A Precoding Method for Multiple Antenna System on the Riemannian Manifold

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Abstract—In this paper, a precoding scheme for multiple antenna systems with space-time coding over correlated Ricean channels is proposed. Based on the channel mean and receive-side spatial correlation, the proposed scheme obtains the solution of the power allocation and beamforming matrix of the precoder iteratively. Unlike most of existing precoder designs in literatures that have expensive computations and cannot guarantee the design algorithms to converge, the convergence of the proposed scheme is guaranteed. By exploiting the special structure of beamforming matrix, a steepest descent method on the Riemannian manifold is proposed for the optimization of beamforming matrix. Simulation results show that the proposed optimal scheme not only achieves superior bit error rate (BER) performance to those of the existing ones, but also provides a guaranteed-convergent solution under different channel conditions.

Index Terms—correlated Ricean channel, transmit precoding, Riemannian manifold, steepest descent method

I. INTRODUCTION

Multiple input multiple output (MIMO) system, which deploys multiple antenna elements at each link-end, can significantly improve cell coverage [1] and system capacity [2] compared with conventional single antenna system. Earlier works on MIMO used space-time block codes (STBC) to exploit spatial diversity [3]. Recently, by exploiting perfect or partial channel state information at transmitter (CSIT), linear precoding with the statistical channel mean and covariance matrix as feedback can improve the channel capacity and bit error rate (BER) performance [4–9].

Different from the precoder design over Rayleigh fading channels [4, 5], the optimal beamforming directions cannot be easily determined only based on the knowledge of transmit-side correlation in a light-of-sight (LOS) environment. In [6], a simplified scheme is proposed to reduce the computational complexity with a cost of performance degradation. A precoder design with structured mean component is proposed using angular domain CSIT in [8]. However, only the asymptotical solution was derived. An optimal precoding scheme proposed in [9] can be applied for MIMO with a LOS component. Like most of design methods, it uses an iterative procedure to obtain the precoding matrix directly. However, the iterative method is implemented with high computational complexity and its convergence is not guaranteed.

The optimization with respect to (w.r.t.) the beamforming unitary matrix can be regarded as a problem under unitary matrix constraint. Most of optimization methods with the unitary matrix constraint are developed on the traditional Euclidean space by using iterative algorithms [10]. Since the unitary matrices are algebraically closed under the multiplication operation rather than under addition, the unitary property is lost after each iteration by using an iterative method based on an additive update. Even though the optimization moves along a straight line pointing in the right direction, departure from the constrained manifold occurs in each step. Actually, optimizing such an objective function on a manifold [11–13], is often considered as a problem of Riemannian geometry endowed with a Riemannian metric [14]. The steepest descent method [11] and the conjugate gradient method [12] have been developed to optimize the real-valued cost function on the Riemannian manifold of unitary matrices.

In this paper, for a downlink cellular MIMO system over a Ricean fading channel, we consider to use both the channel mean and receive-side spatial correlation as CSIT for the beamforming design by minimizing a pairwise error probability (PEP). A steepest descent method on the Riemannian manifold is proposed under unitary matrix constraint. Based on the proposed steepest descent method, the unitary matrix of the design is optimized. Simulation results show that the BER performance of proposed precoder design outperforms the existing schemes under different channel conditions. Moreover, it is guaranteed to converge to the local optimal solution.

II. SYSTEM MODEL

We consider a downlink MIMO system with Nt transmit antenna in the base station (BS), while mobile subscriber (MS) have Nr co-located antennas regarded as receiver. A space time encoder maps data symbols into STBC codewords, then a linear precoding matrix, \( F \in \mathbb{C}^{N_t \times N_r} \), is applied to the codeword before transmission. In a quasi-static and flat fading channel, the received signal can be expressed as

\[
Y = \sqrt{\rho} H F C + n
\]  

(1)
where \( C \) of size \( N_t \times T \) is the block codeword with codeword length \( T \) in the orthogonal STBC (OSTBC) codebook, \( n \) is the \( N_t \times T \) noise matrix with i.i.d. entries modelled as \( \mathcal{CN}(0,1) \), \( p \) is the total transmit power to receive noise ratio, and \( H \) is the \( N_r \times N_t \) MIMO channel matrix.

