# Research on Stability Control Algorithm of Slotted ALOHA 

Fei Fang and WenchunYu<br>NeiJiang Normal University, NeiJiang Sichuan 641112, China<br>Email: \{fangfei_nj, ywclmxx\}@163.com


#### Abstract

Because of its simplification, slotted ALOHA is comprehensive used in the satellite and wireless communication. While slotted ALOHA is essentially unstable. Therefore various kinds of control algorithms are applied in order to keep stable throughput of the communication system. In this paper, pPersistent Control Algorithm (pPCA) of slotted ALOHA is analyzed using binomial distribution model, design of algorithm is finished and throughput is simulated. Markov Model of Binary Exponential Backoff (BEB) Algorithm is established on the basis of which Binary Exponential Backoff Algorithm is designed whose performance is tested through numerical analysis and simulation. Finally, Operating principle of PseudoBayesian Control Algorithm (PBCA) is analyzed utilizing Poisson distribution model of input stream, corresponding algorithm is designed and throughput is done simulation test. Then comparisons are done on the properties of throughput, adjustment process and complexity of computation among Binary Exponential Backoff Algorithm, Pseudo-Bayesian Control Algorithm and p-Persistent Control Algorithm whose results show that overall performance of Pseudo-Bayesian Control Algorithm is better than that of the other two.


Index Terms-Slotted ALOHA; throughput; stability; complexity of computation; Markov Model; adjustment process

## I. Introduction

When multiple users share the communication medium in a communication network, they need to acquire the right of using the channel by the way of competition. As an universal Medium Access Control (MAC) Protocol, ALOHA and slotted ALOHA [1], [2] is widely used in the environment of multiple user nodes competing for one communication medium. Nowadays slotted ALOHA Protocol has been extensively applied to the Random Access Channel (RACH) of Global System for Mobile Communication. Otherwise its operation is simple. It is used in both experimental network developments of ultrawideband wireless networks and dynamic spectrum sharing network based on wireless cognitive technology. While slotted ALOHA is essentially unstable in which channel throughput enlarges with the increasing of load flow when the input load flow gradually increases from 0 . Throughput reaches up to the maximum value when the input flow reaches to one certain threshold value. At this time conflicts among these users would become bigger if input load flow continues increasing in which repetitive

[^0]collision would waste part of band width of system leading throughput to rapidly decline with the increasing of input load flow and making the system be kept in an unstable condition.

In order to solve the problem of instability of slotted ALOHA, literature [3] proposes that different retransmission probabilities should be used to keep system being stable under different overall loads. In literature [4]-[6], Sarker does research on the influences of finite retransmission control mechanism on the stability of slotted ALOHA and studies the range of system stability under a variety of constraints like controlling the formation rate of the new packet, allowing system to keep a certain range of rejecting rate and allowing transmission channel to have error probability. Refernce [7] analyzes the system performance of slotted ALOHA utilizing the collaboration among nodes in the system, introduces game theory into channel competition and hereby studies the throughput and delay characteristics of systems. Literature [8] combining with physical channel characteristics and signal detection into consideration, he researches the throughput and stability of slotted ALOHA on this kind of information channel. Moreover, Pietrabissa. A and others study the stability of standard slotted ALOHA and the one with cache and use convex optimization way to propose a type of conflict resolver algorithm to lead system to acquire stable maximum throughput [9]. Rivest R.L proposes PseudoBayesian Control Algorithm ${ }^{[1]}$ on the basis of analyzing slotted ALOHA principle and takes the new arrival packets as retransmission packets in which all nodes needing to send data use the same probability to transmit data. In literature [10], Milosh Ivanovich and others study the influences of p Persistent Control Algorithm on throughput of slotted ALOHA under the condition of non-Poisson arriving and channel transmission having errors. However, the existing researches on slotted ALOHA mainly concentrate on how to keep system being stable. The number of researches on control algorithms for system stability is so small that just a few literatures do researches on the algorithm of dynamically adjust retransmission probability p and Pseudo-Bayesian Control Algorithm. While as one type of stability control algorithm, Binary Exponential Backoff Algorithm is widely used in the CSMA/CD Protocol of wired Ethernet networks and CSMA/CA Protocol of 802.11 Distributed Coordination Function [11], [12] whose regulation perfor-mances have not aroused high concerns from researchers.

This paper mainly researches the operating principles of Binary Exponential Backoff Control Algorithm, p Persistent Control Algorithm and Pseudo Bayesian Control Algorithm and their performances of ensuring system stability of slotted ALOHA. On the basis of Markov Model in Binary Exponential Backoff of slotted ALOHA, it analyzes the throughputs of system at different nodes. Through acting the node's sending data process as Poisson distribution, it analyzes PseudoBayesian Control Algorithm principle in which mathematical analysis and computer simulation are utilized to test the performance of this algorithm. Packet transmission of active nodes (nodes having data to transmit) in this system is taken as a random variable of binomial distribution to analyze the stability adjustment principle of p Persistent Control Algorithm and do simulation testing on the performance of pPCA. Finally under the environment of multiple types of nodes, comparisons of system throughput, stability adjustment time and algorithm complexity are done among the three control algorithms. (Note: If there are no special instructions, system throughput in this paper means the normalized system throughput namely the proportion of successful transmission during one tile slot.)

