Construction of Irregular QC-LDPC Codes in Near-Earth Communications

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Abstract —High-speed, high-reliability requirements and strict power limitation of near-earth communications yield that the channel coding must have high coding gain and lower error floor. In this paper, an irregular quasi-cyclic low-density parity-check (QC-LDPC) code for specific use in near-earth communications is proposed. In order to eliminate the impact of short-cycles and further improve the performance of codes, a base matrix with girth at least 8 is employed. Then dual-diagonal structure is adopted in parity check matrix to reduce encoding complexity. Besides, circulant permutation matrices and zero matrices with different sizes are used to fill the nonzero positions and zero positions of the base matrix so as to remove the complexity of the selection of cyclic shift coefficients. The adoption of circulant permutation matrices with different sizes ensures that the parity check matrix is full rank, which makes the code rate controllable. Finally, simulation results are given to demonstrate the performance of the constructed codes by comparing with several existing codes.

Index Terms—Low-density parity-check codes, parity check matrix, circulant permutation matrix, near-earth communications

I. Introduction

Due to high-speed, high-reliability requirements and strict power limitation, excellent channel coding becomes one of the key technologies to ensure the reliability and validity of near-earth communications [1]. Low-density parity check (LDPC) codes [2] have received more and more attention in near-earth communications for their outstanding performance, such as faster iterative convergence, larger throughput, lower error floor, etc. Among LDPC codes, the quasi-cyclic LDPC (QC-LDPC) codes are the most promising class of structured LDPC codes due to their ease of implementation and excellent performance over noisy channels when decoded with message-passing algorithms [3].

In general, the overall performance of a LDPC code is described by two different regions, the waterfall region and the error-floor region [4], [5]. The performance in the waterfall region is heavily affected both by the girth (The length of the smallest cycle in the parity check matrix) and by the variable and check nodes degree profiles for irregular LDPC codes. Iteratively decoded LDPC codes demonstrate an abrupt change in their error rate curves. An “error floor” in the performance curve means that the decoding failure rate does not continue to decrease rapidly as the signal to noise ratio (SNR) increases [6]. Eliminating or lowering error floors is particularly important for near-earth applications that have extreme reliability demands. The performance in the error floor region is also affected by the girth. For example, in erasure channels every stopping set contains cycles [7], so that increasing the girth turns out to increasing the smallest stopping set size [8] of the LDPC codes. Short cycles in the Tanner graph, which lead to inefficient decoding and limit the convergence rate of the sum-product algorithm, affect the performance of the LDPC codes. Therefore, design of LDPC codes with large girth is of great interest [4].

There have been several approaches, including direct construction approaches [9]-[12] and cycle removal based approaches [13], to remove short cycles in Tanner graph and construct LDPC codes with large girth. However, these approaches are either not sufficiently flexible in the code parameter selection due to the constraint on girth [14], or unable to preserve the quasi-cyclic structure thereby increasing the encoding complexity [15].

In order to reduce the complexity of parity check matrix construction and to achieve an efficient encoding, we use the parity check matrix with systematic form, and design the check part and the information part independently. The check part of parity check matrix adopts the dual-diagonal structure to achieve iterative encoding. The information part uses the quasi-cyclic structure whose nonzero positions replaced by circulant permutation matrices (CPMs) with different sizes and zero positions replaced by corresponding-size zero matrices to reduce the complexity of parity check matrix construction. The class of LDPC code derived from our construction possesses large girth and much more flexibility in terms of code length and rate. What’s more, the proposed codes also can achieve very good performance while retaining the practicality of quasi-cyclic structured.

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The organization of the rest of this paper is as follows. First, we present the construction method of the check part and the information part of parity check matrix in Section II. Then, we show the performance comparison between the proposed code and several existing codes over binary-input Additive White Gaussian Noise (AWGN) channel in Section III. The paper is summarized in the last section.

II. Construction Method

In this section, we introduce the basic construction of the proposed irregular QC-LDPC codes. Suppose the parity check matrix of QC-LDPC code is systematic form:

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_p & \boldsymbol{H}_k \end{bmatrix} \tag{1}$$

where $\boldsymbol{H}_p$ is an $M\times M$ matrix corresponding to the check part of parity check matrix; $\boldsymbol{H}_k$ is an $M\times(N\text{-}M)$ matrix that indicates the information part. The parity check matrix of irregular QC-LDPC code can be obtained through constructing $\boldsymbol{H}_p$ and $\boldsymbol{H}_k$ independently, where $\boldsymbol{H}_p$ is a dual-diagonal square matrix and $\boldsymbol{H}_k$ keeps the quasi-cyclic form.

