according to the different characteristics and inherent between delays and phase differences. Theoretical analysis and simulation results show that the two methods can both greatly improve the estimation accuracy of the delays. For the given case, the accuracy of delays improves about two orders of magnitude after the least squares filtering. The delay errors can also be greatly reduced after the complementary Kalman filtering. The estimation accuracy can be further improved, if the two methods are properly combined.

Index Terms—Antenna arraying, delay, least squares estimation, complementary kalman filtering

I. INTRODUCTION

As the signal from the deep-space spacecrafts become weaker and weaker, the need arises to compensate for the reduction in signal-to-noise ratio (SNR) [1]-[3]. With maximum antenna apertures and lower receiver noise temperatures pushed to their limits, one effective method for improving the effective SNR is to combine the signals from several antennas. Arraying holds many tantalizing possibilities: better performance, increased operational robustness, implementation cost saving, more programmatic flexibility, and broader support to the science community [4], [5].

The output of an array is a weighted sum of the input signals applied to the combiner. The residual delay estimation accuracy between the signals has a direct impact on the combine performance, and the higher the code rate, the higher the required delay accuracy. With the development of deep space exploration, the demand

for downlink code rate is growing rapidly. Currently, the maximum bit rate of the Deep Space Network can reach 20Mbps in Mars exploration(From the Earth 0.6Au), and may be up to 400Mbps(X-band) and 1.2Gbps(Ka-band) in 2020 [6]. Such a high bit rate requires high precision delay.

For the array composed of a large number of small antennas, which is usually more than one hundred, it is difficult to get enough delay precision only estimated by cross-correlation without high precision spacecraft orbit data. Therefore, the least squares estimation method of delays is presented, considering the geometric relation of delays estimated by cross-correlation. The complementary Kalman filtering method of delays is also presented, according to the different characteristics and inherent between delays and phase differences. Finally, theoretical analysis and simulation of the two methods is presented.

II. THE INFLUENCE OF DELAY ERRORS TO COMBINING PERFORMANCE IN ANTENNA ARRAYING

The delay errors can directly reduce the combining SNR and also affect the communication signal waveform, which affecting the signal demodulation performance, especially in the case of high bit rate.

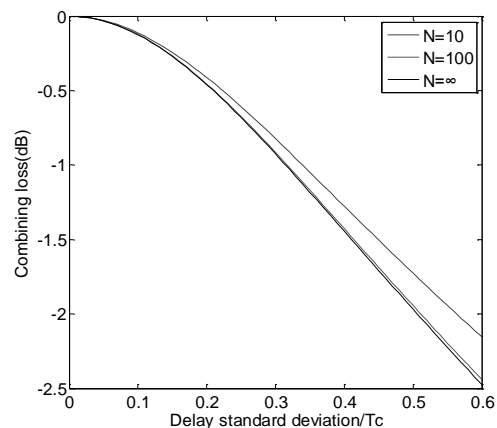


Fig. 1. Combining loss in the case of different delay standard deviation.

The combining loss of uniform array can be estimated as [7]:

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$$D_s = 10 \log \left[\frac{1/L + (1-1/L)/\sqrt{k_0^2 + 1}}{\sqrt{k_0^2 + 1}} \right]$$

where L is the number of antennas, k_0 is the normalized standard deviation of delay error. In particular, when L tends to infinity, the combining loss becomes

$$\lim_{L \rightarrow \infty} D_s = -5 \log(k_0^2 + 1)$$

Fig. 1 shows the combining loss in the case of different delay standard deviation, where the number of array antennas is respectively 10, 100 and infinity. It can be seen from that the number of antenna has some influence on the combining loss, the greater the number of antennas the greater the combining loss, and eventually approaches to the solid line. Calculation results show that the standard deviation of delay should be less than 0.09 times code width (T_c), to ensure the combining loss caused by the delay error less than 0.1dB.

