Optimization and Implementation for the Modified DFT Filter Bank Multicarrier Modulation System

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Abstract—The modified Discrete Fourier Transform (MDFT) filter bank is a Nearly Perfect Reconstruction (NPR) system based on pseudo-QMF filter bank. In this paper, we introduce an optimized algorithm to implement the MDFT filter bank based multicarrier modulation system. The system based on this optimization algorithm has the low error rate and system delay. To verify the designed system we have compared the MDFT multicarrier modulation system with the OFDM modulation system. The results show that at high speed communication the data error rate in the MDFT filter bank multicarrier modulation system is clear improved in comparison with OFDM systems.

Index Terms—MDFT filter bank, NPR, Error rate, Optimization algorithm

I. INTRODUCTION

Digital filter banks have been widely applied and are particularly useful in subband coding and multiple carrier data transmission [1]-[6]. Historically, there are two classes of modulated filter banks, namely DFT filter banks and cosine filter banks. Because the DFT filter bank provides no mechanism to cancel adjacent spectrum aliasing components caused by subsampling the subband signals, this kind of filter bank is considered as nonsuitable for subband coding. This disadvantage has been overcome by introducing a certain modification which leads to the modified DFT (MDFT) filter bank [7]-[12].The modified discrete Fourier transformation is a high efficient nearly perfect reconstruction system based on pseudo-QMF filter bank[8,12]. Fig. 1 shows the polyphase decomposition structure for MDFT filter bank based multicarrier modulator. It consists of the analysis and synthesis filter banks.

The MDFT filter bank is a complex modulated M channel filter bank based on the DFT. There are two steps of decimation. The first step of decimation is the $M/2$ (M must be even) factor. The second step of decimation is the factor two sampling. After total M-decimation, each channel will be divided into two parts, i.e. real part and imaginary part. The time delay is introduced either

into the imaginary part or into the real part. Such a time delay occurs between real part and imaginary part among adjacent channels interchangeably. This structure is able to cancel exactly the adjacent spectrum aliasing. The nonadjacent spectrum aliasing is suppressed by an appropriately low stop band gain of the analysis and synthesis filters.

II. MDFT FILTER BANKS

In order to explain the mechanism to cancel adjacent spectrum aliasing, we first consider the M channel DFT filter bank system. The analysis filter bank can be described by

$$H_i(z) = H(zW_M^i) \ , \ i = 0,1,2,...M-1, \qquad (1)$$

where $W_M^i = e^{-j\frac{2\pi i}{M}}$. The synthesis filter bank is

$$G_i(z) = M \cdot H(zW_M^i) \ , \ i = 0,1,2,...M-1. \qquad (2)$$

The synthesis filter bank output signal, namely $\hat{X}(z)$ can be written as

$$
\begin{aligned}
\hat{X}(z) &= \frac{1}{M}\sum_{i=0}^{M-1} G_i(z)\sum_{k=0}^{M-1} H_i(zW_M^k)X(zW_M^k) \\
&= \sum_{l=0}^{M-1} X(zW_M^l)\sum_{i=0}^{M-1} H(zW_M^i)H(zW_M^{i+1}).
\end{aligned}
\qquad (3)
$$

Supposing the prototype filter to be band-limited to $2\pi/M$, namely

$$H(e^{j\omega}) \approx 0, \ 2\pi/M \le \omega \le \pi,$$

Eq. (3) can be rewritten as

$$
\begin{aligned}
\hat{X}(z) &\approx \sum_{i=0}^{M-1} H(zW_M^i)\left[\sum_{l=-1}^{1} H(zW_M^{i+l})X(zW_M^l)\right] \\
&= \frac{1}{M}\sum_{i=0}^{M-1} G_i(z)[H_{i-1}(z)X(zW_M^{-1}) + H_i(z)X(z) \\
&\quad + H_{i+1}(z)X(zW_M^{+1})],
\end{aligned}
\qquad (4)
$$

where the product $G_i(z)H_i(z)$ describes the transfer function of the ith channel, $G_i(z)H_{i-1}(z)$ and $G_i(z)H_{i+1}(z)$ describes the adjacent-spectrum aliasing. The corresponding frequency performance scheme for ith channel is depicted in Fig. 2.