A. Construction of $\boldsymbol{H}_p$

Dual-diagonal structure of $\boldsymbol{H}_p$ has the following form:

$$\boldsymbol{H}_p = \begin{bmatrix} 1 & & & & \\ 1 & 1 & & & \\ & 1 & & & \\ & & \cdots & & \\ & & & 1 & \\ & & & 1 & 1 \end{bmatrix}_{M\times M} \tag{2}$$

Clearly, $\boldsymbol{H}_p$ is a non-singular matrix. It is well known that degree-2 variable nodes *(VNs)* make it possible to achieve sparse and optimal irregular properties and simplify the encoding process [16]. However, many works show that the *VNs* with low degree are bad for decoding performance [18]. Thus, in order to avoid the bad influence of degree-2 *VNs*, we make the number of *VNs* with degree-2 satisfy the following constraint. The maximum number of *VNs* with degree-2 should satisfy $N_{v,\max}(2) = N - K - 1 = M - 1$, where M is the number of check nodes *(CNs)*, N is the number of *VNs* and K is the number of information bits.

B. Construction of $\boldsymbol{H}_k$.

In this subsection, we use the CPMs with different sizes to construct a class of irregular QC-LDPC codes. Compared with the direct construction methods [9]-[12], this method has a relatively simple structure, and there is no need to search the cyclic shift coefficients of base matrix.

C. Construction of Base Matrix

We design three sub-matrices $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$, combine them into a row and transpose to obtain matrix $\boldsymbol{H}_1$ [17], and then expand $\boldsymbol{H}_1$ into desired matrix $\boldsymbol{H}_k$ using CPMs or zero matrices with different sizes.

1) Construction of sub-matrix $\boldsymbol{B}_1$:

Design a matrix $\boldsymbol{B}_{10}$ with dimension $v\times v^2$, where $\boldsymbol{B}_{10}(1,1)=\boldsymbol{B}_{10}(2,1)=\boldsymbol{B}_{10}(3,1)=\cdots=\boldsymbol{B}_{10}(v,1)=1$, and other elements are "0". Then $\boldsymbol{B}_{10}$ is a $v\times v^2$ matrix of the following form:

$$\boldsymbol{B}_{10} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{v\times v^2} \tag{3}$$

By cyclically shifting the elements of $\boldsymbol{B}_{10}$ one place to right, we can obtain v^2-1 new matrices, denoted by $\boldsymbol{B}_{11}, \boldsymbol{B}_{12}, \boldsymbol{B}_{13}, \cdots, \boldsymbol{B}_{1(v^2-1)}$. Combining $\boldsymbol{B}_{10}$ and the v^2-1 new matrices into a column, then we can obtain the following $v^3 \times v^2$ matrix $\boldsymbol{B}_1$:

$$\boldsymbol{B}_1 = \begin{bmatrix} \boldsymbol{B}_{10} & \boldsymbol{B}_{11} & \boldsymbol{B}_{12} & \cdots & \boldsymbol{B}_{1(v^2-1)} \end{bmatrix}^T \tag{4}$$

2) Construction of sub-matrix $\boldsymbol{B}_2$:

Design a matrix $\boldsymbol{B}_{20}$ with dimension $v\times v^2$, where $\boldsymbol{B}_{20}(1,1)=\boldsymbol{B}_{20}(2,2)=\boldsymbol{B}_{20}(3,3)=\cdots=\boldsymbol{B}_{20}(v,v)=1$, and other elements are "0". Then $\boldsymbol{B}_{20}$ is a $v\times v^2$ matrix of the following form:

$$\boldsymbol{B}_{20} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 \end{bmatrix}_{v\times v^2} \tag{5}$$

Let $\boldsymbol{B}_{21}$ have the following form:

$$\boldsymbol{B}_{21} = \begin{bmatrix} \boldsymbol{B}_{20} & \boldsymbol{B}_{20} & \boldsymbol{B}_{20} & \cdots & \boldsymbol{B}_{20} \end{bmatrix}^T \tag{6}$$