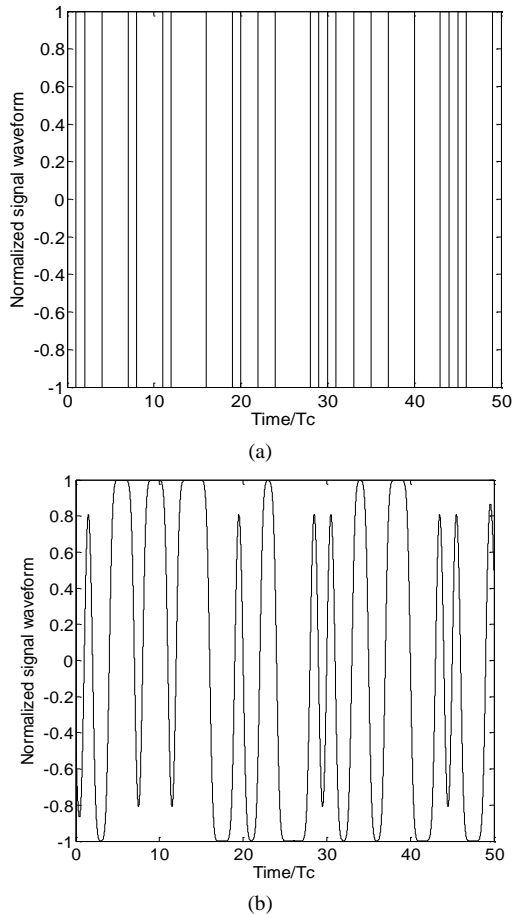


Fig. 2. The impact of delay errors to combined signal waveform. (a) Original signal waveform, (b) Combined signal waveform.

Assuming the array is composed by a large number of antennas, which are all equal in size and performance, the function of combining signal waveform is equivalent to [7]:

$$\tilde{c}(t) = (c * g)(t)$$

where $c(t)$ is the code signal function, $\tilde{c}(t)$ is the

combining signal waveform, $g(t)$ is Gaussian function with the standard deviation of delay error σ_τ , namely

$$g(t) = \left(\frac{1}{2\pi\sigma_\tau^2} \right)^{1/2} \int_{-\infty}^{\infty} e^{-\frac{\gamma^2}{2\sigma_\tau^2}} d\gamma$$

An example of signal waveform before and after combining is shown in Fig. 2. The waveform of original signal which is randomly selected is shown in Fig. 2 (a), and the combined signal waveform is shown in Fig. 2 (b). The delay standard deviation is taken as $0.3T_c$, and the number of antennas is 100.

It can be seen delay errors have an impact on the combined signal waveform. When the array is composed by a large number of antennas, the impact of delay errors to combined signal waveform can be equivalent to Gaussian filtering. Because the Gaussian filter doesn't meet the Nyquist criteria, delay errors can cause intersymbol interference.

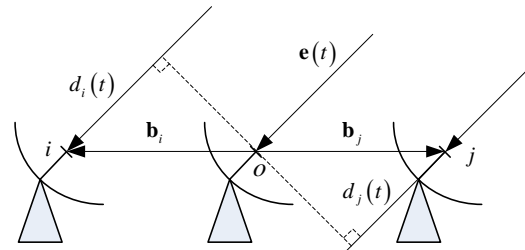


Fig. 3. Signal receiving diagram of antenna array

III. THE LEAST SQUARES ESTIMATION AND PERFORMANCE ANALYSIS OF SIGNAL DELAYS

As the spacecraft is generally very far from Earth, when the distance between antennas is small, the signal can be as far-field. In other words, the signal direction of arrival (DOA) of every antenna is the same. The signal receiving diagram of antenna array is shown in Fig. 3, where \mathbf{b}_i and \mathbf{b}_j represent the baseline vectors of the antenna i and j relative to the phase center o of the array, $\mathbf{e}(t)$ represents the unit vector of DOA at time t , $d_i(t)$ and $d_j(t)$ represent the distance difference of the signal received by the antenna i and j relative to the phase center o . So the distance difference $d_i(t)$, baseline vector \mathbf{b}_i and unit vector of DOA $\mathbf{e}(t)$ satisfy the following formula [8]:

$$d_i(t) = \mathbf{b}_i \cdot \mathbf{e}(t) \tag{1}$$

It can be seen from Eq. (1) that $\mathbf{e}(t)$ is the same for all antennas. As baseline vector \mathbf{b}_i ($i=1, \dots, L$) can be accurately measured in advance, the delay estimation accuracy can be greatly improve by proper filtering method for the delay calculated by correlator, without additional correlator added. Firstly, $\mathbf{e}(t)$ can be

accurately estimated from $d_i(t) (i=1, \dots, L)$ using the least squares estimation. Then more accurate estimates $\hat{d}_i(t)$ of $d_i(t)$ can be calculated using Eq. (1).