## II. Slotted ALOHA System Mode

In the research of this paper, system utilizes a kind of universal slotted ALOHA protocol, is made up of $n$ nodes and shares one channel in which channel access uses slotted ALOHA Protocol. The basic thought is to divide channel time to discrete slots whose length is the sending time one frame needs. Each node cannot send data before slot begins. In addition, the involved nodes in this paper stand for the active nodes who are waiting for transmitting packets (not including nodes without packets). Suppose that active nodes always have packets to transmit. They would become inactive ones. Nodes in the system cater for the following assumptions:

1) Each node could perceive that channel is idle before data transmission begins and immediately acquire information to successfully transmit or conflict through incidentally response ACK way when transmission is ending.
2) Conflicts would appear among nodes when they are transmitting packets. Then packets would be retransmitted. New packets could not be transmitted until blocking packets' transmission finished.
3) Length of the packet of each node should be less than the maximum data length that one slot could transmit.


Fig. 1 System principle diagram of slotted ALOHA


Fig. 2. Throughput $S$ versus G
4) Wireless transmission channel is ideal and it is the conflict that leads errors of packets to appear.

It is seen from Fig. 1 that after one packet arrives at a certain slot, it begins to transmit at the next slot and expects that it would not collide with packets sent by other nodes. If just one packet arrives at one slot (including new arrival packet and retransmission one), this packet's transmission is successful. If two or more packets arrive at one slot, collision would happen because colliding packets would be retransmitted at the following slots. Suppose that time delay of retransmission is enough random. It may be approximately thought that sum of retransmission packet process and new arrival packet process is the Poisson process whose arrival rate is $G(G>\lambda)$. Then only one packet retransmission probability exists during one slot which is

$$
\begin{align*}
& P\left[\text { interval of arrival time }>T_{0}\right]=\int_{T_{0}}^{\infty} a(t) d t \\
& =\int_{T_{0}}^{\infty} \frac{G_{0}}{T} \exp \left(\frac{-G t}{T_{0}}\right) d t=\exp (-G) \tag{1}
\end{align*}
$$

Throughput of system $S=G e_{-\mathrm{G}}$ is defined as the passing rate of system or the rate of leaving system. When $G$ is equal to $1, S$ acquires its maximum value 0.368 as shown in Fig. 2.

## III. Stability Control Algorithm of Slotted ALOHA

It is seen from Fig. 2 that when the arrival rate of system $G$ is more than 1 , throughput of system would decrease with the increasing of $G$. In order to ensure that system would acquire stable throughput which is close to the theoretical maximum value, system needs to use some kind of algorithm to control $G$ at one section close to 1 . Consider that $n$ nodes exist in the system. Data buffer length of each node is unlimited. New packets are always waiting for being transmitted after node successfully transmits one packet.

## A. p-Persistent Control Algorithm

Suppose that $n$ nodes transmit data in the system while $m$ estimate nodes exist in it. Each node transmits data with probability $p=1 / m$ and enters the next slot to
compete with $1-p$ until data is successfully transmitted. Assume that each node's data transmission process is independent. Data transmission processes of $m$ nodes could be treated as random variables of the binomial distribution. When $m$ and $n$ are bigger, the probability of successful transmission at one certain slot is:

$$
\begin{align*}
P_{\text {succ }}=\binom{n}{1} p(1-p)^{n-1} & =n \frac{1}{m}\left(1-\frac{1}{m}\right)^{n-1}  \tag{2}\\
& =\frac{n}{m} e^{-\left(\frac{n-1}{m}\right)} \approx \frac{n}{m} e^{-\frac{n}{m}}
\end{align*}
$$

Do derivation on the right end of formula (2) and make it be 0 . We acquire that system gets maximum throughput 0.368 when $m=n$ namely $p=1 / n$. Aiming at the abovementioned system, its probability of channel idle being is:

$$
\begin{equation*}
P_{\text {idle }}=\binom{n}{0}(1-p)^{n}=\left(1-\frac{1}{m}\right)^{n} \approx e^{-\frac{n}{m}} \tag{3}
\end{equation*}
$$

Take logarithms away from both sides of (3). We acquire that:

$$
\begin{equation*}
\ln P_{\text {idle }}=-\frac{n}{m} \Rightarrow n=-m \ln P_{\text {idle }} \tag{4}
\end{equation*}
$$

It is seen from (4) that actual node number $n$ of current system could be acquired as long as exact value of $P_{\text {idel }}$ is obtained. Then nodes of system transmit data with probability $p=1 / n$ which would help acquire throughput close to maximum theoretical value. In order to do statistics on the idle probability of channel, a statistic window should be set which is noted with variable $S W$. If the window is designed to be too large, adjustable rate would become slower. If it is designed too small, error of probability statistical value would increase.
In the design, our algorithm is as follows:

1) Initialize simulation value $n_{0}$, calculate $p=1 / n_{0}$, define a variable $s$ of statistical slot and initialize it to be 0 .
2) Determine whether $s$ equals to $S W$. If yes, $\mathrm{s}=0$ and turn it to iv).
3) $s$ subtracts 1 which produces even-distributed random number $x$ in the range $(0 \sim 1)$. Decide whether $x<p$. If yes, data would be transmitted and turned to ii).
4) Do statistics on the number of idle slot of one $S W$ and calculate $P_{\text {idle }}$. If $P_{\text {idle }}=0, p=p / 2$. If not, $p=-p / \ln P_{\text {idle }}$ and turn it to c.

In order to test the performance of $p$-Persistent Control Algorithm, we test the performance of the p-persistent algorithm for slotted ALOHA using the MATLAB.

The relevant parameters of simulation scenario one is set as follows: $S W$ is set to be 32 , system estimate number of node is 5, actual node numbers of system are respectively $2,5,20,50,100$ and simulation time is 100 slots and The relevant parameters of scenario two is set as follows: system estimate number of node is 50 , other
parameters is same as scenario one. The simulation result is show as Fig. 3.

Fig. 3 (a) is the simulation result of scenario one. From it we can see that the pPCA can get the steady approximate maximum throughput of slotted ALOHA after definite adjusting time.But, if the initial estimate node is small, the bigger the real number of node ,the throughput of adjustment process is lower. And the time of adjustment is additional when the actual node of system is more greater than the initial number of system node.

Fig. 3 (b) is the simulation result of scenario two. From it we can see that the pPCA can get the steady throughput approximate maximum throughput of slotted ALOHA after definite adjusting time same as scenario 1.But, if the initial estimate node is bigger,i.e. $n_{0}=50$; the smaller the real number of node ,the throughput of adjustment process is lower, and the time of adjust is additional when the actual node of system is more smaller than initial number ofsystem node.


Fig. 3. Throughtput of p-persistent algorithm in every slot
In the interest of appraisal the colligation capability $p$ persistent control algorithm (pPCA), the average throughput was tested using MATLAB to simulate with the parameter as fellow: $S W$ is set to be 32 , system estimate number of node is $5,10,30,100$; actual node numbers of system are respectively 2 to 150 and simulation time is 100 slots. Fig. 4 shows that the system average throughput depends on initial estimate number of node $n_{0}$ and actual number of node $n$. In particular, the

Fig. 4 shows that the difference between the actual number of nodes and initial estimate number of nodes is bigger, the average throughput is lower in 100 slots.


Fig. 4. Average throughput versus actual number of node \& initial estimate number of node.

## B. BEB Algorithm

During the process of users accessing channel, conflicts would appear in the channel if two or more nodes are transmitting packets at the same slot. The conflicting nodes utilize BEB Algorithm to retransmit the conflicting packets. Each node randomly chooses one backoff latency time value (integral value) in an uniform distribution way in the certain range $\left[0, C W_{i}-1\right]$ of Contention Windows Parameter $C W_{i}$ which is defined by the backoff algorithm. Then this value is done an assignment on the Backoff Counter to finish its initialization. In terms of the BEB Algorithm of DCF, its Backoff Counter would subtract 1 every time nodes in its system monitor the last idle slot again after they monitored that duration idle time of the channel was more than DIFS. Comparing with this, Backoff Counter of slotted ALOHA will subtract 1 as long as one slot passes whether or not channel is idle.

In order to guarantee that each node could fairly access to the system, all of them have to backspace to wait for a random backspacing time before packets retransmit or before continuous packets transmit. Utilize a discrete integer to label on the time in which $t$ and $t+1$ stand for the starting points of two continuous slots. Here $b(t)$ stands for the value of Backoff Counter (BC) of a given node. Therefore backoff count value of each node relies on its transmission history and the number of slot during the process. Define that CWmin is the minimum window value, $m$ is the maximum backoff order and $C W_{\text {max }}=2^{L} W$ is the window backoff maximum value. When new packets are transmitted, backoff order of one node initializes to be 0 and its backoff window is $C W_{\text {min }}$. Each time node retransmits; backoff window value $C W_{i}$ would be enlarged to be 2 times of the original one if it did not reach to the maximum value $C W_{\text {max }}$. Otherwise it would maintain to be $C W_{\text {max }}$. When retransmission time reaches to the highest value $m, C W_{i}$ initializes again. $W_{i}$ is used to stand for the backoff window value of a certain node which is in the backoff order $i$. It is

$$
W_{i}= \begin{cases}2^{i} W & 0 \leq i \leq L  \tag{5}\\ C W \max & L<i \leq m\end{cases}
$$

Here $b(t) \in\left(0, W_{i}-1\right)$.Suppose that random process $s(t) \in(0, \cdots, m)$ stands for the number of backoff order at the moment $t$ of the given node. Moreover suppose that conflicting probability of nodes at $s(t)=i, i \in(0, m)$ is the independent parameter $p_{\mathrm{i}}$ meaning the probability of conflict happening of one node at backoff order $i$ when transmitting data. According to independence assumption, two dimensional discrete Markov Model $\{s(t), b(t)\}$ could be used to describe node's backoff process [12] shown in Fig. 5.

