# A Dual-mode Blind Equalization Algorithm for Improving the Channel Equalized Performance

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***Abstract*—**In this paper, we propose a new dual-mode blind equalization algorithm to improve the channel equalization performance. First, a kind of new error function is proposed and a nonlinear function between the step-size factor and the new error function is built to realize the variable step-size constant modulus algorithm (CMA). Then, in order to optimize the channel equalization performance, the variable step-size CMA is combined with decision directed least mean square (DDLMS) algorithm to form the new dual-mode algorithm. The new dual-mode algorithm uses either switching-dual mode or weighted dual-mode to update the coefficient of blind decision feedback equalizer. The simulation results show that in the situation of Gaussian white noise with 30dB signal-to-noise ratio, the proposed algorithm reduces the steady-state error to –12dB and drops inter symbol interference to –31dB in the condition of satellite channel. The new dual-mode algorithm not only increases the rate of convergence but also reduces the steady-state error. The proposed algorithm can be widely used in different communication channels such as the satellite channel.

***Index Terms*—**Blind equalization; constant modulus algorithm; variable step-size; decision directed least mean square.

## I. Introduction

Equalization techniques can effectively reduce inter-symbol interference caused by the limited bandwidth of the channel and multi-path effects; largely solve some problems caused by nonlinear distortion such as spread spectrum, the constellation distortion and channel distortion. Equalization techniques can improve the transmission rate of the system; therefore, the study related to the Equalization techniques for nonlinear distortion channel has urgent requirements and positive meaning.

The traditional adaptive equalization techniques need the help of the training sequence known by the sender and the receiver to train the equalizer adjusting the equalizer tap coefficients to ensure the equalizer quickly enter the convergence state so as to achieve channel estimation and equalization [1]; however the training sequences can be hardly informed of in many applications. Blind equalization techniques use the statistical properties of the data signal itself to achieve balance state. So blind equalization can update the tap coefficients without knowing the training sequences and it is a kind of adaptive equalization techniques and has become a research hotspot in recent years [2], [3]. There are several works in the literature on blind channel estimation and equalization [4]. Blind equalization algorithms can be divided into three categories: Bussgang class blind equalization algorithms, higher-order statistic based blind equalization algorithms and neural network based nonlinear blind equalization algorithms [5]. The theory of Bussgang class blind equalization algorithm is conducting some kinds of nonlinear variable changes in the output of the equalizer to obtain estimate value of the desired signal. Then design appropriate cost functions, using stochastic gradient descent algorithm to find the minimum gradient value to complete adaptive equalization process. In variety of Bussgang class blind equalization algorithms, the techniques respectively proposed by Sato and Godard are the two most widely referenced techniques available for blind equalization of a QAM system [6], [7]. The constant modulus algorithm (CMA) is only related to the amplitude of the receiving signal and is not sensitive to the carrier phase shift, therefore, the robustness of CMA is very excellent. In general, CMA is one of the most popular algorithms in the blind equalization occasions due to the advantages of small amount of calculation and being easily achieved.

One problem in CMA is the low rate of the convergence rate [8] because the convergence rate depends on the eigenvalues of the correlation matrix of the channel output. Especially when compared to the traditional used decision directed least mean square (DDLMS) algorithm [9], the CMA converges slowly and is easy to form a high error rate. However, the DDLMS algorithm has a strict requirement of the stability of the channel to guarantee the low steady-state error. Once there is a sudden interference in the channel, the decision is likely to be wrong and the DDLMS algorithm cannot converge. Considering the pros and cons of these two equalization algorithm, the CMA and DDLMS algorithm can be combined in an appropriate manner to get a new dual-mode algorithm which contains the advantages of the two algorithms to attain better equalization performance [10].

In this paper, by studying the impact of the step-seize on the performance of CMA, a variable step-size idea to

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CMA is applied, where a nonlinear function between the step-size factor and the residual errors is proposed to control the step size in order to improve the convergence precision and convergence rate. Then, DDLMS algorithm is joined, both the switching dual-mode and the weighted dual-mode are applied to combine VS-CMA and the DDLMS algorithm to form the VS-CMA-DDLMS algorithm to improve the performance of the system [11], the rate of convergence of the proposed new dual-mode equalization algorithm is improved and the residual errors have a great degree of reduction.

## II. Variable Step-Size CMA

Blind equalization structure is shown in Fig. 1.

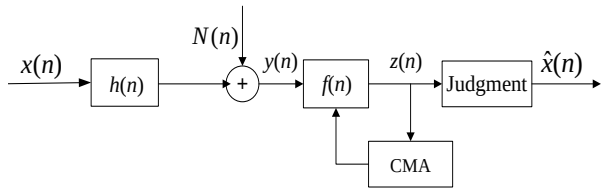



Fig. 1. Blind equalization system diagrams.

