Application of EMD Denoising Approach in Noisy Blind Source Separation

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Abstract**—**Blind Source Separation (BSS) algorithms based on the noise-free model are not applicable when the Signal Noise Ratio (SNR) is low. In view of this situation, our solution is to denoise the mixtures with additive white Gaussian noise firstly, and then use BSS algorithms. This paper proposes a piecewise Empirical Mode Decomposition (EMD) thresholding approach to denoise mixtures with strong noise. This approach can distinguish the noise-dominated IMFs and signal-dominated IMFs, and then respectively apply different thresholdings methods. Simulation results show that compared with the Wavelet denoising, the proposed approach has a better denoising performance, and can remarkably enhance the separation performance of BSS algorithms, especially when the signal SNR is low.

Index Terms**—**Signal denoising; empirical mode decomposition (EMD); wavelet transform (WT); waveshrink algorithm; noisy blind source separation.

I. INTRODUCTION

Blind source separation (BSS) is a well-known domain in signal processing. It deals with the separation of observed sensor signals into their underlying source signals, without knowing the source signals and the mixing process. The only assumption is that the source signals are mutually statistically independent. A lot of BSS models, such as instantaneous linear mixtures and convolutive mixtures, are have been presented in some publications[1][2][3], and some prominent BSS methods with good performance, such as FastICA[3], RobustICA[4] and *etc.*, have been widely applied to telecommunication, speech and medical signal processing.

However, the best performances of these methods are obtained for the ideal BSS model and their effectiveness is definitely decreased with observations corrupted by additive noise. In order to solve the problem of the BSS with additive noise, i.e. Noisy BSS, a good solution is to apply a powerful denoising processing before separation. At present, the denoising techniques mainly include Kalman filtering, particle filtering, wavelet denoising, etc. As for the Noisy BSS, for lack of any apriori information about the observed mixtures, we cannot build the exact model. Wavelet denoising based on wavelet transform (WT) is simple and wavelet thresholding has been the dominant technique in the area of non-parametric signal denoising for many years. Thus, wavelet denoising is suitable for Noisy BSS. Nevertheless, the wavelet approach has a main drawback, that is, its basis functions are fixed and do not necessarily match varying nature of signals[5].

Huang *et al.* [6] proposed Empirical Mode Decomposition (EMD) to analyze data from nonstationary and nonlinear processes. The major advantage of EMD is that the basis functions are derived from the signal itself. Hence, the analysis is adaptive, which is different from the wavelet approach whose basis functions are fixed. Signal denoising based on EMD is a novel denoising technique of non-parametric signal denoising, and it has a wide range of applications, such as in biomedical signals[7], acoustic signals[8] and ionospheric signals[9]. Considering the good performance of EMD denoising, we can apply this technique to Noisy BSS.

This paper aims to combine EMD denoising processing with BSS to improve the performance of BSS algorithms. Since the thresholds using the method explained in [15] decreased so slowly that part of the signal will get lost after thresholding, we propose a piecewise EMD thresholding approach to denoise mixtures with strong noise. This approach can find the noise-dominated IMFs and signal-dominated IMFs, and then use the different thresholds methods respectively. The new thresholds decrease faster than the conventional ones.

This paper is organized as follows: firstly, we introduce the Noisy BSS model. Then in Section 3, we explain the EMD method and its denoising principle; while in Section 4 we analyze the disadvantages of conventional thresholding EMD denoising approach, and propose a new EMD denoising approach which has a better performance than the wavelet denoising approach. Finally, we apply this denoising approach to the Noisy BSS. A short summary concludes this paper.

II. NOISY BSS

A. Noisy BSS Model

Consider a linear instantaneous problem of blind source separation, and the unknown source signals and the observed mixtures are related to:

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{v}(t) = \mathbf{x}(t) + \mathbf{v}(t) \tag{1}$$

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in which $\mathbf{y}(t)=[\mathbf{y}_1(t),\mathbf{y}_2(t),\cdots,\mathbf{y}_m(t)]^T$ is the vector of *m* observed mixtures, and $\mathbf{s}(t)=[\mathbf{s}_1(t),\mathbf{s}_2(t),\cdots,\mathbf{s}_n(t)]^T$ is the vector of *n* source signals which are assumed to be mutually and statistically independent. **A** is an unknown full rank $m\times n$ mixing matrix and $\mathbf{v}(t)$ is an additive noise. This paper focuses on the signals with white Gaussian noise. We call this model Noisy BSS model (Fig. 1).

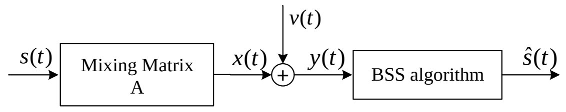


Fig. 1. Noisy blind source separation model

In normal BSS model (without noise), we can find a demixing matrix **W** so that $\mathbf{W}\mathbf{y}(t)=\hat{\mathbf{s}}(t)\approx\mathbf{s}(t)$, i.e. $\mathbf{WA}\approx\mathbf{I}$, and this demixing matrix **W** is optimum. But in Noisy BSS, even if we can get **W**, the result of demixing is $\mathbf{Wy}(t)=\mathbf{WAs}(t)+\mathbf{Wv}(t)\approx\mathbf{s}(t)+\mathbf{Wv}(t)$ which is the mixture of the source signals and the noise. In practice, we cannot find the optimum demixing matrix **W** in noisy BSS at all. Therefore, generally, Noisy BSS is much more difficult to deal with than normal BSS.