It's one-step transition probability is:

$$
\left\{\begin{array}{lll}
P\{i, k \mid i, k+1\}=1 & &  \tag{6}\\
P\{0, k \mid i, 0\}=\frac{\left(1-p_{i}\right)}{W_{0}} & k \in\left(0, W_{i}-2\right) & i \in(0, m) \\
& k \in\left(0, W_{0}-1\right) & i \in(0, m) \\
P\{i, k \mid i-1,0\}=\frac{p_{i-1}}{W_{i}} & k \in\left(0, W_{i}-1\right) & i \in(1, m) \\
& k \in\left(0, W_{i}-1\right) & \\
P\{m, k \mid m, 0\}=\frac{p_{m}}{W_{m}} & &
\end{array}\right.
$$

Here comes
$P\left\{i^{\prime}, k^{\prime} \mid i, k\right\}=P\left\{S(t+1)=i^{\prime}, b(t+1)=k^{\prime} \mid S(t)=i, b(t)=k\right\}$


Fig. 5. Markov model of BEB algorithm.
In this formula, the first line means that backoff counter of node subtracts 1 when every slot begins. The second line means that after finishing a successful transmission, node has new packets to transmit, enters backoff order 0 and randomly chooses a backoff value from [ $0, W_{0}-1$ ] in uniform distribution way. The third line includes information of nodes being at backoff order $i-1$ and conflicting after sending out data. It stands for the probability of nodes' entering backoff order $i$. The fourth line stands for the process during which backoff order keeps staying in the backoff stage after it reaches to m . When the number of node $n$ is bigger and system is under
stable condition, collision probability of each backoff stage can be supposed as an independent constant $p$ namely $p_{0}=\cdots=p_{m}=p$. Then the stable probability of node at each backoff order is acquired:

$$
\begin{gather*}
Q(i, 0)=Q(0,0) p^{i}, \quad 0<i<m \\
Q(m, 0)=\frac{Q(m-1,0) p}{1-p}=\frac{Q(0,0) \cdot p^{m}}{1-p}  \tag{7}\\
Q(i, k)=\frac{W_{i}-k}{W_{i}} \begin{cases}(1-p) \sum_{j=0}^{m} Q(j, 0) & i=0 \\
p Q(i-1,0) & 0<i<m \\
p(Q(m-1,0)+Q(m, 0)) & i=m\end{cases} \tag{8}
\end{gather*}
$$

According to the definition of Formula (6), it is simple to acquire that:

$$
\begin{equation*}
\sum_{i=0}^{m} Q(i, 0)=\sum_{i=0}^{m-1} Q(0,0) p^{i}+Q(m, 0)=\frac{Q(0,0)}{1-p} \tag{9}
\end{equation*}
$$

Aiming at stable system, probabilities of all conditions should cater for:

$$
\begin{align*}
& \sum_{i=0}^{m} \sum_{j=0}^{W_{i}-1} Q(i, j)=\sum_{i=0}^{m} Q(i, 0) \sum_{k=0}^{W_{i}-1} \frac{W_{i}-k}{W_{i}} \\
&=\sum_{i=0}^{m} Q(i, 0) \frac{W_{i}+1}{2}  \tag{10}\\
&=\frac{Q(0,0)}{2}\left[W\left(\sum_{i=0}^{m-1}(2 P)^{i}+\frac{(2 p)^{m}}{1-p}\right)+\frac{1}{1-p}\right]=1
\end{align*}
$$

Solution is

$$
\begin{equation*}
Q(0,0)=\frac{2(1-2 p)(1-p)}{(1-2 p)(W+1)+p W\left(1-(2 p)^{m}\right)} \tag{11}
\end{equation*}
$$

Then transmission probability of a certain node during one slot is

$$
\begin{align*}
\tau & =\sum_{i=0}^{m} Q(i, 0)=\frac{Q(0,0)}{1-p} \\
& =\frac{2(1-2 p)}{(1-2 p)(W+1)+p W\left(1-(2 p)^{m}\right)} \tag{12}
\end{align*}
$$

Under general condition, $\tau$ relies on conditional probability $p$ which means that one node transmits data at a certain slot and at least one node of the others also does data transmission at the same time. Then comes:

$$
\begin{equation*}
p=1-(1-\tau)^{n-1} \tag{13}
\end{equation*}
$$

Convert (13) as $\tau^{*}(p)=1-(1-p)^{1 /(n-1)}$. This function is a monotone increasing one in the range $P \in(0,1)$ and $\tau^{*}(0)=0, \tau^{*}(1)=1 . \tau$ in (12) is the function of $p$ expressed by $\tau(p)$ which is a monotone decreasing function in the range $p \in(0,1)$ and $\tau(0)=2 /(W+1)$ ,$\tau(1)=2 /\left(1+2^{m} W\right)$. As $\tau(0)>\tau^{*}(0)$ and $\tau(1)<\tau^{*}(1)$, there must be intersection between $\tau(p)$ and $\tau^{*}(p)$ in the range $p \in(0,1)$. Therefore there exists simultaneous solution between (12) and (13).

