Uncertain Quadratic Minimum Spanning Tree Problem

Jian Zhou, Xing He, and Ke Wang
School of Management, Shanghai University, Shanghai 200444, China
Email: {zhou_jian, hexing, ke}@shu.edu.cn

Abstract—The quadratic minimum spanning tree problem is to find a spanning tree on a graph that minimizes a quadratic objective function of the edge weights. In this paper, the quadratic minimum spanning tree problem is concerned on the graph with edge weights being assumed as uncertain variables. The notion of the uncertain quadratic α-minimum spanning tree is introduced by using the uncertain chance constraints. It is shown that the problem of finding an uncertain quadratic α-minimum spanning tree can be handled in the framework of the deterministic quadratic minimum spanning tree problem requiring no particular solving methods.

Index Terms—Quadratic minimum spanning tree, uncertainty theory, network optimization, chance-constrained programming

I. Introduction

The minimum spanning tree problem is to find a spanning tree on a graph of which the total edge weight is smallest. As one of the most important network optimization problems, the minimum spanning tree problem has found many applications in telecommunication, power systems, transportation, etc. (see, for instance, [1], [2]). It is well known that the minimum spanning tree problem can be formulated as a linear integer programming problem with some special features, and some efficient solving algorithms are available (see, for instance, [3], [4]).

Assad and Xu [5] proposed in 1992 the quadratic minimum spanning tree problem where a quadratic objective function was involved to characterize the minimum spanning tree. The quadratic minimum spanning tree problem is however NP-hard. Some heuristic or metaheuristic solving techniques have been developed for the quadratic minimum spanning tree problem. For example, to improve the branch-and-bound based exact method in [5], Zhou and Gen [6] adopted the Prüfer number to encode the tree and enforced the genetic algorithm approach to get the solution. Following that, Sundar and Singh [7] proposed an artificial bee colony algorithm based on the swarm intelligence technique, which may obtain the better quality solution than that in [6]. Öncan and Punnen [8] developed a Lagrangian relaxation procedure as well as an efficient local search algorithm to solve this problem. Recently, Cordone and Passeri [9] described a tabu search implementation for the quadratic minimum spanning tree problem. Besides, Gao and Lu [10] presented the fuzzy quadratic minimum spanning tree problem where the edge weights were assumed to be fuzzy variables, and designed a fuzzy simulation based genetic algorithm.

However, it is frequently encountered in practice that some information in a complicated system cannot be properly observed or statistically estimated. Uncertainty theory, founded by Liu [11] and refined by Liu [12], provides an appropriate framework to describe such nondeterministic phenomena, particularly those involving the linguistic ambiguity and subjective estimation. By now, it has been applied to many areas, and has brought many branches such as uncertain programming, uncertain graph, uncertain logic, uncertain inference, uncertain process, and uncertain finance (see, e.g., [13]-[20]).

In this paper, the uncertain quadratic minimum spanning tree problem is concerned where the edge weights of the graph are assumed to be uncertain variables in the sense of Liu [11]. It is clear that a quadratic minimum spanning tree in such a situation cannot be defined in the usual sense. In this paper, the notion of uncertain quadratic-minimum spanning tree is introduced using the uncertain chance constraint. With this notion, an uncertain chance-constrained programming model is developed for the uncertain quadratic minimum spanning tree problem. It turns out that this problem can be transformed into a deterministic quadratic minimum spanning tree problem.

The rest of this paper is organized as follows. Some notions and results in uncertainty theory are briefly introduced in Section II. The uncertain quadratic minimum spanning tree problem is investigated in details in Section III. A numerical example is presented in Sections IV for illustration.

II. Preliminaries

In this section, some notions and results in uncertainty theory are briefly introduced which are indispensable to formulate the uncertain quadratic minimum spanning tree problem. The reader may refer to [11], [12], [21] for more details like the concepts of fuzzy variable, credibility measure, and some other related definitions.

