Performance Analysis and Capacity Issue of IDMA-based Satellite Systems under Imperfect Power Control

Gongliang Liu and Xinrui Fang
Department of Communication Engineering, School of Information and Electrical Engineering
Harbin Institute of Technology (Weihai), Weihai 264209, P.R.China
Email: liugl@hit.edu.cn; happygoole@163.com

Abstract—Considering the advantage of interleave division multiple access (IDMA) technique and the existing technical bottlenecks in satellite systems, IDMA is introduced into satellite communication networks. In multi-beam IDMA satellite systems, combined with unique SINR evolution technology, the system interference value can be accurately estimated, and system capacity can be accurately calculated considering the multi-user detection (MUD). Another important issue in applying IDMA scheme is that the received mobile user power levels at satellite are unequal. Due to inherent long propagation delay, the power fluctuations in mobile satellite channels cannot be perfectly compensated by power control techniques. Consequently, the capacity of the system is reduced, compared to system with perfect power control (PPC). This paper derives the signal power to interference and noise ratio compared to system with perfect power control (PPC). This techniques. Consequently, the capacity of the system is reduced, compared to system with perfect power control (PPC). IDMA adopts the iterative chip-by-chip (CBC) detection scheme to combat with multiple access interference (MAI). Compared to CDMA, IDMA solves the problem of MAI at a lower computational complexity, the decrement of which is linear with the number of users.

In a practical system, we cannot expect perfect power control (PPC), thus the accompanying imperfect power measurement along with the response delay in the power control process will lead to power control error (PCE). Considering the link characteristics, an optimistic assumption is made that the performance of the IDMA on satellite systems is comparable to the performance achieved in terrestrial systems. An outstanding feature of satellite channels is the round trip propagation delay, which reduces the effectiveness of the power control methods and makes PCE inevitable in satellite systems. In [3], the input and output of the power control module for IDMA are studied. The capacity performance of the IDMA system under the perfect power control is examined in [4]. Considering the other cell interference factor, the link capacity of CDMA under imperfect power control is evaluated in [5]. However, the analyses mentioned above did not consider the advantages of IDMA technical or the influences of imperfect power control on the multi-beam satellite systems. A log-normally approximation method presented in reference [6] and [7] is adopted to analyze the capacity of multi-user system under imperfect power control, but they have not made in-depth analysis to the interference impacting on the system performance. The advantage of IDMA technology for future satellite communication systems is the high efficiency to overcome both intra-cell and inter-cells MAI with simple CBC MUD. Due to the analysis above, the capacity analyses of multi-beam IDMA satellite systems with the imperfect power control, combining the IDMA-CBC with MUD, have a practical significance.

Considering the advantage of IDMA technique, this paper derives the expression of system capacity by taking the multi-beam satellite systems into consideration. The other-cell interference in satellite environment is
calculated considering the user number per beam, spot-beam antenna pattern, and the number of interfering beams in cellular layout. What’s more, the outage probability is calculated with different power control error values. It is found that the system capacity may decrease significantly owing to the effect of imperfect power control. The results provide reliable guarantee for the optimization results to be used in real control system.

II. SYSTEM MODE

The interference calculation model, based on the locations in desired and interfering beams, with respect to the satellite and the characteristics of the spot-beam antenna, are shown in Fig. 1. (We assume that all users are uniformly distributed.) From Fig. 1, the cluster considered in the calculations consists of three tiers of surrounding spot-beams (37-beam cluster).

![Satellite multi-beam interference model](image)

Fig. 1. Satellite multi-beam interference model

$G$ is the antenna gain characteristic of the spot-beam, $L_i$ and $L_j$ are the link-loss functions of users, $\theta_i$ and $\theta_j$ are the deflection angles of interfering user signals. $\psi_j$ is the acceptance angle of interfering signal from other spot-beams, which is relative to center of the own spot-beam. The antenna gain of spot-beam can be specified as:

$$ G(\theta) = \left( J_i(u) \right)^2 $$  (1)

where $u = \frac{D \sin \theta}{\lambda}$ and $J_i(*)$ is the Bessel function of the first kind and first order, $D$ is the antenna diameter, $\lambda$ is carrier wavelength. At this point, the other-beam interference factor, which is defined as the interference power produced by users in other beams, divided by the power produced by users in local beam, can be calculated as:

$$ f_{\text{other}} = \frac{\sum_{j \neq \text{other-cell}} \frac{P_j G(\psi_{ij})}{L_j}}{\sum_{i \text{own-cell}} \frac{P_i G(\theta_i)}{L_i}} $$  (2)