Without loss of generality, the flat fading channel \( H \) can be regarded as a sum of two components, i.e., the deterministic LOS component and the stochastic multipath component, respectively. The channel matrix can be expressed as \([15]\)

\[
H = \sqrt{\frac{K}{1 + K}} H_0 + \sqrt{\frac{1}{1 + K}} \frac{1}{R_p^\frac{1}{2}} R_w R_T^\frac{1}{2}
\]  

(2)

where \( K \) is the Ricean \( K \)-factor, \( H_w \) is a complex Gaussian random matrix with i.i.d. entries of zero-mean and unit variance. \( H_0 \) is the deterministic normalized channel mean with \( \mathbb{E}[H_0] \mathbb{E}[H_0]^\dagger = N_s N_t \). The columns of \( H_0 \) can be structured as in \([15]\) but this paper does not consider any structure as in \([9]\) in order to generalize the channel mean. \( R_T \) and \( R_T \) denote the receive and transmit spatial correlation matrices, respectively. In this paper, we assume that transmit antennas are independent due to large antenna spacing in BS. Meanwhile, all the transmit antennas experience the same spatial correlation at MS. Thus \( R_T = I_{N_t} \) as in \([5]\). Let \( \breve{H} = \sqrt{\frac{K}{K + 1}} H_0 \) and \( \bar{H} = \sqrt{\frac{1}{1 + K}} \frac{1}{R_p^\frac{1}{2}} R_w R_T^\frac{1}{2} \). The channel is said to be Rayleigh fading if \( \breve{H} = 0_{N_t \times N_t} \) and Ricean fading if \( \bar{H} \neq 0_{N_t \times N_t} \). Based on the channel model in (2), \( h = \text{vec}(\breve{H}^H) \) is a circularly symmetric Gaussian random vector with mean \( \bar{h} = \text{vec}(\bar{H}^H) \) and covariance matrix

\[
R = \mathbb{E}\{\text{vec}(\breve{H}^H)\text{vec}(\bar{H}^H)^\dagger\} = \frac{1}{1 + K} R_p \otimes R_T^\dagger
\]  

(3)

(4)

where \([ \cdot ]^\dagger\) denotes the Hermitian transpose and \( \text{vec}(\cdot) \) denotes the columnwise vectorization operator. As a result, the probability density function (pdf) of \( h \) can be expressed as

\[
p_h(h) = \frac{1}{\pi^{N_r N_t} \det(R)} \exp\left(-\frac{1}{2} (h - \bar{h})^H R^{-1} (h - \bar{h}) \right)
\]  

(5)

III. OPTIMAL PRECODER DESIGN PROBLEM FORMULATION

In this section, we use PEP as design criterion to formulate the optimization problem. Assuming the receiver knows the CSI perfectly, maximum likelihood (ML) detection can be used to perform OSTBC decoding. With ML detection and applying the Chernoff bound of the PEP, the probability that a transmitted codeword \( C \) is incorrectly decoded as \( \hat{C} \) can be tightly upper-bounded by

\[
P(C \rightarrow \hat{C} | H) \leq \exp\left(-\frac{\mu p ||HF||^2}{4} \right)
\]  

(6)

where \( \mu \) is the factor for the minimum codeword distance matrix, whose value is depending on the modulation format. The PEP bound conditioned on \( H \), denoted by \( f(H, F) \), can be expressed in vectorized form as:

\[
f(H, F) = \exp\left(-\frac{\mu p}{4} \text{tr}(HF^H H^H)\right)
\]  

(7)

\[
xp(-g \text{vec}(H^H) \Phi \text{vec}(H^H))
\]  

(8)

where \( g = -\mu p/4 \) and

\[
\Phi = I \otimes F^H F
\]  

