

Performance Analysis of Multi-Source Multi-Relay with Multiple Antennas Cooperative Networks with Best Relay Selection

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Abstract—In this paper we analyze the system performance of multi-source multi-relay with multiple antennas cooperative networks. We present an amplify and forward (AF) multiple-input multiple-output relay scenario, where finite number of sources communicate with single destination via multiple relay access points equipped with finite number of antennas. In this network a relay selection scheme is adopted, where the best relay is chosen using the instantaneous channel state information (CSI). Maximal ratio combiner (MRC) and maximal ratio transmitter (MRT) are used at the relay to obtain maximum achievable received signal-to-noise ratio (SNR). We derive closed-form and analytical bound expressions for the outage probability and symbol error rate (SER) with joint multiple-sources and cooperative diversity. Further high SNR asymptotic analysis on the outage probability and average SER are performed to characterize the diversity gain. Then, the analytical expressions are validated with Monte Carlo simulations. The results reveal that the diversity gain is determined by the number of the sources, the number of access points in the system and the number of the antennas in each access point.

Index Terms—Cooperative diversity, amplify-and-forward, MIMO, symbol error rate

I. INTRODUCTION

Diversity techniques are presented in wireless distributed network as a mean to cope with the dynamic channel conditions of the wireless medium. Cooperative communication arise as an effective diversity technique solution to major issues such as noise interferences, poor liability, distance expansion and high consumption of the transmission power via number of relays to help the source with its transmission to the destination. In the relaying transmission, there are several proposed relay protocols such as amplify and forward (AF), decode and forward (DF) [1]-[3].

On the other hand, multi-source diversity emerges as an important diversity aspect in multiple-sources systems [4]-[11]. By exploiting the fact that the sources suffer independently from the variation of channel conditions at anytime, there is at least one source whose channel gain is near to the peak, thus the system can serve that source at a time.

Multi-source diversity in single antenna systems is well studied in many works, e.g. in [4]-[5]. Furthermore, the interaction between multi-sources diversity and spatial diversity is extensively addressed in [6], [7]. However, the analysis of the multi-source diversity in cooperative relay networks has been addressed in few works.

The works in [8]-[11] and the references therein proposed joint selection framework, where each source and "best" relay pair are selected at time. Based on more centralized feedback approach these works provide asymptotic analysis to the diversity order of AF, fixed DF, and selective DF, in multi-source multi-relay networks.

In this paper, we investigate the impact of the number of sources, the number of the relays in the system and the number of the antennas equipped in each relay on the diversity performance of MIMO relay cooperative networks. We extend the work on multi-source diversity in [8]-[11] and the single antenna multiple relay in [3] to multi-source multi-relay with multiple antennas networks. We consider an AF relay network consists of finite number of single antenna sources communicate with single antenna destination via finite number of relay access points equipped with number of antennas. The proportional fair scheduling (PFS) algorithm in [12] is adopted in the system to achieve the multi-source diversity. In this network relay selection scheme is utilized to select the relay with the highest SNR to forward the source transmission. However the analysis can be extended to address other selection schemes such as the best harmonic means and the best worse channel selection [13]. We provide exact closed-form and analytical bound expressions for the outage probability and average SER. For high SNR, asymptotic analyses are provided for the outage probability, average SER, and the diversity gain. We validate the analytical expressions by using Monte Carlo simulations, the results reveal that the diversity gain is affected by the number of the sources, the number of the relays in the system and the number of the antenna equipped each relay.

Notations: \mathbf{A}^T and $\|\mathbf{A}\|_{\Phi}$ denotes the conjugate-transpose and the complex norm of matrix \mathbf{A} , respectively. I_Q stands for the $Q \times Q$ is the identity matrix. $Q!!$ is the double factorial of Q . $b \sim CN(0, \sigma^2 I_Q)$ indicates each element of the Q -dimension vector b is circularly

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symmetric complex Gaussian random variable with zero mean and σ^2 variance.

II. NETWORK MODEL

We consider a dual hop MIMO relay-based network with M sources equipped with single antenna ($M \geq 1$), a single antenna destinations and K relay access points equipped with Q antennas ($K \geq 1, Q \geq 1$), which are randomly distributed between the sources and the destination.

All the links are modeled as half duplex, independent, none essential identically distributed, orthogonal and flat-frequency channels; we assume that the destination can obtain the channel state information (CSI) of all the channel hops in the network via feedback links.

