Comparative Performance Analysis for Peak-to-Average Power Ratio Reduction in OFDM System Based on the Fractional Fourier Transform Using Selective Mapping and an Active Constellation Extension Technique

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Abstract—Orthogonal Frequency Division Multiplexing (OFDM) is an attractive modulation technique for transmitting large amounts of digital data over radio waves. One major disadvantage of OFDM is the high peak to average power ratio (PAPR) considered as the main implementation drawback of OFDM system. In this paper, we are discussing about PAPR, which affects the performance and efficiency of Power Amplifier and its influence of utilizing the selective mapping (SLM) technique and Active Constellation Extension (ACE) for PAPR reduction in OFDM systems. The Proposed SLM and ACE techniques for reducing the PAPR are the most promising reduction technique in its non-uniform phase factor for PAPR reduction in OFDM system. In addition, an approximate expression using the complementary cumulative distribution function (CCDF) of PAPR is discussed. The simulated results of comparative analysis demonstrate that OFDMA signals, which have used ACE technique, indeed provide a significant reduction of PAPR in OFDM system.

Index Terms—Orthogonal frequency division multiplexing (OFDM), active constellation extension (ACE), peak to average power ratio (PAPR), selective mapping (SLM), quadrature phase shift keying (QPSK).

I. INTRODUCTION

Recently, Orthogonal Frequency Division Multiplexing (OFDM) is known to be as prominent technology due to its property of robustness against the multipath fading channels. It is an attractive and known to be as high speed digital data transmission scheme. This technique has replaced to use the expensive equalizer. Therefore, due to this reason, many broadband high speed data rate technologies are used as the basic of physical layer including Digital Subscriber Line (XDSL), WiFi (IEEE802.11a/g), Digital Video Broadcasting (DVB) and WiMAX (IEEE802.16a/e). OFDM is similar to Frequency Division Multiple Access (FDMA) such that FDMA access multiple users by subdividing the available bandwidth into multiple channels, which are then allocated to users. However, OFDM uses the spectrum much more efficiently by spacing the channels more closely together.

One of the major problems is the high PAPR of transmitting OFDM signals. Therefore, there is a need to do more research on the characteristic of PAPR reduction, its distribution and stochastic characteristic in OFDM system, which could be expressed by Complementary Cumulative Distribution Function (CCDF). Orthogonal subcarrier of OFDM spectra is shown in Fig. 1. A high PAPR necessitates the linear amplifier to have a large dynamic range, which is difficult to accommodate. Furthermore, an amplifier with non-linear signal behavior produces distortion, which is considered to be an undesired signal in out-of-band and in-band [1]-[5].

Fig. 1. Orthogonal subcarrier signal

However, to overcome this problem, several methods have been proposed such as Active Constellation Extension (ACE), Clipping and filtering, Tone Injection (TI), Tone Reservation (TR), Partial Transmit Sequence (PTS) and Selective Mapping (SLM). Besides, some other methods are used for the transmission of data based on coding schemes. Clipping and filtering can overcome PAPR but it produces in-band clipping noise. Filtering removes the side lobes caused by clipping, but it also produces additional PAPR. While other methods such that TR, does not possess such kind of problem except that it leads to consume more power. In transmitter phase rotation is another approach to reduce PAPR including SLM. These methods do not have disadvantages such as bit error rate performance degradation, Out-of-band noise presents in clipping method and high power consumption. Therefore, constellation extension based method is employed to overcome PAPR efficiently, thereby these days research mainly focus on optimization method. High power amplifier brings PAPR of OFDM signal into the non-linear region. This permits signal distortion and produces bit error rate degradation. Transmitted power must be operated into the linear region to preclude the spectral growth of multicarrier signals such as inter modulation...
among the subcarriers and out-of-band radiations. It is impossible to maintain out-of-band power below to specified limit until high power amplifier does not operate into the linear region with high power back-off. This situation leads to inefficient amplification and expensive transmitters. Therefore, it has been important and necessary to research on the characteristics of the PAPR, including its distribution and reduction. The technical features of the OFDM systems in Fig. 2, shows the process of a typical OFDM system. OFDM has several favorable properties like high spectral efficiency, robustness to channel fading, immunity to impulse interference, uniform average spectral density, capacity to handle very strong echoes, less non-linear distortion, multipath delay spread tolerance, immunity to the frequency selective fading channels, immunity inter-symbol interference and capability of handling very strong multipath fading [6]-[8]. However, there are still many challenging issues which are unresolved in the design of OFDM system. In this paper, firstly we investigate the distribution of PAPR based on the characteristics of the OFDM signals. The PAPR properties of OFDM system are determined in Section II.