The total number of $\boldsymbol{B}_{20}$ is v. By cyclically shifting the elements of $\boldsymbol{B}_{21}$ v places to right, we can get v-1 new matrices, denoted by $\boldsymbol{B}_{22}, \boldsymbol{B}_{23}, \cdots, \boldsymbol{B}_{2v}$. Combining $\boldsymbol{B}_{21}$ and the v-1 new matrices into a column, then we can get a $v^3\times v^2$ matrix $\boldsymbol{B}_2$ of the following form:

$$\boldsymbol{B}_2 = \begin{bmatrix} \boldsymbol{B}_{21} & \boldsymbol{B}_{22} & \boldsymbol{B}_{23} & \cdots & \boldsymbol{B}_{2v} \end{bmatrix}^T \tag{7}$$

3) Construction of sub-matrix $\boldsymbol{B}_3$:

Design a matrix $\boldsymbol{B}_{30}$ also with dimension $v\times v^2$, where $\boldsymbol{B}_{30}(1,\ 1)=\boldsymbol{B}_{30}(2,\ 1+v)=\boldsymbol{B}_{30}(3,\ 1+2v)=\cdots=\boldsymbol{B}_{30}(v,\ 1+v\text{-}v^2)=1$ and other elements are "0". Then we can obtain the following matrix:

$$\boldsymbol{B}_{30} = \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 & \ddots & \vdots & \vdots & \ddots & \vdots \\ \underbrace{0 \quad 0 \quad \cdots \quad 0}_{v\times v} & & & & \cdots & \underbrace{1 \quad 0 \quad \cdots \quad 0}_{v\times v} & & & \end{bmatrix}_{v\times v^2} \tag{8}$$

Cyclically shifting the elements of $\boldsymbol{B}_{30}$ one place to right, then we can get v-1 new matrices, denoted by $\boldsymbol{B}_{31}$,

$\boldsymbol{B}_{32},\cdots,\boldsymbol{B}_{3(v-1)}$. Arranging $\boldsymbol{B}_{30}$ and the v-1 new matrices results in a column of v matrices:

$$\boldsymbol{B}_{3v} = \begin{bmatrix} \boldsymbol{B}_{30} & \boldsymbol{B}_{31} & \boldsymbol{B}_{32} & \cdots & \boldsymbol{B}_{3(v-1)} \end{bmatrix}^T \tag{9}$$

By arranging v $\boldsymbol{B}_{3v}$ into a column, we form the following $v^3 \times v^2$ matrix:

$$\boldsymbol{B}_3 = \begin{bmatrix} \boldsymbol{B}_{3v} & \boldsymbol{B}_{3v} & \boldsymbol{B}_{3v} & \cdots & \boldsymbol{B}_{3v} \end{bmatrix}^T \tag{10}$$

Then we can get $\boldsymbol{H}_1$ as follows:

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \boldsymbol{B}_3 \end{bmatrix}^T_{3v^2 \times v^3} \tag{11}$$

D. Expansion of *the Base Matrix*

Following the above construction, we can construct three $v^3 \times v^2$ arrays $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$. Replace the nonzero elements of the three sub-matrices with CPMs of different suitable sizes, and then replace the zero elements of the three sub-matrices with zero matrices of the corresponding sizes, we can obtain the desired matrix $\boldsymbol{H}_k$.

1) Size selection of CPMs:

Let $\boldsymbol{P}$ be the $p \times p$ CPM defined by:

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{12}$$

Since the three sub-matrices have the same dimension, using different-size CPMs and zero matrices padded will lead them have different dimensions. So in order to make $\boldsymbol{H}_1$ keep the desired matrix form and avoid short cycles, the sizes of CPMs are needed to be selected carefully.

Now, suppose we use CPMs with size p_1, p_2 and p_3 to replace the nonzero elements of $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$, and exploit zero matrices with size p_1, p_2 and p_3 to replace the zero elements of them. For simplicity and without loss of generality, we assume $p_1 \leq p_2 \leq p_3$, then the sizes of the obtained sub-matrices are $(v^3 p_1)\times(v^2 p_1)$, $(v^3 p_2)\times(v^2 p_2)$ and $(v^3 p_3)\times(v^2 p_3)$, respectively. Clearly, the size of row gap between $\boldsymbol{B}_1$ and $\boldsymbol{B}_3$ is $v^3(p_3-p_1)$, and the row size difference between $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$ is $v^3(p_3-p_2)$.