We assume that there are $N$ uniformly distributed users in each beam and the transmitted power from satellite with perfect power control (PPC) to the user-$k$ is $P_0$, thus the inter-cell interference can be calculated as:

$$ I_{\text{inter}} = \sum_{j=1}^{N} \sum_{k=3}^{N} \frac{G(\psi_{jk})}{G(\theta_{jk})} P_j $$  (3)

III. SYSTEM CAPACITY WITH IMPERFECT POWER CONTROL

A. Perfect Power Control

Capacity is defined as the number of traffic channels through which a satellite can simultaneously offer given data rate and bit error rate. Therefore, in order to research the capacity performance of the perfect power control system, we introduce the concept of the overall interference power and signal power to interference and noise ratio (SINR).

Fig. 1 gives the interference model of the given satellite. For a single spot-beam with perfect power control, satellite receives the same power from all uplink signals (user-to-satellite). With the assumption of a 37-beam layout and $N$ users per beam, for convenience, the total interferences are divided into inter beam interferences and intra beam interferences. If $P_0$ and $P_N$ represent signal power and system noise power, respectively, then the total interference power can be defined as:

$$ I_{\text{total}} = I_{\text{intra}} + I_{\text{inter}} + P_N $$  (4)

With the interference power from own beam $I_{\text{intra}}$ is written as $I_{\text{intra}} = (N - 1)P_0$.

The SINR (ratio of signal to interference and noise) in this case is:

$$ \text{SINR} = \frac{P_0}{(N - 1)P_0 + I_{\text{inter}} + P_N} $$  (5)

Combining (3) with (5), we get the overall equation of SINR as follows:

$$ \text{SINR} = \frac{P_0}{(N - 1)P_0 + \sum_{j=1}^{N} \sum_{k=3}^{N} \frac{G(\psi_{jk})}{G(\theta_{jk})} + P_N} $$  (6)

As to an ordinary system, given the values of the threshold $(E_b/I)_{\text{req}}$, the total bandwidth $B$ and the information bit rate $R_b$, $N$ can be calculated.

$$ \text{SINR} = \frac{P_0}{(N - 1)P_0 + \sum_{j=1}^{N} \sum_{k=3}^{N} \frac{G(\psi_{jk})}{G(\theta_{jk})} + P_N} \geq \frac{(E_b/I)_{\text{req}} R_b}{B} $$  (7)
B. IDMA-CBC MUD and SINR Evolution Technique

Combining with the specific IDMA-CBC MUD, we consider the capacity analysis further, which effectively resist the internal MAI. Complete structure and procedure of IDMA-CBC MUD are given in literature [2].

With single path channel synchronous and modulation setting as BPSK in this study, the performance of IDMA-CBC detection scheme is mainly reflected by the decrease in the variable variance, e.g. the variance of \( \{ x_k (j), \forall k, j \} \). It can be written as

\[
V_k = 1 - \tanh^2 \left( \frac{Y_{\text{SINR}}}{2} \right), k = 1, \ldots, K
\]

(8)

As shown in (8), \( V_k \), e.g. the variance of an arbitrary chip from user-\( k \), which is the corresponding power interference factor in the iteration, is the function of \( \text{SINR}_k \). The function \( f(\text{SINR}) \) is referred as the expectation of the interference power, also the anti-interference percentage with fixed SINR, written as

\[
f(\text{SINR})_k = E(V_k) = 1 - E \left[ \tanh^2 \left( \frac{Y_{\text{SINR}}}{2} \right) \right], k = 1, \ldots, K
\]

(9)

When the iteration reaches the iteration convergence point for user-\( k \), equivalently, the system achieves maximum MAI elimination capacity, thus we define:

\[
\text{SINR}_k = \frac{P_0}{\sum_{j \neq k} P_j E(V_j) + \sum_{j = 1}^{N} \sum_{k = 1}^{K} P_j G(\psi_{jk}) + P_N} \geq \gamma_k
\]

