An Improved DOA Estimation Algorithm Based on Wavelet Operator

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Abstract—In array signal processing, direction of arrival (DOA) estimation has typically play a key role. Its task is to find the directions impinging on an array antenna to increase the performance of the received signal. How to let DOA estimation methods are applicable to most environments, has become the key of technique implementation. The previous traditional algorithms can get superior performance for DOA estimation in the rich receiving conditions, but they are not fit for adverse environment such as low SNR, small number of array elements or snapshots, etc. In order to improve the performance of DOA estimation, the new method based on wavelet operator is proposed. Moreover, the method is respectively applied to multiple signal classification (MUSIC) and estimation of signal parameter via rotation invariance techniques (ESPRIT) to get new algorithms named WMUSIC and WESPRIT. Simulation results illustrate that the proposed method can obtain higher resolution and accuracy to compare with traditional MUSIC and ESPRIT in low SNR environment. Undoubtedly, the new method expands the application range of traditional DOA estimation algorithms and has widely practical prospects in future.

Index Terms—first term, second term, third term, fourth term, fifth term, sixth term

I. Introduction

The array signal processing issue gets fast development in the last decades. As one of most critical technology of array signal processing, DOA estimation has been widely applied in many actual domains, such as radar, speech signal processing, mobile communication and so on [1]-[3]. In the application of smart antennas, the main technologies focus on DOA estimation. Generally, the accuracy of DOA estimation will affect the consequence of beamforming immensely [4].

Moreover, the high accurate DOA estimation will bring many significant advantages, such as increasing the capacity, reducing the transmitter power of mobile terminal, resisting multipath effect, decreasing interference from outside, and so on [5]-[7].

Because of the importance of DOA estimation, much of the existing work has been concerted in exploring the high resolution for DOA estimation. The various algorithms are proposed successively in recent years, including capon, high-order cumulate method, propagator method, min-norm, MUSIC and ESPRIT, among which MUSIC and ESPRIT are considered to be high resolution and accuracy methods [8]-[12]. However, much of the previous studies show that the performance of the most DOA estimation algorithms depends on some factor, such as the number of array elements and spatial distribution, the number of snapshots, and signal-to-noise ratio (SNR) of received array signals [13], [14]. Generally, the high resolution and accuracy for estimating DOA should use the larger number of array elements or snapshots, but this will increase greatly equipment overhead and computing complex. Usually, when the communication environment is very poor and we don't employ a larger scale of array elements or snapshots, we can't obtain satisfactory performance of DOA estimation even adopting MUSIC or ESPRIT. But it is difficult to increase arbitrarily the array elements or snapshots in real engineering. Naturally, high performance estimation algorithms of DOA should be based on improving SNR of the receiving signal only in the limited hardware conditions [15-16].

In the light of wavelet transform having the powerful ability on noise reduction, we develop a new signal receive model based on wavelet operator to improve original methods for estimating DOA. In this paper, we propose two new methods called as WMUSIC and WESPRIT which combine wavelet operator with MUSIC or ESPRIT method to estimate DOA. By utilizing wavelet transform to filter lots of noise from the received array signals, DOA of news de-noised signals can be estimated by MUSIC or ESPRIT method. The results show that the proposed WMUSIC and WESPRIT can get superior performance even under weak communication environment. In contrast to the traditional MUSIC or ESPRIT, the proposed method can improve greatly both resolution and accuracy of DOA estimation. Therefore, the problem of DOA estimation can be resolved for WMUSIC and WESPRIT under lower SNR. The following simulation experiments are provided for certifying the effectiveness of the new methods.

The remaining parts of this paper are organized as follows: Section II gives an overview of array signal received model. In the section III, we present the classical

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MUSIC and ESPRIT model. The new proposed methods based on wavelet operator named WMUSIC and WESPRIT are given in section IV. In the section V, the simulation experiments for DOA estimation are performed to verify the proposed WMUSIC and WESPRIT methods and compare respectively them against the classical MUSIC and ESPRIT. Finally, we give conclusions of this paper in section VI.