\[ \text{PAPR} = \max_{n=0}^{N-1} \left\{ \frac{|S_n|^2}{E[|S_n|^2]} \right\} \]  

Fig. 2. Process of a typical OFDM system.

Here the numerator denotes the maximum instantaneous power and the denominator denotes the average power of the signal. Usually, the PAPR performance of a multicarrier system is evaluated in term of complementary cumulative distribution function of PAPR. The CCDF of PAPR is defined as the probability that the PAPR of the transmitted signal exceeds a given threshold \( \text{PAPR}_0 \), i.e. \( \Pr(\text{PAPR} > \text{PAPR}_0) \) such that

\[ \Pr(\text{PAPR} > \text{PAPR}_0) = 1 - \prod_{n=0}^{N-1} \Pr\left[ |S_n| \leq \sqrt{r \times E[|S_n|^2]} \right] \]

We have discussed a new method to reduce the PAPR in OFDM System based on the Fractional Fourier Transform in the next section.

**III. SYSTEM MODEL**

**A. PAPR Reduction Using Selective Mapping**

PAPR reduction techniques are usually available such that clipping technique, coding technique, probabilistic (scrambling) technique and an adaptive pre-distortion technique. Clipping technique provides the nonlinear saturation or clipping around the peak to minimize the PAPR, which is the simple method for implementation. However, it produces interference between the out-of band and in band and it destroys the orthogonality array subcarrier. Further approaches includes clipping and filtering technique, Block scaling technique, peak constellation technique and peak windowing technique. In coding technique the selection of codes words is selected by particular codes words, which reduce or minimize the PAPR. Therefore, no distortion exists and no out-of band radia-
tion comes out. During the selection of particular code words, it suffers from bandwidth efficiency as the code rate is reduced and also it increases the complexity to detect better code and give the storage of large look up tables for encoding and decoding large subcarrier. Reed Muller code, Golay complementary sequence, M-sequence or Hadamard code schemes are used for PAPR reduction. Probabilistic (scrambling) schemes used to scramble the input data block in OFDM symbols and transmit data block with minimum PAPR, so that the probability of high PAPR can be minimized. Even though, it does not suffer from out-of-band power, complexity increases as the number of subcarrier getting high and its spectral efficiency is deceased. SLM, PTS, TR, and TI techniques cannot guarantee that PAPR below the specified level. In OFDM system the nonlinear effect of high power amplifier can be compensated using adaptive pre-distortion technique. In OFDM system the non-linear effect of high power amplifier can be compensated using adaptive pre-distortion technique. Input constellation automatically modifies the time variations of non-linear high power amplifier with less hardware requirement. DFT spreading technique is used to distribute input signals. PAPR of OFDM signal can be reduced to the level of single carrier transmission. This technique is useful for uplink transmission particularly in mobile terminals. It is also called single carrier FDMA (SC-FDMA) also it is widely adopted for uplink transmission such that 3G, LTE standard [9]-[13].

![Block diagram of SLM approach](image)

For OFDM system with $N$, denoted the subcarriers, for each OFDM subcarrier, there will be $M$ independent data generated by multiplying each OFDM subcarrier with a phase offset $P$. The generated data are then compared to choose the OFDM subcarrier with minimum PAPR. Fig. 3 shows the block diagram of SLM approach for reducing PAPR in OFDM signal.

Transmitted OFDM subcarrier is for getting proper reception at the receiver, which carries information based on chosen OFDM mapping such that transmitted OFDM subcarrier can be detected correctly at the receiver. At the transmitter side the extra information has to be generated for getting original information with selected OFDM mapping at the receiver. The modulation process of OFDM system carries binary bit stream which is changed into complex data during modulation process. In this research OFDM system modulation posses QPSK modulation key as data subcarrier $x_m$ and after IFFT the time domain, denoted by $x_k$ [14]-[20]. PAPR in OFDM system may expressed by the ratio of peak power to average power.

