Improvement of Rotation Matrix Based Differential Limited Feedback with DFT Codebook and Its Application in CoMP Environment

Yin Zhu[1], Yusheng Ji[2], Ping Wang[1], and Fuqiang Liu[1]
[1]Broadband Wireless Communications and Multimedia Laboratory, Key Laboratory of Embedded System and Service Computing supported by Ministry of Education, Tongji University, Shanghai, 201804, China
[2]National Institute of Informatics, Tokyo, Japan
Email: yin_zhu74@163.com; kei@nii.ac.jp; shaffer_2001@163.com; liufuqiang@tongji.edu.cn

Abstract**—**Recently, exploiting the temporal correlation of slowly varied multiple input multiple output (MIMO) channels to further improve the system performance or the feedback efficient in MIMO wireless communication systems with limited feedback has been investigated in many papers. Among them, T. Kim et al. proposed a rotation matrix based differential feedback scheme which is shown to have good performance with regard to the system throughput for spatial multiplexing MIMO systems. Although the rotation matrix codebook and the precoding codebook in their scheme are optimally constructed, they are not fit for a practical system due to the structure constraints. In this paper, we propose to replace the optimal codebook in the rotation matrix based differential feedback scheme with discrete Fourier transform (DFT) structured codebook, and propose a search-based method to decrease the performance degradation induced by the un-optimality of the DFT codebook. In addition, the modified algorithm is also applied to a per-cell codebook based coordinated multi-point (CoMP) transmission system to verify its applicability to different MIMO scenarios. Simulation results show the effectiveness of our modified scheme.

Index Terms**—**Temporal correlation, differential feedback, DFT codebook, spatial multiplexing

I. Introduction

Multiple input multiple output (MIMO) technology with channel state information at transmitter (CSIT) is known to greatly increase the data transmission rate or the system reliability in wireless communication systems. In a frequency division duplexing (FDD) system, the CSI only can be obtained by the transmitter with limited feedback. The receiver quantizes the CSI into a codeword of a finite set called codebook that is both known to the transmitter and the receiver, then the index of the codeword is fed back from the receiver via a low rate reverse link [1]. Optimization of the codebook and the feedback scheme to maximize the system performance are important research topics in limited feedback technology. Design of optimal codebook based on the channel spatial condition [2]-[5] can obtain optimal system performance. Recently, many research papers began to focus on exploiting the channel temporal correlation [6]-[14] to further reduce the feedback overhead and improve the system performance.

By leveraging temporal correlation of the channel, [6] used Givens rotations to parameterize spatial channel information and then reduces feedback complexity using Delta Modulation. This method directly compresses the elements of the channel matrix but shows lower efficiency than the method of feeding back the quantized precoding matrix that was adopted in, e.g., [7]-[16]. [7] and [8] modeled the quantized CSI as a first-order finite-state Markov chain and [8] proposed a feedback compression algorithm by truncating the transition probabilities; [9] used a geodesic prediction algorithm to reduce the feedback overhead; [10]-[13] modeled the channel as a first-order Gauss-Markov process. Among them, [12] proposed a rotation based differential limited feedback scheme by constructing a differential precoding codebook to adapt to the channel temporal correlation. This scheme was further systemized in [13] by taking the temporal correlation and error propagation effect into account. Simulation shows a noticeable improvement of the system performance over the traditional feedback methods that adopt fixed precoding codebook and it is also superior to the geodesic tracking algorithm in [11]. However, the differential codebook constructed in [13] is not optimal for a spatially uncorrelated MIMO channel as its codewords are not uniformly distributed. Moreover, though the precoding codebook and the rotation matrix codebook adopted in [13] are optimally constructed, they are not fit for a practical system as their structures are not convenient for efficient computation and storage.

Discrete Fourier transform (DFT) structured codebook is a kind of codebook that is often adopted in many wireless communication standards. Though in many cases this kind of codebook is not optimal for a spatially uncorrelated channel, it has important advantages with regard to its structure. In this paper, we first analyze the system performance of the differential limited feedback scheme proposed in [13] by replacing the optimal codebook with DFT codebook and show there is obvious performance degradation due to the un-optimality of the

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Corresponding author email: yin_zhu74@163.com.
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DFT codebook. To decrease the performance degradation, we then propose a search based method to generate a better differential codebook by exploiting the structure property of the DFT codebook. Simulation results show that our method is effective and it can trade off between the search complexity and the improvement of the system performance.

In the latest version of 3GPP LTE standards, high spectral efficiency is required and the coordinated multi-point transmission and reception (CoMP) technology is considered as a potential solution. In these systems, multiple transmission points or cells are coordinated to transmit data to a single user or multiple users. Coordinated multi-point transmission can effectively increase the system spectral efficiency by avoiding the inter-cell interference or even converting the interference into useful signals [17]. To get the benefits of the coordinated system, high quality CSIT is needed. Besides this, the feedback overhead of the system is multiplied due to the multiplied transmission antennas. Thus feedback overhead reduction and improvement of CSI quantization precision become more urgent in coordinated systems.