II. Array Signal Received Model

Most DOA estimation methods which based on signal processing rely on certain assumptions made on the received array signals. In the section, we take the 2D pattern to describe briefly the model of DOA as follows. As shown in Figure.1, we considered a scenario with D emitting sources from different directions with narrowband property, and we have a linear antenna array with M element to receive each emitting signal. Therefore the signal of direction θ whose the i element is expressed as $e^{j(2\pi/\lambda)d_i \sin\theta}$ and the output signals of received antenna array $x(t)$ at the k sampling time can be showed as in (1):

$$x(k)=\sum_{i=1}^{d} a_i \cdot s_i(k)+n(k)=A\cdot s(k)+n(k) \tag{1}$$

where $A=[a(\theta_1),a(\theta_2),\cdots,a(\theta_k)]$, $s(k)=[s_1(k),s_2(k),\cdots,s_D(k)]^T$ and λ is the signal wavelength. Moreover $n(k)=[n_1(k),n_2(k),\cdots,n_M(k)]$ is assumed to be spatially white Gaussian noise with variance σ_n^2. Generally, the designed antenna arrays require that $M>D$.

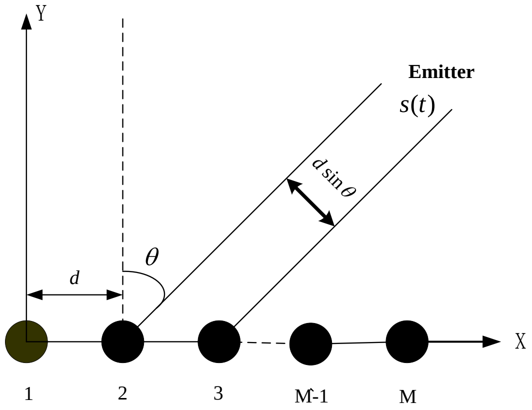

Fig. 1. The M element antenna array with θ arriving signal

III. Classic DOA Estimation Algorithms

A. MUSIC Algorithm

As Eqs.1, the array output is then confined to an M-dimensional subspace of the complex subspace, which is spanned by the steering vectors. The MUSIC algorithm [17] is based on this kind of subspace analysis method. Here, we calculate the covariance matrix R of the received signal $x(k)$ as follows

$$R_{xx}=E[x(k)*x^H(k)]=A\cdot R_{SS}\cdot A+\sigma_n{}^2\cdot I \tag{2}$$

where $R_{ss}=E[s(k)*s^H(k)]$ is the covariance matrix of input signals. Under the condition that the number of signal sources D has been known, there should be D eigenvalues shown as $\lambda_1\ge\lambda_2\ge\ldots\ge\lambda_D$ and corresponding eigenvector shown as e_i, $i=1,2,\cdots,M$. Then the signal subspace and noise subspace are respectively expressed as $U=e_i, i=1,2,\cdots,D$ and $E_N=e_i$, $i=D+1,D+2,\cdots,M$.

To get estimation of DOA $\theta_1,\theta_2,\cdots,\theta_D$, the pseudo-spectrum of MUSIC algorithm is given as

$$P_{MUSIC}=\frac{1}{\left|a(\theta)^H * E_N * E_N^H * a(\theta)\right|} \tag{3}$$

B. ESPRIT Algorithm

The ESPRIT algorithm [18] is very popular technique and widely applicable to a particular class of received arrays whose geometry is shift invariant. Due to no need to exhaustive search, the ESPRIT algorithm has many advantages such as lower computational complexity and less storage resources compared to MUSIC algorithm. The key aim of the ESPRIT is to utilize the rotational invariance of the signal subspace which could be recreated by two sub-arrays. The algorithm can be specifically described as follows.

Considered M array elements that includes two similar subarrays with M-1 array elements each, the first and the second subarrays which are displaced by standard interval d could be expressed by $1,2,\cdots,M\text{-}1$ and $2,\cdots,M$. The received signals for each subarray are given by

$$x(k)=\begin{bmatrix} A_1 \\ A_1\bullet\varphi \end{bmatrix} * s(k)+\begin{bmatrix} n_1(k) \\ n_2(k) \end{bmatrix} \tag{4}$$

where $\varphi=\mathrm{diag}\left\{e^{jkd\sin(\theta_1)},e^{jkd\sin(\theta_2)},\cdots,e^{jkd\sin(\theta_D)}\right\}$ is $D*D$ diagonal unitary matrix.

Constructed two matrices E_1 and E_2 stand for signal subspace. Since the whole antenna arrays E can be decomposed into E_1 and E_2. Due to the arrays are related, the eigenvectors of subspaces are associated with a unique nonsingular transformation matrix ϕ such that

$$E_1\cdot\phi=E_2 \tag{5}$$

Similarly, there must exist a unique nonsingular transformation matrix T such that

$$\begin{cases} E_1=A\cdot T \\ E_2=A\cdot\phi\cdot T \end{cases} \tag{6}$$

Then we can get that

$$T\cdot\phi\cdot T^{-1}=\varphi \tag{7}$$

where the eigenvectors $\lambda_1,\lambda_2,\cdots,\lambda_D$ of ϕ is diagonal elements of φ. The DOA are calculated as

$$\theta_i=\sin^{-1}(\arg(\lambda_i)/(k\cdot d)) \tag{8}$$

Obviously, the ESPRIT algorithm has advantages on calculation efficiency when it used to estimate DOA, but it also exist unstable shortcomings.