B. The Solution of Noisy BSS

A solution of noisy BSS based on wavelet denoising is proposed in [10]. The idea of this solution is to transform Noisy BSS into normal BSS without noise, i.e. to denoise the observed mixtures before BSS, and then directly use normal BSS algorithms (Fig. 2).

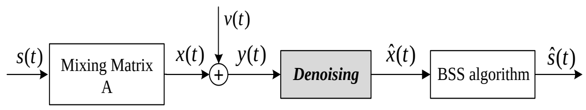


Fig. 2. The principle of the method in [10]

WaveShrink algorithm[14] is used to denoise the observed mixtures in [10]. WaveShrink algorithm is one of the most widely used denoising techniques based on wavelet transform. However, besides its own drawbacks, the wavelet approach is not very efficient when the SNR is low. Therefore, in this paper, we use EMD denoising method to remove the noise in observed mixtures firstly, and then separate them using BSS algorithms.

III. EMPIRICAL MODE DECOMPOSITION

A. The EMD Technique

Empirical Mode Decomposition is an algorithm that can decompose a signal into a series of structural components, known as Intrinsic Mode Functions (IMFs), together with the possibility of providing an estimate of the trend of the data. An IMF is defined as any function having the same number of zero-crossings and extrema, and also having symmetric envelopes defined by the local maxima and minima respectively.

For a discrete time signal $x(n), n=1,2,\cdots,N$, N is the sample number of the signal, the algorithm for the extraction of IMFs from the real world data is called *sifting* and it consists of the following steps:

Step1: $x_0(n)\leftarrow x(n)$, $h_0(n)\leftarrow x(n)$.

Step2: Get the envelopes of the maxima and the minima of $h_0(n)$ using cubic splines interpolation, and denote them as $E_{\max}(n)$ and $E_{\min}(n)$.

Step3: Calculate the mean of the two envelopes as

$$m_1^{(1)}(n)=\frac{E_{\max}(n)+E_{\min}(n)}{2} \tag{2}$$

Step4: Subtract the mean $m_1^{(1)}(n)$ from the original signal $x(n)$ as

$$h_1^{(1)}(n)=x_0(n)-m_1^{(1)}(n) \tag{3}$$

Step5: Examine the residual $h_1^{(1)}(n)$ to see whether it satisfies the definition of IMF.

a) If it doesn't, then $h_0(n)\leftarrow h_1^{(1)}(n)$, repeat the steps from Step2 to Step5 many times until it satisfies the definition of IMF. Thus:

$$IMF_1=h_1^{(k)}(n)=h_1^{(k-1)}(n)-m_1^{(k)}(n) \tag{4}$$

b) If it does, the procedure stops and we get the first IMF, i.e. $IMF_1=h_1^{(1)}(n)$.

Step6: After extracting IMF, the new signal under examination is expressed as:

$$x_1(n)=x_0(n)-IMF_1 \tag{5}$$

Then $x_0(n)\leftarrow x_1(n)$ and $h_0(n)\leftarrow x_1(n)$, and repeat the previous steps until the final residual is a monotonic function.

After completion of EMD the signal can be written as follows:

$$x(n)=\sum_{j=1}^{K} IMF_j+r(n) \tag{6}$$

where *K* is the total number of the IMF components and *r(n)* is the residual.

B. Analysis of Wavelet Transform and EMD

In this section, we compare the decomposition results of Wavelet Transform (WT) with Empirical Mode Decomposition (EMD) at different SNR levels. The signal "Heavysine" obtained using MATLAB software is corrupted by white Gaussian noise, and the SNR levels are 15dB and 0dB respectively (Fig. 3). The sample size of the signals is $N=1024$.

The parameters of WT are set as follows: the chosen wavelet basis function is "sym7" and the number of the decomposition level is 4. The decomposition results of WT and EMD are depicted in Fig. 4 and Fig. 5 respectively. Comparing the decomposition results of WT with those of EMD, we can see that WT is linear transform and WT of the noise is superposed on the corresponding WT of the noise-free signal. The number of decomposition level is fixed no matter the SNR is high or low. On the other hand, EMD is non-linear, and the decomposition level is adaptive. The stopping criterion of EMD is that the final residual is a monotonic function. Therefore, the number of decomposition level increases with the decrease of the SNR level. We can see from Fig. 5(b) that there are four levels at the SNR level of 15dB which contains more noise than signal, compared with decomposition results of the noise-free signal, shown in Fig. 5(a). While the SNR level decreases to 0dB, there are five levels containing more noise than signal. EMD can separate more noise from the noisy signal than WT.