A. The Least Squares Estimation Method

From Eq. (1), delay measurements and the DOA can be expressed as

$$d_{\tau i}(t) = \mathbf{b}_i \cdot \mathbf{e}(t) + n_{\tau i} \quad (2)$$

where $n_{\tau i}$ is delay measurement error and $d_{\tau i}(t)$ delay measurement of the antenna i . Assuming all antennas are located in $x-y$ plane of the coordinate system, $n_{\tau i}$ is normal distribution with zero mean and independent with each other, and $d_{\tau i}(t)$ is expressed in the form of distance. Eq. (2) can be re-written in matrix form that

$$\mathbf{D}_{\tau}(t) = \mathbf{B}\mathbf{e} + \mathbf{N}_{\tau} \quad (3)$$

where

$$\begin{aligned} \mathbf{D}_{\tau}(t) &= [d_{\tau 1} \quad d_{\tau 2} \quad \dots \quad d_{\tau L}]^T \\ \mathbf{B} &= \begin{bmatrix} b_{1x} & b_{2x} & \dots & b_{Lx} \\ b_{1y} & b_{2y} & \dots & b_{Ly} \end{bmatrix}^T \\ \mathbf{e} &= [e_x \quad e_y]^T \\ \mathbf{N}_{\tau} &= [n_{\tau 1} \quad n_{\tau 2} \quad \dots \quad n_{\tau L}]^T \end{aligned}$$

where b_{ix} and b_{iy} are components of baseline vector \mathbf{b}_i with respect to x and y axis, e_x and e_y are the direction cosines with respect to the x and y axes and L is the antenna number of array.

Since there are lots of antennas in array, DOA can be estimated from using the least squares estimation method. Then more accurate delay estimates can be gotten from this DOA estimate.

Assuming the baseline matrix \mathbf{B} is known and measurement error can be ignored, by the least squares estimation method, we get the following estimate of $\mathbf{e}(t)$

$$\hat{\mathbf{e}}(t) = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{D}_{\tau}(t) \quad (4)$$

Replace $\mathbf{e}(t)$ with $\hat{\mathbf{e}}(t)$ of Eq. (3). We get the following estimate of $\mathbf{D}_{\tau}(t)$

$$\hat{\mathbf{D}}_{\tau}(t) = \mathbf{B}\hat{\mathbf{e}}(t) \quad (5)$$

From Eq. (4), the Covariance matrix of $\hat{\mathbf{e}}_{\tau}(t)$ can be expressed as

$$\mathbf{M}_{\hat{\mathbf{e}}_{\tau}} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{C}_{\tau_n} \mathbf{B} (\mathbf{B}^T \mathbf{B})^{-1} \quad (6)$$

where \mathbf{C}_{τ_n} is the covariance matrix of initial delay estimation error, and

$$\mathbf{C}_{\tau_n} = E(\mathbf{N}_{\tau} \mathbf{N}_{\tau}^T)$$

So the Covariance matrix of $\hat{\mathbf{D}}_{\tau}(t)$ can be expressed as

$$\mathbf{M}_{\hat{\mathbf{D}}_{\tau}} = \mathbf{B} \mathbf{M}_{\hat{\mathbf{e}}_{\tau}} \mathbf{B}^T = \mathbf{G} \mathbf{C}_{\tau_n} \mathbf{G} \quad (7)$$

where

$$\mathbf{G} = \mathbf{B} (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T$$

B. Performance Analysis of the Least Squares Estimation

Assuming all the antennas are equal in size and performance, matrix \mathbf{C}_{τ_n} becomes

$$\mathbf{C}_{\tau_n} = \sigma_{\tau}^2 \mathbf{E}$$

where σ_{τ}^2 is the variance of initial delay estimates. From Eq. (7) the covariance matrix $\mathbf{M}_{\hat{\mathbf{D}}_{\tau}}$ becomes

$$\mathbf{M}_{\hat{\mathbf{D}}_{\tau}} = \sigma_{\tau}^2 \mathbf{G}$$

To study the nature of the matrix $\mathbf{M}_{\hat{\mathbf{D}}_{\tau}}$, the matrix \mathbf{F} is defined as