(9)

By taking the expectation of \( f(H, F) \) over the pdf of \( h \) in (5), an upper bound of the average PEP is obtained as \([8]\)

\[
f(F) = \frac{\exp\left(-\bar{h}^H ((g \Phi)^{-1} + R)^{-1} \bar{h} \right)}{\det(I_{N_t N_r} + g R \Phi)}
\]  

(10)

where \( \det(\cdot) \) denotes the matrix determinant. We take the average PEP bound as the performance criterion to design the precoder. Applying the eigenvalue decomposition (EVD), the matrix \( F^H \) and receive correlation matrix can be decomposed as

\[
F^H F = U_F A_F U_F^H
\]  

(11)

\[
\frac{1}{1 + K} R_R = U_F A_R U_R^H
\]  

(12)

where \( U_F = [u_{F1} \ldots u_{FN_t}] \) and \( U_R = [u_{R1} \ldots u_{RN_t}] \) denote the matrices of eigenvectors, and \( A_F = \text{diag}(\lambda_{F1} \ldots \lambda_{FN_t}) \) and \( A_R = \text{diag}(\lambda_{R1} \ldots \lambda_{RN_t}) \) denote the diagonal matrices of eigenvalues. Without loss of generality, the eigenvalues are sorted in a decreasing order. Using (4) and (9), we have

\[
R = \left(U_F^\dagger I_{N_t} \otimes I_{N_t}\right) \left(U_F^\dagger A_F U_F^H\right) \left(I_{N_t} \otimes U_F^H\right)
\]  

(13)

\[
\Phi = \left(I_{N_t} \otimes I_{N_t}\right) \left(I_{N_t} \otimes A_R U_R^H\right) \left(I_{N_t} \otimes U_R^H\right)
\]  

(14)

(15)

(16)

Substituting (13) - (16) into (10), the average PEP can be expressed as

\[
f(F) = \frac{\exp\left(-\bar{h}^H \left(U_F \otimes U_F^H\right) \Omega^{-1} \left(U_R^H \otimes U_F^H\right) \bar{h} \right)}{\det(I_{N_t N_t} + g A_R \otimes A_F)}
\]  

(17)

where \( \Omega = g^{-1}(I_{N_t} \otimes A_F)^{-1} + (A_R \otimes I_{N_t}) \). Taking logarithm of \( f(F) \) and deleting the terms independent of \( F \), the objective function with power constraint can be formulated as

\[
\max_F J(F) = L(F) + \sum_{i=1}^{N_t} \sum_{j=1}^{N_r} \log(1 + g \lambda_{Fi} \lambda_{Rj})
\]  

(18)

s.t. \( \sum_{i=1}^{N_t} \lambda_{Fi} = 1, \lambda_{Fi} \geq 0 \)

where

\[
L(F) = \bar{h}^H \left(U_F \otimes U_F\right) \Omega^{-1} \left(U_R^H \otimes U_F^H\right) \bar{h}
\]  

(19)
Applying the identity $\text{vec}(ABC) = (C^T \otimes A)\text{vec}(B)$ into (19), we get

$$L(F) = \text{vec}^H(U_R^H \bar{U}_R) \Omega^{-1}\text{vec} (U_R^H \bar{U}_R)$$

(20)

$$= \sum_{j=1}^{N_r} b_j^H U_j \bar{\Sigma}_j b_j$$

(21)

$$= \sum_{j=1}^{N_r} \text{tr}\{U_j^H R_{b_j}/U_j \bar{\Sigma}_j\}$$

(22)

where $b_j = \bar{U}_R^H u_{b,j}$, and $R_{b_j} = b_j b_j^H$ is a rank one Hermitian matrix.

The optimization in (18) is a highly nonlinear problem. The design solution could be obtained by nonlinear programming but with poor convergence and high computational complexity [9]. In the following, an efficient precoding scheme is proposed to obtain the power allocation and beamforming unitary matrix, iteratively.