(a) The transmitter(the synthesis filter banks)   (b)The receiver (the analysis filter banks)
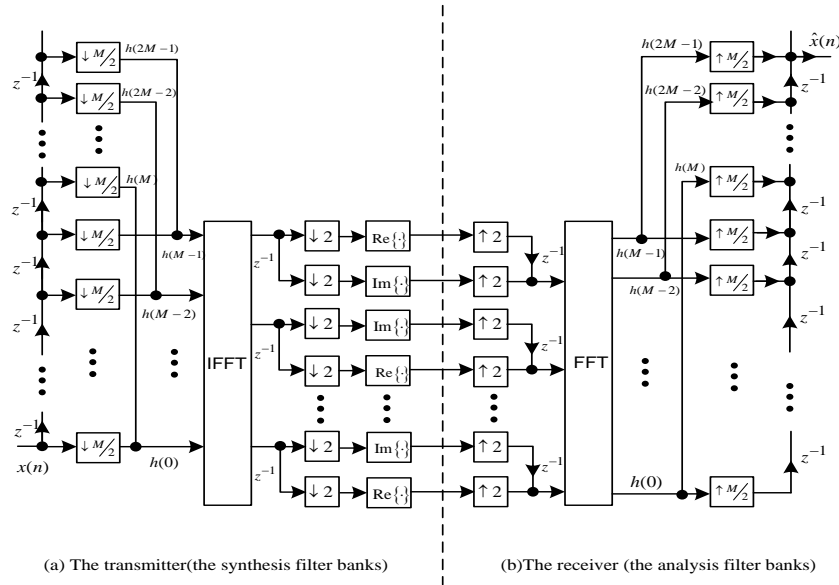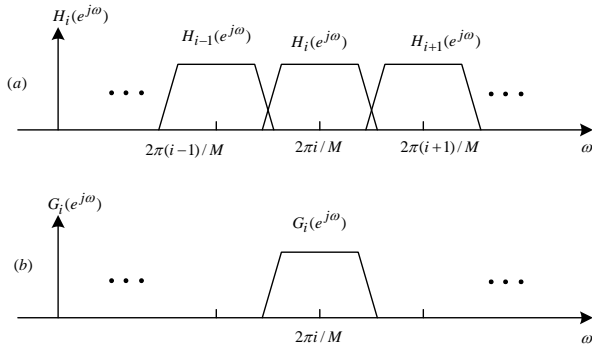
Figure 1.   Polyphase decomposition structure for the MDFT filter bank based multicarrier modulator

Figure 2.   Frequency performance for i-th channel: (a) Components of analysis filter bank; and (b) Components of synthesis filter bank.

In Fig. 1, the real subband signal can be described as

$$
\begin{aligned}
X_i^{(R)}(z) &= \frac{1}{M} Z[h(n)\,\mathrm{Re}\{x(n)W_M^{in}\}] \\
&= \frac{1}{M} Z[h(n)\frac{1}{2}x(n)\{W_M^{in}+W_M^{-in}\}] \\
&= \frac{1}{2M} H(z)[X(zW_M^{-i})+X(zW_M^{i})] \\
&= \frac{1}{2M} H_i(z)[X(z)+X(zW_M^{2i})].
\end{aligned}
\tag{5}
$$

The ith channel output signal in synthesis filter bank reads

$$
\hat{X}_i^{(R)}(z) = \frac{1}{2M}G_i(z)H_i(z)[X(z)+X(zW_M^{2i})],
\tag{6}
$$

where $X(zW_M^{2i})$ is the mirror spectrum lying symmetrically with respect to $X(z)$ about the center frequency of the ith channel. Supposing that the original signal spectrum $X(e^{j\omega})$ lies between i and $i+1$ channel, and considering the ith channel aliasing spectrum, the real-part signal in ith channel is

$$
\begin{aligned}
\hat{X}_i^{(R)}(z) &= \frac{1}{2M}G_i(z)[H_i(z)\cdot X(z)+H_i(z)\cdot X(zW_M^{2i}) \\
&\quad + H_i(zW_M^{-1})X(zW_M^{-1}) \\
&\quad + H_i(zW_M^{+1})X(zW_M^{2i+1})].
\end{aligned}
\tag{7}
$$

