The Inverse Spanning Tree of a Fuzzy Graph Based on Credibility Measure

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Abstract**—**An inverse spanning tree problem is to make the least modification on the edge weights such that a predetermined spanning tree is a minimum spanning tree. In this paper, based on the notion of fuzzy α-minimum spanning tree, the inverse spanning tree problem with fuzzy edge weights is discussed and formulated as a fuzzy programming model with some chance constraints. It shows that when all the edge weights are assumed to be independent fuzzy variables with regular credibility distributions, the proposed model can be reformulated into a traditional nonlinear programming according to the equivalent condition of fuzzy α-minimum spanning tree characterized by a set of constraints on non-tree edges and their tree paths. Moreover, if all the fuzzy weights are triangular fuzzy numbers, a linear programming problem can be obtained and solved efficiently with the help of some well developed software packages.

Index Terms**—**inverse spanning tree, credibility measure, inverse optimization, fuzzy programming

I. Introduction

As a particular sort of inverse optimization problems, the inverse spanning tree problem has been extensively studies in the literature. For a connected graph with edge weights, the inverse spanning tree problem is to modify the weights as little as possible such that a given spanning tree becomes a minimum spanning tree of the graph with respect to the new weights, which means the deviation incurred by the modification is to be minimized.

The inverse spanning tree problem was first studied by Zhang *et al.* [1]. Since then, much work has been done on the inverse spanning tree problem because many practical problems can be handled in this framework (see, e.g., [2]–[4]). For the classical inverse spanning tree problem and its derivatives, some efficient algorithms are available (see, e.g., [5]–[7]). Besides, in some practical circumstances, the edge weights cannot be explicitly determined. Therefore, the inverse spanning tree problem was studied in stochastic environments by Zhang and Zhou [8], in which the edge weights were described as some random variables with probability distributions estimated by the historical statistical data.

Concerning the insufficiency of statistical data in some real applications, or the parameters are subjective or vague, the edge weights cannot be characterized by probability distributions via statistical techniques. In these situations, the problem parameters can be considered as fuzzy variables or uncertain variables by an expert system (see, e.g., [9], [10]), and then solved by fuzzy set theory [11] or uncertainty theory [12]. In the view of fuzzy variables, Zhang *et al.* [13] initiated a notion of fuzzy α-minimum spanning tree by means of credibility measure, and then formulated the fuzzy inverse spanning tree problem as a fuzzy α-minimum spanning tree model and a credibility maximization model, which were solved by some genetic algorithm-based algorithms. Zhou and Chen [14] redefined the conception of fuzzy α-minimum spanning tree, and proved an equivalent condition of this notion, called the α-path optimality condition characterized by some constraints on non-tree edges and their tree paths according to the operational laws on fuzzy variables suggested by Zhou *et al.* [15]. They also provided the other two new definitions, i.e., the expected minimum spanning tree and the most minimum spanning tree, for the fuzzy minimum spanning tree problem. In the field of uncertain variables, Zhang *et al.* [16]-[17] discussed the inverse spanning tree problem with uncertain edge weights, and formulated this problem as various uncertain programming models in terms of different decision-makers' requirements.

In this paper, based on the new definition of fuzzy α-minimum spanning tree developed by Zhou and Chen [14], we investigate the inverse spanning tree problem with fuzzy edge weights. A fuzzy chance-constrained programming model is established to formulate this problem, and then transformed into its deterministic counterpart using the α-path optimality condition in [14].

The rest of this paper is organized as follows. Section II reviews the concept of fuzzy variables and relevant notions in credibility theory. Section III introduces the classic inverse spanning tree problem. After that, Section IV interprets the notion of the fuzzy α-minimum spanning tree as well as its equivalent condition. In Section V, a fuzzy programming model is formulated for the fuzzy inverse spanning tree problem, and consequently transformed into its deterministic equivalent counterpart.

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Besides, an illustrative example is presented in Section VI.

II. Fuzzy Variables

The concept of fuzzy set was initiated by Zadeh [11] via membership function. In order to measure a fuzzy event, Zadeh [18] proposed the concept of possibility measure. However, possibility measure has no self-duality property. In order to improve the weakness on possibility measure, Liu and Liu [19] introduced the conception of credibility measure. In 2004, Liu [20] initiated credibility theory as a branch of mathematics that studies the behavior of fuzzy phenomena via credibility measure. In the following, we briefly review the concepts of fuzzy variable, credibility measure, and some other related definitions.

