Multiuser Binary Rate Scheduling for Fading OFDM Networks with User Fairness Rate Constraints

Xiangyu Gao and Yuesheng Zhu
Communication and Information Security Lab, Shenzhen Graduate School, Peking University, Shenzhen, China
Email: xygao037@gmail.com; zhuys@pkusz.edu.cn

Abstract**—**In this paper, we propose an efficient scheduling scheme for fading orthogonal frequency division multiplexing multiuser networks with user fairness rate constraints. This scheme, termed as binary rate scheduling (BRS), allows at most one user to transmit at a fixed rate if the selected user has its channel gain higher than a certain threshold. BRS scheme avoids the complicated interference cancellations and the variable-rate transmissions involved in the optimal scheduling scheme, hence has very low complexity. The optimization problems of BRS for the related channel gain thresholds and weight factors are formulated with respect to both adaptive and fixed transmit power assumptions. And the optimization algorithm is developed by utilizing the decent structure of quasi-convexity of the problems. With numerical results, it is shown that BRS scheme is capable of achieving near optimal performance in both the homogeneous and heterogeneous multiuser scenarios. Particularly, in the low-rate regime, it is proved that BRS asymptotically achieves the same spectral efficiency as the optimal scheme. The results in this paper indicate that BRS scheme is a promising scheduler solution in practical wireless networks with regard to both hardware implementation and power efficiency.

Index Terms**—**Orthogonal frequency division multiplexing (OFDM), binary rate scheduling (BRS), BRS with adaptive transmit power (BRS-A), BRS with fixed transmit power (BRS-F), minimum transmit sum power (MTSP), water-filling

I. Introduction

The increasing demands for high speed wireless connections have posed a great challenge on the design of next generation wireless communication systems. One promising solution is to employ channel adaptive scheduling techniques, which have been demonstrated to be capable of improving the spectral efficiency in many research studies [1]-[3]. To enable such kind of scheduling techniques, transmitters are required to gain certain amount of knowledge of the channel state information (CSI) and adapt the transmit power and rate accordingly.

For multiuser wireless channels with continuous fading states, information theory predicts that a non-orthogonal variable-rate scheduler achieves the channel capacity [1], [4]. However, the optimal scheduler is deemed as impractical since it involves several high-complexity operations, such as multiuser concurrent transmissions, variable-rate channel coding and infinite amount of CSI feedback [4].

To resolve the restrictions of complexity, many research works have been conducted aiming at designing novel schedulers that can achieve a more efficient performance-complexity trade-off [5]-[13]. One line of these studies focuses on reducing the CSI feedback overhead, which is generally referred as to limited feedback techniques in the literature [5]-[6], [8]-[9]. Unfortunately, most of these proposed schedulers implicitly assume a variable-rate Gaussian coding scheme, and they generally neglect the cost incurred by the transmit rate agreement between the transceivers in each channel block. Note that for a channel block comprised of T_b quadrature phase-shift keying modulated symbols, the overhead incurred by specifying a code rate from the rate set with a cardinality of 2^F is $F/2T_b$. Therefore, in channels where $F/2T_b$ is not sufficiently close to 0, these proposed schedulers may not work as efficiently as reported after considering the overhead from the variable-rate agreements.

For the above concerns, we investigate a fixed rate fading adaption scheme in this paper, aiming at reducing the number of rate agreements to 1 time per scheduling period. The proposed scheme, termed as binary rate scheduler (BRS), is motivated by its simple channel allocation method and implied channel coding structures. Specifically, BRS allocates each sub channel to a unique user according to a linear competitive mechanism, and the allocated user either transmits at a fixed rate when its channel gain is higher than a certain threshold or keeps silent. BRS fully nullifies multiuser interference and only requires a fixed-rate coding and modulation scheme. The related complexity and feedback overhead of BRS can be very low, hence is relatively more implementable than the schedulers utilizing the variable-rate coding schemes.

We study the performance of BRS in the context of Rayleigh fading orthogonal frequency division multiplexing (OFDM) cellular networks. In addition, to support certain level of quality of services (QoS) [10]-[12], a user fairness rate constraint is imposed that the information rate of each user is fixed at the same level. In practice, this constraint of the information rate may be due to requirements of application layers or wireless

Manuscript received April 15, 2014; revised July 28, 2014.
This work was supported by the 973 program #2012CB315904, China and Shenzhen Engineering Laboratory for Broadband Wireless Network Security.
Corresponding author email: zhuys@pkusz.edu.cn.
doi:10.12720/jcm.9.7.572-578.

services payment covenants. The optimized BRS performance is obtained by our developed algorithm and is analyzed by comparing it with the optimal scheduler in terms of power efficiency.

The rest of the paper is organized as follows. Section II introduces the system model and the operating principles of BRS. In Section III, the performance of BRS is analyzed and optimized for both homogeneous and heterogeneous multiple access channels (MACs). Numerical results are provided and discussed in Section IV. Finally, conclusions are drawn in Section V.