In the same way, the imaginary-part signal in i-th channel

$$
\hat{X}_i^{(I)}(z) = \frac{1}{2M}G_i(z)H_i(z)[X(z)-X(zW_M^{2i})].
\tag{8}
$$

Because the imaginary-part subband signal has the phase offset of $M/2$ in i-th channel, the signal must be multiplied by factor $W_M^{lM/2}$. In other words, it is equal to $+1$ if $l=0$ and equal to -1 if $l=\pm1$. So we obtain the imaginary part in the i-th channel as

$$
\begin{aligned}
\hat{X}_i^{(I)}(z) &= \frac{1}{2M}G_i(z)\{H_i(z)X(z)- \\
&\quad H_i(z)X(zW_M^{2i})-H_i(zW_M^{-1})X(zW_M^{-1})+ \\
&\quad H_i(zW_M^{+1})X(zW_M^{2i+1})\}.
\end{aligned}
\tag{9}
$$

Adding (7) and (9) yields

$$
\hat{X}_i(z) = \frac{1}{M}G_i(z)[H_i(z)X(z)+H_i(zW_M^{+1})X(zW_M^{2i+1})].
\tag{10}
$$

Furthermore, using the same way, the output signal in $(i+1)$-th channel can be formulated like

$$
\begin{aligned}
\hat{X}_{i+1}(z) &= \frac{1}{M}G_{i+1}(z)[H_{i+1}(z)X(z) \\
&\quad -H_{i+1}(zW_M^{-1})X(zW_M^{2i+1})].
\end{aligned}
\tag{11}
$$

Adding (10) and (11), and using (1) and (2), we get the reconstruction signal

$$
\hat{X}(z) = X(z)[H^2(zW_M^{i})+H^2(zW_M^{i+1})].
\tag{12}
$$

For the perfect reconstruction only if

$$H^2(zW_M^i) + H^2(zW_M^{i+1}) = 1.$$

Namely, the prototype filter is approximately power complementary.

III. OPTIMIIZATION AND PROTOTYPE FILTER DESIGN

A. Prototype Filter Design

In the MDFT filter bank, the analysis filter bank and the synthesis filter bank are derived from the linear phase FIR prototype filter by uniform frequency shift. So the design of MDFT filter bank can be restricted to the prototype filter design. The following distortions should be taken into account in the designing process.

1) Aliasing distortion: it is produced by the decimator of the analysis filter bank.

2) Amplitude and phase distortion: the reason is that no all-pass function exists in the pass band of analysis and synthesis filter bank. In the practice design the following limitations should be taken into account:

a) The prototype filter should be linear phase, i.e h(n)=h(L-1-n),where L means the filter length.

b) The transfer function should be able to provide high stop band attenuation.

c) The transform function should be able to realize an approximation of the power complementary property. That means: $H^2(zW_M^i) + H^2(zW_M^{i+1}) = 1$.

d) The number of coefficients of the transfer function should be minimized.

B. Square Root Raised-Cosine Function

The raised-cosine function $H^2(e^{j\omega})$ which is equal to unity in the pass band $|\omega| \leq (1-r)\omega_c$ and zero in the stop band $(1+r)\omega_c \leq |\omega|$ .where r $(0 < r \leq 1)$ is the roll-off factor, ω_c is the cut-off frequency. The raised-cosine function is

$$H^2(e^{j\omega}) = \frac{1}{2} + \frac{1}{2}\cos(\frac{\pi}{2r}(\frac{\omega}{\omega_c} + r - 1)) . \quad (13)$$

With $\omega_c = \pi/M$, $\omega = \omega - 2\pi k/M$ and $r = 1$, we obtain

$$H^2(e^{j(\omega - \frac{2\pi k}{M})}) = \frac{1}{2} + \frac{1}{2}(-1)^k \cos(\frac{\omega M}{2}). \quad (14)$$

If M is even

$$\sum_{k=0}^{M-1} H^2(e^{j(\omega - \frac{2\pi k}{M})}) = \sum_{k=0}^{M-1}\left(\frac{1}{2} + \frac{1}{2}(-1)^k \cos(\frac{M\omega}{2})\right) = \frac{M}{2} \quad (15)$$

The adjacent channel transfer function is power complementary. Calculating square root for (13) and using

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x), \quad (16)$$

the square root raised-cosine function is

$$H(e^{j\omega}) = \cos(\frac{\pi}{4r}(\frac{\omega}{\omega_c} + r - 1)) , \ 1 - r \leq \frac{|\omega|}{\omega_c} \leq 1 + r. \quad (17)$$

Finally, we get the square root raised-cosine function transfer function as

$$h(n) = \frac{\frac{4rn}{M}\cos\left(\frac{\pi(1+r)n}{M}\right) + \sin\left(\frac{\pi(1-r)n}{M}\right)}{\left(1 - \left(\frac{4rn}{M}\right)^2\right)\pi n} , \quad (18)$$

where $-\infty < n < \infty$.