Let Θ be a nonempty set, P(Θ) the power set of Θ, and Pos a possibility measure. The triplet (Θ, P(Θ), Pos) is called a possibility space. A fuzzy variable is defined as a function from a possibility space (Θ, P(Θ), Pos) to the set of real numbers.

Suppose that ξ is a fuzzy variable with membership function μ. Then the possibility, necessity, and credibility of a fuzzy event $\{\xi \ \geq r\}$ can be defined by

$$\mathrm{Pos}\{\xi \ \geq r\} = \sup_{x\geq r}\mu(x),$$

$$\mathrm{Nec}\{\xi \ \geq r\} = 1-\sup_{x<r}\mu(x),$$

$$\mathrm{Cr}\{\xi \ \geq r\} = \frac{1}{2}(\mathrm{Pos}\{\xi \ \geq r\}+\mathrm{Nec}\{\xi \ \geq r\}),$$

respectively.

Definition 1: (*Liu [21]*) The credibility distribution $\Phi\colon \Re \to [0,1]$ of a fuzzy variable ξ is defined by $\Phi(x) = \mathrm{Cr}\{\,\theta \in \Theta \mid \xi(\theta) \leq x\,\}$.

In addition, a credibility distribution Φ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1)$. The inverse function Φ^{-1} is called the inverse credibility distribution of ξ by Zhou *et al.* [15].

For example, a triangular fuzzy number ξ is fully determined by the triplet $(r_1,\ r_2,\ r_3)$ of crisp numbers with $r_1 < r_2 < r_3$, whose membership function is given by

$$\mu(x) = \begin{cases} \dfrac{x-r_1}{r_2-r_1}, & \text{if } r_1 \leq x \leq r_2 \\ \dfrac{x-r_3}{r_2-r_3}, & \text{if } r_2 \leq x \leq r_3 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

It follows from Definition 1 that the credibility distribution of $\xi = (r_1,\ r_2,\ r_3)$ is

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq r_1 \\ \dfrac{x-r_1}{2(r_2-r_1)}, & \text{if } r_1 \leq x \leq r_2 \\ \dfrac{x+r_3-2r_2}{2(r_3-r_2)}, & \text{if } r_2 \leq x \leq r_3 \\ 1, & \text{if } x \geq r_3. \end{cases} \tag{2}$$

Moreover, the inverse credibility distribution of the triangular fuzzy number ξ is

$$\Phi^{-1}(\alpha) = \begin{cases} (1-2\alpha)r_1 + 2\alpha r_2, & \text{if } \alpha \leq 0.5 \\ (2-2\alpha)r_2 + (2\alpha-1)r_3, & \text{if } \alpha \geq 0.5. \end{cases} \tag{3}$$

Based on the conceptions of regular credibility distribution and independence of fuzzy variables given by Liu and Gao [22], an important operational law for independent fuzzy variables with regular credibility distributions has been proved by Zhou *et al.* [15] as follows.

Theorem 1: (Zhou et al. [15]) Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent fuzzy variables with regular credibility distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If the function $f(\xi_1, \xi_2, \cdots \xi_n)$ is strictly increasing with respect to $\xi_1,\ \xi_2, \cdots,\ \xi_m$, and strictly decreasing with respect to $\xi_{m+1},\ \xi_{m+2}, \cdots,\ \xi_n$, then

$$\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$$

is a fuzzy variable with inverse credibility distribution

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha)). \tag{4}$$

III. Classic Inverse Spanning Tree Problem

Let $G = (V, E)$ denote a connected graph consisting of the vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and the edge set $E = \{1, 2, \cdots, m\}$. A spanning tree $T = T(V, S)$ of G is a connected acyclic subgraph containing all vertices. For simplicity, we denote a spanning tree T by its edge set S throughout this paper. A spanning tree T^0 is said to be a minimum spanning tree if

$$\sum_{i\in T^0} x_i \leq \sum_{j\in T} x_j \tag{5}$$

holds for any spanning tree T, where $x_i,\ i \in E$, are edge weights.

The classic inverse spanning tree problem is to find some new edge weights such that a given spanning tree T^0 is a minimum spanning tree with respect tothe new edge weights, and the total change of the edge weights is minimized simultaneously.