In this network, we adopt the PFS algorithm in [12] to select one source with the highest SNR out of M sources to transmit at a time. Assuming that the direct link between the sources and the destination are neglected, thus, the destination has to rely on the relays to receive the sources signals. We adopt time division multi access scheme (TDMA) so the communication between the m^{th} source and the destination is completed in two time slots. Furthermore, the destination is also considered to select one "best" relay with the maximal end-to-end SNR to forward the m^{th} source's signal.

In the first time slot, the selected source broadcasts a scaled symbol block with zero means and unit variance to all relays. The received signal at the k^{th} relay is expressed as follows:

$$y_{S_m-R_k} = P_{s_m} H_{S_m-R_k} x_m + n_{S_m-R_k} \quad (1)$$

where x_m is the m^{th} source signal, P_{s_m} is the source's transmitted power, $H_{S_m-R_k} \sim CN(0, \sigma_{S_m-R_k}^2 I_Q)$ is $Q \times 1$ the link gain vector from the m^{th} source to the k^{th} relay with each element of the Q -dimension vector $H_{S_m-R_k}$ is circularly symmetric complex Gaussian random variable with zero mean and $\sigma_{S_m-R_k}^2$ variance, $n_{S_m-R_k} \sim CN(0, \sigma_{n_{S_m-R_k}}^2 I_Q)$ is the additional white Gaussian noise vector (AWGN) with each of $n_{S_m-R_k}$ elements has one-sided spectral density $\sigma_{n_{S_m-R_k}}^2 = N_o$.

In the second time slot, i.e. the relaying phase, the selected relay uses the rule of MRC to obtain the scalar symbol, normalize it and then forward the resulted signal to the destination with the rule of MRT. The received signal at the destination is given by:

$$y_{D_m,k} = P_r H_{R_k-D} w_{R_k-D} \alpha (x + n_{S_m-R_k}) + n_{R_k-D} \quad (2)$$

where P_{R_k} is the k^{th} source transmitted power, $w_{R_k-D} = H_{R_k-D}^T / \|H_{R_k-D}\|_{\Phi}$, $H_{R_k-D} \sim CN(0, \sigma_{R_k-D}^2 I_Q)$ is $1 \times Q$ the

link gain vector from the k^{th} relay to the destination with each element of the Q -dimension vector H_{R_k-D} is circularly symmetric complex Gaussian random variable with zero mean and $\sigma_{R_k-D}^2$ variance,

$\alpha = 1 / \sqrt{P_s \|H_{S_m-R_k}\|_{\Phi}^2}$ is the amplified gain, $n_{R_k-D} \sim CN(0, \sigma_{n_{R_k-D}}^2 I_Q)$ is the additional white Gaussian noise (AWGN) with one-sided power spectral density N_o .

From (2), we express the instantaneous end-to-end received SNR at the destination as follows:

$$\gamma_{D_m,k} = \max_k \frac{\gamma_{S_m-R_k} \gamma_{R_k-D}}{\gamma_{S_m-R_k} + \gamma_{R_k-D}} \quad (3)$$

Defining $a_{m,k} = P_s \|H_{S_m-R_k}\|_{\Phi}$ and $b_{m,k} = P_r \|H_{R_k-D}\|_{\Phi}$ as the received signal power from the m^{th} source to the k^{th} relay and from the k^{th} relay to the destination, which are exponentially distributed with parameter $\bar{a}_{m,k}$ and $\bar{b}_{m,k}$, we can re-express (3) as follows:

$$\gamma_{D_m,k} = \max_k \frac{\varepsilon^2 (a_{m,k} b_{m,k})}{\varepsilon (a_{m,k} + b_{m,k})} \quad (4)$$

where $\varepsilon = 1 / N_o$ is the measure for the average system SNR

Since we employed PFS algorithm to select the source with the highest SNR, the total achieved SNR multi-sources multi-relay system is expressed as follows:

$$\gamma_D = \max_m \gamma_{D_m,k} \quad (5)$$

III. PERFORMANCE ANALYSIS

In this section, we analyze the performance of the multi-source multi-relay with multiple antennas system, thus we derive exact closed-form expressions for the outage probability and SER based on the cumulative density function (CDF) of the total achieved SNR.