According to the power complementary condition, square root raised-cosine function has better performance than raised-cosine function. From (18), we know that the prototype function has infinite impulse response. From the practical respect of view we can get the causal finite impulse response by truncating and time shifting. However, the truncated finite impulse response is no more power complementary. In other words the amplitude distortion always exists. But we could select the appropriate roll-off factor and the filter length to minimize it.

C. Optimization Algorithm

Our earlier study shows that for the QPSK baseband modulation the MDFT filter bank multicarrier modulation system has the similar performance with the OFDM system, even though the prototype filter length is increased. But with the increment of prototype filter roll-off factor the system error rate can be decreased. For the 16QAM and 64QAM, the MDFT filter bank multicarrier modulation system has better performance than the OFDM system. For the 16QAM, the cost is the increment of prototype filter length. For the 64QAM, the MDFT filter bank multicarrier modulation system has better performance than the OFDM system even though the filter length is unchanged. Otherwise, for the 16QAM and 64QAM, increasing the prototype filter roll-off factor also can decrease the system error. However, increasing the length of prototype filter means the increment of system delay, and increasing the roll-off factor of prototype filter means the efficiency reduction of spectrum utilization.

In order to get the better system performance, we will optimize the prototype filter to minimize the system overall delay and improve the efficiency of spectrum utilization.

Suppose that the prototype filter length is L=N*M, where N is the length factor and M means the number of MDFT filter bank channel. The target of optimization process is to find the minimal N to minimize the overall system delay, while keeping the system performance. In the optimization process, we have two variables N (the length factor) and r (the roll-off factor of the prototype filter).The objective function is $h(n)$ ($h(n)$ is the

prototype filter coefficients).The constrain condition is that the error rate in the MDFT filter bank system based modulation system should be less than the error rate of the OFDM system. The optimization flow chart is showed in Fig. 3. The optimization process is to find the optimized square root raised-cosine function in respect with variables *N* and *r*, with the MDFT based modulation system has minimal system delay.
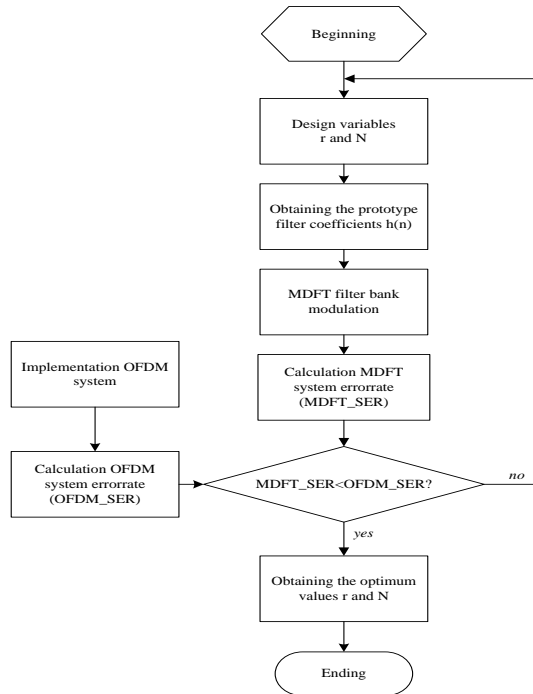

Figure 3. Optimization flow chart for MDFT filter bank multicarrier modulation system.