In Fig. 1, $x(n)$ is the sending sequence, $h(n)$ is the channel impulse response, $N(n)$ is the noise sequence, $y(n)$ is the receiving sequence and it is also the input signal of blind equalizer, $z(n)$ is the output restoration signal of blind equalizer, $\hat{x}(n)$ is the decision output signal.

The structure of transversal filter is shown in Fig. 2.

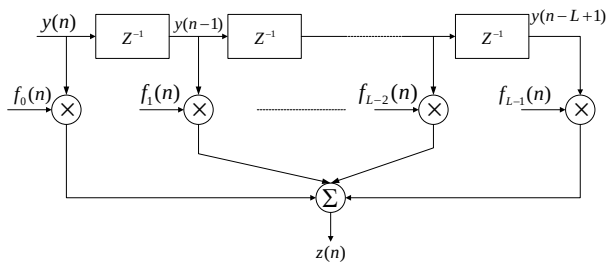



Fig. 2. Transversal filter structure

In Fig. 2, $y(n)$ is the input sequence of the filter, $L$ is the length of transversal filter, $f(n)$is the tap coefficient of the equalizer.

According to the signal processing theory, operator $*$ represents convolution, and

$$\begin{aligned} y(n) &= x(n) * h(n) + N(n) \\ &= \sum_i h(n)x(n-i) + N(n) \end{aligned} \tag{1}$$

The equalizer output is

$$\begin{aligned} z(n) &= f(n) * y(n) \\ &= \sum_i f_i(n) y(n-i) \end{aligned} \tag{2}$$

Let transversal filter input sequences vector $\mathbf{y}$(n) in Fig. 2 is

$$\mathbf{y}(n) = \left[ y(n), y(n-1), \cdots, y(n-L+1) \right]^T \tag{3}$$

Let the tap coefficients vector $\mathbf{f}$(n) of the equalizer in Fig. 2. is

$$\mathbf{f}(n) = \left[ f_0(n), f_1(n-1), \cdots, f_{L-1}(n-L+1) \right]^T \tag{4}$$

So the equalizer output sequence $z(n)$ can be changed as

$$\begin{aligned} z(n) &= f(n) * y(n) \\ &= \sum_{i=0}^{L-1} f_i(n) y(n-i) \\ &= \mathbf{y}^T(n)\mathbf{f}(n) \\ &= \mathbf{f}^T(n)\mathbf{y}(n) \end{aligned} \tag{5}$$

The cost function of CMA is

$$J(n) = E\left[ \left( |z(n)|^2 \right) - R^2 \right] \tag{6}$$

The error function of CMA is

$$e(n) = z(n)\left( |z(n)|^2 - R \right) \tag{7}$$

$$R = \frac{E\left[ |x(n)|^4 \right]}{E\left[ |x(n)|^2 \right]} \tag{8}$$

In the CMA, the iteration formula of the tap coefficient is

$$\mathbf{f}(n+1) = \mathbf{f}(n) - \mu e(n) \mathbf{y}^*(n) \tag{9}$$

where $\mu$ is the step size factor which is usually a quite small positive constant.

The CMA uses a fixed step size. If the step size is small enough, a smaller residual error and a smaller bit error rate after the convergence can be achieved which at the expense of a slow convergence speed. If we increase the step size to obtain a faster convergence rate, it will lead to larger residual error and bit error rate. Even the algorithm cannot converge. Therefore, in the CMA algorithm, the convergence rate and the restraining precision are contradictory.

By utilizing variable step-size to the CMA can solve this contradictory. The basic idea is as follows: the larger step size is used in the algorithm iteration initial period or the time when channel characteristics have a great change to keep the convergence speed. The step size is reduced to ensure a small steady state error when the algorithm has been converged. A nonlinear function between the step-size factor and the residual errors is proposed to control the step size based on these principles. The residual error $d(n)$ is

$$\begin{aligned} d(n) &= \hat{x}(n) - z(n) \\ &= \hat{x}(n) - \mathbf{f}^T(n)\mathbf{y}(n) \end{aligned} \tag{10}$$

Supposing the best real-time variable vector of the equalizer is:

$$\hat{\mathbf{f}}(n) = \left[ \hat{f}_0(n), \hat{f}_1(n), \cdots, \hat{f}_{L-1}(n) \right]^T \tag{11}$$

We can deduce that

$$\hat{z}(n) = \hat{\mathbf{f}}^T(n)\mathbf{y}(n) + \zeta(n) \tag{12}$$

$$\hat{x}(n) = \hat{\mathbf{f}}^T(n)\mathbf{y}(n) \tag{13}$$

In (12), $\zeta(n)$ the Gaussian noise which is independent and identically distributed and has zero mean value. We can put (12) and (13) into (10) to derive the function

$$\begin{aligned} d(n) &= \hat{x}(n) - z(n) \\ &= \hat{\mathbf{f}}^T(n)\mathbf{y}(n) - \mathbf{f}^T(n)\mathbf{y}(n) + \zeta(n) \\ &= (\hat{\mathbf{f}}^T(n) - \mathbf{f}(n))\mathbf{y}(n) + \zeta(n) \\ &= \mathbf{w}^T(n)\mathbf{y}(n) + \zeta(n) \end{aligned} \tag{14}$$

where $\mathbf{w}(n)$ is called weight error vector.