To reduce the feedback overhead, [18] proposes a subspace-based channel quantization method and a feedback overhead reduction scheme that exploiting the temporal correlation of the channel. But its quantization codebook is constructed by treating the coordinated system as a "virtual MIMO system", i.e., the transmitters of different BSs are considered to belong to a "central unit", and the composite channel is assumed to be i.i.d. as in the traditional single cell system. Although this assumption is simple and easy for system analysis, it doesn't fit for the real environment. [19] lists two properties of the coordinated system: *dynamic number of cooperating BSs* and *heterogeneous path loss effects* in the composite channel. It also proposes a per-cell codebook based limited feedback method which is flexible to construct and shows asymptotic optimality to joint-cell codebook design approach. Based on this idea, [20] and [21] proposed a per-cell codebook based feedback method with phase ambiguity compensation for CoMP system in which single data stream is transmitted. Compared with [19],their method has low complexity and better quantization performance by feeding back additional phase ambiguity quantization overhead. In this paper, we apply our modified differential feedback scheme to the above system and compare its performance with that of the method in [20] and [21]. As shown in simulation results, it needs lower feedback overhead and has better performance when the user speed is not very high.

The rest of the paper is organized as following: Section II introduces the single cell single user MIMO system model and the differential limited feedback scheme, then points out the performance degradation induced by using the DFT structured codebook. Section III proposes a search based differential feedback scheme and two reduced searching methods by exploiting the structure of the DFT codebook to improve the system performance. In Section IV is the application of the modified feedback scheme to a multi-cell joint transmission MIMO system. Finally, Section V concludes the work of this paper.

II. System Model and Problem Formulation

A. System Model

Consider a single cell MIMO spatial multiplexing system that transmits multiple data streams. Let the number of the transmit and the receive antenna be n_T and n_R respectively, and the number of the independent data stream be M, $M \leq \min\{n_T, n_R\}$. At the channel instance τ, the received signal is:

$$\mathbf{y}_\tau = \sqrt{\frac{\rho}{M}}\mathbf{H}_\tau \mathbf{F}_\tau \mathbf{s}_\tau + \mathbf{n}_\tau$$

where $\mathbf{s}_\tau \in \mathbf{C}^{M\times 1}$ is the complex vector of transmitted data streams and ρ is the total transmit power. The channel matrix $\mathbf{H}_\tau \in \mathbf{C}^{n_R\times n_T}$ is modeled as spatially uncorrelated Rayleigh flat fading. $\mathbf{F}_\tau \in \mathbf{C}^{n_T\times M}$ is the precoding matrix and $\mathbf{n}_\tau$ represents the noise vector. $\mathbf{F}_\tau$ is constrained to be orthonormal, i.e., $\mathbf{F}_\tau^H\mathbf{F}_\tau = \mathbf{I}_M$. $(\cdot)^H$ denotes the conjugate transpose.

Furthermore, when the speed of the user is low, the temporal correlation between successive channel instances can be modeled as a first-order Gauss-Markov process [13]:

$$\mathbf{H}_\tau = \varepsilon\mathbf{H}_{\tau-1} + \sqrt{1-\varepsilon^2}\boldsymbol{\Delta}_\tau$$

where $\varepsilon=J_0(2\pi f_D T)$ is the time correlation coefficient of the channel. $J_0(\cdot)$ is the zeroth order Bessel function. T represents the interval of the channel instance and f_D is the maximum Doppler frequency which is determined by the user speed and the carrier frequency. $\mathbf{H}_{\tau-1}$ and $\boldsymbol{\Delta}_\tau$ are independent and both have i.i.d. entries with the distribution of $\mathcal{CN}(0,1)$. In addition, $\mathbf{H}_\tau$ can be decomposed through singular value decomposition (SVD) into $\mathbf{H}_\tau = \mathbf{U}_\tau\boldsymbol{\Sigma}_\tau\mathbf{V}_\tau^H$, then the optimal precoding matrix $\bar{\mathbf{V}}_\tau = \mathbf{V}_\tau(1:M)$, i.e., it is formed by taking the first M columns of $\mathbf{V}_\tau$.