As data transmission of slotted ALOHA happens at the moment slot beginning, successful transmission of packets, leisure of channel and time of conflict all take slot as unit. Under the condition of $n$ nodes, probability of each node sending data is $\tau$.Thus probabilities of successful transmission of packets, leisure of channel and conflict are respectively shown as follows:

$$
\begin{align*}
& P_{\text {succ }}=n \tau(1-\tau)^{n-1} \\
& P_{\text {idle }}=(1-\tau)^{n}  \tag{14}\\
& P_{\text {coll }}=1-P_{\text {succ }}-P_{\text {idle }}
\end{align*}
$$

During the backoff process, BEB Control Algorithm of slotted ALOHA does not monitor whether channel is idle. While backoff count subtracts 1 each time it goes through one slot. Therefore throughput rate of system is:

$$
\begin{align*}
S & =\frac{P_{\text {succ }} \cdot \text { Slot }}{P_{\text {succ }} \cdot \text { Slot }+P_{\text {idle }} \text { Slot }+P_{\text {coll }} \text { Slot }}  \tag{15}\\
& =P_{\text {succ }}=n \tau(1-\tau)^{n-1}
\end{align*}
$$

Do derivation on the right side of formula (15) and make it be 0 , we acquire that system gets the maximum value $S_{\text {max }}=\left(1-\frac{1}{n}\right)^{n-1} \approx e^{-1}=0.368$ under the condition of $\tau=1 / n$ which acquires the same conclusion with (1).

In order to test the performance of BEB Algorithm, MATLAB is used to do simulation on it. Here the minimum value of backoff window is $C W \min =32$, $C W \max =1024$ and the maximum number of backoff order is 7 . With the number of node changing from 2 to100, average throughputs of system are respectively simulated. The simulation time is $10^{7}$ slots. Its results are shown in Fig. 6 from which it is seen that throughput of system is less than the maximum theoretical value 0.368 when the number of node in the system is less than 64 . That is, the smaller number of nodes, the lower the throughput.


Fig. 6. Throughput of BEB algorithm.
For the purpose of determining whether it is the setting of window values $C W_{\text {min }}$ and $C W_{\text {max }}$ that leads to the problem, simulations are done with the parameter as fellow:
(a) $C W_{\text {min }}=32, C W_{\text {max }}=1024$
(b) $C W_{\text {min }}=16, C W_{\text {max }}=512$
(c) $C W_{\text {min }}=8, C W_{\text {max }}=1024$.

The acquired simulation results are seen in Fig. 7. Simulation results in Fig. 7 show that the setting of window values $C W_{\text {min }}$ and $C W_{\text {max }}$ is one of the important factors that influences the throughput of system. When the number of node is smaller and $C W_{\text {min }}$ is lower, its throughput is bigger. Otherwise average throughput would be smaller.

Moreover, so as to measure the changing condition of throughput of BEB Algorithm during its stability adjustment process, the adjustment process of BEB Control Algorithm is simulated under both conditions of
(a) $C W_{\text {min }}=32, C W_{\text {max }}=1024$ and
(b) $C W_{\text {min }}=16$, $C W_{\text {max }}=1024$ in which conditions of the number of node $n$ are 5,20,50, 100 and 200. Simulation results are shown in Fig. 8.


Fig. 7. Average throughputs versus $C W$


Fig. 8. Throughputs during adjustment process of BEB

It is seen from simulation results in Fig. 8 that window values of all nodes are set to be 16 or 32 at the beginning moment in which throughput of system in the initial period is close to 0 . If $X$ stands for the random variable in discrete uniform distribution of one node in the range $\left(0, W_{i}-1\right)$, average value of node backoff counter is $E(X)=\left(W_{i}-1\right) / 2$. During adjustment period after initialization, the production of data transmission of node averagely needs to wait for 16 slots if $W_{0}=32$. In addition, as backoff windows of each node are the same, large numbers of collisions would come into being thus leading to the problem of throughput being smaller during adjustment period. After a period of adjustment, throughput of system is basically stable. However, aiming at the condition of low number of node, its throughput is obviously lower than the maximum stable throughput of slotted ALOHA.