Definition 1: (Liu [11]) Let $\mathcal{L}$ be a σ-algebra on a nonempty set Γ. A set function $\mathcal{M}: \mathcal{L} \to [0, 1]$ is called an uncertain measure if it satisfies the following axioms:

Manuscript received December 18, 2013; revised April 15, 2014.
This work was supported in part by grants from the Innovation Program of Shanghai Municipal Education Commission (No. 13ZS065), the Shanghai Philosophy and Social Science Planning Project (No. 2012BGL006), and the National Social Science Foundation of China (No. 13CGL057).
Corresponding author: Ke Wang, email: ke@shu.edu.cn
doi:10.12720/jcm.9.5.385-390

Axiom 1. (Normality Axiom) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ;

Axiom 2. (Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ;

Axiom 3. (Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$$

In uncertainty theory, the triplet (Γ, $\mathcal{L}$, $\mathcal{M}$) is called an uncertainty space. Besides, let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \cdots$. Denote

$$\Gamma = \Gamma_1 \times \Gamma_2 \times \cdots, \quad \mathcal{L} = \mathcal{L}_1 \times \mathcal{L}_2 \times \cdots.$$

Then the product uncertain measure $\mathcal{M}$ on the product σ-algebra $\mathcal{L}$ is defined by the following axiom (Liu [21]).

Axiom 4. (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \cdots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} \le \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \cdots$, respectively.

An uncertain variable is defined as a measurable function from an uncertainty space to the set of real numbers. In order to describe uncertain variables, the uncertainty distribution of an uncertain variable ξ is defined as

$$\Phi(x) = \mathcal{M}\{\xi \le x\} \quad (1)$$

for any real number x.

For example, an uncertain variable ξ is called linear if it has a linear uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \le a \\ \dfrac{(x-a)}{b-a}, & \text{if } a < x \le b \\ 1, & \text{if } x > b \end{cases} \quad (2)$$

denoted by $\mathcal{L}(a, b)$, where a and b are real numbers with $a < b$. The linear uncertainty distribution is illustrated in Fig. 1.

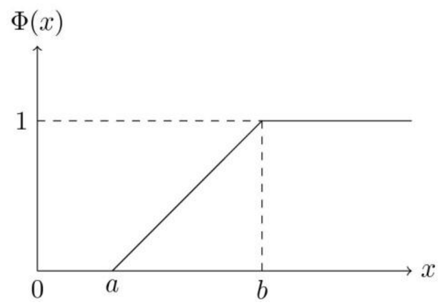

Fig. 1. Linear uncertainty distribution

An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \le a \\ \dfrac{x-a}{2(b-a)}, & \text{if } a < x \le b \\ \dfrac{x+c-2b}{2(c-b)}, & \text{if } b < x \le c \\ 1, & \text{if } x > c \end{cases} \quad (3)$$

denoted by $\mathcal{Z}(a, b, c)$ where a, b, c are real numbers with $a < b < c$. The zigzag uncertainty distribution is illustrated in Fig. 2.

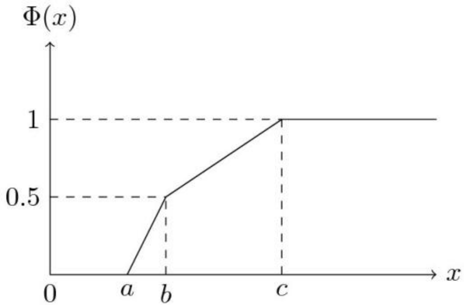

Fig. 2. Zigzag uncertainty distribution

An uncertainty distribution Φ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$. If an uncertainty distribution $\Phi(x)$ is regular, it is continuous and strictly increasing on the domain $\{x | 0 < \Phi(x) < 1\}$. So does its inverse distribution $\Phi^{-1}(\alpha)$ on the domain $\{\alpha | 0 < \alpha < 1\}$.

It is clear that the linear and zigzag uncertainty distributions are both regular. If the uncertain variable ξ has a linear uncertainty distribution, i.e., $\xi \sim \mathcal{L}(a, b)$, its inverse distribution is

$$\Phi^{-1}(\alpha) = (1-\alpha)a + \alpha b. \quad (4)$$

Similarly, if ξ has a zigzag uncertainty distribution, i.e., $\xi \sim \mathcal{Z}(a, b, c)$, its inverse distribution is

$$\Phi^{-1}(\alpha) = \begin{cases} (1-2\alpha)a + 2\alpha b, & \text{if } \alpha \le 0.5 \\ (2-2\alpha)b + (2\alpha-1)c, & \text{if } \alpha > 0.5. \end{cases} \quad (5)$$

Definition 2: (Liu [21]) The uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n} \{\xi_i \in B_i\}\right\} = \bigwedge_{i=1}^{n} \mathcal{M}\{\xi_i \in B_i\} \quad (6)$$

for any Borel sets $B_1, B_2, \cdots, B_n$ of real numbers.

