Energy Efficiency Analysis and Power Allocation of Cooperative Communications in Wireless Sensor Networks

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Abstract—In order to improve the energy saving performance of cooperative communication, a novel power allocation solution, named Half Transmission Power Allocation Solution (HTPAS), is proposed in this paper. In this solution, the transmission power of the relay for forwarding the packet is half of the transmission power of the source. Based on the energy efficiency analysis, HTPAS outperforms the equal transmission power allocation solution in energy saving of relay and cooperation gain, meanwhile the proposed solution is easy to be implemented. The numerical results also show that the SDF cooperative transmission is more energy efficient than the direct and multi-hop transmission when the source-destination distances is larger than a small threshold.

Index Terms—cooperative communications, energy efficiency, power allocation, wireless sensor networks

I. INTRODUCTION

Energy-constrained networks, such as wireless sensor networks, are composed of nodes typically powered by batteries due to constraints in size and cost, for which replacement or recharging is very difficult [1]. With finite energy, only a finite amount of information can be transmitted. Therefore, the energy consumption of the sensors is a key issue in the design and realization of the wireless sensor networks. Multi-input-Multi-output (MIMO) techniques based on antenna arrays can remarkably improve system capacity, and reduce the required transmission power under a certain throughput requirement due to spatial diversity, which is studied to combat the effects of multipath fading in wireless communications. Even though it’s impractical to mount multiple antennas due to the limitation in size of the nodes, multiple nodes could cooperate for forming a virtual antenna array to achieve spatial diversity [2] [3]. Cooperative communication techniques, no doubt, can be regarded as one of the potential solutions to minimize the required transmits energy by exploiting the diversity obtained by the cooperation among the nodes in wireless sensor networks.

Power allocation can further improve the performance of cooperative communication. The energy-efficient cooperative communication based on power control and selective sing-relay has been studied in [4], which confirms that cooperative communication can achieve significant energy savings and prolongs the network lifetime considerably. In [5], the cooperative selective decode-and-forward (SDF) protocol has been investigated, in which cooperative transmission outperforms single-hop and multi-hop transmissions. The energy efficiency of the selective decode-and-forward protocol and the incremental decode-and-forward (IDF) protocol have been studied in [6], where cooperation may be considerably more energy efficient than non-cooperative schemes, by taking into account the energy consumption of the RF circuitry, especially if a feedback channel is available. The power allocation of cooperative communications has been studied in [7]-[10], in which the proposed optimal power allocation strategies outperform the equal power allocation to some extent, except implementing more complicated than the equal power allocation.

Based on the above cited works, the energy efficiency analysis and a novel suboptimal power allocation scheme are presented in this paper. In order to modeling and analyzing accurately, the following characteristics are covered based on [6]: a) the circuitry energy consumption for transmitting and receiving; b) the spectral efficiency loss at the receiver when employing multi-hop and cooperative communications; c) Nakagami-m distribution for describing the radio propagation environment.

The remainder of this paper is organized as follows. Section II presents the system model and the outage analysis. The energy consumption and power allocation analysis of the transmission schemes are studied in Section III. Finally, numerical results are discussed in Section IV, and conclusions are presented in Section V.

II. SYSTEM MODEL

As shown in Fig. 1, the SDF cooperative transmission system model consists of three relevant nodes, which are the source, the relay, and the destination. Cooperative transmission takes advantage of the wireless medium broadcast nature. The SDF cooperative communication is considered in two slots. The first time slot is the broadcast phase, when the packet sent by the source to the destination is also overheard by the relay. The second time slot is the cooperation phase, when the relay
cooperates with the source, forwarding the same packet through a different and independent channel, thus achieving the cooperation diversity. In SDF, the relay cooperates whenever the packet from the source has been correctly received at the relay. Differing from SDF, IDF considers the existence of a feedback channel from the destination, so that the relay cooperates only receiving a negative acknowledgement (NACK). In this paper, SDF cooperative transmission is studied instead of IDF, because that IDF produces delay due to the NACK signal, comparing to SDF. And SDF is easier to realize than IDF. In order to contrastively analysis the energy efficiency of the cooperative communication, another two transmission strategies are considered, including direct transmission and multi-hop transmission. In direct transmission scheme, the source directly communicates with the destination. In the multi-hop transmission scheme, the communication is generally carried out in two different time slots. In the first time slot, the source sends a packet to the relay, and the relay receives the packet. In the second time slot, the relay forwards the packet to the destination.