In the process of implementation of the VS-CMA, $\hat{\mathbf{f}}^T(n)$ is gradually closed to $\mathbf{f}^T(n)$, So the value of weight error vector $\mathbf{w}(n)$ is tending to zero and the value of the residual error also decreases. After the algorithm converged, $d(n)$ tends to be a very small number, it is proper to use $d(n)$ to control the value of step size. However, because of the existence of $\zeta(n)$, $d(n)$ is sensitive to the interference signal. In some unstable channel, $d(n)$ can suddenly be very large if there is a strong interference. In this situation we cannot directly use $d(n)$ to control the step-size, otherwise, the step-size will be too large and the convergence of the algorithm cannot be assured. So we use a nonlinear function and set $d(n)$ as a parameter of the function to make sure the step-size will be influenced by the residue error but will be varied in a reasonable scope.

The step function controlled by the residual error is

$$\mu(n) = \frac{k\left|d(n)\right|^{\alpha}}{(1+\left|d(n)\right|^{\alpha})(1+\exp(-\left|d(n)\right|^{\beta}))} \tag{15}$$

The iteration formula of the tap coefficient is

$$\mathbf{f}(n+1) = \mathbf{f}(n) - \mu(n)e(n)\mathbf{y}^*(n) \tag{16}$$

In (15), $k$, $\alpha$, $\beta$ are all constants. $k$ controls the whole range of values of $\mu(n)$ so that it can adjust the convergence rate of the algorithm. $\alpha$ and $\beta$ are used to change the shape of the function, different values of $\alpha$ and $\beta$ are taken, different curvatures of the function $\mu(n)$ can be achieved. So by taking the appropriate values, we can make that if $d(n)$ is small then $\mu(n)$ is small, if $d(n)$ is increasing then $\mu(n)$ is also increasing at a reasonable speed and amplitude. This heuristic is very useful in a variety of control and signal processing applications [12].

The improved algorithm structure is shown in Fig. 3.

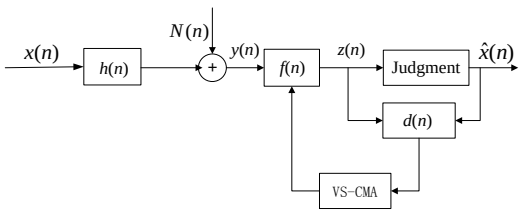



Fig. 3. Improved algorithm structure.

The convergence of the VS-CMA is as follows: when $0 \le \frac{\left|d(n)\right|^{\alpha}}{(1+\left|d(n)\right|^{\alpha})(1+\exp(-\left|d(n)\right|^{\beta}))} < 1$, the value scope of $\mu(n)$ satisfies: $0 \le \mu(n) < k$. In order to guarantee the algorithm restrained, the step size must ensure that [13]

$$0 \le \mu(n) < \frac{2}{3\times \mathrm{tr}\left(\mathbf{R}_{yy}\right)} \tag{17}$$

where $\mathbf{R}_{yy}$ is the autocorrelation matrix of the receiving sequence $y(n)$, $\mathrm{tr}(\mathbf{R}_{yy})$ is the trace of $\mathbf{R}_{yy}$. So we can change the value of $k$ which is often a small constant to ensure the convergence of the formula (16).

Define $\mathbf{L}(n) = E\{\mathbf{f}(n)\}$, we can put formula (5) namely, $z(n) = \mathbf{f}^T(n)\mathbf{y}(n)$ into the iteration formula of the tap coefficient (9). So we can derive that

$$\begin{aligned} \mathbf{L}(n+1) &= E\left\{\left[\mathbf{f}(n) - \mu\mathbf{f}^T(n)\mathbf{y}(n)\left(\left|\mathbf{f}^T(n)\mathbf{y}(n)\right|^2 - R\right)\mathbf{y}^*(n)\right]\right\} \\ &= \mathbf{L}(n) - \mu E\left[\mathbf{f}^T(n)\mathbf{y}(n)\mathbf{y}^*(n)\left(\left|\mathbf{f}^T(n)\mathbf{y}(n)\right|^2 - R\right)\right] \\ &= \mathbf{L}(n)\left[\mathbf{I} - \mu\left(\mathbf{R}_{yy}\mathbf{f}(n)\mathbf{f}^H(n)\mathbf{R}_{yy} - R\mathbf{R}_{yy}\right)\right] \end{aligned} \tag{18}$$

where $\mathbf{R}_{yy}$ is the autocorrelation matrix of received sequence, $\lambda$ is the maximum eigenvalue of $\mathbf{R}_{yy}$. The estimate of convergence time [14] can be obtained by

$$t_{\mathrm{CMA}} \propto \frac{1}{\mu\left(\left\|\mathbf{f}(n)\right\|_2 \lambda^2 - R\lambda\right)} \tag{19}$$