As an illustration, a graph with 5 vertices and 7edges is shown in Fig. 1, where c_i and x_i denote the original and new weights on edge i, respectively. The edges (solid lines) define a spanning tree T^0 and hence called *tree edges* while the rest edges (dash lines), that are not in T^0, are called *non-tree edges*. We denote $T^0 = \{1, 3, 5, 7\}$ for simplicity. It is well-known that a spanning tree induces a unique path between every pair of vertices. Especially, for any non-tree edge j, there must be a unique path between the vertices of edge j containing only the tree edges. This path is called the *tree path of edge j*, and is denoted by P_j. For instance, the tree path of non-tree edge

BC in Fig. 1 is AB-AE-CE, i.e., $P_2 = \{1, 5, 7\}$. An inverse spanning tree problem is to find appropriate new weights $x_i, i \in E$, to minimize $\sum_{1\le i\le 7}|x_i - c_i|$ and at the same time, $x_1 + x_3 + x_5 + x_7 \le \sum_{i\in T} x_i$ holds for any spanning tree T.

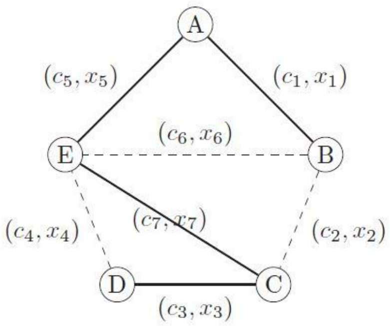

Figure 1. An example of inverse spanning tree problem

In order to formulate the inverse spanning tree problem, an alternative representation of the minimum spanning tree is needed, which is referred to as the path optimality condition. The path optimality condition characterizes the minimum spanning tree by a set of constraints on non-tree edges and their tree paths.

Theorem 2: (*Ahuja et al. [23], Path Optimality Condition*) Given a connected graph $G = (V, E)$ with edge weights $x_i,\ i \in E$, a spanning tree T^0 is a minimum spanning tree if and only if

$$x_i - x_j \le 0, \qquad j \in E\backslash T^0, i \in P_j \tag{6}$$

where $E\backslash T^0$ is the set of non-tree edges and P_j is the corresponding tree path of edge j.

According to Theorem 2, the classic inverse spanning tree problem can be modeled as

$$\begin{cases} \min\limits_{x} \sum\limits_{i=1}^{m} |x_i - c_i| \\ \text{subject to:} \\ \quad x_i - x_j \le 0\,,\ \ j \in E\backslash T^0, i \in P_j \end{cases} \tag{7}$$

where c_i and x_i are the original and new weights of edge $i, i \in E$, respectively. In order to solve model (7), some effective algorithms have been developed in the literature (see, e.g., [5]-[7]). Note that other types of objective functions can be considered as well for the inverse spanning tree problem (see [7]).

IV. Fuzzy α-Minimum Spanning Tree

The notion of fuzzy α-minimum spanning tree has been defined by Zhang *et al.* [13] and then redefined by Zhou and Chen [14]. In this paper, for our purpose, we adopt the following definition.

Definition 2: (*Zhou and Chen [14], Fuzzy α-Minimum Spanning Tree*) Given a connected graph $G = (V, E)$ with fuzzy edge weights $\xi_i, i \in E$, and a predetermined confidence level α, a spanning tree T^0 is called a fuzzy α-minimim spanning tree if

$$\begin{aligned} &\min\{\omega | \mathrm{Cr}\{\mathrm{W}(T^0, \boldsymbol{\xi}) \le \omega\} \ge \alpha\} \le \\ &\min\{\omega | \mathrm{Cr}\{\mathrm{W}(T, \boldsymbol{\xi}) \le \omega\} \ge \alpha\} \end{aligned} \tag{8}$$

holds for any spanning tree T, where $\mathrm{W}(T, \boldsymbol{\xi})$ stands for the weight of spanning tree T, and $\boldsymbol{\xi} = (\xi_1, \xi_2, \cdots, \xi_m)$.

It should be noted that in a graph with fuzzy edge weights, the weight $\mathrm{W}(T, \boldsymbol{\xi})$ of a spanning tree T is also a fuzzy variable. When all the fuzzy edge weights are assumed to be independent with regular credibility distributions, Definition 2 can be described by an equivalent form according to the operational law on independent fuzzy variables (see Theorem 1).