IV. RIEMANNIAN OPTIMIZATION METHOD

In this section, we will develop a steepest descent method on the Riemannian manifold to optimize the beamforming directions of the problem (18) under the unitary matrix constraint. Most of optimization methods with unitary matrix constraint operate on the Euclidean space by using the Gram-Schmidt process to restore the unitary property of the matrix [10]. Actually, all the $n$-th order non-singular unitary matrices, $U \in \mathbb{C}^{n \times n}$, form the manifold of unitary matrices $U(n)$, which is a real differentiable manifold [14]. Thus, it motivates us to develop a steepest descent method based on the Riemannian geometry. Firstly, an overview of the Riemannian optimization method under the unitary matrix constraint is presented.

A. Overview of the Riemannian Optimization Method on $U(n)$

The Riemannian optimization methods are proposed to solve optimization problem posed on a differentiable manifold [14]. Assume a real-valued objective function $L$ of $U$ defined on $U(n)$, i.e., $L: \mathbb{C}^{n \times n} \rightarrow \mathbb{R}$. The goal is to maximize (minimize) the function $L = L(U)$ under the constraint that $UU^H = U^H U = I_n$.

The tangent space $T_U U(n)$ of $U(n)$ is an $n^2$-dimensional real vector space attached to every point $U \in U(n)$. At the group identity $I_n$, the tangent space is the real Lie algebra of skew-Hermitian matrix

$$u(n) = T_I U(n) = \{G \in \mathbb{C}^{n \times n}|G^H + G = 0\}$$

(24)

Since the differential of the right translation is an isomorphism, the tangent space at $U \in U(n)$ may be identified with the matrix space $T_U U(n) = \{X \in \mathbb{C}^{n \times n}|X^H U + U^H X = 0\}$.

Let $X$ and $Y$ be two tangent vectors. Different from the Euclidean metric, the inner product given by [11], [12]

$$\langle X,Y \rangle_R = \frac{1}{2} \text{Re}\{\text{tr}(XY^H)\}, \quad X,Y \in T_U U(n)$$

(25)

induces the bi-invariant Riemannian metric (structure) on $U(n)$, where $\text{Re}\{\cdot\}$ is the real part of a complex number.

Once $U(n)$ is equipped with the Riemannian metric (25), the steepest descent direction of $L$ at a given $U(n)$, denoted by $\Delta \in T_U U(n)$, can be determined.

To satisfy the unitary matrix constraint during the optimization, the search should be proceeded along the manifold surface $U(n)$ rather than along a straight line. The curve emanating from the identity $I_n$ with the tangent vector $G$ is characterized by

$$P_s(t) = \exp(tG), \quad G \in u(n)$$

(26)

where $t$ is a step size and controls the algorithm convergence speed. Low computational complexity is required to implement the matrix exponential function whose argument, $G$, is a skew-Hermitian matrix and translated from $\Delta$.

We choose to follow a surface motion for the steepest descent method due to the desirable property of $U(n)$ that the right multiplication is an isometry with respect to the Riemannian metric (25). Thus, a curve emanating from $U$ can be expressed as

$$P_0(t) = P_s(t) U = \exp(tG) U$$

(27)

where $P_0(t)$ can be regarded as the updated unitary matrix. The general procedure of Riemannian optimization can be applied to the steepest descent method [11], conjugate gradient method [12].

B. Unitary matrix design on the Riemannian Manifold

In this subsection, we will derive a local optimal solution for the unitary matrix $U_F$ in (18). The optimized $U_F$ can be obtained by maximizing $L(F)$ in (22) which is equivalent to maximizing $J(F)$ of (18) because $U_F$ is only related to $L(F)$. With the assumption of fixed power allocation, $L(F)$ in (22) can be regarded as a function of $U_F$. Meanwhile, a unitary matrix constraint is implicitly included in (22), i.e., $U_F U_F^H = U_F^H U_F = I_{N_t}$. Based on (22), the optimization problem w.r.t. $U_F$ under unitary matrix constraint is given by

$$\max_{U_F} L(U_F) = \text{tr}\{U_F^H R_{b_j}/U_F \bar{\Sigma}_j\}$$

s.t. $U_F U_F^H = U_F^H U_F = I_{N_t}$

(28)