Theorem 1: (Liu [18]) Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively, and $f : \mathcal{R}^n \to \mathcal{R}$ a continuous and strictly increasing function. Then the uncertain variable $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)) \quad (7)$$

By Theorem 1, for independent uncertain variables with linear or zigzag uncertainty distributions, they possess some good properties under additive and scalar multiplication. That is, if $\xi_1 \sim \mathcal{L}(a_1, b_1)$, $\xi_2 \sim \mathcal{L}(a_2, b_2)$, and ξ_1, ξ_2 are independent, then for $k_1 > 0$ and $k_2 > 0$,

$$k_1\xi_1 + k_2\xi_2 \sim \mathcal{L}(k_1a_1 + k_2a_2, k_1b_1 + k_2b_2). \quad (8)$$

Analogously, if $\xi_1 \sim \mathcal{Z}(a_1, b_1, c_1), \xi_2 \sim \mathcal{Z}(a_2, b_2, c_2)$, and ξ_1, ξ_2 are independent, then for $k_1 > 0$ and $k_2 > 0$,

$$k_1\xi_1 + k_2\xi_2 \sim \mathcal{Z}(k_1a_1 + k_2a_2, k_1b_1 + k_2b_2, k_1c_1 + k_2c_2). \quad (9)$$

Moreover, it can be verified that for $k_1 > 0$ and $k_2 > 0$,

$$\Phi^{-1}_{k_1\xi_1+k_2\xi_2}(\alpha) = k_1\Phi^{-1}_{\xi_1}(\alpha) + k_2\Phi^{-1}_{\xi_2}(\alpha) \quad (10)$$

whenever ξ_1 and ξ_2 are independent and have linear or zigzag uncertainty distributions. Note that the property (10) holds actually for independent uncertain variables with regular uncertainty distributions, which follows immediately from Theorem 1.

Definition 3: (Liu [11]) Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$\mathrm{E}[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq r\}\mathrm{d}r - \int_{-\infty}^{0} \mathcal{M}\{\xi \leq r\}\mathrm{d}r \quad (11)$$

provided that at least one of the two integrals is finite.

Theorem 2: (Liu [11]) Let ξ be an uncertain variable with uncertainty distribution Φ. If the expected value exists, then

$$\mathrm{E}[\xi] = \int_0^{+\infty} (1 - \Phi(x))\mathrm{d}x - \int_{-\infty}^{0} \Phi(x)\mathrm{d}x. \quad (12)$$

III. Uncertain Quadratic Minimum Spanning Tree Problem

Let $G = (V, E)$ be a connected undirected graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and edge set $E = \{e_1, e_2, \cdots, e_m\}$. A spanning tree $T = (V, S)$ is a connected subgraph of G such that $S \subset E$ and $|S| = n - 1$, where $|S|$ denotes the cardinality of S.

To represent a spanning tree $T = (V, S)$, we may introduce a binary vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_m)^{\mathrm{T}}$ such that

$$x_i = \begin{cases} 1, & \text{if } e_i \in S \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

Conversely, a binary vector $x = (x_1, x_2, \cdots, x_m)^{\mathrm{T}}$ may characterize a spanning tree with the cardinality constraint

$$\sum_{i=1}^{m} x_i = n - 1 \quad (14)$$

and the connection constraints

$$\sum_{e_i \in E(N)} x_i \leq |N| - 1, N \subset V, |N| \geq 3 \quad (15)$$

where $E(N)$ is the set of edges with both vertices in N.

In the quadratic minimum spanning tree problem, there are two types of edge weights involved to evaluate the spanning tree. The first type is associated with each single edge, while the second type characterizes the interactive effect of the edges. Denote by c_i the weight of edge e_i, and by d_{ij} the interactive weight of edge e_i and edge $e_j, i, j = 1, 2, \cdots, m$. Note that $d_{ij} = d_{ji}$ and $d_{ii} = 0$, $i, j = 1, 2, \cdots, m$. The weight of a spanning tree T is defined as

$$C(T, \boldsymbol{c}, \boldsymbol{d}) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_ix_j + \sum_{i=1}^{m} c_ix_i \quad (16)$$

where $\boldsymbol{c} = (c_1, c_2, \cdots, c_m)$ and $\boldsymbol{d} = (d_{ij})^{m\times m}$.