A. Outage Probability

The outage probability is defined as γ_D falls below a certain threshold γ , thus, we express it as follows:

$$P_{out} \triangleq \Pr[\gamma_D \leq \gamma] = F_{\gamma_D}(\gamma) \quad (6)$$

In order to obtain the outage probability we need to derive the CDF of γ_D , which is obtained using the following proposition.

Proposition 1: The CDF of γ_D in multi-sources multi-relays with multiple antennas network is expressed as follows:

$$F_{\gamma_D}(\gamma) = \sum_{t=0}^{y-1} \binom{y-1}{t} (-1)^{y-t-1} \sum_{|g|=y-t-1} (y-t-1)! \varphi_1 \times$$

$$\prod_{q=0}^{Q-1} \varphi_2 \sum_{\xi}^{\mu} \varphi_3 \gamma^{\mu-\xi} e^{\gamma^{(y-t-1)}} \Gamma(i+1, v\gamma) \quad (7)$$

where $y = M \cdot K$, $\varphi_1 = \frac{\varepsilon^{Q(y-t-1)}}{(Q-1)!^{(y-t-1)} \bar{b}_{m,k}^{Q(y-t-1)}}$,

$\mathbf{g} = (g_0, g_1, \dots, g_q, \dots, g_{Q-1})$ is the Q -dimension vector of non-negative integers with the length definition of

$|\mathbf{g}| = \sum_{q=0}^{Q-1} g_q$, the second summation term means that the sum is taken over all \mathbf{g} of length $y-t-1$,

$$\mu = \sum_{q=0}^{Q-1} qg_q, \varphi_2 = \frac{\varepsilon^{\mu}}{(q!)^{g_q} g_q! \bar{a}_{m,k}^{\mu}}, v = (y-t-1) \cdot \left(\frac{\bar{b}_{m,k} + 1}{\bar{b}_{m,k}}\right),$$

$$i = (Q-1)(y-t-1) \text{ and } \varphi_3 = \left(\frac{\mu}{\xi}\right) (1/v)^{i+1}.$$

Proof: see Appendix A

Our expression derived in (10) shows that better outage probability can be determined by either K , M or Q compared to the approaches in [8], [11]. Furthermore, a better P_{out} is obtained as either of them increases.

B. Symbol Error Rate

To compute the SER we use the approach that is adopted in [14], which expresses its using CDF resulted in (10). The resultant expression of SER is valid for all general modulation forms that satisfy the conditional error $Q_c(\sqrt{2\beta\gamma})$ with $Q_c(\cdot)$ is the Gaussian Q-function and $\beta = \sin^2(\pi/M_e)$ for M-ary phase shift keying (M-PSK). According to [14] the SER is given by:

$$P_e = \frac{\sqrt{\beta}}{2\sqrt{\pi}} \int_0^{\infty} \frac{e^{\beta\gamma}}{\sqrt{\gamma}} F_Z(\gamma) d\gamma \quad (8)$$

Substituting (7) in (8) with some algebraic manipulations, we obtain the SER expression as:

$$F_{\gamma_D}(\gamma) = \sum_{t=0}^{y-1} \binom{y-1}{t} (-1)^{y-t-1} \sum_{|g|=y-t-1} (y-t-1)! \prod_{q=0}^{Q-1} \varphi_2 \times \sum_{\xi}^{\mu} \varphi_3 \int_0^{\infty} \gamma^{\mu-\xi} e^{\gamma^{(\beta+y-t-1)}} \Gamma(u+1, v\gamma) d\gamma \quad (9)$$

Then, solving the integral in (9) with the identity in [15, 4.55.1]

$$\int_0^{\infty} t^{\mu-1} e^{pt} \Gamma(r, ct) dt = \frac{c^r \Gamma(u+r)}{r(c+p)^{u+r}} {}_2F_1(1, u+r; u+1; \frac{p}{c+p}) \quad (10)$$

We get the exact SER expression for multiple sources MIMO relay system as in:

$$P_{e_{\gamma_D}}(\gamma) = \sum_{t=0}^{y-1} \binom{y-1}{t} (-1)^{y-t-1} \sum_{|g|=y-t-1} (y-t-1)! \varphi_1 \times \prod_{q=0}^{Q-1} \varphi_2 \sum_{\xi}^{\mu} \varphi_3 \frac{\tilde{z} v^{\mu+1} \Gamma(w_1)}{w_2 (w_3)^{w_1}} {}_2F_1\left(1, w_1; w_2 + 1, \frac{\beta + y - t - 1}{w_3}\right) \quad (11)$$

where $w_1 = M + \mu - \xi + 2$, $w_2 = \mu - \xi + 1$, $w_3 = v + \beta + y - t - 1$, ${}_2F_1(a, b; c, d)$ is the hypergeometric function defined in [16] and $\tilde{z} = \sqrt{\beta} / (2\sqrt{\pi})$.