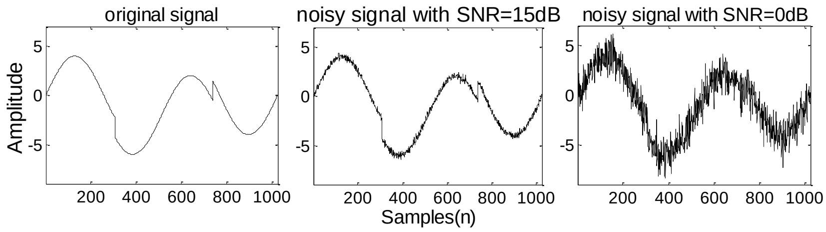
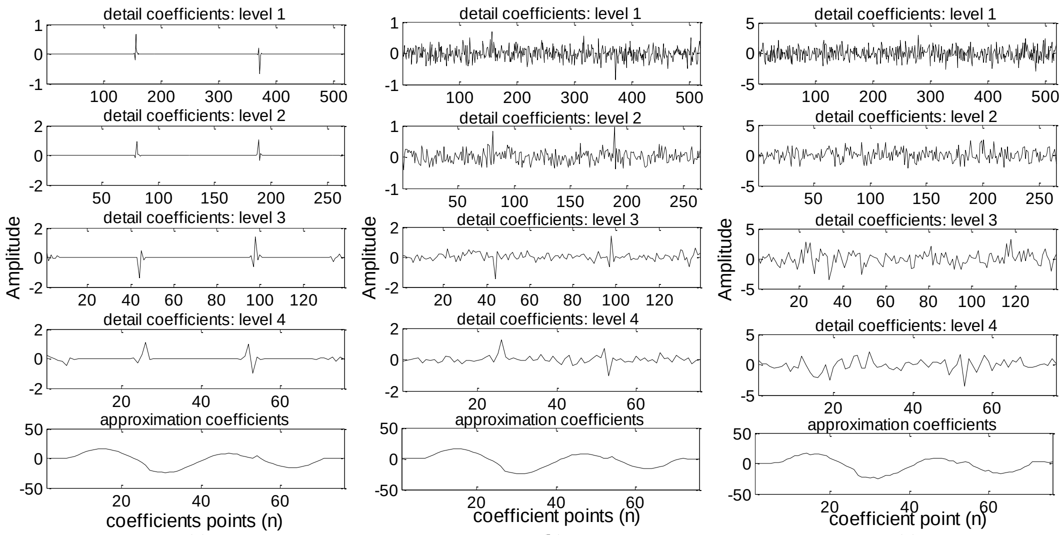
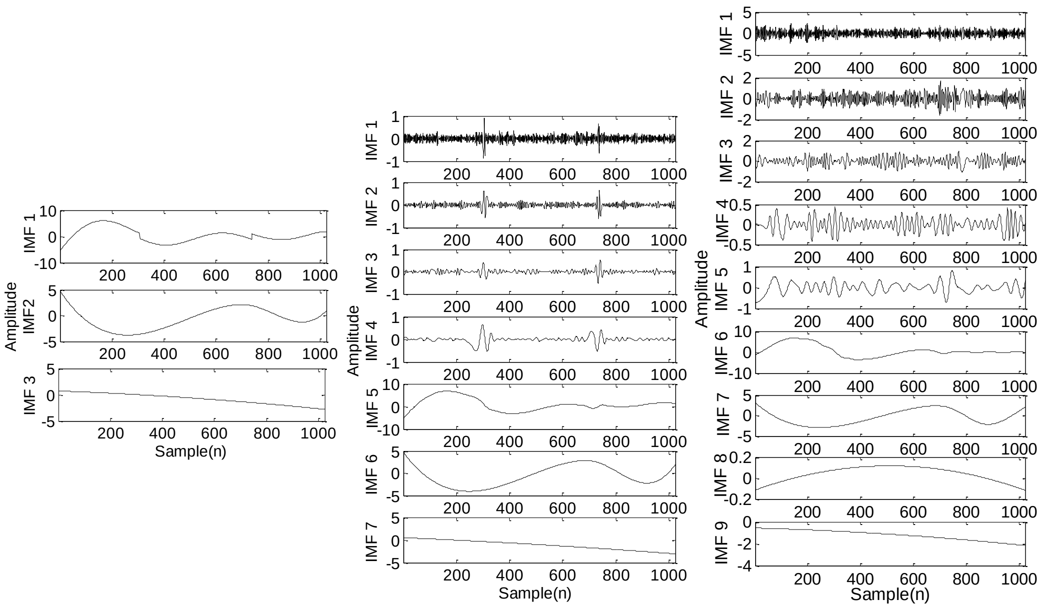


Fig. 3. The signal “Heavysine” at different SNR levels



(a) (b) (c)

Fig. 4. Decomposition results of WT: (a) Original signal, (b) Noisy signal with SNR=15dB, (c) Noisy signal with SNR=0dB.