Since $\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \boldsymbol{B}_3 \end{bmatrix}^T_{3v^2 \times v^3}$, then we can negligent the column size difference among $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$. The size of row gap originated from different size CPMs and zero matrices can be resolved through cyclic copy. The principle of cyclic-copy is shown in Fig. 1, where "Gap1" indicates the row size difference between $\boldsymbol{B}_1$ and $\boldsymbol{B}_3$, and "Gap2" indicates the row size difference between $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$. If we copy a "Gap1" from the head of $\boldsymbol{B}_1$ and place it at the end of $\boldsymbol{B}_1$, then it can make sure that $\boldsymbol{B}_1$ and $\boldsymbol{B}_3$ are with the same row size. In a similar manner, we can maintain $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$ having the same row size. Then we will get the desired nearly quasi-cyclic form matrix $\boldsymbol{H}_1$.

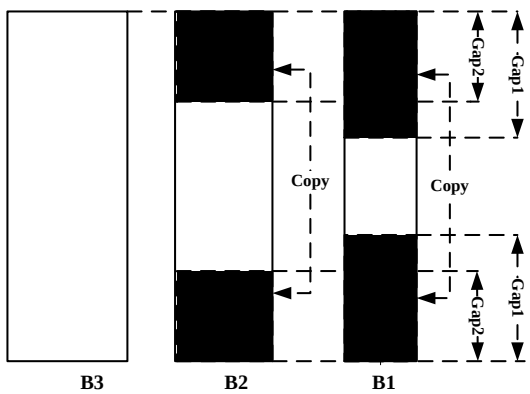

Fig. 1. Principle of cyclic-copy.

The null space of $\boldsymbol{H}_1$ gives an irregular QC-LDPC code of length $v^3 \times p_3$ whose check matrix with column and row weight at least 3 and v, respectively. The corresponding Tanner graph has girth at least 8. Taking a sub-matrix as $\boldsymbol{H}_k$ of the parity check matrix, we can construct a class of irregular QC-LDPC codes.

Based on the above analysis, the conditions that the sizes of the CPMs and the base matrix need to meet are as follows:

a) The sizes of CPMs need to satisfy the condition:

$$\max\{p_1, p_2, p_3\} \leq 2 \cdot \min\{p_1, p_2, p_3\} \tag{13}$$

If $\max\{p_1, p_2, p_3\} > 2 \cdot \min\{p_1, p_2, p_3\}$, then "Gap1" is larger than the row size of $\boldsymbol{B}_1$. Similarly, "Gap2" is larger than the row size of $\boldsymbol{B}_2$. The cyclic copy operation will not be implemented.

b) The number of columns for a common quasi-cyclic matrix should be smaller than the least common multiple (LCM) of any two of its CPMs sizes. That is, the number of columns should satisfy the condition:

$$n \leq \min_{1 \leq i, j \leq 3\{p_i, p_j\}} \mathrm{LCM}\{p_i, p_j\} \tag{14}$$

Next we use an example to illustrate the above proposition *b*. Let 2 and 3 be the sizes of CPMs. Then the LCM of 2 and 3 is 6. When the number of columns of the matrix is greater than 6, there will form cycles of length four. The four bold "1" in matrix $\boldsymbol{A}$ form a 4-cycle.

$$= \left[\begin{array}{cc|cc|cc|cc|cc} \mathbf{1} & 0 & 1 & 0 & 1 & 0 & \mathbf{1} & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ \hline \mathbf{1} & 0 & 0 & 1 & 0 & 0 & \mathbf{1} & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right] \tag{15}$$

c) p_1, p_2 and p_3 are prime. Minimum distance is an important parameter of LDPC code, which determines the LDPC codes detecting and correcting random errors capacity. Gallager has demonstrated that when the degrees of the variable node are less than 3, the minimum distances of LDPC codes grow linearly with block length (for a fixed rate and fixed degree of variable node), and

when the variable node degree is greater than 3, the minimum distance of LDPC codes increase linearly along with block length [2]. So we often choose p_1, p_2 and p_3 are prime and also relatively prime to make sure that the code have large minimum distance, and further make it possess powerful error detection and correction capability.