C. Analytical Bound

Assuming that the total power is equally allocated to the source and the selected relay, we can estimate an upper bound of the average end-to-end SNR γ_D^{up} in multiple-sources multi-relay with multiple antennas systems using the approaches in [3]; thus (4), (5) can be re-expressed as follows:

$$\gamma_{D_{m,k}} \leq \varepsilon \max_i \min(a_{m,k}, b_{m,k}) = \gamma_{D_{m,i}}^{up} \quad (12)$$

$$\gamma_D^{up} = \max_m \gamma_{D_{m,k}} \quad (13)$$

Proposition 3: According to (11), the outage probability and the CDF of γ_D^{up} in multi sources multi relay with multiple antennas network is expressed as follows:

$$P'_{out} \triangleq \Pr[\gamma_D^{up} \leq \gamma] = F_{\gamma_D^{up}}(\gamma) \quad (14)$$

with $F_{\gamma_D^{up}}(\gamma)$ is expressed as follows:

$$F_{\gamma_D^{up}}(\gamma) = \sum_{t=0}^{y_2-1} \binom{y_2-1}{t} (-1)^{y_2-t-1} \times \sum_{|g|=y_2-t-1} (y_2-t-1)! \prod_{q=0}^{Q-1} \psi \gamma^{\mu} e^{-\Omega \gamma^{(y_2-t-1)}} \quad (15)$$

where $y_2 = 2MK$, the second summation term means that the sum is taken over all \mathbf{g} of length y_2-t-1 ,

$$\Omega = \varepsilon^2 (\bar{a}_{m,i} \bar{b}_{m,i}) \text{ and } \psi = \frac{\varepsilon^{\mu}}{(q!)^{g_q} g_q} \left(\frac{1}{\Omega}\right)^{\mu}.$$

Proof: see Appendix B

Substituting (13) in (8) and applying the identity in [15, 381.4] with some algebraic manipulations we get an upper bound on the SER as in:

$$P'_e(\gamma) = \sum_{t=0}^{y_2-1} \binom{y_2-1}{t} (-1)^{y_2-t-1} \sum_{|g|=y_2-t-1} (y_2-t-1)! \times \prod_{q=0}^{Q-1} \psi \frac{z \Omega^{\mu+1}}{[(\beta + y_2 - t - 1)]^{\mu+1}} \Gamma(\mu+1) \quad (16)$$

Our derived expressions and their upper bound are easy to compute expressions compose of finite summations of standard functions, which can be simulated easily in software such as MATLAB.

D. High SNR Evaluation

The closed-form expressions derived previously enable us to estimate the performance of multiple sources MIMO relay systems, However their complex form do not provide a valuable insights on how M , K and Q

affect the overall performance. Thus, we further analyze our expressions for the high SNR regime. At sufficiently high SNR, the diversity gain (outage diversity) G^d defines the slope of the outage probability or the SER against the average SNR.

Proposition 4: In high SNR regime as $\gamma \rightarrow \infty$, the outage probability and SER for γ_D are given by:

$$P_{out}^\infty = \left(\frac{\gamma}{\varepsilon \bar{c}} \right)^N \left[\frac{1}{\eta_1 Q!} + \frac{1}{\eta_2 (Q-1)!} \right]^{\frac{N}{Q}} \quad (17)$$

$$P_e^\infty = \frac{(2N)!}{2^{2N+1} N!} \left[\frac{1}{Q! \eta_1^Q} + \frac{1}{(Q-1)! \eta_2^Q} \right]^{\frac{N}{Q}} \quad (18)$$

where $\bar{a}_{m,k} = \eta_1 \bar{c}_{m,k}$, $\bar{b}_{m,k} = \eta_2 \bar{c}_{m,k}$ and $N = KMQ$.

Proof: see Appendix C.