(a) (b) (c)

Fig. 5. Decomposition results using EMD: (a) original signal, (b) Noisy signal with SNR=15dB, (c) Noisy signal with SNR=0dB.

As we all know, the nature of the sources is not known in BSS. Both the WT basis function and the WT decomposition level are needed to select apriori, i.e. they are predetermined by the user. This affects the quality of the analysis especially when the WT basis and the level are not compatible with the signal parameters. On the other hand, EMD is an adaptive method and its decomposition results are driven by the signal itself. Therefore, EMD is preferable with no predetermined decomposition basis.

It can be obtained from the above preliminary analysis that EMD is more suitable for decomposing noisy signal than WT and EMD is more capable in separating the noise from noisy signal than WT.

C. EMD Denoising

The first attempt to use EMD as a denoising tool emerged from the need to know whether a specific IMF contains useful information or primarily noise. Then the significance IMF test procedures were simultaneously developed by several researchers based on statistical analysis. Just as in wavelet analysis, the lower frequency temporal modes are dominated by the signal, while the higher ones are dominated by the noise. According to this, we can separate the original signal from the noisy signal.

Let C_j be a clean deterministic IMF with a length of L . IMF_j is the corrupted IMF which contains the additive noise n_j with a variance of σ_j^2 , IMF_j can be written as follows.

$$IMF_j = C_j + n_j \tag{7}$$

The purpose of denoising is to get the estimation of the clean deterministic IMF $\hat{C}_j$. We can get the denoising signal as follows:

$$\hat{x}(t) = \sum_{j=1}^{K} \hat{C}_j + r(t) \tag{8}$$

Then, the key problem is how to get $\hat{C}_j$. The simplest approach is to pick out and remove the high frequency IMF only with noise. A denoising method based on the autocorrelation characteristics of white Gaussian noise is proposed by Wang [11]. Moreover Huang *et al.* [12] find that the mean period of any IMF component almost doubles that of the previous one through studying the characteristics of the white noise using EMD. Using this characteristic of the white noise, we can find the IMF only with noise. However, using this method to judge whether the IMF only contains noise is not precise and robust. To solve this problem, Higher Order Statistics criteria [13] can be applied to detect the IMFs which only capture white Gaussian noise, and then they can be safely excluded from the final signal reconstruction process. This is because the Higher Order Statistics of Gaussian signals are equal to zero, which is not the case for non-Gaussian ones. However, in practice, the cumulants estimation of a noisy signal may still be invalid, especially when the samples of the signal are not numerous enough. Besides, the computational cost of this method is much higher.

Although the approach of removing the high frequency IMFs only with noise is very simple, the noise is distributed not only over the high frequency IMFs but also over the other IMFs which contain both the signal and the noise. So this denoising approach cannot remove the noise completely, and we need a more efficient approach. Considering the success of WaveShrink algorithm, we can apply this classical technique to EMD denoising. In the next section, we propose a new denoising approach which combines EMD with WaveShrink algorithm. When the SNR is low, this approach works better than WaveShrink algorithm.

IV. A New EMD Denoising Approach

A. Disadvantages of Conventional Thresholding EMD Denoising

Copsinis and McLaughin[15] proposed a thresholding EMD denoising algorithm which applied the wavelet thresholding principle to EMD denoising, hereafter referred to as EMD thresholding. The threshold they used is as follows:

$$Thr_k = C\sqrt{V_k \cdot 2\ln N} \tag{9}$$

in which C is a constant experimentally found to take the values from 1 to 0.7 depending on the type of the signal, and N is the sample number of the signal. We also have:

$$V_k = \frac{V_1}{\beta}\rho^{-k}, k = 2,3,4\cdots \tag{10}$$

$$V_1 = \frac{1}{N}\sum_{n=1}^{N}\left(IMF_1\right)^2 \tag{11}$$

where V_1 is the energy of the first IMF, and β and ρ are the parameters and Flandrin *et al*. [16] specifically proposed that the values of β and ρ are 0.719 and 2.01 respectively when the noise is white Gaussian noise.

However, we find the thresholds of IMFs decrease so slowly from the first to the last that part of the signal may get lost from some signal-dominated IMFs after thresholding. Therefore, we should first find the IMFs which are dominated by noise and the ones dominated by the signal. Then, we apply the thresholds explained in [15] to the noise-dominated IMFs, and another estimating threshold method which is able to decrease faster to the signal-dominated IMFs.

In this section, we propose a new denoising approach. This approach consists of three key steps: firstly, find the IMFs dominated by the noise, and then compute their thresholds separately; secondly, compute the thresholds of the other IMFs using a new estimating threshold method which is able to decrease faster; lastly, apply the thresholding technique to each IMF, and then reconstruct the signal by adding the thresholded IMFs. We will explain them in detail.