The Rayleigh distribution is one of the most accepted models to describe the behavior of the radio propagation environment. However, the Nakagami-m distribution more closely matches with the practical wireless communication environment than the Rayleigh distribution. Therefore, the Nakagami-m distribution is used for describing the wireless communication environment in this paper. And the channel in long-term quasi-static fading is assumed, which means that the channel is strongly correlated in time, remaining in the deep fade state for a long time.

The system performance is characterized in terms of outage probability in this paper. Outage is defined as the event that the received SNR (Signal-to-Noise Ratio) falls below a certain threshold \( \beta \). Therefore, the probability of outage \( P_O \) is defined as

\[
P_O = P(\text{SNR} \leq \beta). \tag{1}
\]

If the received SNR is higher than the threshold \( \beta \), the receiver is assumed to be able to decode the received message with negligible probability of error. If an outage occurs, the packet is considered lost. Based on the derived outage probability expressions, a constrained optimization problem with power allocation strategies to minimize the total consumed power is illustrated in the next section.

III. ENERGY EFFICIENCY ANALYSIS AND POWER ALLOCATION

In this section, the direct, multi-hop and SDF transmission schemes are formalized. And a novel power allocation solution for SDF cooperative transmission is proposed.

A. Problem Formulation

Shown in the system model, there are three communicating links in the SDF cooperative communication scheme. The received packet for each link can be expressed as

\[
\gamma_{ij, SDF} = \sqrt{\frac{P_{SDF}^i}{\gamma_0}} h_i x + n_j \tag{2}
\]

where \( i \in \{S, R\}, \ j \in \{R, D\}, \ P_{SDF}^i \) is the transmission power for the source and relay respectively, \( \gamma_0 \) is the path loss, \( h_i \) is the Nakagami-m quasi-static fading coefficient, \( x \) is the packet to be transmitted, \( n_j \) is the white Gaussian noise (AWGN) with variance \( N_0 \).

The received SNR at the receivers are given by [6]

\[
\text{SNR}_{ij} = \frac{|h_i|^2 \gamma_0 P_{SDF}^i}{N_0} \tag{3}
\]

where \( i \in \{S, R\}, \ j \in \{R, D\} \). And \( \gamma_0 \) can be expressed as [11]

\[
\gamma_0 = \frac{G \lambda^2}{(4\pi)^2 d_i^2 M N_f} \tag{4}
\]

where \( i \in \{S, R\}, \ j \in \{R, D\} \), \( d_i \) is the distance in meters among the source, the relay and the destination, \( \alpha \) is the path loss exponent, \( G \) is the total gain of the transmit and receive antennas, \( \lambda \) is the wavelength, \( M \) is the link margin and \( N_f \) is the noise figure at the receiver [6].

As discussed before, an outage occurs when the SNR at the receiver falls below a certain threshold \( \beta \), which allows error free decoding. This threshold is defined as \( \beta = 2^\Delta - 1 \), where \( \Delta \) is the system spectral efficiency. However, the loss in spectral efficiency inherent to the cooperative transmission can degrade the system performance since the end-to-end throughput will be reduced to half [6]. Hence, the solution that the nodes operate with a spectral efficiency \( L (L = 2) \) times greater than that of the direct transmission is assumed, which is \( \beta_L = 2^{\Delta L} - 1 \) [6].

Hence, the outage probability for each link, in Nakagami-m fading, is given by [12]

\[
P(SNR_{ij} < \beta_L) = \frac{1}{\Gamma(m)} \Psi \left( m, \frac{m N \beta_L}{P_{SDF}^i \gamma_0} \right) \tag{5}
\]

Fig. 1. Cooperative transmission system model.
where $i \in \{S, R\}, j \in \{R, D\}$, $m$ is the Nakagami-$m$ fading figure, 
\[
\Psi(a, b) = \int_{0}^{b} y^{a-1} \exp(-y) dy
\]
is the incomplete gamma function and 
\[
\Gamma(a) = \int_{0}^{\infty} y^{a-1} \exp(-y) dy
\]
is the complete gamma function. At high SNR, 
\[
\Psi(a, b) \approx (1/a) \cdot b^a \quad [12].
\]
Therefore, the outage probability for each link can be expressed as 
\[
P_{o,i} \approx \frac{1}{\Gamma(m+1)} \left( m \beta_k \right)^m (\gamma_i)^i (\gamma_f)^f
\]
The total outage probability for the SDF cooperative transmission can be given by 
\[
P_{o,SDF} = \left( 1 - \frac{\gamma_i}{\gamma_f} \right) \Gamma(m+1) \frac{m \beta_k}{\Gamma(m+1)} (\gamma_i)^i (\gamma_f)^f
\]