2) Girth of the proposed irregular QC-LDPC codes

It is well-known that girth is one of the designing criteria to optimize the performance of message-passing decoding algorithms for LDPC codes. The girths of QC-LDPC codes have some unique features, as show in Fig. 2.

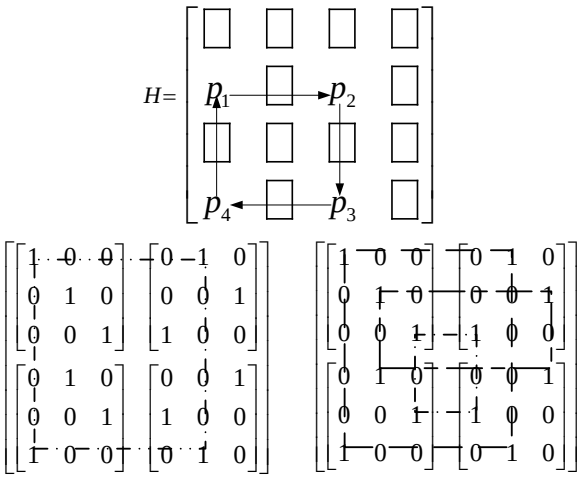

Fig. 2 Cycles in parity check matrix of QC-LDPC code.

A cycle is a path through nodes in the Tanner graph of a code. It alternates between check and variable nodes, and starts and ends at the same node. In terms of the code's parity check matrix, each check node in the Tanner graph corresponds to a row in its parity check matrix, and each variable node corresponds to a column.

However, the systematic form QC-LDPC codes also have some unique structures. Next, we will take advantage of the relationship between Tanner graph and the parity check matrix to show the proposed code of girth 8.

The proposed $\boldsymbol{H}$ matrix has irregular structure and the Tanner graph [17] of the code c is similar to those of (irregular) repeat-accumulate (RA) codes [19]. RA codes are typically systematic. What's more, RA codes provide two important advantages. First, they allow flexibility in the choice of the repetition rate for each information bit, so that high-rate codes may be designed. Second, their irregularity allows operation closer to the capacity limit. In the Tanner graph of the code c, we call the nodes corresponding to the columns of $\boldsymbol{H}_p$ *parity bit* nodes, the nodes corresponding to the columns of $\boldsymbol{H}_k$ *information bit* nodes, and the nodes corresponding to the rows of $\boldsymbol{H}$ *check nodes* [20], [21]. We also call information and parity bit nodes *variable nodes*.

We now use a general example to illustrate the relationship between Tanner graph and the parity check matrix owning systematic form. Suppose the parity check matrix of the basic construction is given by:

$$
= \left[\begin{array}{c|c} \boldsymbol{H}_p & \boldsymbol{H}_k \end{array} \right]
= \left[\begin{array}{cccccc|cccccc}
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 1 \\
0 & 0 & \mathbf{1} & 1 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0
\end{array} \right] \tag{16}
$$

As shown in Fig. 3, the columns of $\boldsymbol{H}_p$ and $\boldsymbol{H}_k$ are numbered from 0 to 5 from the left. The rows of $\boldsymbol{H}_p$ and $\boldsymbol{H}_k$ are numbered from 0 to 5 from the top. The bold lines in Tanner graph represent a 4-cycle, and they correspond to the four bold "1" in the parity check matrix.

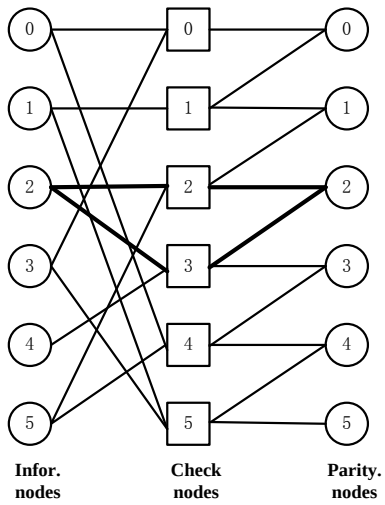

Fig. 3. Tanner graph of $\boldsymbol{H}$ in (16).