At each instance τ, the receiver feeds back the index of the estimated precoding matrix $\hat{\mathbf{F}}_\tau$ chosen according to the capacity selection criterion:

$$\hat{\mathbf{F}}_\tau = \underset{\mathbf{F}_{\tau,i}\in\mathcal{F}_\tau}{\arg\max}\, I(\mathbf{F}_{\tau,i}), \tag{1}$$

where

$$I(\mathbf{F}_\tau) = \log_2\left(\det\left(\mathbf{I}_M + \frac{\rho}{M}\mathbf{F}_\tau^H\mathbf{H}_\tau^H\mathbf{H}_\tau\mathbf{F}_\tau\right)\right)$$

and $\mathbf{F}_{\tau,i}$ is the codeword of the precoding codebook $\mathbf{F}_\tau$. With this selection criterion, it is proved [3] that $\mathbf{F}_\tau$ is optimal if the minimum Fubini-Study distance of its codewords is maximized, i.e.:

$$\mathcal{F}_\tau = \underset{\tilde{\mathcal{F}}_\tau}{\operatorname{argmax}}\, \delta_{\text{FS}}(\tilde{\mathcal{F}}_\tau) \tag{2}$$

where $\delta_{\text{FS}}(\mathcal{F}_\tau) = \min_{1\le i<j\le L} d_{\text{FS}}(\mathbf{F}_{\tau,i}, \mathbf{F}_{\tau,j}), \mathbf{F}_{\tau,i}, \mathbf{F}_{\tau,j} \in \mathcal{F}_\tau$. L is the codebook size and $d_{\text{FS}}(\mathbf{S}_i, \mathbf{S}_j) = \arccos|\det(\mathbf{S}_i^H \mathbf{S}_j)|$ is the Fubini-Study distance. Finding the optimal codebook can be seen as a Grassmannian subspace packing problem.

B. Problem Formulation

Since our modified scheme is highly dependent on the rotation matrix based differential feedback framework proposed in [13], we need to explain it in detail first. By exploiting the temporal correlation of the channel, at each channel feedback instance τ, a differential codebook $\mathcal{F}_\tau$ is developed which centers around the previously selected precoder $\mathbf{F}_{\tau\text{-}1}$. Specifically, the codewords of $\mathcal{F}_\tau$ are generated via perturbing $\mathbf{F}_{\tau\text{-}1}$ by a rotation matrix codebook with a certain radius. The radius is determined by taking the quantization error propagation and channel variation into account. Thus the feedback scheme has a better capability to track the variation of the channel than other differential feedback methods. The ith codeword of $\mathcal{F}_\tau$ is constructed as following:

$$\mathbf{F}_{\tau,i} = proj\left(\sqrt{1-\bar{r}_\tau}\mathbf{I}_{n_T} + \bar{r}_\tau \Theta_i\right)\mathbf{F}_{\tau-1} \tag{3}$$

where $proj(\bullet)$ represents the projection operator that ensures $\mathbf{F}_{\tau,i}$ is still orthonormal after the above operation. This can be done by Procrustes orthonormalization or Gram-Schmidt column orthonormalization. $\bar{r}_\tau$ is the differential radius and $\mathbf{\Theta}_i$ is the ith codeword of a rotation matrix codebook $\mathbf{Q}$, which is constructed according to the criterion

$$\mathcal{Q} = \arg\max_{\tilde{\mathcal{Q}}} \delta\left(\tilde{\mathcal{Q}}\right) \tag{4}$$

in order to optimize the system performance, where $\delta(\mathcal{Q}) = \min_{1\le i<j\le L} d\left(\mathbf{\Theta}_i, \mathbf{\Theta}_j\right)$ and d is a distance measure defined by:

$$d\left(\mathbf{\Theta}_i, \mathbf{\Theta}_j\right) = \sqrt{1 - \frac{1}{n_T}\left|\text{tr}\left(\mathbf{\Theta}_i^H \mathbf{\Theta}_j\right)\right|}$$

To activate the differential process, a traditional Grassmannian codebook is adopted as the initial precoding codebook. Then at each differential step (channel instance τ), a differential codebook $\mathcal{F}_\tau$ is constructed online in which the receiver selects the proper codeword as the estimation of the precoding matrix according to the criterion (1).

In this differential feedback scheme, the initial codebook and the rotation matrix codebook are optimally constructed to maximize the system performance, but these codebooks are hard to be employed in practical systems because their structures are not convenient for computation and storage. In addition, it can be seen that after perturbation to $\mathbf{F}_{\tau\text{-}1}$, the codewords of $\mathcal{F}_\tau$ are not isotropically distributed on the Grassmann manifold anymore [14]. That is to say, the differential codebook constructed according to (3) is actually not optimal for a spatially uncorrelated MIMO channel. This property also inspired us to develop the search based method in our modified feedback scheme.