IV. DOA Estimation Algorithm Based on Wavelet Operator

As many research results have demonstrated, both MUSIC and ESPRIT algorithm can get good performance on DOA estimation when the communication environment is normal (SNR is not very low). However, the two algorithms will gradually deteriorate when the SNR continues to decrease. To solve this problem, we utilize the wavelet operator to denoise for improving the SNR of received signals in this section. And then it is applied to the MUSIC and ESPRIT algorithm named WMUSIC and WESPRIT algorithm. The algorithm can be specifically described as follows.

According to Mallet theory [19], the array signals $x(k)$ in (1) are decomposed to T layer by discrete wavelet. Then the characteristics coefficients $q_{j,t}$ and $p_{T,t}$ of the array signals in the T scale decomposition model can be denoted by

$$\begin{cases} q_{j,t}=\langle x(k),\varsigma_{j,t}(k)\rangle \\ p_{T,t}=\langle x(k),\xi_{j,t}(k)\rangle \end{cases} \quad t\in Z \tag{9}$$

where $\xi(k)$ and $\varsigma(k)$ are respectively scaling function and wavelet function. Furthermore, p_T is the approximation coefficients of T scale, and mainly presents low-frequency property of the array signals. Correspondingly, q_j is the detail coefficients of j scale, and mainly presents high-frequency property of the array signals. t is the time shift exponent.

The array signals $x(k)$ can be refresh expressed by a linear combination with p and q such that

$$x(k)=\sum_t p_{T,t}\cdot\xi_{M,t}(k)+\sum_{j=1}^{T}q_{j,t}\cdot\varsigma_{j,t}(k) \tag{10}$$

Most of white Gaussian noise is included in the components with high-frequency property. The $\xi_{T,t}$ and $\varsigma_{j,t}$ can be obtained by mother wavelet $\xi(k)$ and scaling function $\varsigma(k)$ after extending such as

$$\begin{cases} \xi_{T,t}(k)=2^{-T/2}\cdot\xi(2^{-T}k\text{-}t) \\ \varsigma_{j,t}(k)=2^{-j/2}\cdot\varsigma(2^{-j}k\text{-}t) \end{cases} \tag{11}$$

Thence, the low-frequency coefficient $p_{j,t}$ and the high-frequency coefficient $q_{j,t}$ of j scale can be gotten by firstly convoluting through low-pass filter $h(m)$ and high-filter $g(m)$, then executing sampling processing such that

$$\begin{cases} q_{j,t}=\sum_m h(m\text{-}2t)\cdot p_{j\text{-}1,t} \\ p_{j,t}=\sum_m g(m\text{-}2t)\cdot p_{j\text{-}1,t} \end{cases} \tag{12}$$

According to wavelet denoising principle, most of noise which is included in can be removed by choosing the appropriate threshold of high-filter $g(m)$ to filter high-frequency composition. Based on the empirical formal, the threshold ε can be used as

$$\varepsilon=\sigma_{thr}*\sqrt{2\log(\kappa)} \tag{13}$$

where σ_{thr} is the variance of high-frequency composition in the highest scale M, and κ is the length of high-frequency composition. Completed the filtering process for high-frequency composition, the high-frequency coefficient $q_{j,t}$ is renewed by new coefficient $\tilde{q}_{j,t}$.

Then the low-frequency coefficient $p_{j\text{-}1,t}$ of j-1 scale is reconstructed by $p_{j,t}$ and $\tilde{q}_{j,t}$ such as

$$\tilde{p}_{j\text{-}1,t}=\sum_t \bar{h}(m\text{-}2t)\cdot p_{j,t}+\sum_t \bar{g}(m\text{-}2t)\cdot\tilde{q}_{j,t} \tag{14}$$

where the $\bar{h}(m)$ and $\bar{g}(m)$ are synthesis filter which match with $h(m)$ and $g(m)$, respectively.

Finally, the original array signals $x(k)$ can be replaced by denoising array signals $\tilde{x}(k)$ which is single-reconstructed by the low-frequency coefficient $\tilde{p}_{j\text{-}1,t}$ and the high-frequency coefficient $\tilde{q}_{j,t}$. Then the processed array signals $\tilde{x}(k)$ is used to MUSIC and ESPRIT to build new algorithms named WMUSIC and WESPRIT.