II. System Model and Binary Rate Scheduling

Consider an ideally interleaved K-user block fading OFDM MAC with L i.i.d. subcarriers. The discrete-time model of the received signal in the t-th time slot, at l-th subcarrier, denoted by $y(m)$, can be written as [14]

$$y(m) = \sum_{k=1}^{K} \sqrt{\alpha_k} h_k(m) x_k(m) + n(m), \qquad (1)$$

where $m = t \times L + l$, denotes the subchannel index and $x_k(m)$ and $h_k(m)$ are, respectively, the transmitted signal and Rayleigh fading power coefficient of user k in the m-th subchannel, α_k the corresponding large-scale fading power factor that represents the joint effect of shadow fading and path loss, and $n(m)$ the additive white Gaussian noise (AWGN) sample. Both $\{h_k(m)\}$ and $\{n(m)\}$ follow independent complex symmetric Gaussian distribution with zero mean and unit variance. The large-scale fading parameters $\{\alpha_k\}$ are assumed fixed while the Rayleigh fading coefficients $\{h_k(m)\}$ vary independently for different subchannels. They are assumed perfectly known at receiver sides. In addition, we assume that the long-term average rate of each user is same at C/K nats/subchannel where C is referred to as the average sum rate.

A. Operating Principles of BRS

For the MAC presented above, the optimal scheduling scheme can be obtained by the algorithm proposed in [4]. The optimal scheme involves complicated operations, hence is difficult to be implemented. To reduce complexity, we propose a simpler BRS scheme outlined below:

- In the subchannel m, only the user (denoted by k) with the largest weighted channel gain is selected for transmission, i.e., $k = \text{argmax}_i\, \alpha_i \beta_i |h_i(m)|^2$ where β_i is the weight factor of user i;
- Denote by δ_k the pre-determined threshold for the user k, then the selected user k transmits at a fixed rate only if $\alpha_k \beta_k |h_k(m)|^2 \geq \delta_k$. Otherwise, the system is in silence in this subchannel.

B. Problem Formulation and Minimum Transmit Sum Power of BRS

According to the above operating principles, user k occupies the channel if and only if its weighted channel gain $\alpha_k\beta_k|h_k(m)|^2$ is the largest among those of all users. Hence, the probability that user k with weighted channel gain $g=\alpha_k\beta_k|h_k(m)|^2$ is selected for transmission can be written as

$$\Pr(k = \arg\max_i \alpha_i\beta_i |h_i(m)|^2 \mid g = \alpha_k\beta_k |h_k(m)|^2) = \prod_{i\neq k} F_i(g), \qquad (2)$$

where $F_k(g) = 1 - e^{-g/\alpha_k\beta_k}$ is the cumulative density function of user k's weighted channel gain. Thus, with the given threshold δ_k, the transmit probability of user k, denoted by ε_k, is given by

$$\begin{aligned}\varepsilon_k &= \Pr(\alpha_k\beta_k |h_k(m)|^2 \geq \delta_k, k = \arg\max_i \alpha_i\beta_i |h_i(m)|^2) \\ &= \int_{\delta_k}^{\infty} f_k(g) \prod_{i\neq k} F_i(g)dg,\end{aligned} \qquad (3)$$

where $f_k(g) = e^{-\frac{g}{\alpha_k\beta_k}} \big/ \alpha_k\beta_k$ is the probability density function of user k's weighted channel gain. For clarity, define the function

$$\psi_k(g) = f_k(g)\prod_{i\neq k} F_i(g) = \frac{1}{\alpha_k\beta_k} e^{\frac{-g}{\alpha_k\beta_k}} \prod_{i\neq k}(1 - e^{\frac{-g}{\alpha_i\beta_i}}) \qquad (4)$$

that represents the joint effects of user selection and channel variation of user k. To satisfy the average rate constraint C/K for each user, the fixed active transmit rate of user k, denoted by r_k, should satisfy

$$r_k = \frac{C}{K\varepsilon_k} = \frac{C}{K\int_{\delta_k}^{\infty}\psi_k(g)dg} \qquad (5)$$

This paper is concerned with achieving the minimum transmit sum power (MTSP) of BRS by optimizing the weighted factors $\{\beta_k\}$ and thresholds $\{\delta_k\}$. And we consider two different power adaption schemes as follows. In the first scheme, the transmit power level of active user k, denoted by $p_k^{\text{BRS-A}}$, is adapted to its present weighted channel gain g. Based on Shannon's capacity formula, the transmit power is given by

$$p_k^{\text{BRS-A}} = \beta_k(e^{r_k} - 1)\big/ g \qquad (6)$$

where the coefficient β_k is included to compensate the ratio between weighted channel gain and real channel gain. We refer to this scheme as BRS with adaptive transmit power (BRS-A). The corresponding average transmit power of user k can be written as

$$P_k^{\text{BRS-A}} = \beta_k \int_{\delta_k}^{\infty} \frac{e^{r_k} - 1}{g}\psi_k(g)dg \qquad (7)$$