Since there is no 4-cycle and 6-cycle in the base matrix, the girth of the QC-LDPC code designed by our base matrix is at least 8 [7]. Making the girth of the parity check matrix as large as possible can improve the performance of the code.

III. Simulations

In this section, we present some simulation results to demonstrate the performance of the constructed code by comparing with several existing codes. Simulations are performed on AWGN channel with binary phase-shift keying modulation.

Let v=8, p_1=11, p_2=13 and p_3=17. Following the above construction, we can construct a 192×512 array $\boldsymbol{H}_1$. Replacing the nonzero elements of $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$ with CPMs of sizes p_1, p_2 and p_3 respectively, and then replacing the zero elements of $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$ by zero matrices with corresponding sizes respectively, we can get a class of irregular QC-LDPC codes.

To validate the advantage of different-size CPMs, the bit error rate (BER) performance comparison between the codes with different-size CPMs and same-size CPMs are presented in Fig. 4. In Fig. 4, the "bi-diagonal regular code" represents the code with same size CPMs and also

has systematic form, the "regular code" represents the code with same-size CPMs and has non systematic form, the "bi-diagonal irregular code" means the code with different-size CPMs and has systematic form, and the "irregular code" means the code with different-size CPMs and has non systematic form. The different-size CPMs with sizes p_1, p_2 and p_3, the same-size CPMs with size p=17. Choosing a sub-matrix $\boldsymbol{H}_{qc1}$ with dimension 2624×5248, the null space of $\boldsymbol{H}_{qc1}$ gives an irregular (5248, 2624) QC-LDPC code with rate 0.5. If we let $\boldsymbol{H}_{qc1}$ be $\boldsymbol{H}_k$, then $\boldsymbol{H}=[\boldsymbol{H}_p \quad \boldsymbol{H}_{qc1}]$, thus we can construct a class of systematic QC-LDPC codes. As shown in Fig.4, for the systematic code, when the SNR is 3dB, the BER performance of our proposed bi-diagonal irregular code is 10^{-5}, while the code whose parity check matrix has the same size CPMs is 10^{-4}. The two differ by an order of magnitude. The code whose parity check matrix is non-systematic form has similar performance.

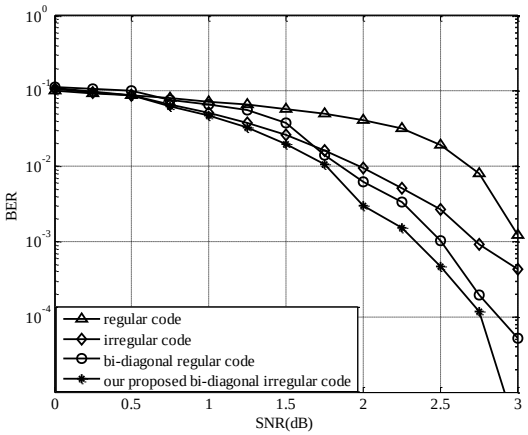

Fig. 4. BER performance of the codes based on same base matrix with various CPMs.

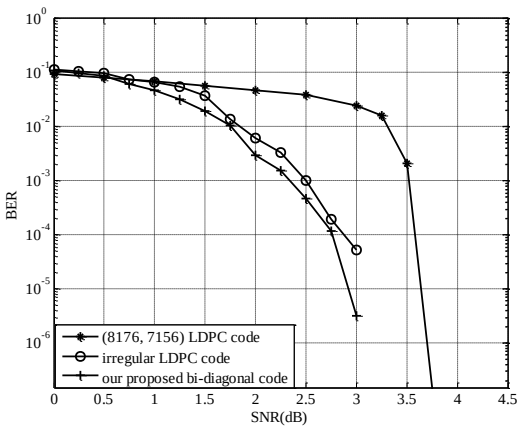

Fig. 5. BER performance of the proposed code and CCSDS standard (8176, 7156) QC-LDPC code.