In many wireless communication standards, a kind of commonly used codebook is DFT structured codebook. DFT codebook has the advantages of [23]: (i) its codeword has equal amplitude so that to avoid high peak to average power ratio (PAPR) at the transmitter; (ii) its codeword entry has finite alphabet which reduces system complexity especially when the number of the antenna elements is large. A DFT precoding matrix codebook $\mathcal{T}_{\text{DFT}}$ with codewords $\mathbf{T}_l \in \mathbf{C}^{n_T \times M}$ (l = 1, …, L) can be constructed as following [22]:

First, we construct a $n_T \times n_T$ DFT matrix

$$\mathbf{A} = \frac{1}{\sqrt{n_T}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{j\frac{2\pi}{n_T}} & \cdots & e^{j\frac{2\pi}{n_T}(n_T-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j\frac{2\pi}{n_T}(n_T-1)} & \cdots & e^{j\frac{2\pi}{n_T}(n_T-1)(n_T-1)} \end{bmatrix}$$

The first codeword $\mathbf{T}_1$ of $\mathcal{T}_{\text{DFT}}$ can be determined by selecting M different columns from $\mathbf{A}$. For example, we can take the first M columns of $\mathbf{A}$ as $\mathbf{T}_1$. Then the remaining L-1 codewords of $\mathcal{T}_{\text{DFT}}$ are determined by:

$$\mathbf{T}_l = \mathbf{W}^{l-1}\mathbf{T}_1 \left(l = 2, 3, \cdots, l\right)$$

where

$$\mathbf{W} = \text{diag}\{e^{j\frac{2\pi}{L}u_1} \quad e^{j\frac{2\pi}{L}u_2} \quad \cdots \quad e^{j\frac{2\pi}{L}u_{n_T}}\}$$

$0 \le u_1, u_2, \cdots, u_{n_T} \le L-1$ can be seen as the row indices of a $L\times L$ DFT matrix and satisfy $u_i \ne u_j, \forall i \ne j$. $u_1, u_2, \cdots, u_{n_T}$ should be selected to maximize the minimum distance of the codewords of the codebook to maximize the system performance. In our feedback scheme, we will replace the initial precoding codebook and the rotation matrix codebook in [13] with this DFT structured codebook. However, in most cases DFT codebook is not optimal apart from some particular combinations of n_T and L (e.g., as pointed out in [24] when the precoder is a vector (M = 1)). In general, a near optimal DFT codebook can be obtained by choosing the row indices set $u_1, u_2, \cdots, u_{n_T}$ via brute force searching. Therefore, the system performance is degraded if we adopt DFT structured codebook as the initial precoding codebook and the rotation matrix codebook in the differential limited feedback scheme. To alleviate this degradation, we propose a search based method by exploiting the structure property of the DFT codebook.

III. Search Based Differential Limited Feedback Scheme

As analyzed above, DFT structured codebook degrades the system performance in the differential limited feedback scheme. On the one hand, the initial DFT precoding codebook can incur some degradation of the system performance due to its un-optimality; On the other hand, at each differential feedback step, the system performance is further degraded as the differential codebook developed with DFT structured rotation matrix codebook also has poor performance. Fortunately, as mentioned previously, we can choose best results of the two codebooks, i.e., the initial precoding codebook and the differential codebook, according to (2) by searching the row indices $0 \le u_1, u_2, \cdots, u_{n_T} \le L-1$, respectively.

Specifically, on the one hand, we find a best initial DFT structured precoding codebook by searching the row indices $u_1, u_2, \cdots, u_{n_T}$ of the diagonal matrix $\mathbf{W}_{\mathbf{init}}$ according to the criterion

$$\mathcal{F}_{\text{init,DFT}} = \arg\max_{\tilde{\mathcal{F}}_{\text{init,DFT}}} \delta_{\text{FS}}\left(\tilde{\mathcal{F}}_{\text{init,DFT}}\right)$$

this can be done offline before transmission; on the other hand, at each differential step we aim to construct a best differential codebook $\mathbf{F}_\tau$ by perturbing the previously selected precoder $\mathbf{F}_{\tau\text{-}1}$ with a DFT structured rotation matrix codebook $\mathbf{Q}_{\tau,\text{DFT}}$. In this step we do not need to generate a best rotation matrix codebook $\mathbf{Q}$ as in [13], but to choose the row indices $0 \le u_1, u_2, \cdots, u_{n_T} \le L-1$ of $\mathbf{W}_\tau$ when constructing $\mathbf{Q}_{\tau,\text{DFT}}$ to obtain a best differential codebook $\mathbf{F}_\tau$. This process must be finished during each differential step. Our focus becomes to find an efficient searching method to obtain a proper set of the row indices $u_1, u_2, \cdots, u_{n_T}$ of the diagonal matrix $\mathbf{W}_\tau$ so as to construct a best differential codebook $\mathbf{F}_\tau$.

A. Random Search Method

A simple searching method is brute force searching. This can surely find a best combination of $u_1, u_2, \cdots, u_{n_T}$ of $\mathbf{W}_\tau$. But it is computationally inhibitive due to the huge searching complexity $\mathbf{C}_L^{n_T}$. We can decrease the search number to trade off between the searching complexity and the performance improvement. A random search can be adopted according to the decreased search number. But to generate the same differential codebooks at the transmitter and the receiver, the transmitter needs to know the search range which must be fed back from the receiver at each differential step. It increases the feedback overhead and is also inconvenient to implement. So in the next subsection we propose a reduced search method by generalizing the method that was proposed in [23].