Definition 4: (Assad and Xu [5]) A quadratic minimum spanning tree T^* is a spanning tree that has the smallest weight, i.e.,

$$C(T^*, \boldsymbol{c}, \boldsymbol{d}) \leq C(T, \boldsymbol{c}, \boldsymbol{d}) \quad (17)$$

holds for any spanning tree T.

Since a spanning tree can be represented by a binary vector with the cardinality and connection conditions, finding a quadratic minimum spanning tree is essentially a binary quadratic integer programming problem, and hence NP-hard in general. However, with the aid of some well-developed optimization software packages, such as LINGO and CPLEX, the quadratic minimum spanning tree problem may be solved to optimality for scenarios of moderate size or even large size. Besides, some heuristic or metaheuristic solving techniques have been developed for the quadratic minimum spanning tree problem (see, for instance, [6]-[9]).

On the other hand, when the edge weights are not deterministic, the problem becomes more complicated. Here we assume that all the edge weights involved are independent uncertain variables with regular uncertainty distributions. Denote by ξ_i the weights of edges e_i with uncertainty distributions Φ_i, and by η_{ij} the interactive weights of edges e_i and e_j with regular uncertainty distributions Ψ_{ij}, $i, j = 1, 2, \cdots, m$.

Consequently, the weight of a spanning tree T becomes

$$C(T, \boldsymbol{\xi}, \boldsymbol{\eta}) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \eta_{ij}x_ix_j + \sum_{i=1}^{m} \xi_ix_i \quad (18)$$

which is also an uncertain variable, where $\boldsymbol{\xi} = (\xi_1, \xi_2, \cdots, \xi_m)$ and $\boldsymbol{\eta} = (\eta_{ij})^{m\times m}$.

Theorem 3: Suppose that ξ_i, η_{ij} are independent uncertain variables with regular distributions Φ_i and $\Psi_{ij}, i, j = 1, 2, \cdots, m$, respectively. Then the weight $C(T, \boldsymbol{\xi}, \boldsymbol{\eta})$ of a spanning tree T has an inverse uncertainty distribution

$$\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \Psi^{-1}_{ij}(\alpha)x_ix_j + \sum_{i=1}^{m} \Phi^{-1}_{i}(\alpha)x_i \quad (19)$$

where Φ^{-1}_i and Ψ^{-1}_{ij}, $i, j = 1, 2, \cdots, m$, are the inverse uncertainty distributions of uncertain weights ξ_i and η_{ij}, $i, j = 1, 2, \cdots, m$, respectively.

Proof: Since $x_i = 0$ or 1 for $i, j = 1, 2, \cdots, m$, we have $x_ix_j = 0$ or 1, $i, j = 1, 2, \cdots, m$. Then the result may be derived immediately from Theore

Definition 5: A spanning tree T^* is called an uncertain quadratic α-minimum spanning tree if

$$\begin{aligned}&\min\{\overline{C} \mid \mathcal{M}\{C(T^*,\boldsymbol{\xi},\boldsymbol{\eta}) \le \overline{C}\} \ge \alpha\} \le \\ &\min\{\overline{C} \mid \mathcal{M}\{C(T,\boldsymbol{\xi},\boldsymbol{\eta}) \le \overline{C}\} \ge \alpha\}\end{aligned} \tag{20}$$

holds for any spanning tree T, where $\alpha \in (0,1)$ is a given confidence level.

By Definition 5, finding an uncertain quadratic α-minimum spanning tree is equivalent to solving the uncertain chance-constrained programming problem

$$\begin{cases}\min \overline{C} \\ \text{subject to:} \\ \qquad \mathcal{M}\{C(T,\boldsymbol{\xi},\boldsymbol{\eta}) \le \overline{C}\} \ge \alpha \\ \qquad T \text{ is a spanning tree}\end{cases} \tag{21}$$

with a predetermined confidence level $\alpha \in (0,1)$.