$$\mathbf{F} = \mathbf{E} - \mathbf{G} \quad (8)$$

where \mathbf{E} is unit matrix. It is easy to verify the following fomula

$$\mathbf{F}^2 = \mathbf{F}$$

It can be seen \mathbf{F} is an idempotent matrix. Because any idempotent matrices are positive semi-definite [9], the following formula can be obtained

$$f_i \geq 0, \quad i = 1, 2, \dots, L \quad (9)$$

where $f_i (i = 1, 2, \dots, L)$ is the diagonal elements of matrix \mathbf{F} . Assuming $g_i (i = 1, 2, \dots, L)$ is the diagonal elements of matrix \mathbf{G} , the following formula is gotten

$$0 < g_i \leq 1, \quad i = 1, 2, \dots, L \quad (10)$$

According to the nature of the matrix trace, we get the following equation

$$\sum_{i=1}^L g_i = \text{trace}(\mathbf{G}) = \text{trace}[\mathbf{B}^T \mathbf{B} (\mathbf{B}^T \mathbf{B})^{-1}] = 2 \quad (11)$$

It can be seen from Eq. (10) and Eq. (11) that the diagonal elements of matrix \mathbf{C}_{τ_n} , the delay variance after the least squares estimation, are greater than 0 and less than σ_{τ}^2 , and the sum of which is equal to $2\sigma_{\tau}^2$. It proves the least squares estimation method can significantly reduce the time delay error compared with the initial estimation error, for the large uniform antenna array.

IV. THE COMPLEMENTARY KALMAN FILTERING OF SIGNAL DELAYS

If the Eq. (1) is described by the phase difference rather than delay, then we get

$$\mathbf{D}_\varphi(t) = \mathbf{B}\mathbf{e} + \mathbf{M}_\varphi\lambda + \mathbf{N}_\varphi \quad (12)$$

where

$$\begin{aligned} \mathbf{D}_\varphi(t) &= [d_{\varphi_1} \quad d_{\varphi_2} \quad \cdots \quad d_{\varphi_L}]^T \\ \mathbf{M}_\varphi &= [m_{\varphi_1} \quad m_{\varphi_2} \quad \cdots \quad m_{\varphi_L}]^T \\ \mathbf{N}_\varphi &= [n_{\varphi_1} \quad n_{\varphi_2} \quad \cdots \quad n_{\varphi_L}]^T \end{aligned}$$

where m_{φ_i} is the carrier-cycle integer ambiguities, n_{φ_i} is phase difference measurement error and d_{φ_i} is phase difference measurement of the antenna i . Assuming all antennas are located in $x-y$ plane of the coordinate system, n_{φ_i} is normal distribution with zero mean and independent with each other, and d_{φ_i} is expressed in the form of distance.

In the previous discussion, two models are established which respectively described by delay and phase difference in Eq. (2) and Eq. (12). The first model is based on the low noise but ambiguous carrier phase difference measurements, and the second one is formed from the unambiguous but noisier delay measurements. The two models of measurements can be combined to produce smoothed and more accurate delay measurements.

Reference the complementary Kalman filter of differential GPS [10], the delay filter equations can be as follows:

$$\begin{aligned} \mathbf{D}_{s_n}^- &= \mathbf{D}_{s_{n-1}}^+ + (\mathbf{D}_{\varphi_n} - \mathbf{D}_{\varphi_{n-1}}) \\ \mathbf{P}_n^- &= \mathbf{P}_{n-1}^+ + \mathbf{Q} \\ \mathbf{K}_n &= \mathbf{P}_n^- (\mathbf{P}_n^- + \mathbf{R})^{-1} \\ \mathbf{D}_{s_n}^+ &= \mathbf{D}_{s_n}^- + \mathbf{K}_n (\widehat{\mathbf{D}}_{\tau_n} - \mathbf{D}_{s_n}^-) \\ \mathbf{P}_n^+ &= (\mathbf{E} - \mathbf{K}_n) \mathbf{P}_n^- \end{aligned} \quad (13)$$

where \mathbf{E} is a unit matrix, $\mathbf{Q} = \text{diag}(q_1 \quad q_2 \quad \cdots \quad q_L)$ is the variance matrix of phase difference measurements and $\mathbf{R} = \text{diag}(r_1 \quad r_2 \quad \cdots \quad r_L)$ is the variance matrix of delay measurements.