B. Mimicing Difference Set Method

It was shown [24] that an optimal DFT beamforming codebook exists only with some particular combinations of (L, n_T). In these cases, the row indices $u_1, u_2, \cdots, u_{n_T}$ of the optimal DFT codebook forms a *difference set*. A (L, n_T, λ) *difference set* is a subset $\{u_1, u_2, \cdots, u_{n_T}\}$ of $\mathbf{Z}_L$ if the $L(L\text{-}1)$ differences (u_k - u_l) mod L, $k \neq l$ take all possible nonzero values 1, 2, …, L-1, with each value exactly λ times. Here $\mathbf{Z}_L = \{0, 1, \ldots, L\text{-}1\}$ [24]. In most cases, brute force searching must be implemented to find a near optimal DFT beamforming codebook. In [23], it was found that by mimicing the row indices as a difference set, the searching complexity can be greatly reduced. Specifically, for a given combination of (L, n_T), a difference set of (L', n_T): $\mathcal{D} = \{u_1, u_2, \cdots, u_K, u_{K+1}, \ldots, u_{n_T}\}$ can be picked where L' is independent of L. Then by fixing the values of u_1, u_2, …, u_K as those in the difference set and varying the values of $u_{K+1}, \ldots, u_{n_T}$ within the range of {0, 1, …, L-1}, a L-K dimensional search can be implemented. Experiments in [23] showed good results even in one dimensional search when n_T is not very large and the precision can be improved by properly increasing the search dimension. It is not very clear why this reduced searching method can obtain good results. However, we find it can be generalized to the case when the element of the DFT codebook is a matrix. Thus by using this method, we can find a good differential precoding codebook with low searching complexity. Comparing to the random search with decreased search number mentioned before, the mimicing difference set method does not need to feedback any extra parameters. The transmitter and the receiver only need to set the same search rule in advance such as the difference set and the search dimension, the identical search results can be obtained.

In our modified scheme, we can pick a set of the row indices $u_1, u_2, \cdots, u_{n_T}$ of $\mathbf{W}_\tau$ of the DFT rotation matrix $\mathcal{Q}$ satisfying (4) as the mimicing difference set before transmission. At each differential step, by varying part of the row indices according to the same dimension, the same differential codebook $\mathcal{F}_\tau$ with maximum $\delta(\mathcal{F}_\tau)$ (the minimum Fubini-Study distance of the codebook) can be found both at the transmitter and the receiver with reduced searching complexity.

C. Complexity Analysis

In our search based DFT differential feedback scheme, brute forcing searching is optimal to find a best differential codebook $\mathbf{F}_\tau$. But the searching complexity is very high. For a system with n_T transmit antennas and let the size of the codebook be L, the search space of the row indices $u_1, u_2, \cdots, u_{n_T}$ of the rotation matrix codebook $\mathbf{Q}_{\tau,\text{DFT}}$ can achieve $C_L^{n_T}$. Besides this, during each differential step, the complexity of computing the minimum distance of the differential codebook $\mathbf{F}_\tau$ is $L(L$-1)/2. Thus the total complexity can be as high as $\frac{1}{2}L(L-1)C_L^{n_T}$. If we adopt reduced search method, the complexity can be greatly decreased. For example, in the mimicing difference set method we can reduce the search dimension to one, i.e., we only let the last row index u_{n_T} vary within its range. Then the search complexity can be decreased from $C_L^{n_T}$ to (L-n_T+1).

D. Analysis of the Simulation Results

Fig. 1 and Fig.2 show the achievable user throughputs comparing our method with that proposed in [13] and with the traditional feedback method. We assume a spatial multiplexing MIMO system with $n_T = n_R = 4$ antennas at both the transmit and the receive ends. The number of the transmission data stream $M = 2$, and the codebook size $L = 16$. For clearly comparison, we take the same time correlation coefficient ε as in [13]. We adopt one dimension search when using the mimicing difference set method (also called "rapid search" in the figures). Then the search number is L-n_T+1 = 13. The random search number is also taken as this value. From the two figures, we can find when using the differential limited feedback proposed in [13], the user throughput obviously increased comparing to the traditional Grassmannian codebook feedback. This improvement is decreased when we take DFT structured codebooks as the initial precoding codebook and the rotation matrix codebook. However, the reduction of the user throughput can be alleviated by using our modified differential feedback method. We can see that the random search and the rapid search have almost the same result, and even one dimension rapid search can compensate almost half of the performance gap between the feedback scheme in [13] and the DFT codebook based scheme.