As a result, the convergence time of the improved algorithm VS-CMA can be estimated as

$$t_{\mathrm{VS-CMA}} \propto \frac{1}{\mu(n)\left(\left\|\mathbf{f}(n)\right\|_2 \lambda^2 - R\lambda\right)} \tag{20}$$

when the algorithm has not converged, the value of $\mu(n)$ is large. Compared with the CMA the step size of which is a fixed small constant, the convergence rate of the improved VS-CMA is quicker.

## III. VS-CMA JIONT DDLMS DUAL-MODE ALGORITHM

The convergence rate of the VS-CMA is quicker than CMA, and its steady state error is also smaller, but it is still based on the CMA. Hence, compared with the DDLMS algorithm which is commonly used in the adaptive equalizer, the performance of the VS-CMA is unsatisfactory. We can combine VS-CMA with DDLMS algorithm to form a new dual-mode blind equalization algorithm which has a better equalization performance and the convergence rate is fast while the steady state error is smaller.

The basic principle of DDLMS algorithm is the same as the least mean square (LMS) algorithm. If we know the training sequence $\tilde{z}(n)$, we can use the mean square error between the filter outputs and the expected receive values as the cost function, namely

$$\begin{aligned} J(n) &= E\left[e(n)^2\right] \\ &= E\left\{\left[\tilde{z}(n) - \mathbf{y}^T(n)\mathbf{f}(n)\right]^2\right\} \\ &= E\left[\tilde{z}^2(n)\right] - 2E\left[\tilde{z}(n)\mathbf{y}^T(n)\right]\mathbf{f}(n) \\ &\quad + \mathbf{f}^T(n)E\left[\mathbf{y}^T(n)\mathbf{y}(n)\right]\mathbf{f}(n) \end{aligned} \tag{21}$$

Define R=$E[\mathbf{y}^T(n)\mathbf{y}(n)]$ which is a $L \times L$ matrix as the autocorrelation matrix of equalizer input sequence. $\mathbf{P} = E\left[\tilde{z}(n)\mathbf{y}^T(n)\right]$ is the correlation matrix of the equalizer system. So the cost function $J(n)$ is

$$J(n) = E\left[\tilde{z}^2(n)\right] - 2\mathbf{P}\mathbf{f}(n) + \mathbf{f}^T(n)\mathbf{R}\mathbf{f}(n) \tag{22}$$

According to the minimum mean square error criterion, make gradient of (22) to $\mathbf{f}(n)$ is zero, namely

$$\nabla = \frac{\partial J(n)}{\partial \mathbf{f}(n)} = 2\mathbf{R}\mathbf{f}(n) - 2\mathbf{P} = 0 \tag{23}$$

we can obtain the best value $\hat{\mathbf{f}}(n)$ of f($n$) which should satisfy the equation

$$\hat{\mathbf{f}}(n) = \mathbf{R}^{-1}\mathbf{P} \tag{24}$$

The LMS algorithm uses the square of the error between the equalizer output and the ideal response instead of the mean square error as the cost function.

It makes that

$$\begin{aligned} \nabla &= \frac{\partial J(n)}{\partial \mathbf{f}(n)} \\ &\approx \left[\frac{\partial e^2(n)}{\partial f_0(n)}, \frac{\partial e^2(n)}{\partial f_1(n)}, \cdots, \frac{\partial e^2(n)}{\partial f_{L-1}(n)}\right] \\ &= 2e(n)\mathbf{y}^*(n) \end{aligned} \tag{25}$$

The algorithm uses the steepest descent method that is along the opposite direction of the gradient vector of the cost function to adjust the equalizer tap coefficient vector. The iteration formula of the tap coefficient is

$$\begin{aligned} \mathbf{f}(n+1) &= \mathbf{f}(n) + \mu\left(-\nabla(n)\right) \\ &= \mathbf{f}(n) - 2\mu e(n)\mathbf{y}^*(n) \\ &= \mathbf{f}(n) - \mu_{LMS} e(n)\mathbf{y}^*(n) \end{aligned} \tag{26}$$

The error of the DDLMS algorithm is defined as the difference between the received signal of the decision judgment and the output signal of the judgment.

$$e(n)_{\mathrm{DDLMS}} = z(n) - \hat{x}(n) \tag{27}$$

The iteration formula of the tap coefficient is the same as that of the LMS algorithm.

$$\mathbf{f}(n+1) = \mathbf{f}(n) - \mu_{DDLMS} e(n)\mathbf{y}(n) \tag{28}$$

The initial idea of combining VS-CMA with DDLMS algorithm is using the switching dual-mode. In the beginning of the equalization, the error between the received signal and the hoped signal is large, VS-CMA the convergence rate and the property of which are good can be used at this stage. When the equalizer working for a while, the eye diagram of the channel is opening, it is time to switch DDLMS mode to obtain faster convergence rate and smaller steady state error.