The total consumed power in the SDF cooperative transmission is 
\[
P_{SDFtot} = P_{TX} + P_{RX} + 2P_{SDF} + 2P_{RX} + 2P_{SDF} + 2P_{RX}
\]
where $P_{SDF}^{s}$ and $P_{SDF}^{r}$ are the transmission power of the source and the relay respectively, $P_{TX}$ and $P_{RX}$ are the power consumed by the internal circuitry for transmitting and receiving, respectively. Minimizing the total consumed power of the SDF cooperative transmission is equivalent to minimize the transmission power of the source and the relay due to the values of $P_{SDF}^{s}$ and $P_{SDF}^{r}$ are fixed.

B. Half Transmission Power Allocation Solution (HTPAS)

As shown in the equation (8), the above optimization problems of cooperative schemes are nonlinear and cannot admit a closed form solution. Even though the optimization solution can lead to the best performance, it is difficult to implement such a complex optimization problem due to the limitation by the low computing power of the nodes in a practical scenario. One of the sub-optimization solutions is that the source and relay nodes utilize the same power for transmission. In [6] and [9], it has been proved that the solution of the equal transmission power of the source and relay nodes could meet the performance of energy efficiency.

However, mathematically speaking, the proportional relation between the transmission power $P$ and the distance $d$ are given by [13] 
\[
P = d^K, 2 < K < 4
\]
where $K$ is a constant that depends on the propagation medium and antenna characteristics. This equation means the farther communicating distance, the more transmission power consumed.

In the cooperative transmission scenario, the relay node generally locates between the source node and the destination node. Hence, the transmission power of the relay node should be less than that of the source node. It is assumed that there is a linear relation between the transmission power of the source node and that of the relay node as follows 
\[
P_{SDF}^{r} = a \cdot P_{SDF}^{s}, 0 < a \leq 1
\]

Fixing the outage probability $P_{o}^*$, which is assumed to meet the QoS requirement, such that $P_{o} \leq P_{o}^*$, and substituting in equation (7). An equation about $P_{SDF}^{s}$ is obtained. The detailed derivation work is given in the Appendix.

\[
a^m P_{o}^*(P_{SDF}^{s})^m - (a^m k_1 k_2 k_3) + (k_1 k_2 k_3)(P_{SDF}^{r})^m = 0
\]

Then, the smallest real and positive solution of the equation (11) is regarded as the optimal transmission power of source. And the optimal transmission power of relay is obtained using the equation (10). In the next section, the numerical results prove that the proposed solution performs best when $a = 0.5$, and is better than equal transmission power allocation solution. The solution when $a = 0.5$ is named as Half Transmission Power Allocation Solution (HTPAS) in this paper.

In equal transmission power solution, the transmission power of source and relay is assumed to be same. Similarly, the optimal transmission power can be obtained as the small real and positive solution of 
\[
P_{o}^*(P_{SDF}^{s})^m - (k_1 k_3 + k_2 k_3)(P_{SDF}^{r})^m + (k_1 k_2 k_3) = 0
\]

From the equations above, the computing complexity of the HTPAS and equal transmission power solution can be derived. The computing complexity of the HTPAS is $O(3m)$, and the computing complexity of the equal transmission power solution is also $O(3m)$. And in the process of implementation, the space complexity of the HTPAS is $O(1)$, the same as the equal transmission power solution. Then the conclusion can be obtained that the proposed HTPAS is also easy to implement.

C. Direct Transmission

In the direct transmission, the source communicates directly with the destination, without any intermediate
nodes. The outage probability of direct transmission can be expressed as
\[ P_{o,d} \geq \frac{1}{\Gamma(m+1)} \left( \frac{mN\beta}{P_{dt}/\delta_d} \right)^m \] (13)
where \( P_{dt} \) is the transmission power, and \( \beta = 2^\alpha - 1 \).