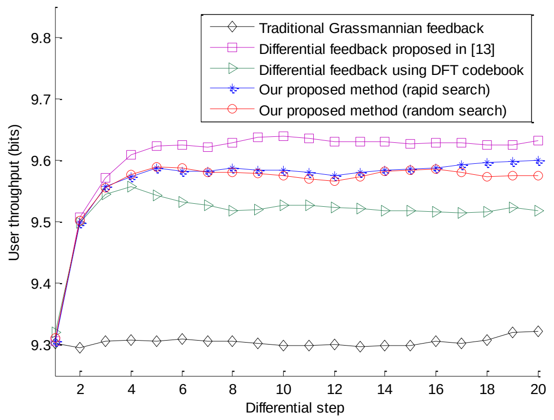

Fig. 1. Comparison of the user throughput between different feedback methods ($n_R = n_T = 4$, $M = 2$, $L = 16$, SNR = 10dB, $\varepsilon = 0.936$)

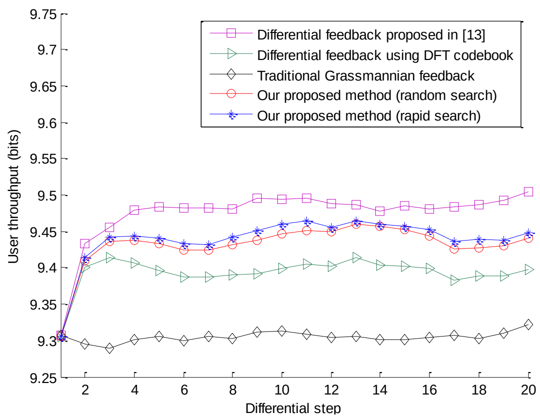

Fig. 2. Comparison of the user throughput between different feedback methods ($n_R = n_T = 4$, $M = 2$, $L = 16$, SNR = 10dB, $\varepsilon = 0.872$)

In addition, we also compare in Fig. 3 the Fubini-Study distance of the differential codebooks generated before and after the searching process. It is clearly observed that we obtain a better differential codebook which has larger minimum distance than the one before searching. This result corresponds to the codebook construction criterion (2), i.e., we should adopt the codebook with its minimum distance as maximum as possible. It also can be seen that the resulting codebooks obtained by random search and the rapid search have almost the same minimum distance.

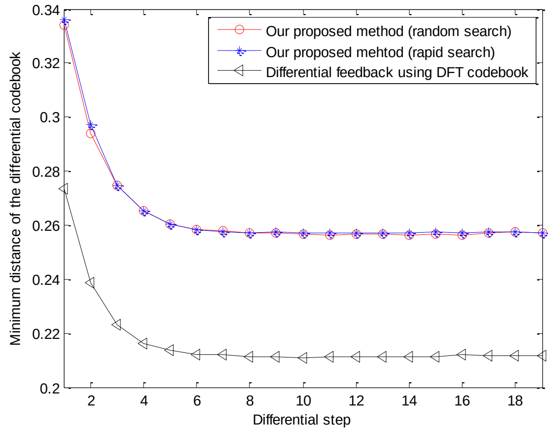

Fig. 3. Minimum Fubini-Study distance of the differential codebook at each differential step ($n_R = n_T = 4$, $M = 2$, $L = 16$, SNR = 10dB, $\varepsilon = 0.936$)

IV. Application to Per-Cell Codebook Based Joint Transmission System

A. System Model

Assuming there are N base stations (BS), each with n_T transmit antennas, jointly transmit single data stream to one user with n_R receive antennas under the coordination of a "central unit", which forms a joint transmission system. The composite channel is composed of N corresponding single cell channels. Here we take into account the heterogeneous path loss effect, which means that the channel formed by different coordinated BSs and the user may have different strengths. The composite channel is represented as

$$\begin{aligned}\mathbf{H}_c &= [\alpha_1\mathbf{H}_1 \quad \alpha_2\mathbf{H}_2 \quad \cdots \quad \alpha_N\mathbf{H}_N] \\ &= \mathbf{H}\begin{bmatrix}\alpha_1\mathbf{I}_{n_T} & & \\ & \ddots & \\ & & \alpha_N\mathbf{I}_{n_T}\end{bmatrix} \\ &= \mathbf{H}\mathbf{R}^{1/2}\end{aligned}$$

where $\mathbf{H}_i \in \mathbf{C}^{n_R \times n_T}$ is the small scale fading of the channel between the ith BS and the user and α_i is the corresponding large scale fading coefficient assumed to be known both at the transmitter and the receiver. The elements of $\mathbf{H}_i$ are i.i.d. and each has distribution of $\mathcal{CN}(0,1)$. $\mathbf{H}_c$ can also be viewed as a special "spatial correlated" channel with the spatial correlation matrix $\mathbf{R}$ determined by the large scale fading coefficient of each coordinated channel. In addition, $\mathbf{H}_c$ can be decomposed via single value decomposition (SVD) into:

$$\mathbf{H}_c = \mathbf{U}_c \mathbf{\Sigma}_c \mathbf{V}_c^H$$

When single data stream is transmitted, the receiver needs to feedback the estimation codeword of the precoding vector $\mathbf{v}_c = \mathbf{V}_c(:,1)$, the first column of $\mathbf{V}_c$ to the central unit:

$$\mathbf{f}_c = \arg\min d_{\text{chord}}\left(\mathbf{f}_c, \mathbf{v}_c\right)$$

where d_{chord} is the chordal distance which is proved to be a proper measurement with regard to the quantization error in the case of single data transmission [2]. $\mathbf{f}_c, \mathbf{v}_c \in \mathbf{C}^{Nn_T \times 1}$ represent the estimated codeword and the optimal precoding vector, respectively. The above criterion can also be easily transformed into the following formation:

$$\mathbf{f}_c = \arg\max\left|\mathbf{f}_c^H \cdot \mathbf{v}_c\right|$$

B. Per-Cell Codebook based Feedback Scheme

In coordinated multi-cell transmission system, optimal codebook for the precoding estimation of the composite channel can be designed by viewing the coordinated system as a “virtual” big MIMO system, and the index of the corresponding codeword is sent back to the transmit end. However, this scheme is hard to implement in the practical system due to its inflexibility and complexity. Considering the main properties of the composite channel mentioned in Section I, [19] proposes a per-cell codebook based limited feedback scheme in which the receiver selects the proper sub-codeword from each per cell codebook and the composite precoder is constructed by those sub-codeword. It is flexible to implement and is asymptotically optimal when n_T is sufficiently large. Based on the per-cell codeword selection idea, [21] proposes an independent codeword selection with phase ambiguity compensation feedback method and it shows lower complexity than that of [19]. In [20], several codeword selection methods based on per-cell codebook are compared with regard to the quantization performance. In the following, we list three codeword selection methods introduced in [20] and compare our modified feedback scheme with them.

- Method 1: Optimal joint codeword selection

The codeword selection is determined by the criterion:

$$\mathbf{f}_{i_1}, \cdots, \mathbf{f}_{i_N} = \arg\min d_{\text{chord}}\left(\mathbf{v}_c, \left[\alpha_1 \mathbf{f}_{i_1} \quad \cdots \quad \alpha_N \mathbf{f}_{i_N}\right]\right)$$

where $\mathbf{f}_{i_k}$ is the codeword selected from the kth per-cell codebook. In this method, the codewords $\mathbf{f}_{i_1}, \cdots, \mathbf{f}_{i_N}$ should be jointly selected to obtain optimal results. It has the best quantization precision compared with the other two methods but the complexity is high when the codebook size is large.

- Method 2: Independent codeword selection

The codeword selection is determined by the criterion:

$$\mathbf{f}_{i_k} = \arg\min d_{\text{chord}}\left(\mathbf{v}_{1,k}, \mathbf{f}_{i_k}\right)$$
$$= \arg\max\left|\mathbf{f}_{i_k}^H \cdot \mathbf{v}_{1,k}\right|$$

where $\mathbf{v}_{1,k}$ is obtained by letting $\mathbf{v}_c = [\mathbf{v}_{1,1}^H \ldots \mathbf{v}_{1,N}^H]^H$. As the codeword $\mathbf{f}_{i_k}$ is independently selected, the complexity is fairly low compared with the above joint selection method, but the quantization error is increased as well.

- Method 3: Independent codeword selection with phase ambiguity compensation

To improve the quantization precision of the independent codeword selection method, the phase ambiguity between $\mathbf{v}_{1,k}$ and $\mathbf{f}_{i_k}$ is also fed back to the transmitter using a few additional bits, i.e.,

$$\mathbf{f}_{i_k}^H \cdot \mathbf{v}_{1,k} = \left|\mathbf{f}_{i_k}^H \cdot \mathbf{v}_{1,k}\right| \cdot e^{j\xi_k}$$

Besides feeding back the index of $\mathbf{f}_{i_k}$, the receiver also quantizes the phase ambiguity $e^{j\xi_k}$ and feeds back its index to the transmitter. Simulation results in [20] show an obvious improvement of the quantization performance than the other two methods. However, it needs more feedback overhead.

To compare with the above three feedback methods, we also use our differential feedback scheme based on the per-cell codebook. We adopt Method 1 to select the codeword. Assume there are two coordinated BSs jointly transmit single data stream to one user. Each BS has n_T = 4 transmit antennas and the user has n_R = 2 receive antennas. We set the size of the codebook to 16 for the above three feedback methods and 8 for our modified feedback method and use DFT structured codebook for all these methods. In addition, Method 3 needs an extra 3 bits to feedback the phase ambiguity. Thus the feedback overhead of the four methods are: 4bits for Method 1, 4 bits for Method 2, 7 bits for Method 3 and 3 bits for our modified method. Fig. 4 and Fig. 5 show the results of the quantization error performance.