Moreover, since all uncertain variables involved are independent uncertain variables with regular distributions, we can obtain the following theorem.

Theorem 4: Given a predetermined confidence level $\alpha \in (0,1)$, the chance constraint

$$\mathcal{M}\{C(T,\boldsymbol{\xi},\boldsymbol{\eta}) \le \overline{C}\} \ge \alpha \tag{22}$$

holds if and only if

$$\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\Psi_{ij}^{-1}(\alpha)x_i x_j + \sum_{i=1}^{m}\Phi_i^{-1}(\alpha)x_i \le \overline{C} \tag{23}$$

Proof: Suppose that $C(T,\boldsymbol{\xi},\boldsymbol{\eta})$ has an inverse uncertainty distribution Υ^{-1}. Then it follows from Theorem 3 that

$$\Upsilon^{-1}(\boldsymbol{x},\alpha) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\Psi_{ij}^{-1}(\alpha)x_i x_j + \sum_{i=1}^{m}\Phi_i^{-1}(\alpha)x_i. \tag{24}$$

Thus (22) holds if and only if $\Upsilon^{-1}(\boldsymbol{x},\alpha) \le \overline{C}$.

For example, assume that ξ_i are linear uncertain variables $\mathcal{L}(a_i,b_i)$, $i = 1,2,\cdots,m$, respectively, η_{ij} are zigzag uncertain variables $\mathcal{Z}(c_{ij},d_{ij},f_{ij})$, $i,j = 1,2,\cdots,m$, respectively, and ξ_i and η_{ij} are all independent. Then it follows from Theorem 4 and Eqs. (4)~(5) that the chance constraint $\mathcal{M}\{C(T,\boldsymbol{\xi},\boldsymbol{\eta}) \le \overline{C}\} \ge \alpha$ holds if and only if

$$\begin{cases}\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\left((1-2\alpha)c_{ij}+2\alpha d_{ij}\right)x_i x_j + \\ \sum_{i=1}^{m}((1-\alpha)a_i+\alpha b_i)x_i \le \overline{C}, \qquad \text{if } \alpha \le 0.5 \\ \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\left((2-2\alpha)d_{ij}+(2\alpha-1)f_{ij}\right)x_i x_j + \\ \sum_{i=1}^{m}((1-\alpha)a_i+\alpha b_i)x_i \le \overline{C}, \qquad \text{if } \alpha > 0.5.\end{cases} \tag{25}$$

As a result, the uncertain chance-constrained programming model (21) can be transformed to the following deterministic equivalent formulation

$$\begin{cases}\min_{x}\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\Psi_{ij}^{-1}(\alpha)x_i x_j + \sum_{i=1}^{m}\Phi_i^{-1}(\alpha)x_i \\ \text{subject to:} \\ \quad \sum_{i=1}^{m}x_i = n-1 \\ \quad \sum_{e_i\in E(N)}x_i \le |N|-1, N \subset V, |N| \ge 3 \\ \quad x_i \in \{0,1\}, i = 1,2,\ldots,m.\end{cases} \tag{26}$$

It should be noted that model (26) is nothing but a deterministic quadratic minimum spanning tree problem. So far we have demonstrated that the problem of finding the uncertain quadratic α-minimum spanning tree can be handled eventually within the framework of the deterministic quadratic minimum spanning tree problem and requires no particular solving methods in such an uncertain environment.

IV. A Numerical Example

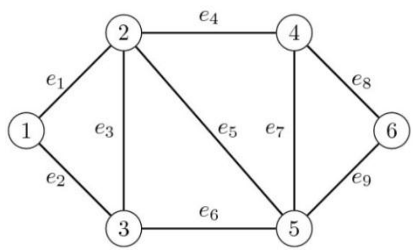

Fig. 3. An uncertain graph for the numerical example

TABLE I. THE DISTRIBUTIONS OF WEIGHTS ξ_i IN FIGURE 3

ξ_i	Φ_i	$\Phi_i^{-1}(0.8)$
ξ_1	$\mathcal{L}(10,12)$	11.6
ξ_2	$\mathcal{L}(7,11)$	10.2
ξ_3	$\mathcal{L}(5,13)$	11.4
ξ_4	$\mathcal{L}(6,12)$	10.8
ξ_5	$\mathcal{L}(9,13)$	12.2
ξ_6	$\mathcal{L}(6,12)$	10.8
ξ_7	$\mathcal{L}(3,13)$	11
ξ_8	$\mathcal{L}(7,13)$	11.8
ξ_9	$\mathcal{L}(3,13)$	11