The signals seem a rounded distribution around the judgment signals in the constellation diagram after equalized. The distribution is compacter and the radius is smaller compared with the judgment distance $r$ mean that the performance of the equalization is better and the inter symbol interference is smaller. So we can set a confidence interval according to the judgment distance. When the equalized points fall on the interval $C_{\min}$ which means $|d(n)| = |\hat{x}(n) - z(n)| \le C_{\min}$, the error is small, DDLMS algorithm can be used to control the tap coefficients iterative formula. If the points fall out of the interval $C_{\min}$, VS-CMA can be chosen. The large value of $C_{\min}$ may lead to larger steady state error or the algorithm does not converge. However, if the value of $C_{\min}$ is too small, the convergence process will have a big time delay, lead to the loss of the superiority of the joint algorithm [15]. In order to overcome the contradictions of such a choice, two option radiuses can be set, namely $C_{\min}$ and $C_{\max}$. When $|d(n)| \ge C_{\max}$, VS-CMA algorithm can be chosen. When $|d(n)| \le C_{\min}$, we use DDLMS. So the iteration formula of the tap coefficient is

$$\begin{cases} \mathbf{f}(n+1) = \mathbf{f}(n) - \mu(n)_{\mathrm{VS-CMA}} e(n)_{\mathrm{VS-CMA}} \mathbf{y}^*(n) \\ \qquad \text{for } |d(n)| \ge C_{\max} \\ \mathbf{f}(n+1) = \mathbf{f}(n) - \mu_{\mathrm{DDLMS}} e(n)_{\mathrm{DDLMS}} \mathbf{y}^*(n) \\ \qquad \text{for } |d(n)| \le C_{\min} \end{cases} \tag{29}$$

In the formula (29), $e(n)_{\mathrm{VS-CMA}} = z(n)(|z(n)^2 - R|)$, $e(n)_{\mathrm{DDLMS}} = z(n) - \hat{x}(n)$.

However, according to the above principles, when $C_{\min} \le |d(n)| \le C_{\max}$, the equalizer cannot determine using which algorithm in the switching mode. In this case, the paper proposes the principle of using the weighted dual-mode. The principle of weighted dual-mode without having to switch is selecting appropriate weighting function to weighted two kinds of error function to form a new error function, so the equalizing process does not need to distinguish the mode [16]. In the weighted mode, the error function is defined in this article is

$$\bar{e}(n) = g(n)e(n)_{\mathrm{VS-CMA}} + (1 - g(n))e(n)_{\mathrm{DDLMS}} \tag{30}$$

where $g(n)$ is the weighted function. It is as follows

$$g(n) = 1 - \exp\left(-\gamma \frac{|d(n)| - C_{\min}}{C_{\max} - C_{\min}}\right) \tag{31}$$

The weighted function is a nonlinear function of the absolute value of residual error $|d(n)|$. It satisfies that when the $|d(n)|$ is closer to the lower limit $C_{\min}$, the value of $g(n)$ is smaller, the proportion of the error of DDLMS algorithm in the weighted error $\bar{e}(n)$ should be larger; when the residual error $|d(n)|$ is closer to the upper limit $C_{\max}$, the value of $g(n)$ is larger, the proportion of the error of VS-CMA in $\bar{e}(n)$ should be larger. Therefore the

weighted function can make the connection between the weighted mode and the switching mode smooth, so the advantages of the two joint modes can be more effectively combined. In (31), $\gamma$ is a constant to adjust the shape of the function $g(n)$. The change degree of $g(n)$ is different corresponded to different $\gamma$. As a result, the value of $\gamma$ can be changed to obtain the most appropriate weighted function $g(n)$ when faced with different channels and noises. In this process, the iteration formula of the tap coefficient is

$$\begin{aligned}\mathbf{f}(n+1) = \mathbf{f}(n) - [g(n)\mu(n)_{\text{VS-CMA}}\, e(n)_{\text{VS-CMA}} \\ + (1-g(n))\mu_{\text{DDLMS}} e(n)_{\text{DDLMS}}]\mathbf{y}^*(n)\end{aligned} \tag{32}$$

The structure of blind equalization system based on the VS-CMA joint DDLMS algorithm is shown in Fig. 4.