## Pseudo-Bayesian Control Algorithm

Suppose that one system has $n$ nodes and each node has just one buffer. New packets may generate only after blocking packets finish transmitting. The arrival of new packets is the Poisson process whose parameter is $\lambda$. When packets collide, packet data needing retransmitting would be chosen to send out with the probability $p$ in the next slot and enters the next slot with the probability $1-p$ and so on. Packets of each slot include two parts: packets of new arrival nodes and the ones of retransmission nodes. Assume that time delay of retransmission is random enough. It may be approximately thought in this system that sum of the arrival process of retransmission packets and that of new arrival packets is one Poisson process whose arrival rate is $G(G>\lambda)$. Suppose that $b(t)$ stands for the number of retransmission node in the system at the moment of slot $t$. Behavior of slotted ALOHA could be described by discrete Markov chain in which status value is $b(t)=\{0,1, \cdots, n\}$ shown in Fig. 9


Fig. 9. Markov model of slotted ALOHA
The arrival packets of transmission nodes are also treated as retransmission nodes in order to simplify the system design,. According to the nature of slotted ALOHA, system can acquires the maximum throughput $S=0.368$ with $G=1$. Namely, all the $n$ nodes transmit data with the probability $p=1 / n$.It suppose that $N_{i}$ denoted the number of nodes which transmit data at $i$ slot. Therefore probability of $k$ nodes transmitting data at $i$ slot is:

$$
\begin{equation*}
P\left(N_{i}=k\right)=\frac{n^{k}}{k!} e^{-n} \tag{16}
\end{equation*}
$$

When each node sends data with probability $p=1 / n$, probability of slot being idle is:

$$
\begin{align*}
& P(\text { idle })=\sum_{k=0}^{\infty} P\left(N_{i}=k\right)\left(1-\frac{1}{n}\right)^{k} \\
& \quad=\sum_{k=0}^{\infty} P\left(N_{i}=k\right)\left(1-\frac{1}{n}\right)^{k} e^{-n}=e^{-1} \tag{17}
\end{align*}
$$

Under the condition of $i$ slot being idle, probability of $k$ terminals waiting for sending data is:

$$
\begin{equation*}
P\left(N_{i}=k \mid i d l e\right)=\frac{P(i d l e \mid k) P\left(N_{i}=k\right)}{P(i d l e)}=\frac{(n-1)^{k}}{k!} e^{-(n-1)} \tag{18}
\end{equation*}
$$

Its probability is one Poisson distribution whose average value is $n-1$. Similarly, here come $k$ terminals needing to send data. Probability of successful transmission is:

$$
\begin{align*}
P(\text { succ })=\sum_{k=0}^{\infty} P\left(N_{i}\right. & =k) k \frac{1}{n}\left(1-\frac{1}{n}\right)^{k-1} \\
& =e^{-N} e^{N-1}=e^{-1} \tag{19}
\end{align*}
$$

When one request packet is successfully transmitted at slot $i$, probability of $k+1$ packets being in this system is:

$$
\begin{align*}
P\left(N_{i}\right. & =k+1 \mid \operatorname{succ})=\frac{P\left(\operatorname{succ} \mid N_{v}=k+1\right)\left(N_{v}=k+1\right)}{P(s u c c)} \\
& =\frac{(k+1)\left(1-\frac{1}{n}\right)^{k} \frac{1}{n} \frac{N^{k+1}}{(k+1)!} e^{-n}}{e^{-1}}=\frac{(n-1)^{k}}{k!} e^{-(n-1)} \tag{20}
\end{align*}
$$

It is visible that the number of packets $k$ which are waiting for being transmitted in the system is one random variable of Poisson distribution whose average value is $n-1$. Considering that new arrival request number during one slot in the system is $\lambda(\leq 0.368)$, a prior probability $p=1 / N_{i}$ is given which is related to $N_{i}\left(N_{i} \geq 1\right)$. When slot is idle or successful, $N_{i+1}$ is a random variable of Poisson distribution whose average value is $N_{i}+\lambda-1$. If collision happens, $N_{i+1}$ could be approximately treated as a random variable of Poisson distribution whose average value is $N_{i}+\lambda+1 /(e-2)$. Therefore sending probability of the next slot may be adjusted according to the condition of the present slot whose implementation steps are as follows:

1) Slot 1 is the beginning one. Its node number of initialized system is $n_{0}$ in which each node transmits data with probability $p=1 / n_{0}$;
2) At slot $i$, each node sends data with probability $p=\min \left\{1,1 / N_{i}\right\}$;
3) Terminal number needing to send data array at the next slot may be estimated by the following formula:
$N_{i+1}= \begin{cases}\max \left(\lambda, N_{i}+\lambda-1\right), & \text { channel is idle or success } \\ N_{i}+\lambda+(e-2)^{-1}, & \text { channel collission }\end{cases}$
4) Each terminal at $i+1$ slot sends packets with probability $1 / N_{i+1}$.

(a) Initial number of node $n_{0}=5$

(b) Initial number of node $n_{0}=10$

(c) Initial number of node $n_{n}=50$

(d) Initial number of node $n_{0}=100$

Fig. 10. PBCA throughput of slotted ALOHA.

This algorithm is simulated under MATLAB platform. Part of terminals change to be active nodes or communication of some active nodes finishes leading them to become inactive terminals which makes number of nodes change. The supposed changing moment $t$ in the system is the beginning slot 0 . Simulation scenarios are set as follow:

1) The estimated nodes in the system at the beginning slot $\left(t_{0}=0\right)$ are supposed to be $5,10,50,100$.
2) The actual node numbers $n$ of system are respectively $5,20,50,100,200$.
3) Simulation time range is from 1 to 200 whose time unit is slot.