(10)

where \( \gamma_k \) represents \((E_b/I_0)_{eq} R_b/B\), and the total interference can be expressed as:

\[
I_{\text{total}} = \sum_{i \neq k} P_i f(\text{SINR})_i + \sum_{j = 1}^{K} \sum_{k = 1}^{K} P_j G(\psi_{jk}) + P_N
\]

(11)

\( f(\text{SINR}) \) is negatively correlated with \( \text{SINR} \), ranging from 0 to 1, which has been verified in the literature [8]. Here we assume that the input signal power to interference and noise ratio of the decoder is approximately the same for an arbitrary user-\( k \), namely, all users share the same power interference factor \( f(\gamma) \), that is:

\[
f(\gamma) = f(\text{SINR}), i = 1, 2, \ldots, N
\]

(12)

Supposing \( \text{SINR}_{k, \text{new}} \) and \( \text{SINR}_{k, \text{old}} \) are SINR-\( k \) values after and before all iterations of CBC (chip-by-chip) algorithm, respectively, IDMA technology involves SINR evolution technique which can obviously improve the system capacity through iteration, concluded as:

\[
\text{SINR}_{k, \text{new}} = \frac{P_k}{\sum_{j \neq k} P_j f(\text{SINR}_{j, \text{old}}) + \sum_{j = 1}^{K} P_j G(\psi_{jk}) + P_N}
\]

(13)

Therefore with the aid of this SINR evolution technique developed from the IDMA systems, the following expressions can be derived for the certification of capacity enhancement in IDMA-based satellite communication system:

\[
\text{SINR} = \frac{P_0}{(N-1)f(\gamma)P_0 + \sum_{j = 1}^{N} \sum_{k = 1}^{K} P_j G(\psi_{jk}) + P_N}
\]

\[
\geq \left( \frac{E_b}{I_{\text{eq}}} \right) \frac{R_b}{B}
\]

(14)

where \( R_b \) is the channel bit rate, \( B \) is the system bandwidth and \( f(\gamma) \) is the iterative factor of power control depending on SINR evolution.

Through (14), we can evaluate bit energy to total noise (thermal noise and MAI) power spectral density ratio, and (14) can be modified as:

\[
\frac{E_b}{(N_0 + I_0)} = \frac{B / R_b}{(N-1)f(\gamma)P_0 + \sum_{j = 1}^{N} \sum_{k = 1}^{K} G(\psi_{jk}) + P_N} \leq \left( \frac{E_b}{I_{\text{eq}}} \right) \frac{R_b}{B}
\]

(15)

where \( P_b = E_b / N_0 = B \).

C. Power Control Error

In satellite communications, the length of the round trip propagation delay makes power control error (PCE) inevitable. PCE is assumed to be log-normally distributed and the effect of it can be studied by multiplying the transmitted power by a log random variable \([9] \). The transmitted power to user-\( k \) becomes:

\[
P'_k = P_k 10^{\gamma/10}
\]

(16)

where \( P_k \) is the transmitted power to user-\( k \) with PPC, \( \gamma \) is a zero-mean Gaussian random variable with a standard deviation \( \sigma \), and the typical value of which is 1~2dB. When \( \sigma = 0 \) dB, the case corresponds to PPC.

With the assumption that received power is log-normally distributed, the SINR in this case is expressed as:

\[
\text{SINR} = \frac{P_0 10^{\gamma/10}}{(N-1)f(\gamma)P_0 10^{\gamma/10} + \sum_{j = 1}^{N} \sum_{k = 1}^{K} P_j 10^{\gamma/10} G(\psi_{jk}) + P_N}
\]

(17)

Similar to PPC case, we can define this ratio in terms of bit energy to total interference power density ratio as:

\[
\frac{E_b}{(N_0 + I_0)} = \frac{B / R_b 10^{\gamma/10}}{(N-1)f(\gamma)P_0 10^{\gamma/10} + \sum_{j = 1}^{N} \sum_{k = 1}^{K} 10^{\gamma/10} G(\psi_{jk}) + P_N} \leq \left( \frac{E_b}{I_{\text{eq}}} \right) \frac{R_b}{B}
\]