In order to illustrate the effectiveness of the model proposed above, in this section, the quadratic minimum spanning tree problem is considered on a graph with 6 vertices and 9 edges as shown in Fig. 3. For each edge $e_i (i = 1,2,\cdots,9)$, its weight ξ_i is assumed to be a linear uncertain variable with distribution Φ_i, while the interactive weight η_{ij} $(i, j = 1, 2, \cdots, 9)$ of edges e_i and e_j is assumed to be with zigzag uncertainty distribution ψ_{ij}, and $\psi_{ij} = \psi_{ji}$ holds. The distributions of ξ_i and η_{ij} are listed in Tables I and II, respectively. Note that Table II only shows the distributions of partial interactive weights in Fig. 3, the rests of which do not appear in Table II are set to zero for simplicity.

TABLE II. THE DISTRIBUTIONS OF INTERACTIVE WEIGHTS η_{ij} IN FIGURE 3

η_{ij}	ψ_i	$\psi_{ij}^{-1}(0.8)$
η_{12}	$\mathcal{Z}(12, 14, 20)$	17.6
η_{13}	$\mathcal{Z}(13, 16, 21)$	19
η_{19}	$\mathcal{Z}(11, 13, 16)$	14.8
η_{24}	$\mathcal{Z}(14, 18, 20)$	19.2
η_{25}	$\mathcal{Z}(12, 13, 22)$	18.4
η_{26}	$\mathcal{Z}(9, 10, 18)$	14.8
η_{35}	$\mathcal{Z}(10, 16, 20)$	18.4
η_{36}	$\mathcal{Z}(10, 15, 18)$	16.8
η_{39}	$\mathcal{Z}(13, 15, 20)$	18
η_{45}	$\mathcal{Z}(11, 12, 21)$	17.4
η_{47}	$\mathcal{Z}(11, 16, 20)$	18.4
η_{56}	$\mathcal{Z}(13, 14, 24)$	20
η_{67}	$\mathcal{Z}(12, 16, 18)$	17.2
η_{89}	$\mathcal{Z}(12, 14, 23)$	19.4

According to model (26), if we want to find the uncertain quadratic minimum spanning tree with a given confidence level $\alpha = 0.8$, we have the following model:

$$\begin{cases} \min\limits_{x} \dfrac{1}{2}\sum\limits_{i=1}^{9}\sum\limits_{j=1}^{9} \Psi_{ij}^{-1}(0.8)x_i x_j + \sum\limits_{i=1}^{9} \Phi_i^{-1}(0.8)x_i \\ \text{subject to:} \\ \qquad \sum\limits_{i=1}^{9} x_i = 5 \\ \qquad \sum\limits_{e_i \in E(N)} x_i \leq |N| - 1, N \subset V, |N| \geq 3 \\ \qquad x_i \in \{0,1\}, i = 1,2,\ldots,9. \end{cases} \tag{27}$$

In model (27), the values of $\Phi_i^{-1}(0.8)$ and $\psi_{ij}^{-1}(0.8)$ can be calculated according to Eqs. (4)~(5), which have been given in Tables I and II. Consequently, model (27) is equivalent to a deterministic quadratic minimum spanning tree problem. The optimal solution of this model can be obtained as

$$\boldsymbol{x}^* = (1, 0, 0, 1, 0, 1, 0, 1, 1)^{\mathrm{T}}$$

by using LINGO, and the minimum spanning tree is shown in Fig. 5 (denoted by solid lines).

The predetermined confidence level α is an important parameter in the formulation. The numerical example is further considered for different confidence levels in order to investigate the influence of this parameter. It is observed that α has an effect on the optimal minimum spanning tree found, and the total weight of the minimum spanning tree increases while the confidence level is increasing. Table III demonstrates the changes of the optimal solutions and the total weights of the corresponding uncertain quadratic minimum spanning tree for different confidence level α. Fig. 4, Fig. 5 and Fig. 6 show the different minimum spanning trees.