Proposition 5: The outage Probability and SER for γ_D^{up} can be computed in high SNR regime as $\gamma \rightarrow \infty$, which are given by:

$$P_{out}^{r\infty} = \left(\frac{\gamma}{\Omega} \right)^N \left(\frac{2}{Q!} \right)^{\frac{N}{Q}} \quad (19)$$

$$P_e^{r\infty} = \frac{(2N)!}{2^{2N+1} N!} \left(\frac{2}{Q! \Omega^Q} \right)^{\frac{N}{Q}} \quad (20)$$

Proof: see Appendix D.

From (18), (19), (20) and (21) it is obvious that the outage diversity gain is $G^d = N$.

Using the approach in [17, Proposition 1], the asymptotic SER can be re-written as $P_e^\infty \propto (\beta^{G^d})$, which reflects the diversity gain, i.e. $G^d = N$.

IV. NUMERICAL RESULTS

In this section, numerical results are provided to demonstrate the impact of M , K and Q on the average SNR of the multi-source multi-relay with multiple antennas systems. Both the symbol error rate and the outage probability reflect the system performance and similar conclusion can be obtain from both parameters, therefore, only SER results are presented here. Monte Carlo simulations are performed to investigate and compare the reliability of the exact and the bound expressions derived previously. In these simulations, we have utilized data with symbol block size $N_f = 512$, quadrature phase shift keying (QPSK) modulation and equal power allocation between the m^{th} source and the k^{th} relay, i.e. $P_{S_m} = P_{R_k} = P/2$ (P is the total power in the system). Furthermore, we assume that the channels are generated with normalized noise gain and variance conditions, i.e. ($N_o = \sigma_{S_m R_k}^2 = \sigma_{R_k D}^2 = 1$).

Fig. 1 to Fig. 3 plots the SER P_e in terms of K and Q for a fixed value of M . Assuming $M = 1$, the expression in (11) and the simulations are given in Fig. 1, while the bound in (16) is compared with the simulations in Fig. 2. The results presented in the figures validate the correctness of (11) and (16) respectively. The figures also reveal the role that K and Q play in determining SER, in which better SER performance is obtained by increasing either K or Q . For example, in Fig. 1, there is about 1.5dB difference between $(Q = 4, K = 3)$ and $(Q = 3, K = 2)$, while 2dB difference is observed from $(Q = 2, K = 4)$. Furthermore, In Fig. 2, at 10^{-5} SER, $(Q = 3, K = 2)$ leads with 1dB performance improvement over $(Q = 2, K = 3)$ Similarly at 10^{-5} SER $(Q = 2, K = 4)$ has 1dB better than $(Q = 3, K = 4)$.

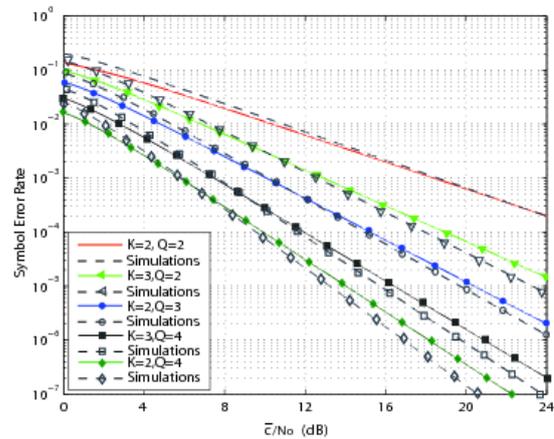


Fig. 1. The Exact in (11) and simulations of SER versus $\varepsilon \bar{c}$ for $M = 1$ with QPSK modulation

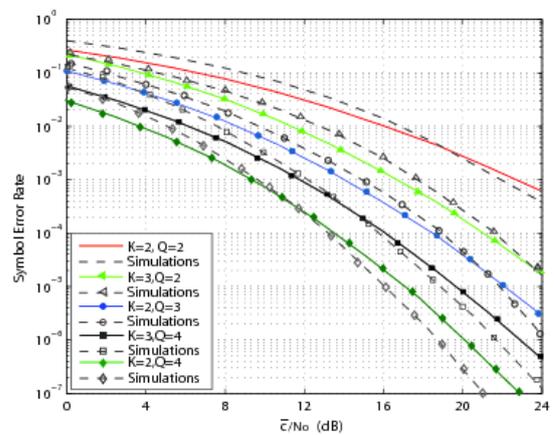


Fig. 2. The upper bound in (16) and simulations of SER versus $\varepsilon \bar{c}$ for $M = 1$ with QPSK modulation

Fig. 3 plots a comparison between the expressions (11) and (16) for $M = 2$. It can be seen from the figure that (11) and (16) achieve relatively close performance as both curves converge to the same region in high SNR regime.