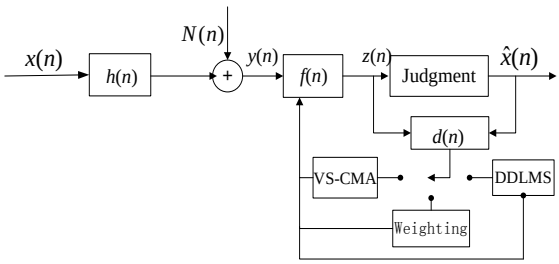



Fig. 4. Blind equalizer structure based on VS-CMA + DDLMS algorithm.

Combining switching dual-mode with weighted dual-mode, the final iteration formula of the tap coefficient is

$$\begin{cases}\mathbf{f}(n+1) = \mathbf{f}(n) - \mu(n)_{\text{VS-CMA}}\, e(n)_{\text{VS-CMA}}\, \mathbf{y}^*(n) \\ \qquad \text{for } |d(n)| \ge C_{\max} \\ \mathbf{f}(n+1) = \mathbf{f}(n) - [g(n)\mu(n)_{\text{VS-CMA}}\, e(n)_{\text{VS-CMA}} \\ \qquad + (1-g(n))\mu_{\text{DDLMS}} e(n)_{\text{DDLMS}}]\mathbf{y}^*(n) \\ \qquad \text{for } C_{\min} < |d(n)| < C_{\max} \\ \mathbf{f}(n+1) = \mathbf{f}(n) - \mu_{\text{DDLMS}} e(n)_{\text{DDLMS}}\, \mathbf{y}^*(n) \\ \qquad \text{for } |d(n)| \le C_{\min}\end{cases} \tag{33}$$

In (33), $\mu_{\text{DDLMS}}$ is an order of magnitude lager than the maximum value of $\mu(n)$. $C_{\min}$ and $C_{\max}$ can find the appropriate values based on the different channel and noise characteristics in the simulation. The parameters in $\mu(n)$ and $g(n)$ can also select the appropriate values according to different channel environment parameters.

For DDLMS algorithm, define $\mathbf{L}(n) = E\{\mathbf{f}(n)\}$, we can put formula (5) namely, $z(n) = f(n) * y(n) = \mathbf{f}^{\mathbf{T}}(n)\mathbf{y}(n)$ into the iteration formula of the tap coefficient (28). So we can derive that

$$\begin{aligned}&\mathbf{L}(n+1) \\ &= E\left\{\left[\mathbf{f}(n) - \mu_{DDLMS}(\mathbf{f}^T(n)\mathbf{y}(n)\mathbf{y}^*(n) - \hat{x}(n)\mathbf{y}^*(n)\right]\right\} \\ &= \mathbf{L}(n) - \mu_{DDLMS} E\left[\mathbf{f}^T(n)\mathbf{y}(n)\mathbf{y}^*(n) - \hat{\mathbf{f}}^T(n)\mathbf{y}(n)\mathbf{y}^*(n)\right] \\ &= \mathbf{L}(n) - \mu_{DDLMS} E\left[(\mathbf{f}^T(n) - \hat{\mathbf{f}}^T(n))\mathbf{y}(n)\mathbf{y}^*(n)\right] \\ &= \mathbf{L}(n)\left[\mathbf{I} - \mu_{DDLMS} \cdot p \cdot \mathbf{R}_{\mathbf{yy}}\right]\end{aligned} \tag{34}$$

where $\mathbf{R}_{yy}$ is the autocorrelation matrix of received sequence, $p$ is a small constant which satisfies $0<p<1$, $\lambda$ is the maximum eigenvalue of $\mathbf{R}_{yy}$,. The estimate of convergence time can be obtained by

$$t_{DDLMS} \propto \frac{1}{\mu_{DDLMS}\lambda} \tag{35}$$

Generally the value of $\mu_{\text{DDLMS}}$ is two orders of magnitude lager than the value of $\mu_{\text{CMA}}$, compared with (19), it is easily seen that the convergence rate of DDLMS algorithm is faster than the convergence rate of CMA. The VS-CMA-DDLMS algorithm is the joint of VS-CMA and DDLMS, so the convergence time of VS-CMA-DDLMS is between $t_{\text{VS-CMA}}$ and $t_{\text{DDLMS}}$, namely

$$t_{VS-CMA} \le t_{VS-CMA-DDLMS} \le t_{DDLMS} \tag{36}$$

Both $t_{\text{VS-CMA}}$ and $t_{\text{DDLMS}}$ are smaller than the $t_{\text{CMA}}$, so the convergence time of VS-CMA-DDLMS algorithm is smaller than CMA .It means the VS-CMA-DDLMS algorithm has a faster convergence rate compared with CMA.