TABLE III. RESULTS FOR NUMERICAL EXAMPLES USING DIFFERENT CONFIDENCE LEVELS

$\boldsymbol{\alpha}$	Total weight	Optimal solution $\boldsymbol{x}^*$
0.5	62	$(1, 1, 0, 0, 1, 0, 1, 1, 0)^{\mathrm{T}}$
0.8	79.4	$(1, 0, 0, 1, 0, 1, 0, 1, 1)^{\mathrm{T}}$
0.95	97.9	$(1, 0, 0, 1, 0, 1, 1, 1, 0)^{\mathrm{T}}$

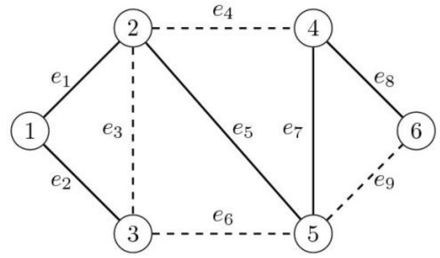

Fig. 4. Uncertain quadratic 0.5-minimum spanning tree

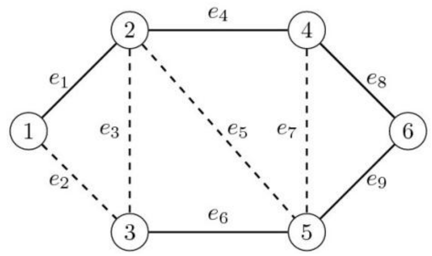

Fig. 5. Uncertain quadratic 0.8-minimum spanning tree

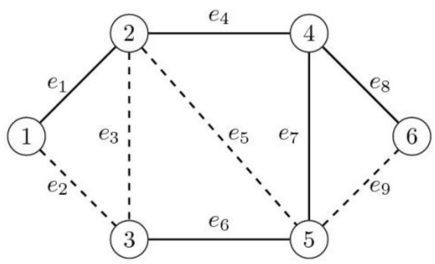

Fig. 6. Uncertain quadratic 0.95-minimum spanning tree

V. CONCLUSIONS

As one of the most important network optimization problems, the minimum spanning tree problem has found many applications, and also has been extensively discussed in the literature. However, the applications of the minimum spanning tree problem encountered in practice usually involve some uncertain issues so that the edge weights cannot be explicitly determined.

Thus, an uncertain model based on uncertainty theory founded by Liu [11] is proposed in this paper to deal with such situations. Moreover, the quadratic objective function is involved to characterize the minimum spanning tree. It is shown that finding an uncertain quadratic α-minimum spanning tree is equivalent to solving an uncertain chance-constrained programming problem which can be further transformed into a deterministic quadratic integer programming model and then be solved with the aid of some well-developed optimization software packages. That is, an uncertain quadratic minimum spanning tree problem can be handled within the framework of the deterministic quadratic minimum spanning tree problem and requires no particular solving methods.

ACKNOWLEDGMENT

The authors are grateful to the anonymous referees for their valuable comments and suggestions to improve the presentation of this paper.

REFERENCES

[1] T. C. Chiang, C. H. Liu, and Y. M. Huang, "A near-optimal multicast scheme for mobile ad hoc networks using a hybrid genetic algorithm," *Expert Systems with Applications*, vol. 33, no. 3, pp. 734-742, 2007.

[2] A. Kershenbaum and W. Chou, "A unified algorithm for designing multidrop teleprocessing networks," *IEEE Transactions on Communications*, vol. 22, no. 11, pp. 1762-1772, 1974.

[3] H. N. Gabow, Z. Galil, T. Spencer, and R. E. Tarjan, "Efficient algorithms for finding minimum spanning trees in undirected and directed graphs," *Combinatorica*, vol. 6, no. 2, pp. 109-122, 1986.

[4] R. L. Graham and P. Hell, "On the history of the minimum spanning tree problem," *Annals of the History of Computing*, vol. 7, no. 1, pp. 43-57, 1985.

[5] A. Assad and W. Xu, "The quadratic minimum spanning tree problem," *Naval Research Logistics*, vol. 39, no. 3, pp. 399-417, 1992.