Fig. 4 and Fig. 5 show the SER results of (11) and (16) compared with the asymptotic analysis in (20) and (22) with different number of M . From the figures, we can see the significant variation in the slope of the curve which is the reflection of the diversity order. The figures also reveal that the expressions in (11) and (16) are in accordance with their asymptotic approximation as they converge to the same value in high SNR.

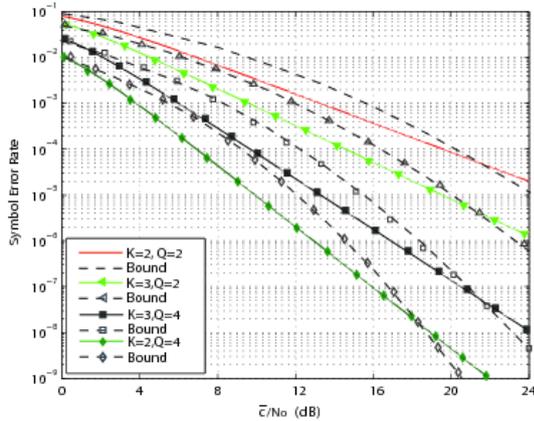


Fig. 3. The Exact in (11) and The bound in (16) of SER versus $\bar{\mathcal{E}}C$ for $M = 1$ with QPSK modulation

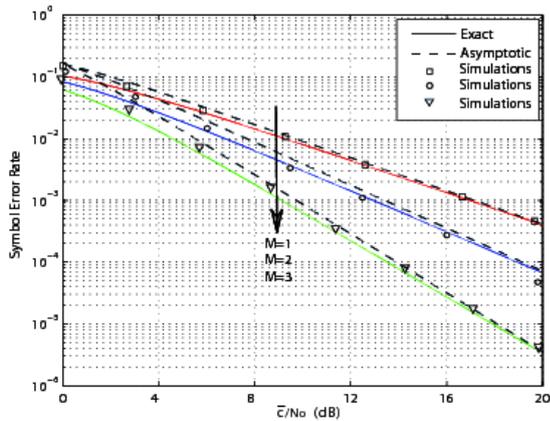


Fig. 4. The Exact in (11) with the asymptotic and simulations of SER versus $\bar{\mathcal{E}}C$ for $M = 1$ with QPSK modulation

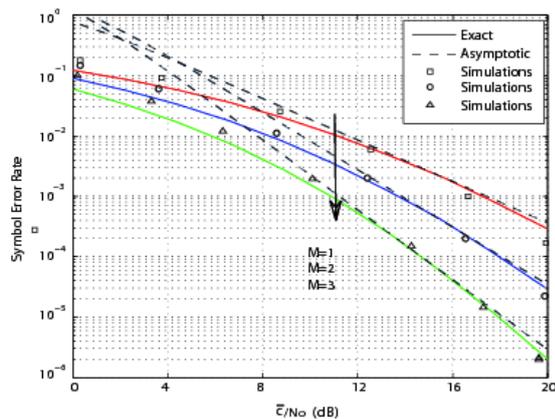


Fig. 5. The Exact in (11) with the asymptotic and simulations of SER versus $\bar{\mathcal{E}}C$ for $M = 1$ with QPSK modulation

Furthermore, the obtained results show the impact of number of the sources M on the improvement of SER performance.

Fig. 6 plots the corresponding diversity gain analysis of the expression in (11). According to [18], the diversity gain is estimated by the slope of $\log_{10} P_e$ and $\log_{10} \bar{c}_{m,k} / N_o$. According to SNR values $(0, \dots, 20)$, the total diversity gain $M.K.Q$ can be obtained.