According to the above, design of the prototype filter come down to the follow optimization problem:

$$\min_{N,r}\{h(n)\}$$
$$s.t, \ MDFT\_SER - OFDM\_SER < 0$$

During the optimization process we take the difference of error rate between MDFT multicarrier modulation system and OFDM system as criterion (*MDFT_SER − OFDM_SER*). With the increasing SNR, the difference of error rate between the two systems will gradually decrease. The error rate of two systems will tend to be equal when the SNR increases to a certain degree. So, in our optimization process we firstly search the position in the SNR-coordinate, where the error rate of two systems is equal. Then, taking this SNR-point as reference we look at the error rate values at the SNR-point before the reference point, and check if the error rate of the MDFT based modulation system is larger than the error rate of OFDM system at this SNR-point. If the condition is not met, we increase the N and r values and calculate again the error rate for both modulation systems and compare them again, until the condition is satisfied. At last we get the values of variable N and r, with them we get the prototype filter.
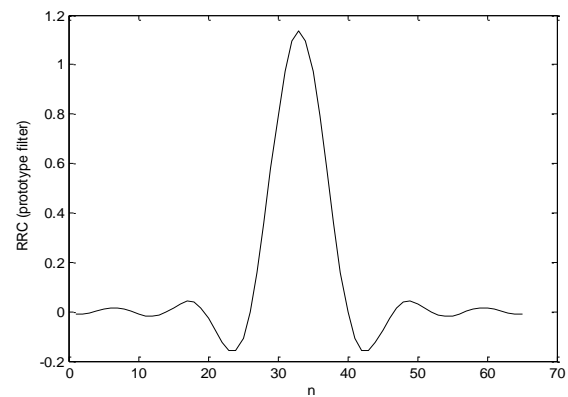

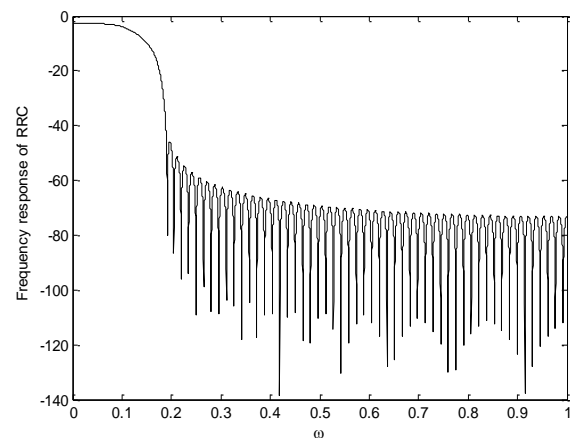Figure 4. Time-domain characteristic of square root raised-cosine function (L=128, r=0.5)

Figure 5. Frequency characteristic of square root raised-cosine function (L=128, r=0.5)
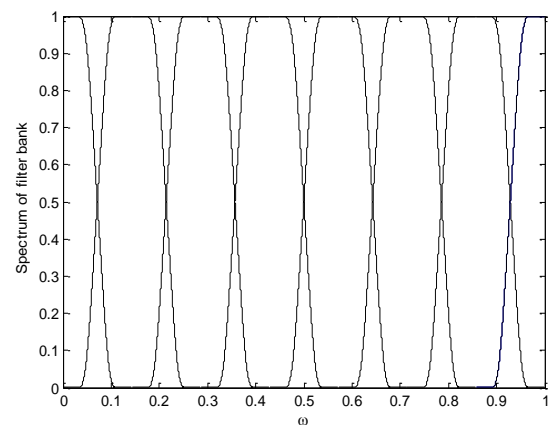

Figure 6. Power complementary for square root raised-cosine function

Here, we want to point out that the initialization values of N and r have clear influence on the results. Based on our extensive experiments we found that N=16 and r=0.5 are reasonable initialization values. Fig. 4 and Fig. 5 show the square root raised-cosine function obtained by the initialization values N and r in the time-domain and the frequency-domain, respectively. Its highest stop band attenuation is -40dB. The frequency response of an 8-channel MDFT filter bank based on this prototype function is showed in Fig. 6, where the power