The first equation of Eq. (13) propagates the smoothed delay $\mathbf{D}_{s_{n-1}}^+$ to the current time epoch n using the previous epoch $n-1$ and the difference of the phase difference $(\mathbf{D}_{\varphi_n} - \mathbf{D}_{\varphi_{n-1}})$ across the current and past epochs. The estimate $\mathbf{D}_{s_n}^+$ centers the averaging of the delay \mathbf{D}_τ , and the \mathbf{D}_φ (phase difference) difference adds the latest low-noise information. Note that differencing two phase difference across an epoch removes the integer ambiguity, thus the estimate of delay remains unambiguous, but the measurement noise is greatly reduced. The estimation error variance \mathbf{P}_n^- is brought forward in the second equation, using its previously estimated value plus the variance matrix of the

phase difference \mathbf{Q} . In the third equation the Kalman gain \mathbf{K}_n is calculated in preparation for weighting the effect of the current delay measurement. It can be seen that when the delay variance approaches zero, the Kalman gain tends to unity. Because the higher the accuracy of a measurement, the greater is its effect on the outcome of the process. In the last two equations, the estimate of the smoothed delay $\mathbf{D}_{s_n}^+$ and estimation error variance \mathbf{P}_n^+ are propagated to the current epoch n in preparation for repeating the process in the next epoch. $\mathbf{D}_{s_n}^+$ involves the sum of the current value of the smoothed delay $\mathbf{D}_{s_n}^-$ and its difference from the current delay \mathbf{D}_{τ_n} weighted by the Kalman gain. Intuitively, if the prediction is accurate, then there is little need to update it with the current measurement.

From the above analysis, the least squares estimation method can significantly improve the estimation accuracy of delays. If the complementary Kalman filter is combined with the least squares estimation method, the estimation accuracy can be further improved, then the expressions of Eq. (13) become

$$\begin{aligned} \mathbf{D}_{s_n}^- &= \mathbf{D}_{s_{n-1}}^+ + (\mathbf{D}_{\varphi_n} - \mathbf{D}_{\varphi_{n-1}}) \\ \mathbf{P}_n^- &= \mathbf{P}_{n-1}^+ + \mathbf{Q} \\ \mathbf{K}_n &= \mathbf{P}_n^- (\mathbf{P}_n^- + \mathbf{R})^{-1} \\ \mathbf{D}_{s_n}^+ &= \mathbf{D}_{s_n}^- + \mathbf{K}_n (\widehat{\mathbf{D}}_{\tau_n} - \mathbf{D}_{s_n}^-) \\ \mathbf{P}_n^+ &= (\mathbf{E} - \mathbf{K}_n) \mathbf{P}_n^- \end{aligned} \quad (14)$$

where $\widehat{\mathbf{D}}_{\tau_n} = \mathbf{G}\mathbf{D}_{\tau_n}$.

V. SIMULATION ANALYSIS

Fig. 4(a) shows an antenna array composed of 275 antennas, which are distributed in 10 concentric circles centered on the coordinate origin. The distance between adjacent concentric circles is 50 meters, and the arc length between adjacent antennas on the same circle is 62.8 meters. Assuming all antennas are equal in size and performance, the delay variances of all antennas are one, and the phase center is the coordinate origin.

The delay variances after the least squares filtering are showed in Fig. 4(b), where the greater the serial number of antenna is, the closer it is from the phase center. It can be seen that the least squares method can significantly reduce the delay errors and the delay variances of the antennas which are near the phase center are smaller than those far away. Fig. 4(c) and Fig. 4(d) respectively show the delay errors of the No. 250 antenna before and after filtering. The variance before filtering is $0.2T_c$ in Fig. 4(d), where T_c is the code wide. It can be seen from Fig. 4(c) and 4(d) the estimation errors are significantly reduced after filtering, and the precision is improved about two orders of magnitude.