[6] G. Zhou and M. Gen, "An effective genetic algorithm approach to the quadratic minimum spanning tree problem," *Computers and Operations Research*, vol. 25, no. 3, pp. 229-237, 1998.

[7] S. Sundar and A. Singh, "A swarm intelligence approach to the quadratic minimum spanning tree problem," *Information Sciences*, vol. 180, no. 17, pp. 3182-3191, 2010.

[8] T. Öncan and A. P. Punnen, "The quadratic minimum spanning tree problem: A lower bounding procedure and an efficient search algorithm," *Computers and Operations Research*, vol. 37, no. 10, pp. 1762-1773, 2010.

[9] R. Cordone and G. Passeri, "Solving the quadratic minimum spanning tree problem," *Applied Mathematics and Computation*, vol. 218, no. 23, pp. 11597-11612, 2012.

[10] J. Gao and M. Lu, "Fuzzy quadratic minimum spanning tree problem," *Applied Mathematics and Computation*, vol. 164, no. 3, pp. 773-788, 2005.

[11] B. Liu, *Uncertainty Theory*, 2nd ed., Springer-Verlag, Berlin, 2007.

[12] B. Liu, *Uncertainty Theory: A Branch of Mathmaticics for Modeling Human Uncertainty*, Springer-Verlag, Berlin, 2010.

[13] B. Liu, *Theory and Practice of Uncertain Programming*, 2nd ed., Springer-Verlag, Berlin, 2009.

[14] X. Zhang, Q. Wang, and J. Zhou, "Two uncertain programming models for inverse minimum spanning tree problem," *Industrial Engineering and Management Systems*, vol. 12, no. 1, pp. 9-15, 2013.

[15] X. Zhang, Q. Wang, and J. Zhou, "A chance-constrained programming model for inverse spanning tree problem with uncertain edge weights," *International Journal of Advancements in Computing Technology*, vol. 5, no. 6, pp. 76-83, 2013.

[16] J. Zhou, Z. Li, and K. Wang, "A multi-objective model for locating fire stations under uncertainty," *Advances in Information Sciences and Service Sciences*, vol. 5, no. 7, pp. 1184-1191, 2013.

[17] B. Liu, "Uncertain set theory and uncertain inference rule with application to uncertain control," *Journal of Uncertain Systems*, vol. 4, no. 2, pp. 83-98, 2010.

[18] J. Zhou, F. Yang, and K. Wang, "Multi-objective optimization in uncertain random environments," *Fuzzy Optimization and Decision Making*, 2014,

[19] J. Zhou, F. Yang, and K. Wang, "An inverse shortest path problem on an uncertain graph," *Journal of Networks*, 2014, accepted.

[20] Y. Chen, S. Zhong, and J. Zhou. (2014). An interactive satisficing approach for multi-objective optimization with uncertain parameters. [Online]. Available: http://orsc.edu.cn/online/131222.pdf

[21] B. Liu, "Some research problems in uncertainty theory," *Journal of Uncertain Systems*, vol. 3, no. 1, pp. 3-10, 2009.

Jian Zhou is an associate professor at the School of Management, Shanghai University, Shanghai, China. She received the B.S. degree in applied mathematics, the M.S. and Ph.D. degrees in computational mathematics from Tsinghua University, Beijing, China in 1998 and 2003, respectively. Her research interests include network optimization, uncertainty theory, fuzzy clustering, and supply chain finance. She has published more than 40 papers in national and international conferences and journals. For the other information about her, please visit her personal webpage at http://zhou-jian.jimdo.com.

Xing He is currently a master student and working towards her M.S. degree in logistics engineering at Shanghai University, Shanghai, China. She received the B.S. degree in information management and information system from Tianjin Agriculture University, Tianjin, China in 2010. Her current research interests include network optimization and supply chain finance.

Ke Wang is a lecturer at the School of Management, Shanghai University, Shanghai, China. He received the B.S. degree in information management and information system from Chongqing University, Chongqing, China in 2005, the M.S. degree in management science and engineering from Beihang University, Beijing, China in 2007, and the Ph.D. degree in management science and engineering from Tongji University, Shanghai, China in 2011. His research interest include location-allocation problem, uncertainty theory, fuzzy clustering and resource allocation. For the other information about him, please visit his personal webpage at http://k-wang.weebly.com.