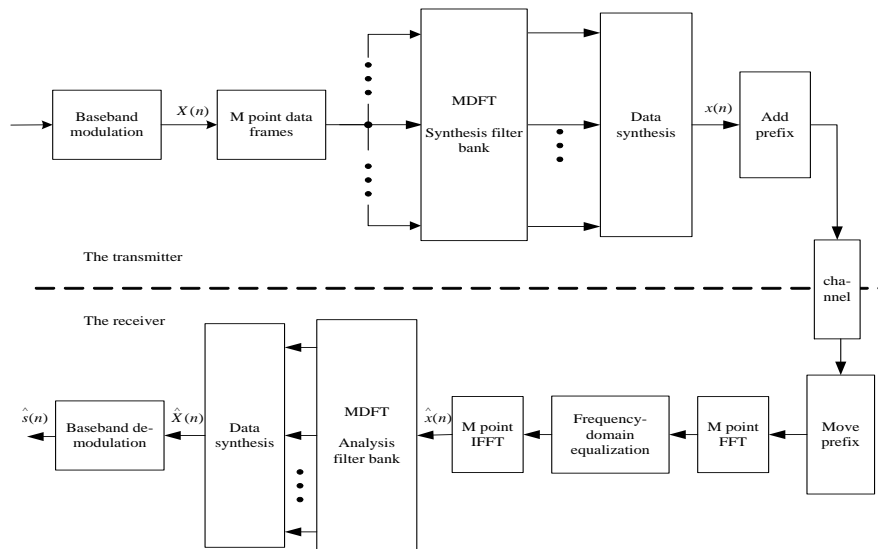complementary property of square root raised-cosine   function is depicted.

Figure 7.   Implementation structure of MDFT filter bank multicarrier modulation system.

IV.   MDFT FILTER BANK MULTICARRIER MODULATION SYSTEM

A.   MDFT Modulator

The polyphase decomposition structure for MDFT filter bank multicarrier modulator is showed in Fig. 1. The transmitter is composed of synthesis filter bank, and the IFFT algorithm is used to implement IDFT. The purpose of transmitter is to modulate each subcarrier with input signal using synthesis filter bank (IFFT). The receiving terminal is composed of analysis filter bank, where we use the FFT algorithm to implement it, because the purpose of the receiver is to reconstruct the input signal. With polyphase structure we have possibility to use FFT algorithm to implement the MDFT filter bank multicarrier modulation system.

In order to complete the polyphase decomposition, the baseband modulated signal $x(n)$ will be shifted and then decimated by sampling factor $M/2$ ($M/2$ is used to produce the factor $(-1)^k$). The resulted polyphase decomposition signal will be IFFT transformed, then divided into real part and imaginary part, which is a key step to cancel adjacent spectrum aliasing. In the receiver, the inversed operations will be carried. In this system implementation process, the most important point is to buffer the inputted and outputted signal when implementing the system because of inherent delay in the system.

B.   MDFT Filter Bank Multicarrier Modulation System

Fig. 7 shows the MDFT filter bank multicarrier modulation system structure. Firstly, the input signal will be baseband modulated to map the input signals into the complex samples. In the simulation we use QPSK, 16QAM and 64QMA as baseband modulation. Secondly, baseband modulated samples will be processed by the filter bank modulator to get the multicarrier modulated signal that will be added with cycle prefix before transmitted to channel. The cycle prefix is the copy version of the last part of the current output data frame and for the channel equalization. In this paper, the length of cycle prefix is the last twenty samples and the resulted samples stream with CP will be transmit into channel. The channel selected here is based on 3GPP TS 25.104 and added with AWGN in order to simulate the practical communication environment. At the receiver, the reverse processing will be done to reconstruct the input signal streams.

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V.   SIMULATION RESULTS

Under multipath channel, we use a 32-channel filter bank. The baseband modulations are 16QAM and 64QAM. Detailed simulation parameters are listed in Table I.