From the simulation results, we can see except for some initial differential steps, the quantization error of our modified differential feedback method is lower than Method 1 and Method 2. When the user speed is low enough (corresponding to high value of ε), the quantization performance of our scheme is even superior to Method 3. Notice that we use fewer bits for our scheme to feedback the codeword. This is easy to explain because the differential codebook centers around the previous precoder and its quantization range is much smaller than the traditional codebook (the traditional codebook covers the whole quantization range). So, high quantization precision can be obtained with fewer codewords. This advantage is even more obvious when the user speed is low, i.e., when the channel changes much slowly. Considering that when we adopt coordinated multi-cell transmission, the user speed usually is much slow to ensure good system performance, so that the differential feedback scheme is also suitable for this scenario. Though the search complexity of joint codeword selection adopted by the differential scheme is fairly high, it can somehow be traded off by smaller codebook size.

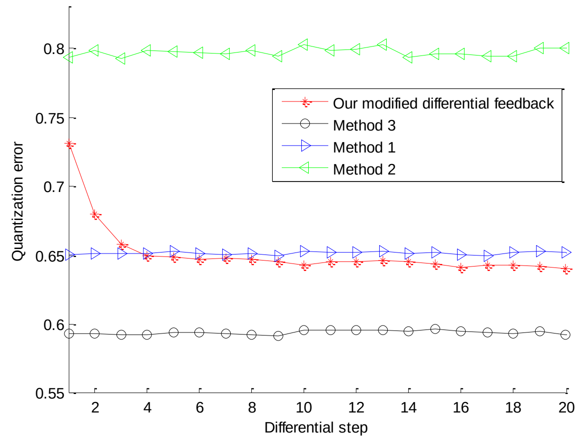

Fig. 4. Quantization error comparison between different feedback methods based on per-cell codebook (n_R = 4, n_T = 2, M = 1, codebook size = 16 (for Method 1, 2 and 3) and codebook size = 8 (for our modified method), ε = 0.872)

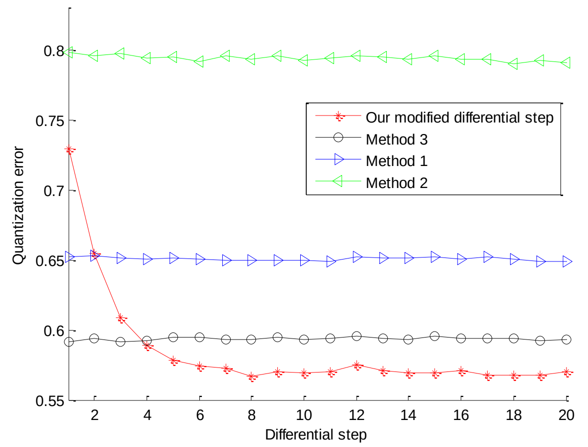

Fig. 5. Quantization error comparison between different feedback methods based on per-cell codebook (n_R = 4, n_T = 2, M = 1, codebook size = 16 (for Method 1, 2 and 3) and codebook size = 8 (for our modified method), ε = 0.936)

V. Conclusion

In this paper, we propose a modified differential limited feedback scheme. Based on the analysis of the differential feedback scheme proposed by T. Kim et al. in [13], we point out its advantage in the improvement of the system performance and its implementation constraint in the practical systems. Then we propose to implement this differential feedback scheme by substituting the optimal codebooks with the DFT structured codebooks and analyze the performance degradation induced by this substitution. Based on this, we propose a search based differential feedback scheme by generalizing the mimicing difference set searching method to alleviate this degradation. Simulation results show the effectiveness of our scheme. In addition, this modified differential limited feedback scheme is also applied to a per-cell codebook based multi-cell coordinated system, and the effect of the improvement of the quantization error is verified.

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Yin Zhu now is a Ph.D. candidate at BM&W Lab of Tongji University. She is also a lecturer at College of Electronic and Information Engineering, Suzhou University of Science and Technology. Her research area is multiple antenna transmission in the next generation wireless networks.

Yusheng Ji received B.E., M.E., and D.E. degrees in electrical engineering from the University of Tokyo. She joined the National Center for Science Information Systems, Japan (NACSIS) in 1990. Currently, she is a professor at the National Institute of Informatics, Japan (NII), and the Graduate University for Advanced Studies (SOKENDAI). Her research interests include network architecture, resource management, and performance analysis for wired and wireless communication networks. She is also a member of IEEE and IPSJ.

Ping Wang is an associate professor in the department of information and communication engineering at Tongji University. He graduated from the department of computer science and engineering at Shanghai Jiaotong University and received Ph.D. degree in 2007. His main research interests are in routing algorithms and resource management in wireless networks and video transcoding.

Fuqiang Liu is a professor in the Department of Information and Communication engineering at Tongji University. He graduated from the department of automation at China University of Mining and received Ph.D. degree in 1996. His main research interests are in technologies in wireless broadband access and image manipulation.