Secondly, we consider a fixed transmit power BRS scheme (BRS-F). Specifically, the transmit power is fixed at the power level that is required when the weighted channel gain is equal to the threshold. Thus for user k, the fixed transmit power is given by

$$p_k^{\text{BRS-F}} = \beta_k(e^{r_k} - 1)\big/\delta_k \qquad (8)$$

Similarly, the average transmit power of user k in BRS-F can be written as

$$P_k^{\text{BRS-F}} = \beta_k \int_{\delta_k}^{\infty} \frac{e^{r_k} - 1}{\delta_k}\psi_k(g)dg \qquad (9)$$

The minimum transmit sum powers (MTSP) of BRS-A and BRS-F are, respectively, the solutions of the following two minimizations:

$$\min_{\{\beta_k\},\{\delta_k\}} \; P^{\text{BRS-A}} = \sum_k \beta_k \left(e^{\frac{C}{K\int_{\delta_k}^{\infty}\psi_k(g)dg}} - 1\right)\int_{\delta_k}^{\infty}\frac{\psi_k(g)}{g}dg \quad (10)$$

$$\min_{\{\beta_k\},\{\delta_k\}} \; P^{\text{BRS-F}} = \sum_k \beta_k \left(e^{\frac{C}{K\int_{\delta_k}^{\infty}\psi_k(g)dg}} - 1\right)\int_{\delta_k}^{\infty}\frac{\psi_k(g)}{\delta_k}dg \quad (11)$$

Henceforth, the solutions of (10) and (11) are respectively denoted by $P_{\text{MTSP}}^{\text{BRS-A}}$ and $P_{\text{MTSP}}^{\text{BRS-F}}$, and we say explicitly "BRS" when referring both BRS-A and BRS-F.

III. Analysis of the Performance of BRS

In this section, we analyze the performance of BRS. We first consider the homogeneous MAC case where the large-scale fading factors $\{\alpha_k\}$ are identical as α for all users. The design of BRS in general heterogeneous MACs will be discussed later in this section.

A. MTSP of BRS-A in Homogeneous MACs

Due to the symmetry property of users' channels in homogeneous MACs, the optimal $\{\delta_k\}$ and $\{\beta_k\}$ in (10) and (11) should satisfy $\{\beta_k = 1/\alpha\}_{k=1}^{K}$, $\{\delta_k = \delta\}_{k=1}^{K}$, where δ represents the identical user threshold. The minimization (10) then reduces to the single variable minimization problem given below:

$$\min_{\delta} \; P^{\text{BRS-A}} = \frac{1}{\alpha}\left(e^{\frac{C}{\int_{\delta}^{\infty}\psi(g)dg}} - 1\right)\int_{\delta}^{\infty}\frac{\psi(g)}{g}dg \quad (12)$$

where $\psi(g) = Ke^{-g}(1-e^{-g})^{K-1}$. For convenience, we define $r = C\big/\int_{\delta}^{\infty}\psi(g)dg$ as the instantaneous rate of each user for a given threshold δ. The corresponding optimal rate that minimizes $P^{\text{BRS-A}}$ is denoted by r^*.

Next, we show that the target function in (12) is a strictly quasiconvex function of δ under certain conditions. Its minimum is given by a closed-form expression of the optimal δ that can be easily obtained via a binary search method. We compute the first derivative as

$$\frac{\mathrm{d}P^{\text{BRS-A}}}{\mathrm{d}\delta} = \frac{e^{\frac{C}{\int_{\delta}^{\infty}\psi(g)dg}}\psi(\delta)}{\alpha\delta\int_{\delta}^{\infty}\psi(g)dg} \times \left(\frac{C\delta}{\int_{\delta}^{\infty}\psi(g)dg}\int_{\delta}^{\infty}\frac{\psi(g)}{g}dg - \frac{\left(e^{\frac{C}{\int_{\delta}^{\infty}\psi(g)dg}} - 1\right)\int_{\delta}^{\infty}\psi(g)dg}{e^{\frac{C}{\int_{\delta}^{\infty}\psi(g)dg}}}\right) \quad (13)$$

and denote the part in the bracket of right hand side of (13) by $A(\delta)$. Note that while $\psi(g) = Ke^{-g}(1\text{-}e^{-g})^{K-1}$, $A(\delta)$ is a strictly increasing function of δ. We can prove this by separately establishing the monotonicity of the two parts of $A(\delta)$ in δ. For the first part, its monotonicity can be easily proved through infinite series expansion whereas the monotonicity of the second part can be established by the positivity of the first derivative with respect to δ. Combined with the monotonicity of $A(\delta)$, the following boundary conditions

$$\lim_{\delta\to 0} A(\delta) = -(e^{c}-1)/e^{c} < 0 \quad (14)$$

$$\lim_{\delta\to\infty} A(\delta) = C > 0 \quad (15)$$

guarantee that there exits one unique positive root for the equation $A(\delta)$=0, which establish that $P^{\text{BRS-A}}$ is a quasiconvex function of δ. Denote the root by δ^{A}. Then, the MTSP of BRS-A can be written as

$$P_{\text{MTSP}}^{\text{BRS-A}} = \frac{\left(e^{C/[1-(1-e^{-\delta^{\text{A}}})^K]} - 1\right)^2\left(1-(1-e^{-\delta^{\text{A}}})^K\right)^2}{\alpha C\delta^{\text{A}} e^{C/[1-(1-e^{-\delta^{\text{A}}})^K]}} \quad (16)$$