Fig. 10 shows the throughputs of system during each slot. Simulation results show that system could be basically stable on theoretical maximum throughput going through a period of adjustment. Meanwhile the figure shows that with difference between actual node number and initial value increasing, time for adjustment becomes longer.

## IV. Performance Comparison of Three Control Algorithms

All the three algorithms of BEB Algorithm, PBCA and pPCA can keep system stable. However, a type of optimal control algorithm is needed in physical design. Therefore analytical research should be done among the three algorithms in the aspects of average throughput of system, adjustment time and computation complexity.

## A. Average Throughput

Channel throughput is the most important index for evaluating channel utilization. Simulation results of throughputs of the three algorithms in the second section show that throughputs of BEB Algorithm and pPCA are smaller than that of PBCA under the condition of node number being small. In order to test specific differences of the three algorithms, simulation testing is done on them according to the simulation environment set by Table I and Table II under MATLAB Platform.

Table. I. Parameter of Simulation Scenarios

| BEB Algorithm | PBCA | pPCA |
| :---: | :---: | :---: |
| $C W_{\min }=32$ | $n_{0}=20$ | $n_{0}=20$ |
| $C W_{\min }=1024$ |  | $R P W=32$ |
| Maximum backoff order=7 |  |  |
| Simulation time $=10000$ slots |  |  |
| Actual number of node: $2 \sim 150$ |  |  |

Table II. Parameter of Simulation Scenarios

| BEB Algorithm | PBCA | pPCA |
| :---: | :---: | :---: |
| $C W_{\min }=16$ | $n_{0}=20$ | $n_{0}=20$ |
| $C W_{\min }=512$ |  | $R P W=16$ |
| maximum backoff times is 7 |  |  |
| Simulation time $=10000$ slots |  |  |
| Actual number of node: $2 \sim 150$ |  |  |

Simulation results of Fig. 10 show that throughput of BEBA is obviously smaller than the ones of PBCA and
pPCA when the number of node is small, while performance of pPCA is a little more than that of PBCA. When the number of node is big, throughput of BEB Algorithm is a little less than those of PBCA and pPCA. When the number of node is more than 5, the performance of PBCA is higher than those of BEB Algorithm and pPCA.

(a) Throughputs Based on Parameter Setting of Table. I

(b) Throughputs Based on Parameter Setting of Table. II

Fig. 11. Average throughputs of three algorithms

## B. Stability Adjustment Time

Average throughput reflects the channel utilization of system during a long time. As network data is a type of burst stream, adjustment time is a system index to measure the ability of control algorithm adapting environment with high volatility. In order to test the time three algorithms need from node changing to achieve stable again, simulation is done on them under the environment set in Table III and Table IV whose simulation results are shown in Fig. 12.

Table. III. Parameter of Simulation Scenarios

| BEB Algorithm | PBCA | pPCA |
| :---: | :---: | :---: |
| $C W_{\min }=16$ | $n_{0}=5$ | $n_{0}=5$ |
| $C W_{\max }=512$ | $R P W=32$ |  |
| Maximum backoff times is 7 |  |  |
| Simulation time is 200 slots |  |  |
| Actual number of node $=30$ |  |  |

Table. IV. Parameter of Simulation Scenarios

| BEB Algorithm | PBCA | pPCA |
| :---: | :---: | :---: |
| CWmin $=16$ | $\mathrm{n}_{0}=30$ | $\mathrm{n}_{0}=30$ |
| CWmax $=512$ | $R P W=32$ |  |
| Maximum backoff times is 7 |  |  |
| Simulation time is 200 slots |  |  |
| Actual number of node $=5$ |  |  |



Fig. 12. Stability adjustment time of three algorithms
Simulation results of Fig. 12 (a) show that simulation time of PBCA is smaller than that of BEB and pPCA when the number of system node is big and bigger than initial estimated node number. As limited by update window, adjustment time of pPCA is the longest and that of BEB presents twice of fluctuation. Results of Fig. 12 (b) show that simulation time of pPCA is still the longest while that of BEB is the shortest when the number of system node is small and smaller than initial estimated node number. However, stable throughputs of BEB and pPCA are obviously smaller than that of PBCA.