In time $x_k$ represent the baseband signal where PAPR is expressed by

$$\text{PAPR (dB)} = 10 \log_{10} \frac{\max_{0 \leq n < N} \left| x_n \right|^2}{E \left| x_k \right|^2}, \quad 0 \leq n < N - 1 \quad (3)$$

OFDM symbol complex value is based on real part $x_r$ and imaginary part $x_d$ of $x_m = x_r + jx_d$ and its Gaussian distribution becomes

$$S(t_0) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(t-m_0)^2}{2\delta^2}} \quad (4)$$

Based on probability theory the amplitude value for OFDM is determined by

$$C_R = \sqrt{x_r^2 + x_d^2} \quad (5)$$

$C_R$ is the Releigh distribution, the pdf value may be represent by

$$P(c) = 2ce^{-c^2} \quad (6)$$

OFDM power symbol is represented by $p \left| x \right|^2$, the pdf of Power

$$P_{\text{power}} \left( X \right) = e^{-X} \quad (7)$$

The probability distribution of PAPR is known by cumulative distribution function (CDF). The value of PAPR below the threshold may be expressed as

$$P(\text{PAPR} \leq X) = (1 - e^{-X})N \quad (8)$$
The probability of PAPR above the threshold value, its complementary cumulative distribution probability (CCDF) express below

\[ P(\text{PAPR} > X) = 1 - P(\text{PAPR} \leq X) = 1 - (1 - e^{-X^N})^N \tag{9} \]

### B. PAPR Reduction Using Active Constellation Extension

To overcome PAPR using ACE technique, the working principle can be employed by considering all sub-channels with QPSK modulation scheme that is used to describe the OFDM system, which distributes individual symbol in each sub-channel such that the probabilities are mapped into four constellation points. Maximum likelihood decision rule is applied in the receiver after white symbol in each sub-channel such that the probabilities are describe the OFDM system, which distributes individual channels with QPSK modulation scheme that is used to consider the parallel component. Ignoring the vertical component of the element is much smaller than the signal peak signal. According to this basic principle to adjust the position of the constellation point, not only the PAPR value will be decreased but also the system performance improved with better efficiency. Fig. 4 shows the different ACE constellation by adjusting the position of the point of these regions to achieve the purposed PAPR reduction. The choice of mapping constellation point does not reduce the distance between adjacent constellation points so this method can guarantee the system has less bit error rate performance.

![Fig. 4. QPSK modulation valid constellation extension (ACE) principle](image)

This method increases the transmitted power with relatively large amplitude existence in the OFDM system. Basically, it does not affect on transmitted power. Although it may increase the complexity of the system PAPR is greatly decreased. However, it maintains a certain system performance. In order to accelerate the convergence speed, the gradient mapping algorithm iscommonly introduced to ACE algorithm and its convergence speed depends on whether it is the best choice for the selecting. \( c' \) is obtained after the \( i \)th iteration by optimizing the following convex programming to select the optimal value of the gradient [21]-[33].

\[ \mu' = \arg \min_{\mu} \| \mu' + \mu \|^2 \tag{10} \]

Corresponding to this optimum \( \mu' \), there will be several usually of two samples of equal value, but it is difficult to decide which sample values. A practical solution to be closer to the optimal \( \mu' \) is to assume that the maximum amplitude of the sample point is one of the two samples and then use a peak detection method to determine the other. If the two samples are precise equality, then we can obtain the following equation:

\[ \| x[n_{\text{max}}] + \mu c[n_{\text{max}}] \|^2 = \| x[n_i] + \mu c[n_i] \|^2 \tag{11} \]

\( \mu \) can be obtained by this equation, we find a simple method to make equations approximate equal. We can simplify the quadratic equation to simple equation.