B. MTSP of BRS-F in Homogeneous MACs

For BRS-F, (11) can be transformed as the solution of the following single variable minimization:

$$\min_{\delta} \; P^{\text{BRS-F}} = \frac{1}{\alpha\delta}\left(e^{\frac{C}{\int_{\delta}^{\infty}\psi(g)dg}} - 1\right)\int_{\delta}^{\infty}\psi(g)dg \quad (17)$$

With similar arguments, we can prove that $P^{\text{BRS-F}}$ is a quasiconvex function of δ as well. Thereby, the solution of (17) can be written as

$$P_{\text{MTSP}}^{\text{BRS-F}} = \frac{\left(e^{C/[1-(1-e^{-\delta^{\text{F}}})^K]} - 1\right)\left(1-(1-e^{-\delta^{\text{F}}})^K\right)}{\alpha\delta^{\text{F}}} \quad (18)$$

where δ^{F} represents the unique positive root of the following function

$$B(\delta) = C - \frac{\left(e^{\frac{C}{\int_{\delta}^{\infty}\psi(g)dg}} - 1\right)\int_{\delta}^{\infty}\psi(g)dg}{e^{\frac{C}{\int_{\delta}^{\infty}\psi(g)dg}}}\left(\frac{\int_{\delta}^{\infty}\psi(g)dg}{\delta\psi(\delta)} + 1\right) \quad (19)$$

Note that the significance of BRS-F is that it requires only one bit CSI feedback for each subchannel. Note that this feedback reduction is achieved at the expense of some power waste. Therefore, $P_{\text{MTSP}}^{\text{BRS-A}} < P_{\text{MTSP}}^{\text{BRS-F}}$ holds under same C and K.

C. Design of BRS in Heterogeneous MACs

For general heterogeneous MACs, where large-scale coefficients $\alpha_1, \alpha_2, \ldots, \alpha_K$ do not equal to each other, the performance of BRS has to be improved with the joint optimization of the channel thresholds $\{\delta_k\}$ and weight factors $\{\beta_k\}$. For clarity, we define the joint threshold vector and joint weight vector as $\boldsymbol{\delta} = [\delta_1, \delta_2, \cdots, \delta_K]$ and $\boldsymbol{\beta} = [\beta_1, \beta_2, \cdots, \beta_K]$.

Our first step for solving the minimizations of (10) and (11) is to study the relationship between the optimal $\boldsymbol{\delta}$ and $\boldsymbol{\beta}$, and it can be identified that for a given joint weight vector $\boldsymbol{\beta}$, the optimal user k's threshold that minimizes (10) or (11) is the unique root of the first partial derivative of (10) or (11) with respect to δ_k. We can prove this by establishing that each part in the sum of (10) or (11) is a strictly quasi convex function as discussed above. This property of the problem provides us a way of mapping **β** to **δ**. And utilizing the quasi convexity, the mapping relationship from **β** to **δ** can be numerically computed with an efficient binary search method.

After establishing the mapping between the two groups of variables, the variable dimensions of the problems are reduced and both $P^{\text{BRS-A}}$ and $P^{\text{BRS-F}}$ can be represented as functions of **β** which needs to be optimized according to the joint large-scale fading coefficients vector (denoted by **α**). We propose a simple gradient descent algorithm (GDA) outlined below to optimize **β**. Although GDA does not necessarily converge to the optimal point due to the non-convexity of the problems, we can show by numerical results that the GDA-optimized BRS performance is sufficiently close to the MTSP.

Gradient Descent Algorithm (GDA) for BRS-A (or BRS-F):

- Initialization: set $\boldsymbol{\beta}^{(0)}=1/\boldsymbol{\alpha}$, and calculate the threshold vector $\boldsymbol{\delta}^{(0)}$ that minimizes $P^{\text{BRS-A}}$ (or $P^{\text{BRS-F}}$) by binary search. Denote the optimized average transmit power achieved in the first iteration by $P^{(0)}$;
- Given $\boldsymbol{\beta}^{(l)}$, $\boldsymbol{\delta}^{(l)}$ and $P^{(l)}$, numerically calculate the gradient $\Delta\boldsymbol{\beta} = -\nabla P^{\text{BRS-A}}(\boldsymbol{\beta}^{(l)})$ (or $\Delta\boldsymbol{\beta} = -\nabla P^{\text{BRS-F}}(\boldsymbol{\beta}^{(l)})$);
- Choose a step length t with backtracking line search method given in[15];
- Update $\boldsymbol{\beta}^{(l+1)}= \boldsymbol{\beta}^{(l)}+t\Delta\boldsymbol{\beta}$. Calculate $\boldsymbol{\delta}^{(l+1)}$ and $P^{(l+1)}$;
- If $\left|P^{(l+1)} - P^{(l)}\right| < \kappa$, go to 6); Else, go to 2);
- Output $\boldsymbol{\beta}=\boldsymbol{\beta}^{(l+1)}$, $\boldsymbol{\delta}=\boldsymbol{\delta}^{(l+1)}$ and the optimized performance $P^{\text{BRS-A}} = P^{(l+1)}$ (or $P^{\text{BRS-F}} = P^{(l+1)}$).