(18)
To simplify the analysis, it is essential to discuss a multi-beam system which offers a single business only. With \( k \) users transmitting signals and their transmission power independent from each other which is controlled individually, the total interference power received from the target beam \( I(k) \) is approximately equal to the summation of \( k \) independent and identically distributed lognormal random variables denoted by \( P_i \). Considering the convolution properties of the lognormal distribution, sum of multiple independent and lognormal random variables are approximately log-normal random [8]. Therefore, as a lognormal distributed random variable, the total received interference power of the system can be expressed as [9]:

\[
I(k) = \sum_{i=1}^{k} P_i \tag{19}
\]

The pdf of \( P_i \) for \( k \) users is approximately log-normal with the following logarithmic mean \( m_i(k) \) and variance \( \sigma_i^2(k) \):

\[
m_i(k) = \ln(k) + k + \frac{\sigma^2}{2} \ln \left( \frac{1}{k} e^{\sigma^2} \right) + \frac{k-1}{k} \tag{20}
\]

\[
\sigma_i^2(k) = \ln \left( \frac{1}{k} e^{\sigma^2} + \frac{k-1}{k} \right) \tag{21}
\]

Therefore, we can conclude that user SINR in a beam would be the ratio of two log-normal random variables, which is another log-normal random variable

\[
SINR = \frac{10^{\frac{\gamma - \xi}{10}}}{10^{\frac{\gamma - \bar{\xi}}{10}}} \tag{22}
\]

where \( I(k) = 10^{\bar{\xi}} \), \( \xi \) and \( \gamma - \bar{\xi} \) are Gaussian distributed.

The user capacity of each beam, namely \( N \), can be calculated according to (13) and (16). To make the system communication link quality satisfy the required BER, the available \( \frac{E_c}{N_0 + I_0} \) for a user should be more than \( \left( \frac{E_c}{I} \right)_{req} \), which is the required level of energy to interference power density at the level of bit error performance required for digital transmission. However, since IDMA scheme is the first time to be applied in satellite communication, the setting of \( \left( \frac{E_c}{I} \right)_{req} \) has no certain standards. In most systems this implies a BER of \( 10^{-3} \) or smaller.

**D. The Other-beam Interference Factor \( f_{\text{other}} \)**

Combing with the interference calculation model, we will mainly concentrate on the effects of the other-beam interference factor \( f_{\text{other}} \), because \( f_{\text{other}} \) perfectly represents the MAI characteristics of the multi-beam system.

According to (2), the other-beam interference factor is defined as the ratio of the interference power received from the other beams \( I_{\text{other}} \) to the interference power produced by users in local beam \( I_{\text{local}} \). Consequently, with the equations (2) and (3), we can represent the \( f_{\text{other}} \) as:

\[
f_{\text{other}} = \frac{I_{\text{other}}}{I_{\text{local}}} = \frac{1}{(N-1)} \sum_{i=1}^{N-1} \sum_{j=1}^{N} P_i 10^{\frac{\gamma_i}{10}} \frac{G(\psi_{i,j})}{G(\theta_{j,k})} \tag{23}
\]

With a given shadow fading condition and the loss of distance, some literature assume that \( f_{\text{other}} \) is equivalent to a constant, whose value is 0.55. Because \( f_{\text{other}} \) is dependent on the user number per beam, spot-beam antenna pattern, and the number of interfering beams in cellular layout, the assumption that \( f_{\text{other}} \) is a constant is quite inaccurate, as we see from equation (23).

In the interference model, we calculated the \( f_{\text{other}} \) considering 7, 19 and 37 spot-beam layouts respectively, with users distributed uniformly. Assuming \( \left( \frac{E_c}{I} \right)_{req} = 3 \text{dB} \), the number of IDMA-based sub-band is 125. Table I illustrates relationship between the values of \( f_{\text{other}} \) with the ring number of beam layout structure around the nadir beam. In the case of ring 1 only, we find that \( f_{\text{other}} = 0.55 \) after many times iterative computations, which is approximately equal to that reported in literature [11]. Accordingly, considering 19 and 37 beam cluster, converging values of the other-beam interference factor after several times of iterative, are approximately 0.98 and 1.34.