V. Simulation Results and Analysis

In order to certify the effectiveness of proposed algorithms on DOA estimation, we design three simulations to compare the performance of MUSIC, ESPRIT, WMUSIC and WESPRIT, respectively.

A. Resolution Test

In this simulation example, we provide numerical examples to compare the resolution of forgoing algorithms, respectively. Under the poor receiving conditions (low snapshot, low array elements and low SNR environment), we consider the same simulation situation that there are 6 uniform linear array elements (M=6) with 2 near narrowband sources (D=2) impinging on it from DOA ($\theta_1=5^\circ,\theta_2=8^\circ$), the number of snapshot is 300 in each interval, and the SNR is 3dB.

Moreover, the wavelet base is ‘db10’ and decomposition scale T is 2 in the WMUSIC and WESPRIT.

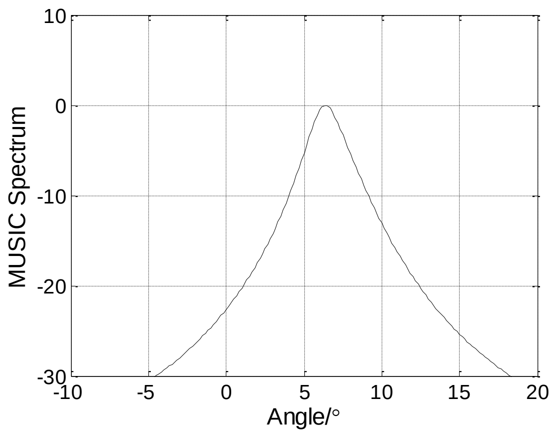

Fig. 2. MUSIC spectrum on resolution test

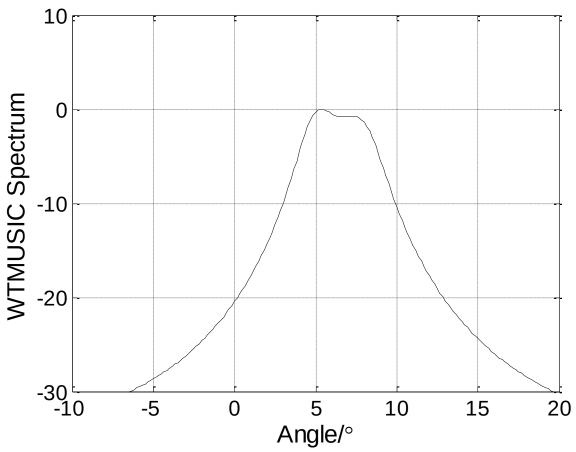

Fig. 3. WMUSIC spectrum on resolution test

Fig. 2 displays the DOA estimation result of MUSIC, the spectrum obviously shows that it is not able to distinguish between 5° and 8°. Moreover, the same situation occurs on the test of ESPRIT that the result is non-robust. Compared with MUSIC and ESPRIT, the proposed WMUSIC and WESPRIT algorithm have a much better performance. As shown in the Fig. 3, the near desired angles 5° and 8° can be distinguished clearly by WMUSIC algorithm, and the real estimation angles are 4.83° and 7.34°. Analogously, the two angles can be computed stably by WESPRIT, and the corresponding estimation angles are 4.67° and 7.45°.

From the above simulations results, it illustrates that the proposed WMUSIC and WESPRIT can get higher resolution than MUSIC and ESPRIT when the communication conditions is very poor. Although there are still some deviations on DOA estimation, the two algorithms have obtained certain progress in resolution, and they have analogous accuracy.

B. Accuracy Test

In the first simulation example, we use the four algorithms for detecting the accuracy of multiple DOA estimation, and the performance of each algorithm can be determined by the comparison of average deviation of DOA estimation. Under the same simulation conditions, the number of uniform linear array elements is set to 8 (M=8) with 4 narrowband sources (D=4), the angles are θ_1=-15°, θ_2=-5°, θ_3=5°, θ_4=15°, the number of snapshot is 300 in each interval, Moreover, the wavelet base is ‘db10’ and decomposition scale T is 2 in the WMUSIC and WESPRIT (SNR=0).