The convergence of this algorithm is obvious, and the parameter κ in step 5) is a small positive number that determines the convergence speed and precision of the optimization algorithm.

In order to illustrate the optimized performance of BRS by our proposed GDA, we introduce a lower bound for the MTSP of BRS. This lower bound is based on the simple idea that all users in the system are assumed interference free. By applying the obtained results in subsection *B*) and *C*), the lower bounds for BRS-A and BRS-F can be respectively written as

$$P_{\text{LB}}^{\text{BRS-A}} = \frac{\left(e^{C/e^{-\delta^{\text{A}}}} - 1\right)^2 e^{-2\delta^{\text{A}}}}{C\delta^{\text{A}} e^{C/e^{-\delta^{\text{A}}}}} \sum_k \frac{1}{\alpha_k}, \tag{20}$$

$$P_{\text{LB}}^{\text{BRS-F}} = \frac{\left(e^{C/e^{-\delta^{\text{F}}}} - 1\right) e^{-\delta^{\text{F}}}}{\delta^{\text{F}}} \sum_k \frac{1}{\alpha_k}, \tag{21}$$

where δ^{A} and δ^{F} are respectively the solutions of the equations $A(\delta)=0$ and $B(\delta)=0$ with $K=1$, i.e., $\psi(g) = e^{-g}$. .

D. BRS Performance in the Low-Rate Limit

In this subsection, we study the BRS performance in the low-rate limit, i.e., C asymptotically approaches 0, which is written as $C\rightarrow 0$ in our notation. For practical systems, this corresponds to the scenarios with large allocated bandwidth. We derive the close-form performance of BRS in such a special limit.

As $C\rightarrow 0$, $\delta_k\rightarrow\infty$. Then the following is shown that

$$e^{\frac{C}{K\int_{\delta_k}^{\infty}\psi_k(g)dg}} - 1 \rightarrow \frac{C}{K\int_{\delta_k}^{\infty}\psi_k(g)dg}. \tag{22}$$

By substituting (22) into (11), we have

$$P^{\text{BRS-F}} \rightarrow \sum_k \frac{C\beta_k}{K\delta_k}. \tag{23}$$

As discussed above, the average power of BRS-F in (23) should be strictly lower bounded by the average power of the corresponding interference free networks. The lower bound can obtained by summing up K single user water-filling average power in the low-rate limit which is explicitly derived in [13] where it is shown that

$$P(\Theta) \rightarrow \frac{\alpha e^{-\Theta/\alpha}}{\Theta^2}, \tag{24}$$

$$C(\Theta) \rightarrow \frac{\alpha e^{-\Theta/\alpha}}{\Theta}, \tag{25}$$

where $P(\Theta)$ and $C(\Theta)$ are respectively the average power and capacity, Θ the water-level. By solving the constraint $C(\Theta) = C$, we obtain the optimal water-level as

$$\Theta \rightarrow \alpha \ln(K/C) - \alpha \ln\ln(K/C). \tag{26}$$

After applying (26) into (24) and combining the result with idea of the lower bound, the lower bound $P_{\text{LB}}^{\text{BRS-F}}$ is shown to converge to

$$P_{\text{LB}}^{\text{BRS-F}} \rightarrow \frac{C}{K\ln(K/C)} \sum_{k=1}^{K} \frac{1}{\alpha_k}. \tag{27}$$

From (27) and (23), it is shown that the performance of BRS-F can converge to the optimal performance (27) in the low-rate limit by setting $\boldsymbol{\beta}=1/\boldsymbol{\alpha}$ and $\delta_k=\ln(K/C)$.

Since BRS-A can be thought as a lower bound of BRS-F, the MTSP of BRS-A in the low-rate limit converges to (27) as well. This result implies that in wideband communication scenarios, BRS achieves the optimal variable-rate performance.

IV. Numerical Results and Discusstion

This section presents some numerical results to illustrate the performance of BRS. For clear intuition, the average sum rate C is expressed in bits/subchannel.

Fig. 1 and Fig. 2 depict the MTSPs of BRS and the optimal scheme in homogeneous MACs with respect to C and K. It is observed from Fig. 1 that the performance of BRS-A is fairly close to that of the optimal scheme even under a substantially high sum rate C=4 bits/subchannel, indicating that in the case where relatively accurate CSI feedback is available, fixed rate channel adaption is sufficiently effective. Thus, complicated variable-rate channel coding structure is not necessary in this case.