<table>
<thead>
<tr>
<th>Number of iterations</th>
<th>The other-beam interference factor ( f_{\text{other}} ), considering different beam cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5740 1.0477 1.2496</td>
</tr>
<tr>
<td>50</td>
<td>0.5558 0.9867 1.3187</td>
</tr>
<tr>
<td>100</td>
<td>0.5542 0.9846 1.3374</td>
</tr>
<tr>
<td>500</td>
<td>0.5550 0.9847 1.3428</td>
</tr>
<tr>
<td>1000</td>
<td>0.5548 0.9847 1.3427</td>
</tr>
</tbody>
</table>

Fig. 2. The other-beam interference factor \( f_{\text{other}} \) considering different number of users
The value of $f_{\text{other}}$, considering different number of users, is illustrated in Fig. 2. Focusing on the 7 spot-beam layouts, it is clear that the value of the other-beam interference factor is inversely correlated with the number of users while the number of users is less than 50. When the user exceeds the threshold whose value is 50, as Fig. 2 illustrates, the other-beam interference factor isn’t relevant to the number of users.

E. The Outage Probability Analysis

The outage probability calculation is necessary in performance evaluation of any multiuser system [12]. A user is not offered with a service when its $-\frac{E_b}{N_0 + I_0}$-level can’t reach requirement. Based on the definition, the outage probability can be calculated by the different locations using the equation:

$$P_{out} = \mathbb{P}\left[ \frac{E_b}{N_0 + I_0} \leq \frac{E_b}{I_{req}} \right]$$

$$= \mathbb{P}\left[ \frac{I}{P_0} = \frac{(N-1)I(\gamma)}{10^{10}} + \sum_{i=1}^{\text{all-subs}} \sum_{j=1}^{N} G(\psi_{ij}) \right] - \frac{E_b}{I_{req}} = \frac{E_b}{I_{req}}$$

(24)

$$= \mathbb{P}\left[ \frac{I}{P_0} = \frac{(N-1)I(\gamma)}{10^{10}} + \sum_{i=1}^{\text{all-subs}} \sum_{j=1}^{N} \frac{G(\psi_{ij})}{G(\theta_{ij})} \right]$$

(25)

$$\zeta$$ is a function of $\psi_{ij}$ and it can be calculated as

$$\zeta = \frac{B}{R_0} \left[ 10^{\frac{E_b}{I_{req}}} \left( \frac{E_b}{I_{req}} \right) \right]$$

(26)

The outage probability calculation is necessary for performance evaluation of any multiuser system. The total multiple access interference $I$ with Gaussian Distribution can be calculated from equation (22). Therefore, $I/P_0$ can be approximately interpreted as Gaussian distribution. Literature [9] showed that

$$P_{out}(\gamma) = \frac{1}{2} \text{erfc} \left( \frac{\mu - \mu}{\sqrt{2}\sigma^2} \right)$$

(27)

where $\mu$ and $\sigma^2$ are the expectation and variance of $I/P_0$. $P_{out}$ can be obtained from formula (26) and (27) as:

$$P_{out} = \int_{-\infty}^{\infty} P_{out}(\gamma) f(\gamma) \, d\gamma$$

(28)

where $f(\gamma)$ represents the probability density function.

IV. PERFORMANCE EVALUATION

The processing gain is assumed to be 307, with the chip rate 1.2288 K chips/s. Here a multi-beam satellite with 37 spot-beam cluster is considered. Values of system parameters are as follows: Antenna diameter $D = 20m$, and signal bit-energy to thermal noise power spectral density ratio $E_b/N_0 = 10dB$. Supposing radius of satellite trajectory $R_s = 42164km$ and beam radius $r = 400km$. Required bit energy to interference power spectral density is assumed to be $E_b/10 = 3dB$, which is sufficient for satisfactory channel quality for a user in highway with QPSK modulation and rate 1/2 constraint length 9 convolution coding strategy[11].