Since (28) is a unitary matrix constrained problem, we can derive optimal $U_F$ employing the steepest descent method on the manifold of unitary matrices $U(N_t)$ described in Section IV-A. The corresponding Riemannian optimization procedure to derive optimal $U_F$ is presented as follows
1) Initialization: \( k = 1 \), and \( U_F^{(k)} = I_{N_t} \).

2) Compute the gradient of \( J(U_F) \) w.r.t. \( U_F \) on the Euclidean space at \( U_F^{(k)} \):
\[
\Gamma_k = \frac{\partial J(U_F^{(k)})}{\partial U_F} = \sum_{j=1}^{N_r} R_{BJ} U_F^{(k)} \Sigma_f 
\]

3) Compute the gradient direction on \( U(N_r) \) with the Riemannian metric (25):
\[
G_k = \Gamma_k U_F^{(k)H} + U_F^{(k)} \Gamma_k^H \tag{30}
\]

4) Determine \( P_k \) emanating from identity \( I_{N_r} \) on the Riemannian manifold: \( P_k = \exp(tG_k) \) with \( t > 0 \). Since \( G_k \) is a skew-Hermitian matrix, \( P_k \) will be unitary matrix.

5) Update the curve emanating from \( U_F^{(k)} \): \( U_F^{(k+1)} = P_k U_F^{(k)} \), \( k = k + 1 \). Iterate the step 2-5 until convergence.

Actually, \( P_k \) can be regarded as a rotation matrix without changing the unitary property.

C. Procedure of Precoding Design

Once \( U_F \) is determined based on the proposed steepest descent method on the Riemannian manifold, by substituting the obtained \( U_F \) into (19), \( L(F) \) becomes a problem only related to the power allocation \( \lambda_F = [\lambda_{F1} \cdots \lambda_{FP}]^T \). For easy derivation, \( L(F) \) in (19) is rewritten as
\[
L(\lambda_F) = \sum_{i=1}^{N_t} \sum_{j=1}^{N_r} \left( \frac{1}{g\lambda_{Fi}} \right) - 1 \sum_{i=1}^{N_t} b_i^H u_{Fi}^H u_{Fi} \tag{31}
\]

Substituting (31) into (18), the optimization problem is reduced to
\[
\max_{\lambda_F} J(\lambda_F) = \sum_{i=1}^{N_t} \sum_{j=1}^{N_r} \left( \frac{1}{1 + g\lambda_{Fi}} \right) + \log(1 + g\lambda_{Fi} \lambda_{Rj}) \tag{32}
\]

where \( \alpha_{ij} = u_{Fi}^H b_j b_j^H u_{Fi} \geq 0 \). The second order derivative of \( J(\lambda_F) \) w.r.t. \( \lambda_{Fi} \) is
\[
\frac{\partial^2 J(\lambda_F)}{\partial \lambda_{Fi}^2} = \sum_{j=1}^{N_r} \left( \frac{2 \alpha_{ij} b_j b_j^H \lambda_{Rj}}{(1 + g\lambda_{Fi} \lambda_{Rj})^2} \right) \tag{33}
\]

The second order derivative is always negative implying that \( J(\lambda_F) \) is a concave function of \( \lambda_{Fi} \). Since the equality constraint is linear, (32) is a convex problem w.r.t. \( \lambda_{Fi} \). Thus, we can always find a valid solution \( \{\lambda_{Fi}\} \) using well-established interior-point methods [16]. Actually, we can employ the low-complexity iterative method described in [4, Appendix C] to solve the optimization problem in (32). Once \( \{\lambda_{Fi}\} \) are obtained, we can take them into the problem (28) again to refine \( U_F \) until convergence.