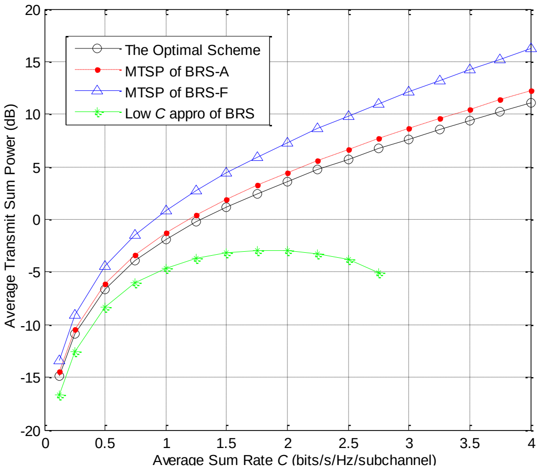


Fig. 1. Average transmit sum power versus average sum rate C for BRS-A, BRS-F in a 2-user homogeneous MAC.

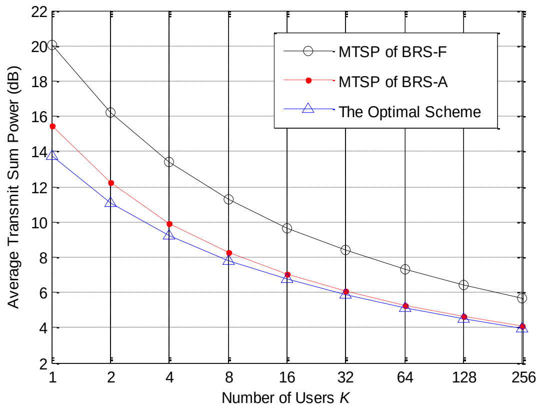


Fig. 2. Average transmit sum power versus the number of users K for BRS-A, BRS-F in a homogeneous MAC with average sum rate C=4 bits/subchannel.

In addition, the performance gap between BRS and the optimal scheme is reduced as C diminishes. This phenomenon suggests that great potentials of binary rate adaption can be exploited in the low-rate regime. Moreover, we can see that when C is near 0.25 bit/sub channel, BRS-F renders nearly the same performance with BRS-A. Hence, feedback reduction potentials can also be exploited in the low-rate regime. This observation is in line with the asymptotic optimality of BRS in the low rate limit as we proved in the previous section. Therefore, BRS-F is preferable when C is near or smaller than 0.25 bit/sub channel. In contrast, when C grows larger, the performance gap between BRS-A and BRS-F is more distinct and BRS-F becomes less efficient.

Fig. 3 shows the impact of the choice of active rate r on BRS-A and BRS-F. It can be seen that the performances of both BRS-A and BRS-F are not sensitive to the choice of active rate. For example, when C=1 bit/s/Hz, if we double the active rate (the corresponding transmit probability is reduced by 50% as well), only 1.7 dB and 2.3 dB performance losses are observed respectively for BRS-A and BRS-F. This observation reveals another merit in the design of BRS that we can conveniently choose an active rate in the favor of the channel code and modulation configurations without incurring much performance loss. Note that BRS-A is less sensitive than BRS-F particularly when C grows large. Hence, BRS-A is more flexible to the requirements of channel codes and modulation schemes.

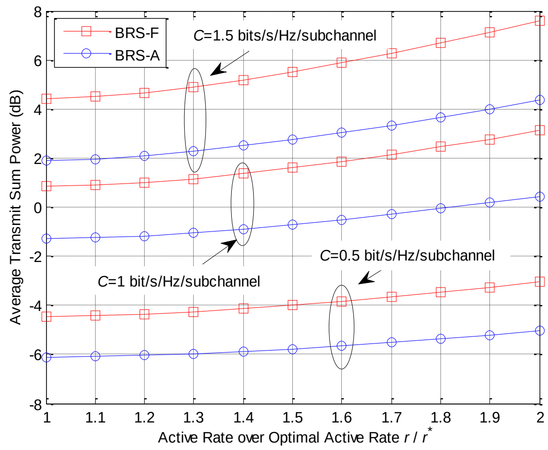


Fig. 3. Average transmit sum power versus the ratio r/r^* for BRS-A and BRS-F in a 2-user homogeneous MAC. The system parameters are K=2, C=0.5, 1, 1.5 bits/subchannel.

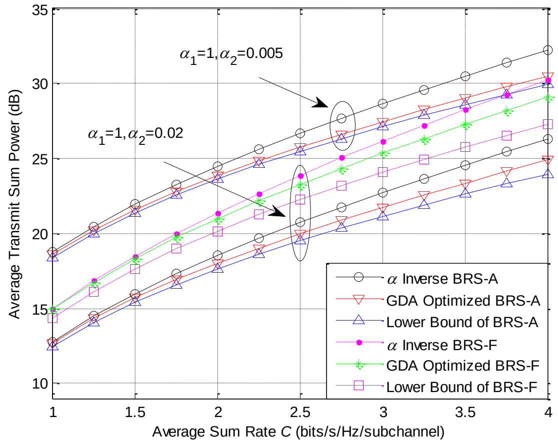


Fig. 4. Average transmit sum power versus average sum rate C for the GDA optimized BRS, α-inverse BRS and the lower bound (20) in 2-user heterogeneous MACs with α={1, 0.02} and {1, 0.005}.