Theorem 3: (*Zhou and Chen [14]*) Given a connected graph $G = (V, E)$ with fuzzy edge weights $\xi_i, i \in E$, and a predetermined confidence level α, a spanning tree T^0 is a fuzzy α-minimum spanning tree if and only if

$$\sum_{i\in T^0} \Phi_i^{-1}(\alpha) \le \sum_{j\in T} \Phi_j^{-1}(\alpha) \tag{9}$$

holds for any spanning tree T, where Φ_i^{-1} are the inverse credibility distributions of $\xi_i, i \in E$, respectively.

In addition, Zhou and Chen [14] showed that a fuzzy α-minimum spanning tree problem can be transformed into a corresponding classical minimum spanning tree problem due to Theorem 3, referred to as the α-path optimality condition for fuzzy α-minimum spanning tree as follows.

Theorem 4: (*Zhou and Chen [14], Fuzzy α-Path Optimality Condition*) Given a connected graph $G = (V, E)$ with fuzzy weights $\xi_i, i \in E$, and a predetermined confidence level α, a spanning tree T^0 is a fuzzy α-minimum spanning tree if and only if

$$\Phi_i^{-1}(\alpha) - \Phi_j^{-1}(\alpha) \le 0, \qquad j \in E\backslash T^0, i \in P_j \tag{10}$$

where $E\backslash T^0$ is the set of non-tree edges, and P_j is the set of tree path of non-tree edge j.

V. Model for Fuzzy Inverse Spanning Tree Problem

In this section, the inverse spanning tree problem is discussed on a graph with fuzzy edge weights. Let $G = (V, E)$ be a connected graph with fuzzy edge weights $\xi_i,\ i \in E$. For each ξ_i, its distribution is determined by a parameter c_i, which can be modified accordingly. For example, in a LAN reconstruction problem, the traveling time on each bridge is with respect to the bandwidth, which will be modified after reconstruction. The fuzzy inverse spanning tree problem is to find new parameters $x_i, i \in E$, such that a given spanning tree becomes a fuzzy α-minimum spanning tree with respect to the new fuzzy edge weights, and the modification $\sum_{i=1}^{m}|x_i - c_i|$ is a minimum.

Based on conception of fuzzy α-minimum spanning tree in Definition 2, the fuzzy inverse spanning tree problem can be formulated as follows,

$$\begin{cases} \min\limits_{x} \sum\limits_{i=1}^{m} |x_i - c_i| \\ \text{subject to:} \\ \quad \min\{\omega | \mathrm{Cr}\{\mathrm{W}(T^0, \boldsymbol{\xi}) \le \omega\} \ge \alpha\} \le \\ \quad \min\{\omega | \mathrm{Cr}\{\mathrm{W}(T, \boldsymbol{\xi}) \le \omega\} \ge \alpha\} \end{cases} \quad (11)$$

where α is a predetermined confidence level, T^0 is a given spanning tree, and $\mathrm{W}(T, \boldsymbol{\xi})$ is the weight of spanning tree T.

Using the fuzzy α-path optimality condition in Theorem 4, the fuzzy programming model (11) can be transformed to the following equivalent formulation,

$$\begin{cases} \min\limits_{x} \sum\limits_{i=1}^{m} |x_i - c_i| \\ \text{subject to:} \\ \quad \Phi_i^{-1}(x_i, \alpha) \le \Phi_j^{-1}(x_j, \alpha), \\ \qquad j \in E \backslash T^0, i \in P_j \end{cases} \quad (12)$$

where Φ_i^{-1}, $i \in E$, represent the inverse credibility distributions of fuzzy weights ξ_i, with their distributions determined by the new parameters $x_i, i \in E$.