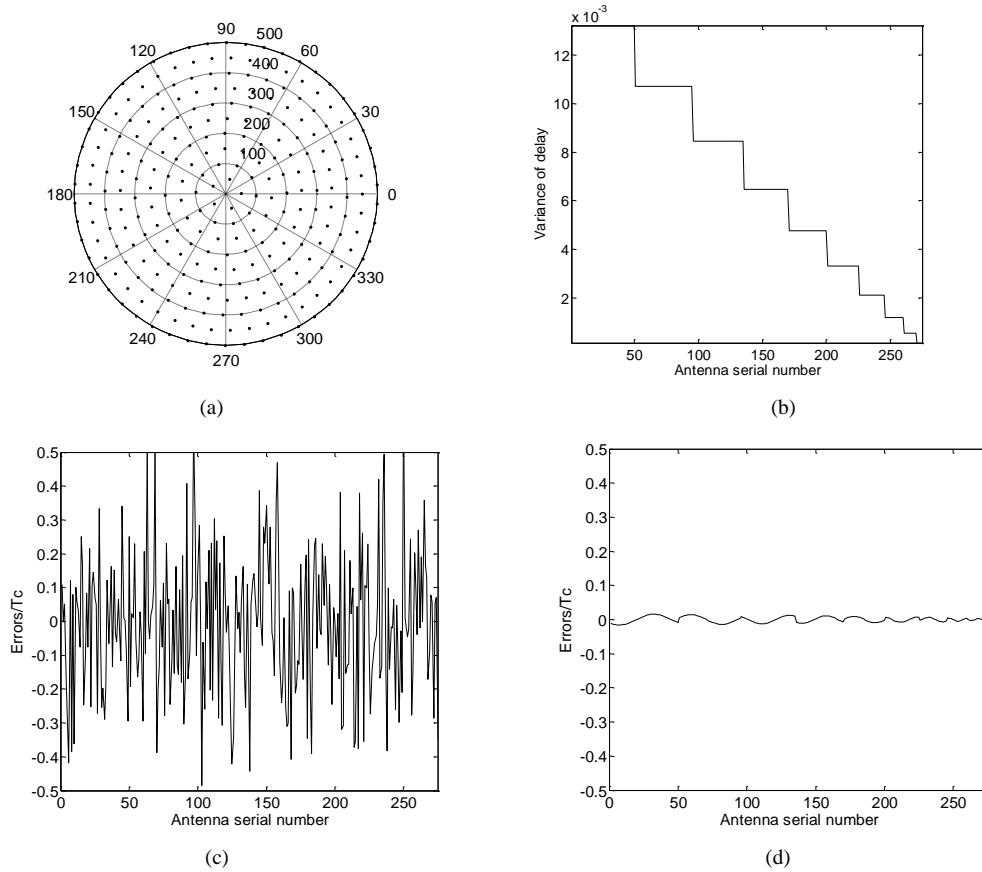


Fig. 4. The estimation errors of antenna No. 250 before and after the least squares filtering. (a) The antenna distribution of array, (b) The variances of delay after filtering, (c) The delay errors before filtering, (d) The delay errors after filtering