Choosing a sub-matrix $\boldsymbol{H}_{qc2}$ with size 1088×8704, the null space of $\boldsymbol{H}_{qc2}$ gives an irregular (8704, 7616) QC-LDPC code with rate 0.87. The BER performance comparison between the proposed bi-diagonal code and consultative committee for space data systems (CCSDS) standard (8176, 7156) QC-LDPC code depicts Fig. 5, where "bi-diagonal code" indicates the code whose parity check matrix is systematic form and "irregular code" represents the code whose parity check matrix is not systematic form. It can be seen that our proposed bi-diagonal code outperforms the CCSDS standard (8176, 7156) QC-LDPC code. Specially, it can achieve 0.5dB SNR improvement at BER of 10^{-5}. In addition, the designing of our base matrix without using another algorithm to select the cyclic shift coefficients simplifies the construction process of the parity check matrix.

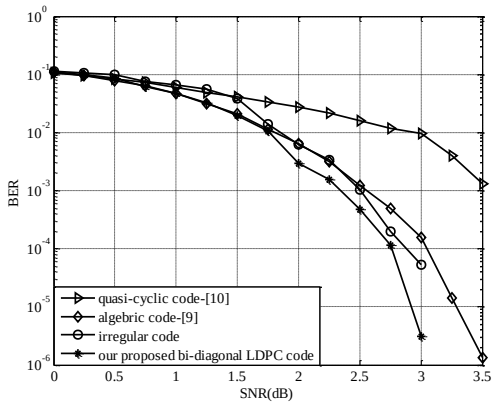

Fig. 6. BER performance of the proposed code and the code presented in [9] and [10].

To further illustrate the performance of the code in this paper, the BER performance of the code constructed by the null space of $\boldsymbol{H}_{qc1}$ and the codes constructed by the methods given in [9] and [10] is shown in Fig.6, where the proposed "bi-diagonals LDPC code" indicates the code whose parity check matrix is systematic form and "irregular code" represents the code whose parity check matrix is not systematic form. The structure of parity check matrix in [9] is also systematic form. However, the information part of parity check matrix $\boldsymbol{H}_k$ filled with the same dimension CPM, and need related algorithms to determine the cyclic shift coefficients of CPMs. Compared with the construction method in [9], using different sizes CPM filled the nonzero elements of $\boldsymbol{H}_k$, can ensure it is a full rank matrix. That is, there is no redundant row in $\boldsymbol{H}_k$. When there are redundant rows in the parity check matrix, the actual code rate is greater than the theoretical design code rate, which make the code rate is not controlled. The construction of parity check matrix in [10] is also employing systematic form. And the notable feature is reflected on the structure of the base matrix, which is constructed by masking method. When the highly structured arrays are densely packed, the density of such an array can be reduced by replacing a set of CPMs by zero matrices. This replacement of CPMs by zero matrices is referred to as masking. Masking an array of CPMs can result in an array of CPMs and zero matrices whose Tanner graph have fewer edges and hence have fewer short cycles and possibly larger girth. However, how to construct the masking matrix with large girth and minimum distance is a very challenging problem. As seen in Fig. 6, compared with the code in [9],

the proposed bi-diagonal code can achieve 0.5dB SNR performance improvement when BER is 10^{-5}. Besides, when the SNR is 3.5dB, the BER performance order of the proposed bi-diagonal code is 10^{-6}, while the code in [10] is 10^{-3}, the gap between them is three orders of magnitude. What's more, the convergence of the code constructed in [10] is slow.

IV. Conclusion

In summary, we have presented a new method to construct irregular QC-LDPC code based on CPMs of different sizes. The main advantages of the proposed QC-LDPC codes are with large girth and irregular structure. What's more, our parity check matrix is quasi-cyclic and composed by zero matrix and CPMs of different sizes. Encoding process can be achieved in a recursive way with few calculations. Simulation results show that the proposed QC-LDPC codes outperform the CCSDS standard code and several existing codes with same size CPMs under AWGN channel.

References

[1] Experimental Specifications, Low Density Parity Check codes for Use in Near-Earth and Deep Space Applications, CCSDS 131.1-O-2, 2007.

[2] R. G. Gallager, "Low-density parity-check codes," *IRE Trans. Information Theory*., vol. IT-8, pp. 21-28, Jan. 1962.

[3] L. Zhang, Q. Huang, S. Lin, K. Abdel-Ghaffar, and I. F. Blake, "Quasi-cyclic LDPC codes: An algebraic construction, rank analysis, and codes on Latin squares," *IEEE Trans. Commun.*, vol. 58, pp. 3126-3139, Nov. 2010.