B. Three Key Steps

1) Find the IMFs dominated by the noise

We have introduced several methods to find the noise-dominated IMFs in section 3. In practice, the signal samples we can get are always limited. Under this condition, some characteristics of white Gaussian noise cannot be satisfied strictly. For example, theoretically, the kurtosis of the white Gaussian noise is always equal to zero, but in practice, kurtosis estimation may still be invalid, especially when the signal samples are not numerous enough to ensure convergence. Through studying the white noise using EMD, it can be found that the mean period of IMF almost exactly doubles that of the previous IMF. Therefore, in practice, in deciding whether the IMF is noise dominated or not, judging the mean period of the IMFs is more suitable than others methods. We just need to get the number of the peak of each IMF $NP_1, NP_2, \cdots, NP_K$, where K is the total number of the IMF components. Then, the ratio of the mean period of the IMFs is equivalent to the following expression.

$$R_i = \frac{NP_i}{NP_{i+1}} \quad i = 1, 2, \cdots, K-1 \tag{12}$$

when $R_{k-1} \le 2 \pm \delta, R_k > 2 \pm \delta$, where δ is a small number (such as $\delta = 0.1$), the first k IMFs are considered to be noise-dominated.

Then we use (9) and (10) to get the thresholds of these IMFs. However, the method of estimating the threshold in [15] is based on the assumption that the total noise energy is captured by the first IMF. But, generally, this assumption is not valid. Therefore, the noise variance of the first IMF can be estimated using the better estimator proposed in [17], as is shown in the following equation.

$$V_1 = \left(\frac{madian\left(\left|IMF_1 - median\left(IMF_1\right)\right|\right)}{0.6745} \right)^2 \tag{13}$$

A series of simulations conclude that this estimator performs better for all types of signals. Then we get the thresholds of the first k IMFs $\{Thr_1, \cdots, Thr_k\}$.

2) Get the thresholds of the other IMFs

We use (9) and (10) to get the thresholds of each noise-dominated IMF, and we have to find another estimation method to get the thresholds of the signal-dominated IMFs. Since the estimation method of (9) and (10) decreases slowly, we need one which can decrease faster for signal-dominated IMFs.

First, through studying the threshold in [15], and substituting (10) into (9), we obtain:

$$Thr_i = C\sqrt{\left(V_1/\beta\right) \cdot \rho^{-i} \cdot 2\ln N}, \quad i = 2, 3, 4 \cdots \tag{14}$$

Removing the constant term in the above formula, and substituting $\rho \approx 2$ into the above equation, we get:

$$Thr_i \propto \left(\sqrt{2}\right)^{-i}, \quad i = 2, 3, 4 \cdots \tag{15}$$

In order to make the threshold decreased faster, the form of improved threshold that we propose remains exponential function. Then we set

$$Thr_i \propto \alpha^{-i}, \quad i = 2, 3, 4 \cdots \tag{16}$$

The new threshold function needs to be proportional to the threshold of the last noise-dominated IMF with the constant term C the same as in (9). Thus, the expression of the threshold is set as:

$$Thr_i = C \cdot \frac{Thr_k}{\alpha^{i-k}}, \quad i = k+1, \cdots, K \tag{17}$$

The value of α is determined by the following experiment. In order to make the threshold decrease faster $\alpha > \sqrt{2}$ has to be satisfied. For this, α is selected as 1.6, 1.8, 2, 2.2, and 2.4 for noisy signal denoising. The experiments select “Heavysine” and “Bumps” and SNR from 5dB to 30dB. The experiments are repeated 100 times for each test point, and the signal mean square error of the signal is calculated, as is shown in Fig. 6.

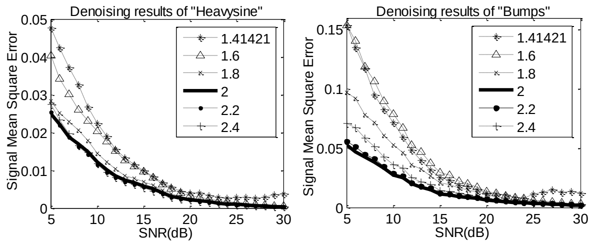


Fig. 6. Denoising results of “Heavysine” and “Bumps” at different values of α

We can see from Fig. 6 that the best results are obtained when $\alpha = 2$, so the threshold is expressed as follows:

$$Thr_i = C \cdot \frac{Thr_k}{2^{i-k}}, \quad i = k+1, \cdots, K \tag{18}$$

where C is a constant as explained in section *A*. Then, we can get the thresholds of the rest IMFs $\{Thr_{k+1}, \cdots, Thr_K\}$. For example, $K = 10, k = 3$, comparing the conventional threshold with the new threshold (Fig. 7), we can see that when the IMF is signal-dominated, the new thresholds decrease faster than the conventional ones.