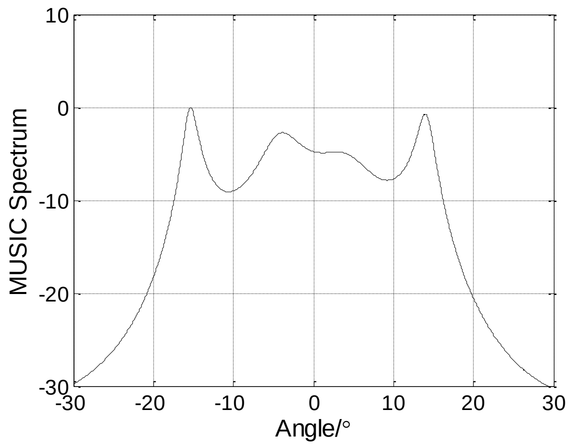

Fig. 4. MUSIC spectrum on accuracy test

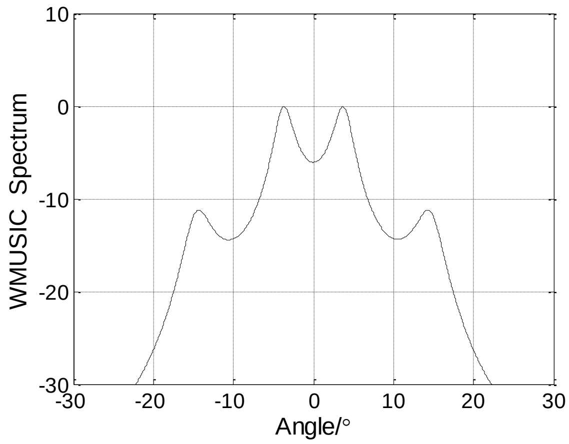

Fig. 5. WMUSIC spectrum on accuracy test

Fig. 4 and Fig. 5 respectively give the DOA estimation of four angles by the MUSIC and WMUSIC algorithms on the poor communication condition (SNR=0). Apparently, the MUSIC algorithm cannot get significant spectrum peaks at the -5 and 5 degrees DOA, but the WMUSIC is able to obtain visible spectrum peaks at all four DOA. Then, the similar test is executed on the rest algorithms. Compared with ESPRIT, the WESPRIT is more responsive to noise to get higher accuracy. Obviously, this example fully illustrates that the DOA estimation algorithm based on wavelet operator can obtain better performance in the adverse communication environment.

C. Comparison Test

In order to fully compare the performance, we test the root mean square error (RMSE) of DOA estimation for the four algorithms, i.e. MUSIC, ESPRIT, WMUSIC and WESPRIT. By repeating each test under varied SNR (-6dB~9dB), the average RMSE results of DOA estimation for the four angles as simulation B are averaged over 30 times as Fig.6. With the increase in SNR, the average RMSE of each algorithm decline accordingly for DOA estimation. Compared against MUSIC and ESPRIT algorithm, the proposed WMUSIC and WESPRIT algorithm can obtain higher accuracy in lower SNR (-6,-3,0,3). However, when the SNR reach enough (SNR>6), the advantage of the new algorithms may be lost, even disadvantage appears sometimes. To analyze the reason, we conclude that the wavelet denoising in the WMUSIC and WESPRIT makes the effective component missing, so the performance is even worse than the traditional MUSIC and ESPRIT in the high SNR environment.

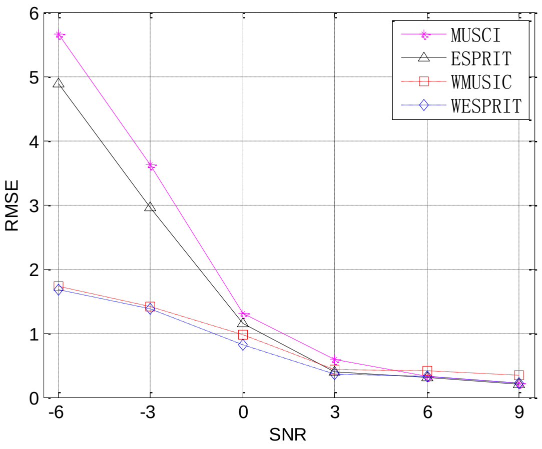

Fig. 6. Average RMSE (in degrees) versus SNR of four algorithms

VI. Conclusions

In this paper, we have attempted a new method based on wavelet operator to solve the DOA estimation problems on poor communication conditions, hence the corresponding algorithms named WMUSIC and WESPRIT are proposed. The simulation results show that the new algorithms can greatly improve the performance on resolution and accuracy to compare with traditional MUSIC and ESPRIT in low SNR environment. But the new method is not recommended under good communication conditions because of having not significant advantage. The proposed method can be widely applied in the design of smart antenna system.

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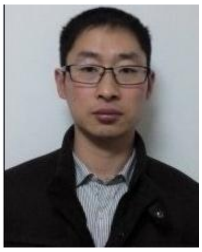

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