Fig. 1 gives the interference model of given satellite. Suppose that there are 37 beams, $N_e$ sub-bands per beam, and $L_c$ traffic channels per sub-band in a given satellite system. Therefore, the whole system capacity $C$ can be calculated as: $C = I N_c L_c$. Now $L_c$ is analyzed under the assumption of the above parameters and imperfect power control. For example, a GEO satellite can provide 16 beams $J$ and a beam can provide 13 sub-bands $N_e$.

Fig. 3 gives the multi-beam model, based on desired and interfering mobile locations in beams. The cluster considered in the calculations consists of three tiers of surrounding spot-beams (37-beam cluster) and the users are assumed to be uniformly distributed. For example, the multi-beam model including ten randomly distributed users is established in Fig. 3.
Fig. 4 shows that the analysis of the multi-beam IDMA system comparing the system capacity with PPC and the capacity degradation with PCE. As to the system with PPC, all mobile signal powers are the same and they satisfy the system requirements. If we consider PCE, some mobile users do not meet the necessary condition for good communication link. The simulation result demonstrates that the system capacity can be maximized by increasing the SNR. Furthermore, the larger the power control error, the greater the impact on the system capacity.

In Fig. 7, the system capacity is increased significantly by decreasing the required threshold of \((E_b/I)_\text{req}\). The result shows that the outage probability will be over 10^{-2} when the SNR threshold gets to 4dB.

As shown in Fig. 7, the fact that \((E_b/I)_\text{req}\) can severely influence the outage probability motivates the use of coding, diversity, and interference cancellation as much as receiver complexity in multi-beam IDMA satellite systems. In other words, a proportional increasing in capacity can also be obtained by decreasing the value of \((E_b/I)_\text{req}\). Additionally, the enhancement of the outage probability conducted by imperfect power control reflects the necessity and significance of PCE analysis in multi-beam IDMA satellite systems.

Based on the SINR evolution technique, the system can make accurate estimation of available resource. \(f(\gamma)\) is the iterative factor of power control depending on SINR evolution and illustrate the high efficiency of CBC MUD. Fig. 5 shows that considering the effect of MUD, the system can lead to the accurate system capacity and the high efficiency of anti-interference ability.

In Fig. 6, the outage probability is plotted for different power control error considered in this paper. As we can see, when the power control error increases, the outage probability of IDMA-based sub-band rapidly reduces. Fig. 6 shows that when the outage probability is 10^-2 and the PCE standard deviation (\(\sigma\)) is 1dB, the number of users per sub-band reach to 45, while with the PCE standard deviation (\(\sigma\)) increase to 3dB, the number of users per sub-band dropped nearly to 25. That is to say, the sub-band capacity of IDMA decline by 44.4%.

V. CONCLUSIONS

The interference calculation model is proposed in this paper to evaluate the uplink performance of mobile-satellite multi-beam IDMA system and power control error performance with specified outage probability, \(P_{out}\). The other-cell interference in satellite environment and the SINR evolution is calculated and included in capacity calculations for this particular IDMA multi-beam satellite system, respectively. What’s more, the outage probability is calculated with different power control error values. It is found that the system capacity may decrease significantly owing to the effect of imperfect power control.

However, in a multi-beam model, capacity reduction is not only caused by power control imperfection but also by the interference from the users within the overlapped region which belongs to the nearest adjacent beam. Based on this, the overall system capacity for multi-beam satellite system should be considered by taking the overlapped regions into account.

REFERENCES

[1] K. Li, X. D. Wang, and Li Ping, “Analysis and optimization of interleave-division multiple-access communication systems,”


Gongliang Liu was born in Shandong, P.R. China, in 1979. He received the B.S. degree in measuring & control technology and instrumentations, and the M.Eng. and Ph.D. degrees in electrical engineering in 2001, 2003 and 2007, respectively, all from Harbin Institute of Technology, Harbin, China. He is now with the Harbin Institute of Technology at Weihai as an associated professor. His current interests include satellite communications and underwater sensor networks.

Xinrui Fang was born in Shandong, P.R. China, in 1987. She received the B. S. degree in Communication Engineering from Weifang University in 2011. She is currently working towards her M.Eng. degree in the Communication Engineering department in Harbin Institute of Technology. Her research interests are in the field of wireless communications and networking.