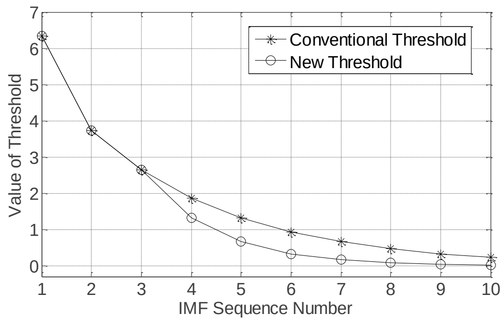


Fig. 7. Comparison of the two different threshold methods

3) Use the estimated thresholds to each IMF

Now with the thresholds of each IMF $\{Thr_1,\cdots,Thr_K\}$ being determined, the WaveShrink algorithm can be used. However, due to the nature of the IMF, directly applying WaveShrink algorithm to each IMF is incorrect in principle and can lead to catastrophic consequences for the continuity of the reconstructed signal[18]. In order to maintain the nature of the IMF, the thresholding operation for EMD in [15] is used. The thresholding operation can be expressed as follows: for every two successive zero crossings interval of the i th IMF $z_j^{(i)} = \left[z_j^{(i)} \quad z_{j+1}^{(i)} \right]$, we can get the thresholded interval $\tilde{z}_j^{(i)}$ for the hard thresholding case, as is shown in the following.

$$\tilde{z}_j^{(i)} = \begin{cases} z_j^{(i)} & \left|r_j^{(i)}\right| > Thr_i \\ 0 & \left|r_j^{(i)}\right| \le Thr_i \end{cases} \tag{19}$$

where $r_j^{(i)}$ is the extremum of the interval $z_j^{(i)}$, i.e. the jth extremum of the ith IMF, and $j = 1,2,\cdots,\left(N_z^{(i)} - 1\right)$, $N_z^{(i)}$ is the number of the zero crossings of the i th IMF. Similarly, for the soft thresholding case, we can get

$$\tilde{z}_j^{(i)} = \begin{cases} z_j^{(i)} \dfrac{\left|r_j^{(i)}\right| - Thr_i}{r_j^{(i)}} & \left|r_j^{(i)}\right| > Thr_i \\ 0 & \left|r_j^{(i)}\right| \le Thr_i \end{cases} \tag{20}$$

And then the thresholded IMF is formed by concatenating the thresholded intervals, i.e.

$$I\tilde{M}F_i = \left[\tilde{z}_1^{(i)} \quad \tilde{z}_2^{(i)} \quad \cdots \quad \tilde{z}_{N_z^{(i)}-1}^{(i)} \right] \tag{21}$$

C. A New EMD Denoising Approach

The above EMD denoising approach, hereafter referred to as piecewise EMD thresholding (EMD-PieThr) is summarized in the following steps and depicted in flow chart in Fig. 8.

Step1: Apply the EMD and decompose the noisy signal in the IMFs.

Step2: Calculate the mean period of the IMFs, and get the ratio R_i using equation (12).

Step3: Find the first k IMFs which are considered noise dominated when $R_{k-1} \le 2 \pm \delta, R_k > 2 \pm \delta$, $\delta = 0.1$.

Step4: Evaluate the thresholds of the first k IMFs $\{Thr_1,\cdots,Thr_k\}$ using equation (9), (10), and (13).

Step5: Evaluate the thresholds of the rest IMFs $\{Thr_{k+1},\cdots,Thr_K\}$ using equation (18).

Step6: Apply the thresholding technique explained in section 4, use the estimated thresholds to every IMF, and get $I\tilde{M}F_i$, i.e. the thresholded IMF.

Step7: Reconstruct the signal by adding $I\tilde{M}F_i$, i.e. $\hat{x}(t) = \sum_{i=1}^{K} I\tilde{M}F_i + r(t)$.

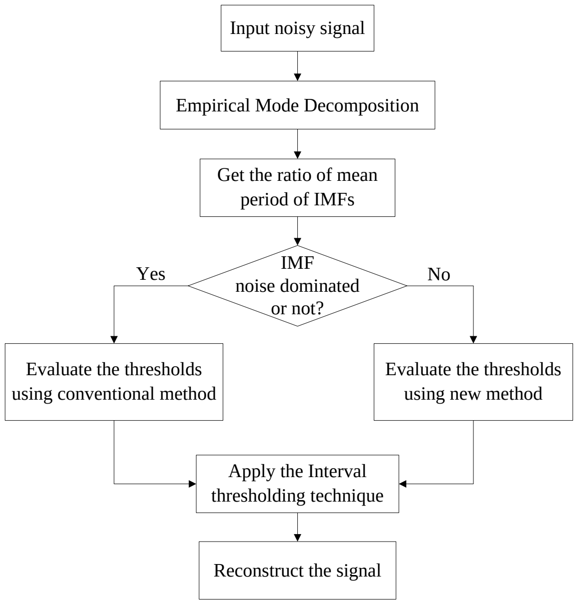


Fig. 8. New EMD denoising approach scheme

V. Experimental Results

The experimental analysis of this section aims at objectively evaluating the denoising performance of denoising algorithms and the separation performance of FastICA and RobustICA after denoising preprocessing.