The total consumed power of the direct transmission is
\[ P_{tot} = P_{dt} + P_{tx} + P_{rx} \] (14)

Similarly, fixing the outage probability \( P_{o,d}' \) and substituting in (13) leads to the optimal transmit power for the direct transmission:
\[ P_{gt} = \frac{mN\beta}{\gamma_{sd}^{m/(m+1)}P_{o,d}'} \] (15)

D. Multi-Hop Transmission

The outage probability for the multi-hop transmission is given by
\[ P_{o,mp} = P(SNR_{sk} < \beta_{l}) + [1-P(SNR_{sk} < \beta_{l})] \cdot P(SNR_{md} < \beta_{l}) \] (16)

Therefore, the total consumed power in the multi-hop transmission is
\[ P_{tot} = P(SNR_{sk} < \beta_{l}) + P_{md} + P_{tx} + P_{rx} + [1-P(SNR_{sk} < \beta_{l})] \cdot (2P_{md} + 2P_{tx} + 2P_{rx}) \] (17)

where \( P_{md} \) is the transmission power of multi-hop transmission scheme.

Similar to the direct transmission, the outage probability \( P_{o,mp}' \) is assumed to be fixed, then substituting in the equation (16). The optimal transmission power for the multi-hop transmission can be obtained as the smallest real and positive solution of
\[ P_{o,mp}'(P_{md})^{m} - (k_{1} + k_{2})(P_{md})^{m} + (k_{1}k_{2}) = 0 \] (18)

IV. SIMULATION RESULTS

In this section, some simulations are presented to prove the theoretical analysis presented in the previous section. Referring to [6], the link margin and the noise figure are assumed to be \( M_{l} = 40 \text{ dB} \) and \( N_{f} = 10 \text{ dB} \), respectively. The total antenna gain is \( G = 5 \text{ dBi} \), the carrier frequency is \( f_{c} = 2.5 \text{ GHz} \), and the path loss exponent is \( \alpha = 2.5 \). Based on [14], The overall power consumption of the circuit for transmitting and receiving are consumed as \( P_{tx} = 97.9 \text{ mW} \) and \( P_{rx} = 112.2 \text{ mW} \), respectively. The Nakagami-\( m \) fading figure is \( m = 1 \), and the outage probability is fixed as \( P_{o,d}' = 10^{-3} \).

The total consumed power of SDF cooperative transmission with the proposed power allocation solution is simulated while assuming \( a \in \{0.2, 0.3, \ldots, 1.0\} \). In order to observe the simulation result clearly, a part of the simulating plot is enlarged, as shown in Fig. 2. The reason that the situation of \( a = 0.1 \) is not shown, is that the proposed power allocation solution performs worst when \( a = 0.1 \), hence, there is not necessary to show this simulation result. When \( a = 1.0 \), the proposed sub-optimization solution is equal to the equal transmission power allocation solution.

From this simulation result, it can be observed that the total consumed power for the SDF cooperative transmission is least when \( a = 0.5 \). In other words, the HTPAS performs better than equal transmission power allocation solution. In the following simulations, the equal transmission power allocation solution and the HTPAS of the cooperative transmission are considered.

Fig. 3 shows the optimal transmission power of relay for the proposed power allocation solution. As mentioned, the proposed sub-optimization solution when \( a = 1.0 \) is equal to the equal transmission power allocation solution. As expected, the transmission power of relay for HTPAS is less than that for equal transmission power allocation solution. In other words, HTPAS decreases the energy consumption of relay and prolong the useful life of relay.
than the direct and multi-hop transmission schemes, while the multi-hop transmission scheme performs worse than all the other schemes. The direct transmission performs better than SDF cooperative transmission only when the distance between the source and the destination is less than about 25 m. The numerical results reveal that for short distance between the source and the destination, for example, below a threshold 25 m, the direct transmission is more efficient than the cooperative transmission. Above the threshold, the SDF cooperative transmission is useful for decreasing the total consumed power in energy-constrained networks. The reason that the multi-hop transmission consumes more power than the direct transmission is considered that the relay consumed a part of power for forwarding the packets. Consumed power and hence the cooperation gain tends to saturate. In addition, this simulation result obviously shows that the proposed power allocation solution HTPAS performs a little better than the equal transmission power allocation solution. Hence, the conclusion is obtained that the SDF cooperative transmission with the proposed power allocation solution HTPAS, in which the transmission power of the relay is half of that of the source, can be energy efficient in energy-constrained networks when the distance between the source and the destination is larger than 25 m.