For general heterogeneous MACs, the proposed GDA-optimized BRS performance and the lower bound (21) are compared in Fig. 4. We also include the performance curve with $\boldsymbol{\beta}=1/\boldsymbol{\alpha}$ and refer to this scheme as α-inverse BRS. From Fig. 4, we can observe that the GDA optimized performance is very close to the lower bound, which indicates that GDA almost achieves the BRS MTSP. Moreover, compared with the α-inverse scheme, the GDA optimized $\boldsymbol{\beta}$ achieves an evident performance gain when C grows large and when the large-scale fading coefficients among users become more distinct. On the other hand, when C becomes small, the performance gain by the optimization of $\boldsymbol{\beta}$ is marginal. In this case, simply inversing the large-scale fading with $\boldsymbol{\beta}=1/\boldsymbol{\alpha}$ is sufficient.

In Fig. 5, we investigate the performance of BRS in a practical single-cell uplink channel where the large-scale fading coefficients in $\{\alpha_k\}$ are produced by the multiplication of two random factors, namely normalized lognormal fading with log-variance σ_s=8 and path-loss in an edge-length-1 single hexagon cell with uniform user distribution. The path loss factor model follows a fourth power path-loss law. To avoid the average dominated by minority extremes and guarantee its convergence, each user is not allowed to transmit if its large-scale fading coefficient is below a given threshold. And the user outage probability is set to be 1% [14].

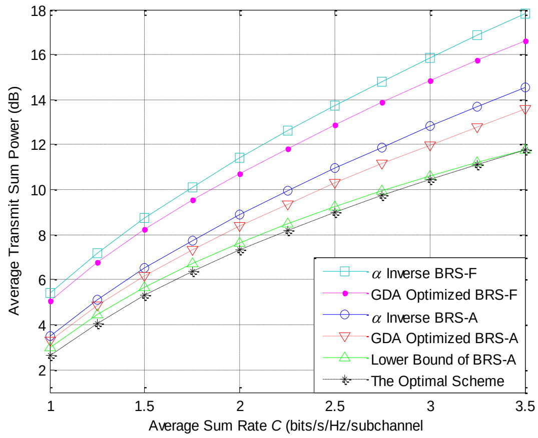


Fig. 5. Average transmit sum power versus average sum rate C for BRS-A, BRS-F, lower bounds (20) of BRS-A and optimal scheme in a 4-user general heterogeneous single-cell uplink over 10000 $\{\alpha_k\}$ realizations with 1% outage.

From Fig. 5, the performance gap between BRS and the optimal scheme in heterogeneous MACs is averagely enlarged compared with that in homogeneous MACs. Particularly for BRS-F, the gap becomes almost unacceptable. The situation for BRS-A is better and we can see that it's still close to the optimal performance especially with the optimization by proposed GDA. For example, at the typical rate C = 2 bits/sub channel, the GDA optimized BRS-A is only 1 dB away from optimality which is quite impressive. When sum rate is higher than 2 bits/s/Hz, the proposed GDA optimization over $\boldsymbol{\beta}$ is very effective.

V. Conclusion

In this paper, we identified that a simple BRS scheme with very low complexity can achieve near optimal variable-rate performance in the multiuser OFDM Rayleigh fading networks. Compared to the variable-rate scheme, BRS reduces the number of handshakes between transceivers in each scheduling period, hence it is more desirable in systems where frequent handshakes may incur much system overhead.

The performance of BRS was analyzed and optimized with respect to the channel gain thresholds and weight factors. For homogeneous networks where users share symmetric channel conditions, we obtained the closed-form expression of the MTSP of BRS after identifying the decent structure of quasi-convexity of the problem.

On the other hand, for general heterogeneous scenarios where users experience asymmetric large-scale fading, a numerical gradient descent algorithm was proposed to approach the MTSP of BRS. Through comparisons between optimized performance and the BRS MTSP lower bound, the proposed GDA was shown to be efficient and effective. In addition, we showed that the potentials of BRS can be further exploited in low-rate wireless environments that its performance can converge to that of the optimal scheme as the sum rate diminishes. The theoretical proof of the asymptotical optimality of BRS was given, which was also confirmed by the numerical results. This observation is in line with the one obtained in [13] for point-to-point fading channels.