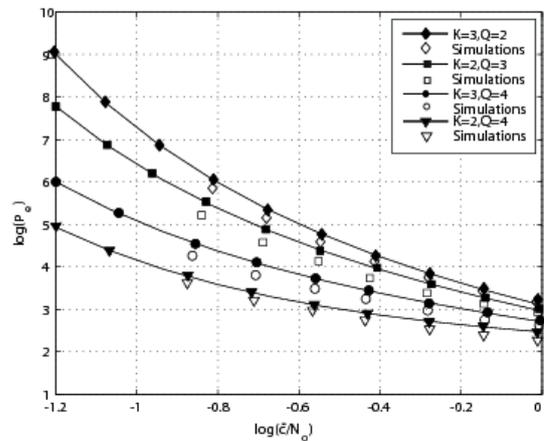


Fig. 6. Diversity gain analysis with P_e in terms of K and Q for $M=1$

For example, in $(M = 1, K = 2, Q = 1)$, $G_d \approx 12$, while $(M = 1, K = 4, Q = 2)$, $G_d \approx 8$ and $(M = 2, K = 4, Q = 3)$ $G_d \approx 24$. Since we utilize large symbol block size, the error related to $G_d \approx 1$ become rare. Furthermore, combining with PFS algorithms, the total achieved gain became $M.K.Q$ thus, the maximum diversity gain can be achieved.

V. CONCLUSIONS

This study presents the performance analysis of multi-source multi-relay with multiple antennas networks. During the analysis, we employ an AF MIMO relay scheme, where the best relay access point is selected using the full CSI at the destination. We derive closed-form and analytical bound expressions for the outage probability and SER as well as high SNR asymptotic expressions to obtain the diversity gain. The analytical expressions are validated with Monte Carlo simulations. The asymptotic analysis together with Monte Carlo simulations have revealed that the diversity gain is determined by the number of the sources, number of the relay access points and the number of antennas equipped in each relay. These observations have provided that the diversity gain is simply given by the multiplication of the multiuser diversity gain and the multipath diversity gain.

APPENDIX A

According to order statistics, for independently distributed random variables $\gamma_{D,m,k}$, the CDF of γ_D is estimated:

$$\begin{aligned}
 F_{\gamma_D}(\gamma) &= \Pr[\gamma_D \leq \gamma] = [\Pr[\gamma_{D_{m,k}} \leq \gamma]]^M \\
 &= \left[\int_0^\infty F_X(\gamma + \gamma_{R_k D}) f_Y(\gamma_{R_k D}) d\gamma \right]^y \\
 &= \sum_{t=0}^{y-1} \binom{y-1}{t} (-1)^{y-t-1} [1 - F_1(\gamma)]^{y-t-1} \quad (23)
 \end{aligned}$$

where $F_X(\gamma_{S_m R_k})$ is the CDF of $\gamma_{S_m R_k}$ and $f_Y(\gamma_{R_k D})$ is the PDF of $\gamma_{R_k D}$, which are expressed by:

$$F_X(\gamma_{S_m R_k}) = 1 - e^{-\frac{\gamma_{S_m R_k}}{\varepsilon \bar{a}_{m,k}}} \sum_{q=0}^{Q-1} \frac{\gamma_{S_m R_k}}{q! (\varepsilon \bar{a}_{m,k})^q} \quad (24)$$

$$f_Y(\gamma_{R_k D}) = \frac{e^{-\frac{\gamma_{R_k D}}{\varepsilon \bar{b}_{m,k}}} \gamma_{R_k D}^{Q-1}}{(Q-1)! (\varepsilon \bar{b}_{m,k})^Q} \quad (25)$$

Substituting (24) and (25) in (23), with the help of the multinomial theorem yields:

$$\begin{aligned}
 F_{\gamma_D}(\gamma) &= \sum_{t=0}^{y-1} \binom{y-1}{t} (-1)^{y-t-1} \sum_{\xi} (y-t-1)! \varphi_1 \prod_{q=0}^{Q-1} \varphi_2 \times \\
 &\quad \sum_{\xi} \varphi_3 \gamma^{\mu-\xi} e^{-(y-t-1)\xi} \int_{\gamma}^{\infty} \gamma_{R_k D}^i e^{-\gamma_{R_k D}} d\gamma_{R_k D} \quad (26)
 \end{aligned}$$

Solving the integral with [15, 3.381.3], yields to the outage probability expression in (7).