## IV. Analysis of the Complexity of Each Algorithm

The complexity of the algorithm is an important indicator to measure the merits of the algorithm, and also an important indicator to determine whether the algorithm is conducive to hardware implementation. In the VS-CMA-DDLMS algorithm, we notice from the iteration formula of the tap coefficient that the process of the calculation of the weighted dual-mode is most complex. So we compared the amount of the weighted part of calculation of VS-CMA-DDLMS with the traditional CMA, the amount of calculation of the new dual-mode algorithm can be accepted if the complexity of the weighted part is reasonable. Table I shows the amount of the calculation of the CMA, VS-CMA as well as the weighted part of the VS-CMA-DDLMS. In the table, $N$ represents the number of iterations.

TABLE I. The Complexity of Each Algorithm.

| Algorithm | The number of addition operations | The number of multiplications | The number of exponentiations |
|---|---|---|---|
| CMA | $2N$ | $5N$ | 0 |
| VS-CMA | $5N$ | $(\alpha+\beta+8)N$ | $N$ |
| VS-CMA-DDLMS (weighted dual-mode) | $9N+1$ | $(\alpha+\beta+14)N$ | $2N$ |

In the general channel, usually the values of parameters $\alpha$ and $\beta$ are between 1 and 3. It can be seen from the Table I that the total number of calculation of CMA is $7N$, the total number of calculation of VS-CMA is about $16N$ ~ $20N$. The total number of calculation of

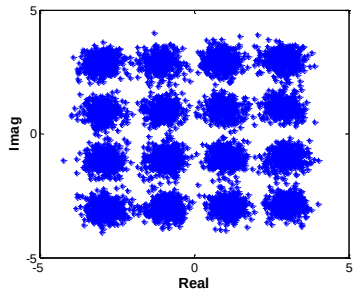
VS-CMA is 2 to 3 times to CMA. If the VS-CMA-DDLMS algorithm uses the most complex weighted dual-mode in the whole process, the total number of calculation steps is about $27N \sim 31N$ which is 4 to 5 times to CMA. In fact, due to the existence of switching dual-mode in the VS-CMA-DDLMS, the total number of calculation will be smaller. Therefore, the complexity of VS-CMA-DDLMS algorithm is acceptable and VS-CMA-DDLMS algorithm is practical in communication system.

## V. Simulation Result

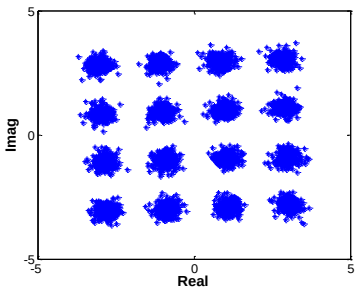
Matlab is used for the simulation to verify the performance of VS-CMA and VS-CMA-DDLMS algorithm. The simulation results are compared with that of traditional CMA. Rectangular 16 QAM signal is used for simulation, and the SNR is set to 30 dB. In the simulation, the real part of the satellite channel impulse response is [17]

$$\mathbf{h} = [0.7631, 0.2567, -0.1343, 0.0592, -0.0267, 0.0098]$$

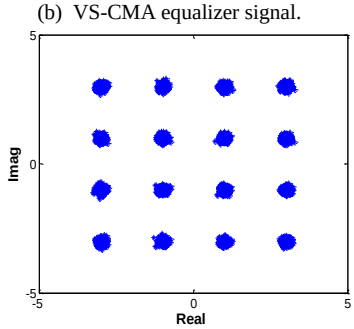
The length of the equalizer is 25. In the initial time, the center tap coefficient is 1, the rest of the tap coefficient is 0. In other functions, after some tests, we select the value of the parameters shown in Table II.




(a) CMA equalizer signal.




(b) VS-CMA equalizer signal.




(c) VS-CMA-DDLMS equalizer signal.

Fig. 5. The output constellations of 3 algorithms.

In order to compare the performance of the algorithms, the two frequently used parameters are the mean square error (MSE) and the inter symbol interference (ISI) of the signal obtained after the balance of the equalizer and the inter symbol interference. MSE is defined as (37) and ISI is defined as (38).

$$\mathrm{MSE} = 10\lg\left[\sum_{n=0}^{N-1}\left|z(n)-\hat{x}(n)\right|^2 \Big/ N\right] \tag{37}$$

$$\mathrm{ISI} = 10\lg\frac{\sum_n\left|\mathbf{h}(n)*\mathbf{f}(n)\right|^2 - \max_n\left|\mathbf{h}(n)*\mathbf{f}(n)\right|^2}{\max_n\left|\mathbf{h}(n)*\mathbf{f}(n)\right|^2} \tag{38}$$

10 Monte Carlo simulations of CMA, VS-CMA, VS-CMA-DDLMS are carried out. The simulation results are shown in Figs. 5–7.

TABLE II. The Value of Each Parameter.