Once the credibility distributions are specified, model (12) becomes a deterministic programming problem, and can be solved by some well developed algorithms or software packages. Generally, it is a nonlinear programming problem and may require much computational effort. However, when the fuzzy edge weights ξ_i, $i \in E$, are triangular fuzzy numbers, for example, $\xi_i = (r_{1i} - x_i,\ r_{2i} - x_i\ ,\ r_{3i} - x_i)$, it follows from (3) that the inverse credibility distribution of ξ_i is

$$\Phi_i^{-1}(x_i, \alpha) = \begin{cases} (1-2\alpha) r_{1i} + 2\alpha r_{2i} - x_i, \\ \qquad \text{if } \alpha \le 0.5 \\ 2(1-\alpha) r_{2i} - (1-2\alpha) r_{3i} - x_i, \\ \qquad \text{if } \alpha \ge 0.5. \end{cases} \quad (13)$$

Thus,

$$\Phi_i^{-1}(x_i, \alpha) - \Phi_j^{-1}(x_j, \alpha) = -x_i + x_j + K_{ij} \quad (14)$$

where

$$K_{ij} = \begin{cases} (1-2\alpha)(r_{1i} - r_{1j}) + 2\alpha(r_{2i} - r_{2j}), \\ \qquad \text{if } \alpha \le 0.5 \\ 2(1-\alpha)(r_{2i} - r_{2j}) + (2\alpha - 1)(r_{3i} - r_{3j}), \\ \qquad \text{if } \alpha \ge 0.5. \end{cases} \quad (15)$$

Consequently, model (12) reduces to the following deterministic programming problem,

$$\begin{cases} \min\limits_{x} \sum\limits_{i=1}^{m} |x_i - c_i| \\ \text{subject to:} \\ \quad -x_i + x_j \le -K_{ij}, j \in E \backslash T^0, i \in P_j \end{cases} \quad (16)$$

Furthermore, by introducing auxiliary variables $y_i^+, y_i^-,\ i \in E$, with

$$y_i^+ = \begin{cases} x_i - c_i, & \text{if } x_i \ge c_i \\ 0, & \text{if } x_i < c_i \end{cases},$$

$$y_i^- = \begin{cases} 0, & \text{if } x_i \ge c_i \\ c_i - x_i, & \text{if } x_i < c_i \end{cases},$$

it is easy to obtain

$$|x_i - c_i| = y_i^+ + y_i^-, \qquad x_i = y_i^+ - y_i^- + c_i. \quad (17)$$

Taking (17) into model (16), it becomes

$$\begin{cases} \min \sum\limits_{i=1}^{m} (y_i^+ + y_i^-) \\ \text{subject to:} \\ \quad y_j^+ - y_j^- - y_i^+ + y_i^- \le c_i - c_j - K_{ij}, \\ \qquad j \in E \backslash T^0, i \in P_j \\ \quad y_i^+,\ y_i^- \ge 0, \end{cases} \quad (18)$$

which is a linear programming model and can be efficiently solved.

VI. NUMERAICAL EXAMPLES

In order to illustrate the effectiveness of the models proposed above, in this section, a LAN reconstruction problem with 6 service centers and 10 bridges is considered (see Fig. 2), where the solid lines represent a predetermined spanning tree T^0. For each bridge i, $i \in E = \{1, 2, \cdots, 10\}$, its traveling time ξ_i is assumed to be a triangular fuzzy number $(r_{1i} - x_i, r_{2i} - x_i, r_{3i} - x_i)$ where x_i is a bandwidth parameter with an original value c_i and r_{1i}, r_{2i}, r_{3i} are constants. The values of c_i and ξ_i, $i \in E$, are given in Table I. Without loss of generality, we assume that all the fuzzy traveling times are independent with each other.

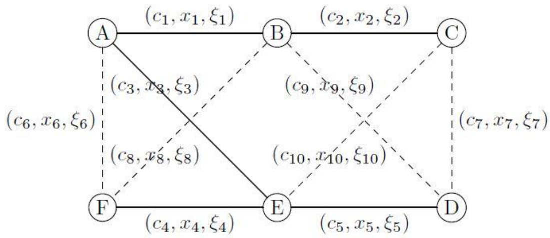

Figure 2. A LAN reconstruction problem

TABLE I. PARAMETER VALUES AND FUZZY WEIGHTS IN FIGURE 2

Edge i	Original parameter $\boldsymbol{c_i}$	Fuzzy weight $\boldsymbol{\xi_i(x_i)}$
1	10	$(20-x_1, 22-x_1, 26-x_1)$
2	4	$(20-x_2, 21-x_2, 24-x_2)$
3	6	$(20-x_3, 21-x_3, 22-x_3)$
4	11	$(20-x_4, 23-x_4, 24-x_4)$
5	15	$(20-x_5, 23-x_5, 27-x_5)$
6	6	$(20-x_6, 22-x_6, 23-x_6)$
7	6	$(20-x_7, 23-x_7, 25-x_7)$
8	8	$(20-x_8, 22-x_8, 23-x_8)$
9	9	$(20-x_9, 24-x_9, 25-x_9)$
10	12	$(20-x_{10}, 24-x_{10}, 26-x_{10})$