## C. Complexity Analysis of Algorithms

As MAC Layer lies in the bottom position of network communication system, DSP Chip or Microcontroller are generally used to finish the relevant algorithms during the physical design process of MAC Protocol. Limited by resources of DSP or Microcontroller, functional operation and floating-point calculation are obviously lower than multiplication and division. During the adjustment process of BEB Algorithm, packets are transmitted when backoff count value $B C_{i}$ is 0 . After conflicts happen, multiply its backoff window value by 2 and randomly choose a value from [ $0, W_{\mathrm{i}}-1$ ] in even-distributed way which is treated as backoff count value $B C_{i}$. Then $B C_{i}$ subtracts 1 each time it goes through one slot. Therefore this algorithm mainly does simple subtraction. PBCA corrects the estimated value of nodes in the system according to the last-time detected channel condition
which needs once of addition and subtraction and once or twice of multiplication. During the adjustment process of pPCA, logarithm operation and at least once of division operation should be done to count the estimated node number of system. In the design of MAC which is based on dynamic spectrum sharing communication system, driving function library based on Linux does not provide logarithmic function in which Taylor series needs using to do approximation which also increases computation complexity. Therefore in terms of calculation complexity, BEB Algorithm is the most simple, the second PBCA and pPCA is the most complex.

## V. Conclusions

Through numerical analysis and simulation testing, it is shown that all of BEB Algorithm, PBCA and pPCA can realize the stability of slotted ALOHA Protocol. When node number is small, average throughput and adjustment time of PBCA are both better than those of BEB and pPCA. When node number is big, average throughputs of three algorithms are basically the same while adjustment time of pPCA is longer. In terms of the computational complexities of the three algorithms, pPCA is obviously more complex than the other two. Comprehensively considering, PBCA is a more excellent type of stability control algorithm of slotted ALOHA when node number is small, while BEB Algorithm is better when node number is large.

## References

[1] R. L. Rivest, "Network control by bayesian broadcast," IEEE Trans. on Information Theory. vol. 33, no. 3, pp. 323-328, Mar. 1993.
[2] T. B. M. Richard, M. Vishal, and R. Dan, "An analysis of generalized slotted-aloha protocols," IEEE/ACM Trans. on Networking. vol. 17, no. 3, pp. 936-949, Mar. 2009.
[3] P. C. Loren, "Control procedures for slotted Aloha systems that achieve stability," ACM SIGCOMM Computer Comm. Review. vol. 16, no. 3, pp. 302 -309, Mar. 1986.
[4] J. H. Sarker and H. T. Mouftah, "A retransmission cut-off random access protocol with multi-packet reception capability for wireless networks," in Proc. 3th Int. Conf. on Sensor Tech. and App., Athens, 2009, pp. 217-222.
[5] J. H. Sarker, "Stability of random access protocol with newly generated packet rejection and retransmission cut-off," in Proc. Wireless Tele. Symp., Pomana, 2006, pp. 1-7.
[6] J. H. Sarker, "Stable and unstable operating regions of slotted ALOHA with number of retransmission attempts and number of power levels," IEEE Proceedings Communications, vol. 153, no. 3, pp. 355-364, Mar. 2006.
[7] G. Liva, "Graph-based analysis and optimization of contention resolution diversity slotted aloha," IEEE Trans. on Commu0nications. vol. 59, no. 2, pp. 477-487, Feb. 2011.
[8] P. Jaeok and M. van der Schaar, "Medium access control protocols with memory," IEEE/ACM Transactions on Networking, vol. 18, no. 6, pp. 1921-1934, Jun, 2010.
[9] S. H. Wang and Y. P. Hong, "Transmission control with imperfect CSI information," IEEE Trans. on Wireless Communications, vol. 8, no. 10, pp. 5214-5224,Oct. 2009.
[10] M. Ivanovich, M. Zukerman, and F. Cameron, "A study of deadlock models for a multiservice medium access protocol
employing a slotted aloha signalling channel," IEEE/ACM Transactions on Networking, vol. 8, no. 6, pp. 800-811, June 2000.
[11] F. Calì, M. Conti, and E. Gregori, "Dynamic tuning of the IEEE 802.11 protocol to achieve a theoretical throughput limit," IEEE/ACM Trans. on Networking, vol. 8, no. 6, pp. 785-799, June 2000.
[12] G. Bianchi, "Performance analysis of the IEEE 802.11 distributed coordination function," IEEE Journal on Selected Areas in Communications, vol. 18, no. 3, pp. 535-547, Mar. 2000.


Fei Fang was born in Sichuan Province, China, in 1974. He received the B.S. degree from the Southwest University of China (SWU), ChongQing, in 1997 and the M.S. degree from the ChongQing University of Posts and Telecommunications of China (CQUPT), ChongQing, in 2004. He is currently pursuing the Ph.D. degree with the School of Communication and Information Engineering,

University of Electronic Science and Technology of China (UESTC). His research interests include Wireless Local Area Network, CognCognitive radio Wireless network.


Wen-Chun Yu was born in Sichuan Province, China, in 1974. He received the B.S. degree from the Sichuan Normal University of China (SICNUU), ChenQu, in 1997 and the M.S degree from the Southwest Jiaotong University of China (WJITU), Chendu, in 2009. His research interests include Wireless Local Area Network Large data processing of network,Data Mining.


[^0]:    Manuscript received January 26, 2014; revised November 20, 2014.
    This work was supported by the Sichuan Education Department under Grant No.13ZA0005.

    Corresponding author email: fangfei_nj@163.com.
    doi:10.12720/jcm.9.11.805-814