Therefore, the optimization problem of (18) can be solved by a two-step iterative procedure which is summarized in Algorithm 1.

Input: \( U_F^{(1)} = I_{N_r}, \lambda_F^{(1)} = \eta = \text{unit vector} \)

Output: \( U_F \) and \( \lambda_F \)

For \( l = 1:N_{\text{iter}} \)

With fixed \( U_F^{(l-1)} \), derive the optimal \( \lambda_F^{(l)} \) by solving the problem in (32) using the iterative method proposed in [4];

With fixed \( \lambda_F^{(l)} \), obtain the unitary matrix \( U_F^{(l)} \) by solving the problem in (28) using the Riemannian steepest descent method;

If \( ||\lambda_F^{(l)} - \lambda_F^{(l-1)}||^2 < 10^{-4} \) then break

End

End

Return \( U_F = U_F^{(l)}, \lambda_F = \lambda_F^{(l)} \)

Algorithm 1: Procedure of the Precoding Scheme

V. SIMULATION RESULTS

In this section, we compare the symbol error rates (SER) with different antenna configurations for different precoder design methods. The signal constellation is 4-QAM. The antenna configurations of MIMO system and the Ricean factor are shown in the captions. Monte-Carlo simulation is employed for performance evaluation. Equal power scheme, one-dimensional (1D) beamforming [7], precoder proposed in [6] and precoder proposed in [9] are used for performance comparison with the proposed schemes.

Scenario 1: In Fig. 1, a rank-one channel mean, i.e., \( H_0 = I_{N_r \times N_t} \) is used for the evaluation under different channel conditions with \( K = 3 \) and \( N_r = N_t = 2 \). The spatial correlation of the \( p \)th and \( q \)th receive antennas is given by \( [R_{pq}]_{p,q} = 0.9^{p-q}, p, q = 1, \ldots, N_r \). The experimental results show that the proposed scheme and the schemes in [6] and [9] outperform the equal power scheme and 1-D scheme. For different antenna configurations and Ricean factors, the proposed scheme achieves almost the same SER performance as the scheme in [9] and the simplified scheme in [6].

![Fig. 1. Scenario 1: SER versus SNR, K = 3, N_r = N_t = 2](image-url)
Scenario 2: A none-rank-one channel mean is used for the evaluation with \( K = 3 \) and \( N_r = N_t = 3 \). The channel mean is

\[
H_0 = \begin{bmatrix}
1.6 & 0.5 - 0.6i & 0.7 + 0.5i \\
-0.8 + 0.4i & -0.7 + 0.6i & -0.5 + 0.3i \\
-0.3 - 0.9i & 0.6 - 0.2i & 0.6 - 1.2i
\end{bmatrix}
\]

where \( i = \sqrt{-1} \). The receive-side correlation matrix \( R_R \) is assumed a complex Toeplitz matrix with the first column being

\[
[R_R]_{1,1} = \begin{bmatrix} 1 & -0.6 - 0.5i & 0.5i \end{bmatrix}^T
\]

As shown in Fig. 2, the performance of proposed method is superior to that of the other schemes. Performance degradation of the simplified solution in [6] can be observed at moderate SNR region. However, the performance of the scheme in [9] is quite sensitive to the spatial correlation matrix \( R_R \) and the channel mean \( H_0 \). For this case, the scheme in [9] cannot always converge to an optimal solution, thus significant performance degradation can be observed for both moderate and high SNR.

![Fig. 2. Scenario 2: SER versus SNR. \( K = 3, N_t = N_r = 3 \)](image)

VI. CONCLUSION

In this paper, an efficient precoding design for MIMO system over Ricean fading channels is proposed. By exploiting the unitary property, a steepest descent method operated on the Riemannian manifold of unitary matrices is developed to obtain the desired beamforming directions. Simulation results show that the BER performance of the proposed scheme is superior to those of the existing solutions, and the convergence of the iterative design is guaranteed under different channel conditions.

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