| Parameter | $k$ | $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|---|---|
| Value | $2\times10^{-4}$ | 3 | 1.5 | 1 |
| Parameter | $C_{\min}$ | $C_{\max}$ | $\mu_{\mathrm{DDLMS}}$ | $\mu_{\mathrm{CMA}}$ |
| Value | 0.4 | 0.8 | $2\times10^{-3}$ | $5\times10^{-5}$ |

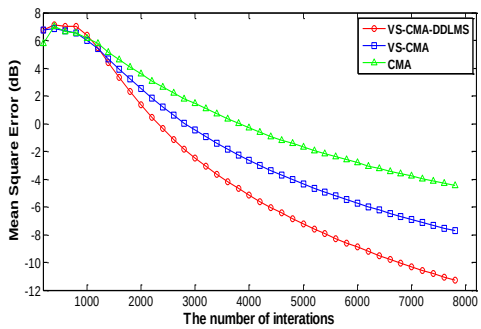



Fig. 6. The MSE of the outputs of the 3 algorithms.

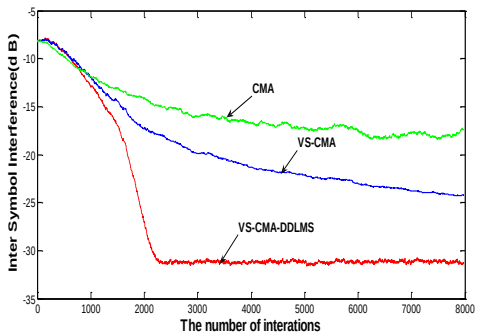



Fig. 7. The ISI of the outputs of the 3 algorithms.

Fig. 5 shows the output signal constellation diagrams after the convergence of the three kinds of algorithms. It can be seen from the Fig. 5 that the constellation diagram of the CMA is the least concentrated, the constellation points of the VS-CMA is more concentrated than the CMA. The constellation diagram of VS-CMA-DDLMS is the most compact which is due to a smaller residual error after the convergence of the algorithm. So the convergence precision of the VS-CMA-DDLMS is the highest.

Fig. 6 shows the mean square errors of the output signals of the three algorithms. It can be seen from the Fig. 6 that the mean square errors of the three algorithms decrease with the number of iterations increasing. The mean square error of CMA is only −4dB when the algorithm is iterated 8000 times. In that time, the mean square error of VS-CMA can reach -8dB and the mean square error of VS-CMA-DDLMS algorithm is reduced to −12dB. Therefore, the mean square error performance is optimal.

Fig. 7 is the inter symbol interference of the three algorithms when the algorithms are iterated 8000 times. In Fig. 7, the final inter symbol interference of CMA is about −17dB. The final inter symbol interference of VS-CMA is reduced to −24dB while the final inter symbol interference of VS-CMA-DDLMS algorithm can be as low as −31dB. It can be seen from the Fig.7 that the CMA is convergent until the iteration time is up to 6500 times. The VS-CMA is always using a nonlinear function to control the step size, so there is not a patent convergent straight line. But when the iteration time is about 2000, the inter symbol interference of VS-CMA is patently lower than that of CMA and then the iteration number is up to 5000, the change of inter symbol interference of VS-CMA is very small. According to the red line of VS-CMA-DDLMS, we can see that when the iteration time is between 1000 and 2000, there is a very steep descent of the inter symbol interference. Compared with the value of the inter symbol interference of CMA and VS-CMA when the iteration number is 2500, the VS-CMA-DDLMS algorithm has already been convergent and the inter symbol interference is 14dB smaller than that of VS-CMA and 18dB smaller than that of CMA. It is the DDLMS algorithm played a considerable role in the improvement of the convergence rate of the joint algorithm. In general, the convergence rate and convergence precision of the VS-CMA-DDLMS is better than those of the CMA and VS-CMA, so the VS-CMA-DDLMS algorithm performance is significantly better than the other two algorithms after comprehensive comparison.

## VI. Conclusion

The paper first makes improvements on the traditional CMA by introducing a nonlinear function of the residual error to control the step size of CMA in order to realize the variable step-size CMA. The obtained VS-CMA has a better equalization performance compared with traditional CMA. Then VS-CMA is combined with DDLMS algorithm with the idea of using both switching dual-mode and weighted dual-mode to get an excellent VS-CMA-DDLMS dual-mode blind equalization algorithm. Theoretical analysis and simulation results show that compared with traditional CMA, the new dual-mode algorithm has a smaller residual error and a quicker convergence rate. What is more, the inter symbol interference of VS-CMA-DDLMS algorithm is smaller that makes the tracking time-varying channel ability of the algorithm stronger. We can conclude that VS-CMA-DDLMS algorithm is a practical blind equalization algorithm with an excellent overall performance.

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