For \( c' \) is modified the clipping noise the value of each element is much smaller than the signal \( x' \). So we can ignore the vertical component of \( c' \) and \( x' \) and only consider the parallel component. Ignoring the vertical component, the elements on both sides of the equation become the same phase angle and it can be obtained separately with the modulus value of the element. The elements on both sides of the equation modulus value of summed squares could be simplified to:

\[ \| x[n_{\text{max}}] + \mu c[n_{\text{max}}] \|^2 = \| x[n_i] + \mu c[n_i] \|^2 \tag{12} \]

Seeking algorithm for \( \mu \) is given below

1) Find the biggest sampling point \( x' \) and the corresponding position:

\[ E = \max_{n} |x'[n]|, n_{\text{max}} = \arg \max_{n} |x'[n]| \tag{13} \]

2) Calculate the projection of \( c[n] \) in the direction of \( x'[n] \) for each sample point

\[ c_{\text{proj}}[n] = \text{Re}\{x'[n]c'[n]\} / x'[n] \tag{14} \]

If \( c_{\text{proj}}[n] \geq 0 \) (expressed gradient growth), by solving the above equation

\[ \mu[n] = \frac{E - |x[n]|}{c_{\text{proj}}[n] - c_{\text{proj}}[n_{\text{max}}]} \tag{15} \]

Selecting a minimum gradient of the most optimal, if the gradient is negative then the algorithm terminated. ACE algorithm essentially uses a limiter and multiple constellation point adjustment to obtain the optimal constellation points instead of the original constellation.
points. Whether, it can search in an iterative process to the optimal constellation point depends on two aspects. First whether the optimal constellation points within the region can be extended. Second the optimal constellation points within the region can be expanded in limited iterations to converge the optimum. The first aspect can be expanded by increasing the area to achieve, such as ACE distortion control algorithm, but it can increase the extended area to be limited by using protocol. Therefore the second aspect without increasing, the availability on the basis of the extended area is more effectively converged to the optimal constellation point.

IV. SIMULATION RESULT

SLM technique carries computational complexity. However, it provides good PAPR reduction. SLM technique is very flexible as it does not impose any restriction on modulation applied in subcarrier. In this technique input data block is given by

\[ X = [X_0, X_1, \ldots, X_{N-1}]^T \]  \hspace{1cm} (16)

which is multiplied with \( U \) at different phase \( \phi \) sequence

\[ P^\phi = [P_0^\phi, P_1^\phi, \ldots, P_{N-1}^\phi] \]  \hspace{1cm} (17)

After applying SLM technique the complex envelope of the transmitted OFDM signal becomes

\[ X_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n t/N} \] \hspace{1cm} \( 0 \leq t \leq NT \)  \hspace{1cm} (18)

NT represents the direction of OFDM data block. Output data of lowest PAPR is selected to transmit, PAPR reduction effect could be better as copy block number \( U \) is increased. SLM based technique effectively reduce PAPR without any signal distortion. The higher computational complexity can be reduced by decreasing the IFFT block. The simulation result for IFFT using random signal 128 points are generated and coded by different coding rates for the SLM technique. The coded signal is QPSK modulated to simply or ease the process that CCDF of PAPR for 128 points using IFFT, as shown in Fig. 5, while Fig. 6 shows that, the PAPR reduction using ACE technique provides a better reduction in OFDM system than its conventional PAPR of OFDM system technique.

V. CONCLUSION

We have proposed comparative analysis between SLM and ACE technique for PAPR reduction in OFDM systems. OFDM provides invaluable to next generation communication systems, with the rising demand for efficient frequency spectrum utilization. Basic requirement of practical PAPR reduction techniques include the high spectral efficiency and low complexity. In this paper, we describe several important aspects, as well as provide a mathematical analysis, including PAPR characteristics in OFDM systems. We have discussed the PAPR performance of multicarrier systems based on the FrFT. Which are more suitable for multicarrier transmission over rapidly time varying channels with respect to the traditional FFT-OFDM system. The main advantage of this ACE method is that it can meet target PAPR. The proposed ACE method can reduce PAPR without any data loss. Moreover, it has better PAPR reduction performance than SLM technique with the lower system complexity and provides 5dB PAPR reduction at third iteration using the ACE method as it shows in Fig. 6.

REFERENCES


[8] R. W. Bauml, R. F. H. Fischer, and J. B. Huber, “Reducing the peak-to-average power ratio of multicarrier modulation by select-


[31] T. Tsiligkaridis and D. L. Jones, “PAPR reduction performance by active constellation extension for diversity MIMO-OFDM sys-


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