TABLE I. SIMULATION PARAMENTERS

PARAMETERS	MDFT	OFDM
Subcarriers	32	32
Cyclic prefix	20	20
Baseband modulation	16QAM 64QAM	16QAM 64QAM
Bandwidth	10 MHz	10MHz
Equalization	Zero-Forcing	Zero-Forcing
Prototype Function	SRRC	Rectangular
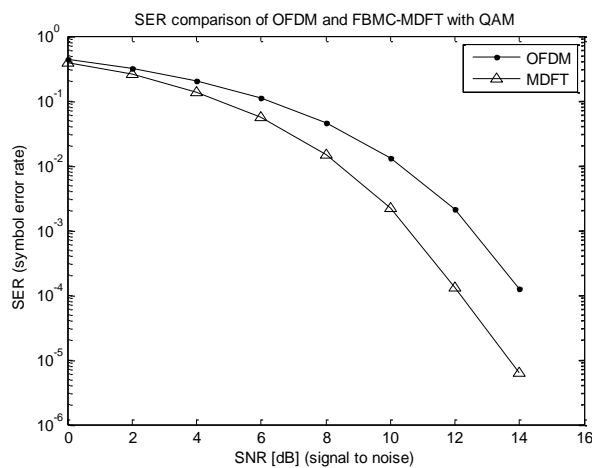
Figure 8. The optimized error rate comparison between MDFT filter bank system and OFDM system for 16QAM (N=7, r=0.2)
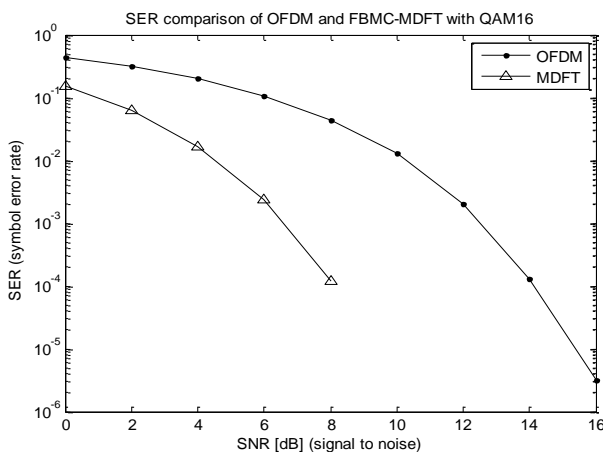
Figure 9. Error rate comparison between OFDM system and MDFT system (16QAM, N=16, r=0.5)

For the 16QAM, we get the optimization parameters of N=7 and r=0.2. With these parameters of N and r we can achieve the minimal system delay with a reasonable performance improvement for the error rate showed in Fig. 8. From this figure, we can see that the MDFT filter bank multicarrier modulation system is stronger against the symbol errors than the OFDM system. If we want to further improve the system performance we can simply increase the length factor N to 16. In this case, as showed in Fig. 9, the error rate of MDFT modulation system is clear better than OFDM system. This means that the longer are the filter bank coefficients, the better error robustness the MDFT filter bank multicarrier modulation system has. Nevertheless, this costs the system delay.
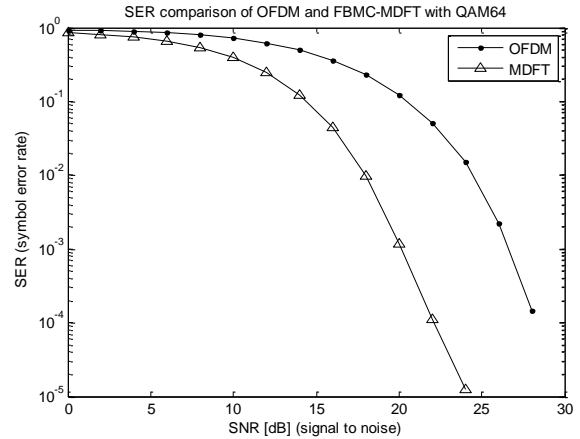
Figure 10. Error rate comparison between MDFT filter bank system and OFDM system (64QAM N=4, r=0.5)

For the 64QAM, we get the optimized parameter of N=4 and r=0.5. Fig. 10 shows the error rate comparison between the MDFT filter bank multicarrier modulation system and the OFDM system for 64QAM at the optimized parameters. From figure 10 we can see that the symbol error rate of the MDFT filter bank multicarrier modulation system is much lower than the OFDM system.

Comparing Fig. 8 and Fig. 10, it can be seen that the error rate improvement for 64QAM is larger than for 16QAM. The system delay for 64QAM is also lower. So, we can say that the MDFT based modulation system is more suitable for high speed data transmission, because 64QAM is faster than 16QAM.

VI. CONCLUSIONS

In this paper we introduced an optimization algorithm to design the prototype function for the MDFT filter bank based multicarrier modulation system. In comparison with OFDM system, the designed MDFT filter bank modulation system has clear performance improvement in respect with system error rate. In particular, for the high speed baseband modulation like 64QAM, the designed MDFT filter bank modulation system is much better than OFDM system. Such property is quite useful for the design of modulation systems in the next mobile communication network.

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[12] T. Karp and N. J. Fliege, " Modified DFT filter banks with perfect reconstruction," *IEEE Trans.on CS-ii: Analog and Digital Signal Processing*, vo1. 46, pp. 1404-1414, Nov.,1999.

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