The variation trend of the total consumed power when the outage probability reduces to $P_o = 2 \times 10^{-4}$ is shown in Fig. 6. According to the figure, the SDF cooperative transmission performs much better than the direct transmission when the source-destination distance is larger than 13 m. Hence, the conclusion can be obtained that the SDF cooperative transmission is more and more energy efficient while the outage probability decreases. When the required system spectral efficiency is increased to $\Delta = 4$ b/s/Hz, the variation trend of the total consumed power is shown in Fig. 7. From this simulation result, it is illustrated that the advantage of energy efficiency of the SDF cooperative transmission is reduced with the system spectral efficiency increased.

Referring to [9], the cooperation gain that the ratio between the power required for the direct transmission and the SDF cooperative transmission is defined. Fig. 5 shows the numerical results for the cooperation gain of two different power allocation solutions. When the source-destination distances is less than 25 m, the simulation result reveals that the direct transmission is more energy efficient than SDF cooperative transmission. For $\gamma_{SD} = 25$ m, the cooperation gain increases as the transmission power starts taking up a significant portion of the total consumed power. This ratio increases until the transmission power becomes the main portion of the total

**V. CONCLUSIONS**

![Fig. 4. Total consumed power of each transmission schemes ($P_o = 10^{-1}$, $\Delta = 2$ b/s/Hz).](image4)

![Fig. 6. Total consumed power of each transmission schemes ($P_o = 2 \times 10^{-4}$, $\Delta = 2$ b/s/Hz).](image6)

![Fig. 7. Total consumed power of each transmission schemes ($P_o = 10^{-1}$, $\Delta = 4$ b/s/Hz).](image7)
In this paper, the energy efficiency of direct, multi-hop, and SDF transmission schemes are compared in energy-constrained networks. The numerical results constrain the transmission schemes to have the same end-to-end throughput and outage probability show that the SDF cooperative transmission is more energy efficient than the direct and multi-hop transmissions when the source-destination distances is larger than a certain threshold. A novel power allocation solution (HTPAS) for the SDF cooperative transmission is proposed. The simulations of the transmission power of the relay and the cooperation gain demonstrate HTPAS performs a little better than the equal transmission power allocation solution, meanwhile it is also easy to be realized. This is significant for applying to the energy-constrained networks, such as wireless sensor networks.

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APPENDIX

From the equation (6), the outage probability of the S-D link, S-R link and R-D link can be derived respectively.

\[ P(SNR_{SD} < \beta_s) = \frac{1}{\Gamma(m+1)} \left( \frac{mN\beta_s}{\gamma_{SD} P_{SD}^5} \right)^m \]

\[ P(SNR_{SR} < \beta_s) = \frac{1}{\Gamma(m+1)} \left( \frac{mN\beta_s}{\gamma_{SR} P_{SR}^5} \right)^m \]

\[ P(SNR_{RD} < \beta_s) = \frac{1}{\Gamma(m+1)} \left( \frac{mN\beta_s}{\gamma_{RD} P_{RD}^5} \right)^m \]

Fixing the outage probability \( P^*_o \), and substituting in equation (7).

\[ P^*_o = P(SNR_{SR} < \beta_s) \cdot P(SNR_{RD} < \beta_s) \]

\[ + (1-P(SNR_{SR} < \beta_s)) \cdot P(SNR_{RD} < \beta_s)) \]

\[ \frac{1}{\Gamma(m+1)} \left( \frac{mN\beta_s}{\gamma_{SR} P_{SR}^5} \right)^m \cdot \frac{1}{\Gamma(m+1)} \left( \frac{mN\beta_s}{\gamma_{RD} P_{RD}^5} \right)^m \]

Simplifying the above equation with

\[ k_1 = \frac{(mN\beta_s)^m}{\Gamma(m+1)\gamma_{SR}^m}, \quad k_2 = \frac{(mN\beta_s)^m}{\Gamma(m+1)\gamma_{RD}^m}, \quad k_3 = \frac{(mN\beta_s)^m}{\Gamma(m+1)\gamma_{SD}^m} \]

and replacing \( P_{SD}^5 \) with \( a \cdot P_{SD}^5 \), \( 0 < a \leq 1 \).

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