Moreover, two different power allocation schemes for BRS were discussed and compared. BRS-A requires accurate CSI available at the transmitter whereas BRS-F requires only 1 bit CSI per. For BRS-A, we found that it can operate quite close to the optimal scheme in both homogeneous and heterogeneous networks. This implies that, when the feedback budget is sufficient, we can aggressively simplify the channel coding modules without much performance loss. In contrast, BRS-F generally incurs considerable power waste. It only can approach the optimal scheme either when the sum rate is low or the number of users is large. This observation indicates that great potentials of complexity reduction exist in the low-rate regime and the systems with large number of users. Therefore, we can greatly simplify the overall communication complexities and in the meanwhile still maintain a good performance in certain special wireless communication scenarios, e.g., ultra wideband wireless networks or centralized sensor networks with the nodes installed at nearly symmetric positions. Although BRS was studied in MAC in this paper, the obtained results can be extended to broadcast channel by utilizing the duality principle [16].

Acknowledgement

The authors would like to thank Professor Ping Li from the City University of Hong Kong for the inspiring discussion with him, and also Dr Yali Zheng for her kind help on proofreading of this manuscript.

References

[1] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. Cambridge: Cambridge University Press, 2005.

[2] A. J. Goldsmith and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. on Inform. Theory*, vol. 43, no. 6, pp. 1986-1992, 1997.

[3] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. on Inform. Theory*, vol. 45, no.5, pp. 1468-1489, 1999.

[4] D. N. C. Tse and S. V. Hanly, "Multiaccess fading channels. I. Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Trans. on Inform. Theory*, vol. 44, no. 7, pp. 2796-2815, 1998.

[5] S. Sanayei and A. Nosratinia, "Opportunistic downlink transmission with limited feedback," *IEEE Trans. on Inform. Theory*, vol.53, no. 11, pp. 4363–4372, 2007.

[6] X. Zhao, J.-B. Wang, M. Feng, and J.-Y. Wang, "Energy-efficient scheduling over fading channels with transmission modulation and deadline constraints," *AEU-International Journal of Electronics and Communications*, vol. 66, no. 6, pp. 491-494, 2012.

[7] M. Torabi and D. Haccoun, "Performance analysis of joint user scheduling and antenna selection over MIMO fading channels," *Signal Processing Letters, IEEE*, vol. 18, no. 4, pp. 235-238, 2011.

[8] S. Sanayei and A. Nosratinia, "Opportunistic downlink transmission with limited feedback," *IEEE Trans. on Inform. Theory*, vol.53, no.11, pp. 4363–4372, 2007.

[9] S. Mondal and V. Sharma, "Scheduling for fading channels under partial channel information," in *Proc. Wireless Communications and Networking Conference*, 2013, pp. 244-249.

[10] G. Wunder, C. Zhou, H. E. Bakker, and S. Kaminski, "Throughput maximization under rate requirements for the ofdma downlink channel with limited feedback," *EURASIP Journal on Wireless Communications and Networking*, vol. 2008, no. 11, pp. 11–24, 2008.

[11] M. C. Gursoy, Q. Deli, and S. Velipasalar, "Analysis of energy efficiency in fading channels under QoS constraints," *IEEE Trans. on Wireless Communications*, vol. 8, no.8, pp. 4252-4263, 2009.

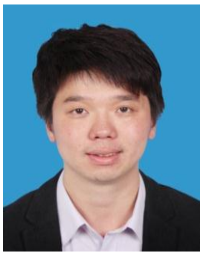
[12] L. Xiaochen, D. Xihua, and W. Dapeng, "On optimal power control for delay-constrained communication over fading channels," *IEEE Trans. on Inform. Theory*, vol. 57, no. 6, pp. 3371-3389, 2011.

[13] S. Borade and Z. Lizhong, "Wideband fading channels with limited feedback," *IEEE Trans. on Inform. Theory*, vol. 56, no.12, pp. 6058-6065, 2010.

[14] L. Liu, J. Tong, and L. Ping, "Analysis and optimization of CDMA systems with chip-level interleavers," *IEEE Journal on Selected Areas in Communications*, vol. 24, no.1, pp. 141-150, 2006.

[15] S. P. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge: Cambridge University Press, 2004.

[16] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. on Inform. Theory*, vol. 49, no. 10, pp. 2658-2668, 2003.



Xiangyu Gao received his B.S. degree from the Department of Microelectronics of Peking University, Beijing, in 2007. He is currently pursuing his PhD degree with the Department of Microelectronics in Shenzhen Graduate School of Peking University, Shenzhen. His research interests include: information theory, information security, wireless system design, QoS provisioning techniques, etc. He won the Zhongying Scholarship and the third prize of Challenging Cup Innovation Competition of Peking University.

Yuesheng Zhu received his B.Eng. degree in Radio Engineering, M.Eng. degree in Circuits and Systems and Ph.D. degree in Electronic Engineering in 1982, 1989, and 1996, respectively. He is currently working as a Professor at the Communication and Information Security Lab, Shenzhen Graduate School, Peking University. He is a senior member of IEEE, fellow of China Institute of Electronics, and senior member of China Institute of Communications. His research interests include: digital signal processing in communications, wireless communications, cryptography and internet security, digital home networking, and multimedia technology. He is listed in the Marquis Whos Who in the World.