In order to precisely describe the performance of the algorithms, we employ signal mean square error (SMSE), a contrast-independent criterion defined as

$$\mathrm{SMSE} = \frac{1}{N}\sum_{j=1}^{N} \mathrm{E}\left\{\left|\mathbf{x}_j - \hat{\mathbf{x}}_j\right|^2\right\} \tag{22}$$

where $\mathbf{x}_j$ is the source signal or the noise-free signal, $\hat{\mathbf{x}}_j$ is estimated signal, and N is the sample number of the signal. The performance is better when the value of SMSE is smaller.

A. Denoising Experiment

In order to test the EMD denoising method, we performed numerical simulations for two test signals: “Heavysine” and “Blocks” obtained using MATLAB Software. The sample size of the signals is $N = 1024$. The denoising performance of four denoising approaches is evaluated: WaveShrink, removing the IMFs noise-only based on EMD (EMD-ReIMF), EMD Thresholding proposed in [15] (EMD-Thr), piecewise EMD thresholding we proposed (EMD-PieThr).

The signal $x(n), n = 1,2,\cdots,1024$ is corrupted by i.i.d zero-mean white Gaussian noise, $v(n) \sim N\left(0,\sigma^2\right)$, and σ^2 is unknown. The parameters of WaveShrink are set as follows: the chosen wavelet is “sym7”; the number of decomposition level is 4; and the data-adaptive threshold selection rule is SureShrink[19]. We evaluate the performance of the four denoising approaches at different SNR levels, and for each SNR level the performance criteria SMSE are averaged over 100 Monte Carlo simulations.

The test signals (noise free) and noisy signals are depicted in Fig. 9. The SNR of the signal "Heavysine" is -5dB and the SNR of the signal "Blocks" is 3dB. Fig. 10 displays the outcomes of applying the four denoising approaches to the two signals. Each reconstructed signal plot (black line) is superposed on the corresponding noise-free signal (red line). We can see that the denoising result of applying piecewise EMD thresholding is much closer to their corresponding original signals than the other three approaches. Table I and Table II compare the SMSE values of the four denoising approaches to the two signals respectively. As indicated in Table I and Table II, the EMD-PieThr outperforms the other approaches at different SNR levels. Comparing Table I with Table II, it can be got that the denoising result of the signal "Heavysine" is much better than the signal "Blocks" at the same SNR. The reason is that the oscillations of "Blocks" is more rapid than "Heavysine", and the same problem is seen in WaveShrink.

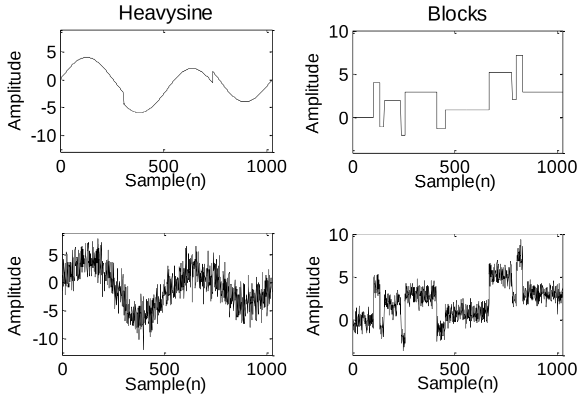


Fig. 9. Test signals with $N = 1024$ and Noisy signals (Heavysine: SNR=-5dB; Blocks: SNR=3dB)

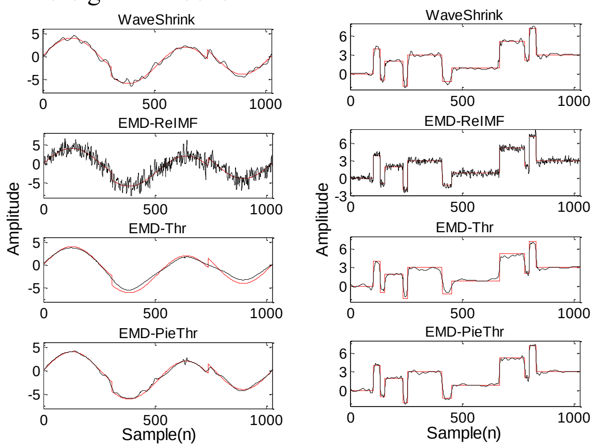


Fig.10. Denoising results of the four approaches. The noise-free signals (red line). The reconstructed signals (black line). (Heavysine: SNR=-5dB; Blocks: SNR=3dB)

TABLE I: DENOISING RESULTS OF "HEAVYSINE" AT DIFFERENT SNR LEVELS

SNR(dB)	-10	-7	-5	-3	0	3	5	7	10
WaveShrink	0.5874	0.3209	0.2020	0.1309	0.0742	0.0411	0.0303	0.0220	0.0144
EMD-ReIMF	0.8262	0.4344	0.3585	0.1980	0.1173	0.0635	0.0456	0.0339	0.0250
EMD-Thr	0.3773	0.2284	0.1821	0.1544	0.0941	0.0607	0.0483	0.0344	0.0226
EMD-PieThr	**0.2891**	**0.1605**	**0.1228**	**0.1062**	**0.0644**	**0.0341**	**0.0248**	**0.0176**	**0.0121**