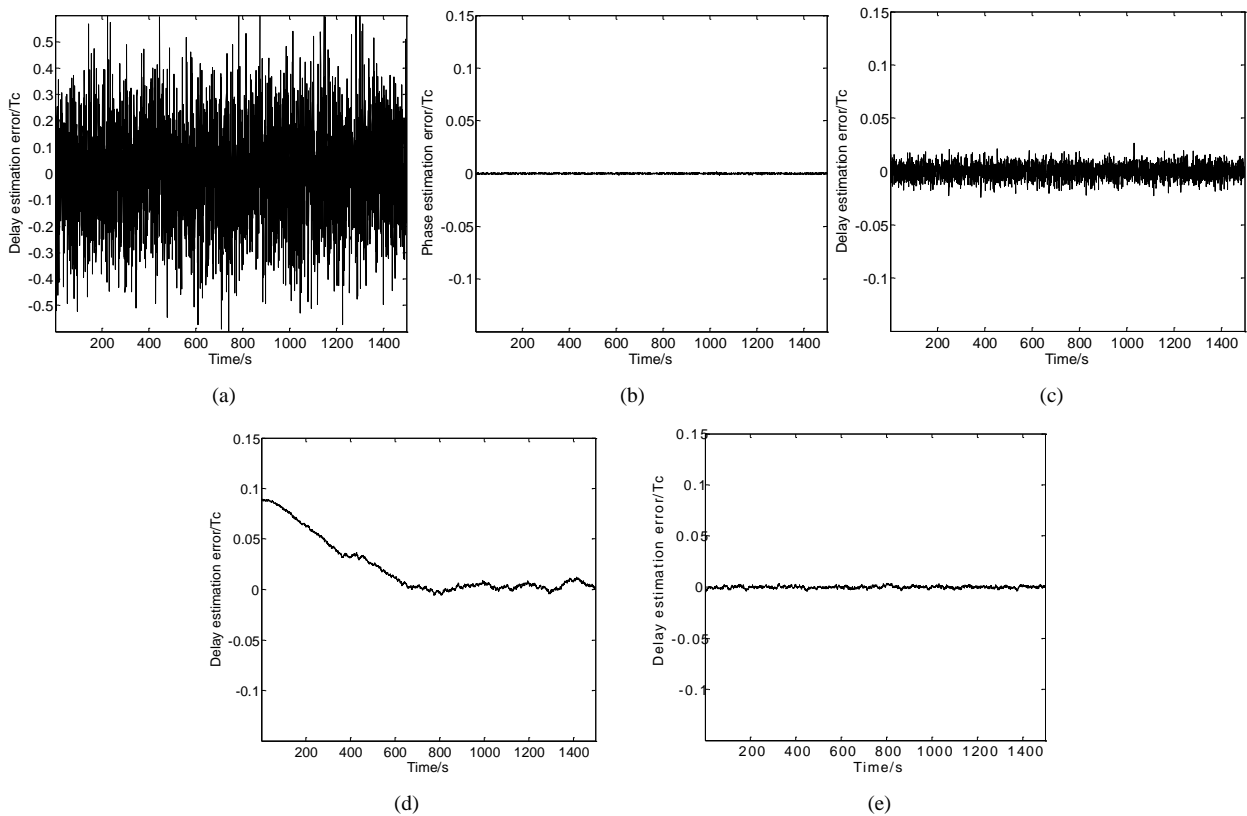


Fig. 5. The delay errors of the No. 250 antenna before and after the complementary Kalman filtering. (a) The initial delay estimation errors, (b) The phase estimation errors, (c) The delay errors after the least squares filtering, (d) The delay errors after the complementary Kalman filtering using Eq. (13), (e) The delay errors after the complementary Kalman filtering using Eq. (14).

The simulation is presented to verify the effect of the complementary Kalman filtering using the same array showed in Fig. 4(a). Assuming the signal code rate is 10Mbps, the center frequency is 2GHz, the initial variance of delay obtained by correlation is $0.2T_c$ and the variance of phase is 30° . The delay errors of the No. 250 antenna before and after the complementary Kalman filtering are shown in Fig. 5.

The initial delay and phase estimation errors are shown in Fig. 5(a) and (b), which are all expressed as a multiple of the code width. The delay errors after the least squares filtering are shown in Fig. 5(c), the delay errors after the complementary Kalman filtering using Eq. (13) are shown in Fig. 5(d) and the errors after the complementary Kalman filtering using Eq. (14) are shown in Fig. 5(e). From Fig. 5(d) and 5(e), it can be seen the delay errors can be greatly reduced after the complementary Kalman filtering with the use of the phase difference estimation which has higher accuracy. Comparing Fig. 5(d) and Fig. 5(e), the convergence of delay errors is slow and there are significant fluctuations in stable, when only using the complementary Kalman filtering. The errors converge rapidly and the fluctuations are greatly reduced, when the complementary Kalman filter is combined with the least squares estimation method, which shows that the estimation accuracy of delay is further improved.

The least squares estimation method of delays is presented, considering the geometric relation of delays estimated by cross-correlation, and theoretical analysis of the method is also presented. The complementary Kalman filtering method of delays is also presented, according to the different characteristics and inherent between delays and phase differences.

VI. CONCLUSIONS

Theoretical analysis and simulation results show that the two methods can both greatly improve the estimation accuracy of the delays. For the given case, the accuracy of delays improves about two orders of magnitude after the least squares filtering. The delay errors can also be greatly reduced after the complementary Kalman filtering with the use of the phase difference estimation which has higher accuracy. The estimation accuracy can be further improved, when the two methods are properly combined.

It doesn't consider the error of baseline in the least squares estimation method of delays. In practice, the baseline error is inevitable, so it is necessary to study the influence of the baseline to the estimation of delays in the future.

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