APPENDIX B

For independently distributed $\gamma_{S_m R_k}$ and $\gamma_{R_k D}$, in which $\gamma_D^{up} = \min(\gamma_{S_m R_k}, \gamma_{R_k D})$, the CDF is expressed as:

$$\begin{aligned}
 P'_{out} &= F_{\gamma_D}^{up}(\gamma) = \Pr[\gamma_D^{up} \leq \gamma] = [\Pr[\gamma_{D_{m,k}}^{up} \leq \gamma]]^M \\
 &= [1 - \Pr(\gamma_{S_m R_k} \leq \gamma) \Pr(\gamma_{R_k D} \leq \gamma)]^{y_2} \\
 &= [1 - F_X(\gamma) F_Y(\gamma)]^{y_2} \\
 &= [F_X(\gamma) + F_Y(\gamma) - F_X(\gamma) F_Y(\gamma)]^{y_2} \quad (27)
 \end{aligned}$$

Then

$$P'_{out} = \sum_t \binom{y_2}{t} (-1)^{y_2-t-1} [1 - F_2(\gamma)]^{y_2-t-1} \quad (28)$$

where

$$F_Y(\gamma_{R_k D}) = 1 - e^{-\frac{\gamma_{R_k D}}{\varepsilon \bar{b}_{m,k}}} \sum_{q=0}^{Q-1} \frac{\gamma_{R_k D}}{q! (\varepsilon \bar{b}_{m,k})^q} \quad (29)$$

is the as the CDF of $\gamma_{R_k D}$

Submitting the (24) and (29) in (28) and Solving with the help of the multinomial theorem yields the expression in (15).

APPENDIX C

Let us define $\bar{a}_{m,k} = \eta_1 \bar{c}_{m,k}$, and, with η_1 and η_2 are real positive integer. By using the identity:

$$e^t \sum_{l=0}^{L-1} \frac{-t^c}{c!} \sum_{l=0}^{L-1} \frac{t^l}{l!} = 1 - \frac{t^L}{L!} + o(t^L) \quad (30)$$

We re-write the CDF of $\gamma_{S_m R_k}$ as:

$$F_X(\gamma_{S_m R_k}) = \frac{\gamma_{S_m R_k}^Q}{Q! (\varepsilon \eta_1 \bar{c}_{m,k})^Q} + o(\gamma_{S_m R_k}^Q) \quad (31)$$

Substituting in (23) we get:

$$\begin{aligned}
 P_{out}^\infty &= \left[\int_{\gamma}^{\infty} \frac{\gamma_{R_k D}^{Q-1}}{(\varepsilon \eta_1 \eta_2 \bar{c}_{m,k})^Q (Q-1)!} d\gamma_{R_k D} \right. \\
 &\quad \left. - \int_{\gamma}^{\infty} \frac{(\gamma_{R_k D} + \gamma)^Q \gamma_{R_k D}^{Q-1}}{(\varepsilon \bar{c}_{m,k})^{2Q} \eta_1^Q \eta_2^Q Q! (Q-1)!} e^{-\frac{\gamma_{R_k D}}{\varepsilon \bar{b}_{m,k}}} d\gamma_{R_k D} \right]^y \quad (32)
 \end{aligned}$$

Solving the first integral by the identities [15, 3.381.3] and [15, 8352.4] we get the expression:

$$1 - \frac{\gamma^Q}{\varepsilon \eta_2 \bar{c}_{m,k}^Q Q!} \quad (33)$$

The second integral is solved with the help of the generalized power series expansion for the exponential function in [15, 1.211.1] and for the incomplete gamma function. We obtain the first order Taylor series expansion as:

$$1 - \frac{\gamma^Q}{\varepsilon \eta_1 \bar{c}_{m,k}^Q Q! (Q-1)!} \quad (34)$$

Substituting (33), (34) in (32) and applying the identity in (30) with some algebraic manipulation we get (17).

APPENDIX D

The CDF in (15) can be re-written as:

$$P_{out}^{\prime\infty} = [1 - (e^{-\frac{\gamma}{\Omega}} \sum_{q=0}^{Q-1} \frac{\gamma}{q! \Omega^q})^2]^{y_2} \quad (35)$$

By using 15, 8.352.2] and [15, 8.354.2] for the incomplete gamma function yields:

$$\begin{aligned}
 P_{out}^{\prime\infty} &= \left[1 - \frac{\Gamma(Q, \frac{\gamma}{\Omega})}{(Q-1)!} \right] \\
 &= \left[1 - \frac{\left((Q-1)! - \left(\frac{\gamma}{\Omega} \right)^Q \frac{1}{Q} (1 + o(\gamma^Q)) \right)^2}{(Q-1)!} \right]^{y_2} \quad (36)
 \end{aligned}$$

By selecting the first order terms, we get (21).

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