TABLE II: DENOISING RESULTS OF "BLOCKS" AT DIFFERENT SNR LEVELS

SNR(dB)	-10	-7	-5	-3	0	3	5	7	10
WaveShrink	0.8908	0.6185	0.4879	0.4074	0.3048	0.2225	0.1714	0.1319	0.0810
EMD-ReIMF	1.4097	1.0493	0.7193	0.5932	0.3764	0.2592	0.2246	0.1880	0.1150
EMD-Thr	1.8524	1.3625	1.0755	0.8522	0.5347	0.3411	0.2579	0.1980	0.1332
EMD-PieThr	**0.8239**	**0.6079**	**0.4590**	**0.3776**	**0.2633**	**0.1839**	**0.1459**	**0.1152**	**0.0721**

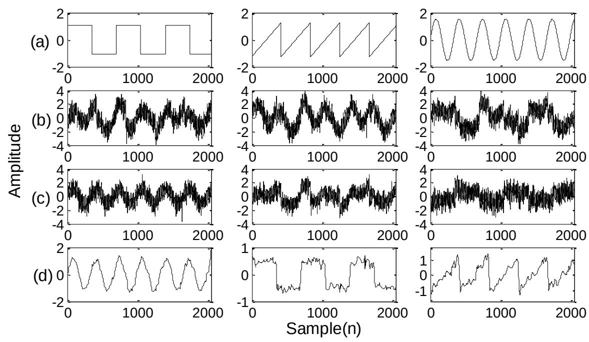


Fig. 11. Comparison of the separation results (SNR=3dB): (a) Original sources, (b) Noisy mixtures, (c) Separation results of FastICA only, (d) Separation results of FastICA with EMD denoising preprocessing

B. BSS Experiment

In the following, the case of three original source signals $x_i(n)$, $i=1,2,3$, $n=1,2,\cdots,2048$ mixed by a 3×3 mixing matrix is considered. Assuming that the mixed source signals are corrupted by additive white Gaussian noise, $v(n)\sim N(0,\sigma^2)$, and σ^2 is unknown.

In order to visualize the performance improvement in restoring the original source waveforms, the three original sources, the noisy mixture (SNR=3dB) and the estimated sources from denoising the mixtures (with FastICA) are depicted in Fig. 11. We can see that the separation waveforms without EMD denoising preprocessing almost cannot be recognized compared with the original sources and the denoising preprocessing provides more accurate waveforms for the estimated sources.

Then, the denoising preprocessing using Wave-Shrink approach and EMD-PieThr approach we proposed are performed individually for each noisy mixture. The parameters of WaveShrink are the same as in Section *A*. And then the separation performances of two prominent BSS algorithms: FastICA[3] and RobustICA[4] are evaluated. Assuming different SNR levels for the observed mixtures, for each SNR level the performance criteria SMSE are averaged over 100 Monte Carlo simulations. The comparison of separation performance is depicted in Fig. 12.

As indicated in Fig. 12, denoising preprocessing is very efficient for improving the performance of BSS algorithms in the presence of strong noise. Moreover EMD denoising preprocessing outperforms Wavelet denoising preprocessing, especially in the cases where the signal SNR is low.

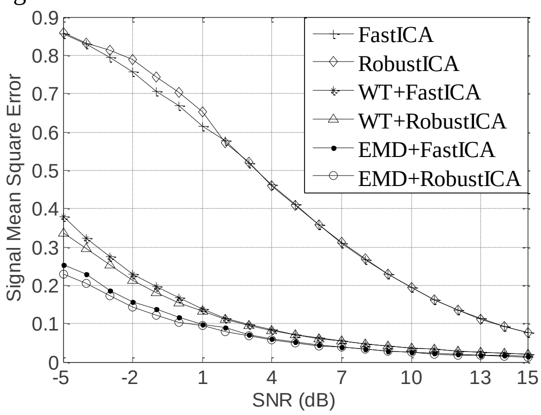


Fig. 12. Comparison of the separation performance of two prominent BSS algorithms with two denoising preprocessing approaches

VI. Conclusions

Noise strongly reduces the separation performance of BSS algorithms, which is known as Noisy BSS problem. A direct and simple solution is to denoise the noisy mixtures before BSS. In this paper, a new signal denoising approach which is called piecewise EMD thresholding approach is proposed. This denoising scheme, based on EMD, is simple and fully data-driven. Moreover, this approach does not use any apriori information. Since the thresholds using the conventional method decrease so slowly that part of the signal will get lost after thresholding, the approach we proposed is able to distinguish the noise-dominated IMFs and the signal-dominated IMFs, and then apply different thresholds methods respectively. The new thresholds decrease faster than the conventional ones. The novel denoising approach exhibits an enhanced performance compared with wavelet denoising in the cases where the signal SNR is low. Simulation results show that denoising preprocessing before BSS is an efficient solution, especially for strong noisy mixtures, and EMD denoising preprocessing outperforms Wavelet denoising preprocessing.

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