[4] T. Richardson, "Error floors of LDPC codes," in *Proc. 2003 Allerton Conf. Commun., Control and Computing*, The University, 2003, pp. 1426-1435.

[5] G. Liva, W. E. Ryan, and M. Chiani, "Quasi-cyclic generalized LDPC codes with low error floors," *IEEE Trans. Commun.*, vol. 56, pp. 49-57, Jan 2008.

[6] Y. Wang, S. C. Draper, and J. S. Yedidia, "Hierarchical and high-girth QC-LDPC codes," *IEEE Trans. Inf. Theory*, vol. 59, pp. 4553-4583, July 2013.

[7] T. Tian, C. R. Jones, J. D. Villasenor, and R. D. Wesel, "Selective avoidance of cycles in irregular LDPC code construction," *IEEE Trans. Commun.*, vol. 52, pp. 1242-1247, Aug. 2004.

[8] J. Wang, L. Dolecek, and R. D. Wesel, "The cycle consistency matrix approach to absorbing sets in separable circulant-based LDPC codes," *IEEE Trans. Inf. Theory*, vol. 59, pp. 2293-2314, April 2013.

[9] J. Chen, R. M. Tanner, J. Zhang, and M. P. Fossorier, "Construction of irregular LDPC codes by quasi-cyclic extension," *IEEE Trans. Inf. Theory*, vol. 53, pp. 1479-1483, April 2007.

[10] S. J. Johnson and S. R. Weller, "A family of irregular LDPC codes with low encoding complexity," *IEEE Commun. Lett.*, vol. 7, pp. 79-81, Feb. 2003.

[11] L. Wang, X. Zhang, F. Yu, Y. Fu, and Y. Wang, "QC-LDPC codes with girth eight based on independent row-column mapping sequence," *IEEE Commun. Lett.*, vol. 17, pp. 2140-2143, Nov. 2013.

[12] S. Kim, J. No, H. Chung, and D. Shin, "On the girth of tanner (3, 5) quasi-cyclic LDPC codes," *IEEE Trans. Inf. Theory*, vol. 52, pp. 1739-1744, April 2006.

[13] P. Yang, X. Bao, and H. Zhao, "Construction of quasi-cyclic LDPC codes with large girth based on circulant permutation matirx," in *Proc. 22nd Wireless and Optical Communication Conference (WOCC)*, Chong Qing, 2013, pp. 564-567.

[14] K. Liu, Z. Fei, J. Kuang, and X. Li, "A novel algorithm for removing cycles in quasi-cyclic LDPC codes," in *Proc. 20th International Symposium on Personal, Indoor and Mobile Radio Communications*, Bei Jing, 2009, pp.1054-1058.

[15] J. Zhang and G. Zhang, "Deterministic girth-eight QC-LDPC codes with large column weight," *IEEE Commun. Lett.*, vol. 18, pp. 656-659, April 2014.

[16] M. Yang, W. E. Ryan, and Y. Li, "Design of efficiently encodable moderate-length high-rate irregular LDPC codes," *IEEE Trans. Commun.*, vol. 52, pp. 564-571, April 2004.

[17] J. Fan and Y. Xiao, "A design of LDPC codes with large girth based on the sub-matrix shifting," in *Proc. IET International Conference, Wireless, Mobile and Multimedia Networks*, Hang Zhou, 2006, pp. 1-4.

[18] R. M. Tanner, "A recursive approach to low complexity codes," *IEEE Trans. Inf. Theory*, vol. 27, pp. 533-547, Sep 1981.

[19] H. Jin and R. J. McEliece, "Coding theorems for turbo code ensembles," *IEEE Trans. Inf. Theory*, vol. 48, pp. 1451-1461, Jun 2002.

[20] Y. Zhang and W. E. Ryan, "Structured IRA codes: performance analysis and construction," *IEEE Trans. Commun.*, vol. 55, pp. 837-844, May 2007.

[21] J. Wang, J. Lei, S. Ni, X. Tang, and G. Ou, "QC-IRA-d codes based on circulant permutation matrices," *IEEE Commun. Lett.*, vol. 15, pp. 1224-1227, Nov. 2011.

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