According to model (11), if we want to minimize the total modification of bandwidths with a given confidence level $\alpha = 0.8$ so as to diminish the total cost of reconstruction, we have the following fuzzy programming model,

$$\begin{cases} \min_{x} \sum_{i=1}^{10} |x_i - c_i| \\ \text{subject to:} \\ \quad \min\{\omega | \mathrm{Cr}\{\mathrm{W}(T^0, \boldsymbol{\xi}) \le \omega\} \ge 0.8\} \le \\ \quad \min\{\omega | \mathrm{Cr}\{\mathrm{W}(T, \boldsymbol{\xi}) \le \omega\} \ge 0.8\} \end{cases} \tag{19}$$

where the non-tree edge set $E\backslash T^0 = \{6,7,8,9,10\}$. By means of the fuzzy α-path optimality condition (see Theorem 4), it can be transformed into

$$\begin{cases} \min_{x} \sum_{i=1}^{10} |x_i - c_i| \\ \text{subject to:} \\ \quad \Phi_i^{-1}(x_i, 0.8) - \Phi_j^{-1}(x_j, 0.8) \le 0, \\ \qquad\qquad j = 6,7,8,9,10, i \in P_j. \end{cases} \tag{20}$$

Taking (14), (15) and (17) into model (20), we get

$$\begin{cases} \min \sum_{i=1}^{10} (y_i^+ + y_i^-) \\ \text{subject to:} \\ \quad y_j^+ - y_j^- - y_i^+ + y_i^- \le \\ \quad c_i - c_j - 0.4(r_{2i} - r_{2j}) - 0.6(r_{3i} - r_{3j}), \\ \qquad\qquad j = 6,7,8,9,10, i \in P_j \\ \quad y_i^+, y_i^- \ge 0, i \in E. \end{cases} \tag{21}$$

Using Matlab, an optimal solution is found at

$$\boldsymbol{x}^* = (10, 5.9, 6.7, 11, 15, 6, 6, 8.3, 9, 15.7)$$

with an optimal objective value 6.6.

In the example above, the confidence level $\alpha = 0.8$ is given by the decision-maker according to different requirements. Some numerical experiments are further considered for different confidence levels from 0.9 to 0.5 in order to investigate the influence of this parameter. The settings and results are summarized in Table II including the optimal solutions and the minimum objective values. The original edge weights $c_i,\ i \in E$, are also included to compare with the new edge weights $x_i, i \in E$.

TABLE II. RESULTS FOR NUMERICAL EXAMPLES USING DIFFERENT CONFIDENCE LEVELS

α	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	OBJ
0.9	10	6	6.4	11	15	6	6	8.6	9	15.8	6.8
0.8	10	5.9	6.7	11	15	6	6	8.3	9	15.7	6.6
0.7	10	5.9	6.6	11	15	6	6	8.4	9	15.5	6.4
0.6	10	5.7	6.7	11	15	6	6	8.3	9	15.5	6.2
0.5	10	5.5	6.7	11	15	6	6	8.3	9	15.5	6.0
c_i	10	4	6	11	15	6	6	8	9	12	---

Results in Table II show that different confidence levels lead to different costs. When α decreases from 0.9 to 0.5, the corresponding optimal objective value, i.e., the minimum reconstruction cost, decreases accordingly. In other words, the higher confidence level the decision-maker requires, the more cost is needed.

VII. CONCLUSIONS

In this paper, the inverse spanning tree problem with fuzzy edge weights is investigated. Based on the notion of fuzzy α-minimum spanning tree defined in [14], the fuzzy inverse spanning tree problem is modeled as a fuzzy programming model with a set of chance constraints. When the fuzzy parameters are assumed to be independent fuzzy variables with regular credibility distributions, the proposed fuzzy programming model can be reformulated as a traditional linear programming problem with some specified inverse credibility distributions in it. As an illustration, a LAN reconstruction problem is given to show the performance of the proposed models. However, in this paper, we only focus on the case when the fuzzy edge weights have regular credibility distributions. As a future work on this topic, an effective algorithm should be designed for solving model (11) for general cases.

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