Superposition Modulated Cooperative Diversity for Half-duplex Scenario

Koji Ishii
Department of of Reliability-based Information Systems Engineering, Kagawa University, Japan
Email: kishii@eng.kagawa-u.ac.jp

Abstract—In this paper, the cooperative diversity with superposition modulation, which has been proposed as an instance of “dirty paper coding”, is theoretically analyzed using the outage probability. However, the conventional system with superposition modulation cannot ideally perform dirty paper coding. Thus, we propose to apply constellation rotation technique and iterative processing to the superposition modulated system, which yields a performance close to the ideal one.

Index Terms—cooperative diversity, dirty paper coding, superposition modulation, constellation rotation, iterative processing

I. INTRODUCTION

In sensor networks, wireless transmitters and receivers, which we shall call nodes throughout this paper, may not be able to support multiple antennas owing to size, complexity, power, or other constraints. As a consequence, a great deal of effort has been, recently, put on the ideal of cooperative communication. Cooperative transmission between pairs of nodes has been suggested [1]-[3], [6]-[11],[16] as a means to achieve diversity gain. In the existing cooperative transmission, several protocols have been considered in the literature, and these are largely classified into amplify-and-forward schemes and decode-and-forward schemes [1] [2].

In this paper, we restrict our attention to decode-and-forward schemes and consider the half-duplex environment that the node cannot transmit and receive simultaneously. Specially we focus on the superposition modulated cooperative transmission which has been proposed in [3] as an embodiment of relay channel theory or even as an instance of dirty paper coding theory [4]. In the superposition modulated cooperative transmission system, a node transmits its own signal superposed to other node’s signal to the destination node. Therefore the node transmits one signal which consists of its own information and other node’s information, simultaneously. The other node’s information can be considered as an interference factor for its own information. Here, the transmit-node knows this interference factor. If the system can ideally perform dirty paper coding theory and the transmitter knows the interference factor, the capacity at the destination node does not decrease even if the destination node (decoding side) does not know the interference factor [4] [5]. In [3], the cooperative transmit diversity has been proposed using one-dimensional superposition modulation and evaluated by only computer simulations. This paper analyzes the superposition modulated cooperative transmission with outage probability and expands to two-dimensional modulation. Moreover we apply a constellation rotation technique to this system in order to achieve maximum coding gain.

A similar idea from the view point that the transmitted symbol is combining its own information and other user’s information, has been proposed in [6] [7]. During inter-user communications phase, each node communicates in a different sub-band in a full-duplex scenario. During cooperative communications phase, the nodes transmit space-time coded QAM symbols combining their own information and other user’s information.

The aim of this paper is to realize dirty paper coding theory. In order to realize it, we utilize a constellation rotation technique and iterative processing to the superposition modulated cooperative transmission. The cooperative transmission using iterative processing has been proposed in [8]. This system is only for full-duplex scenario. The source node transmits coded data to relay and destination nodes, while the relay node simultaneously forwards its estimate for the previous coded block to the destination after decoding and re-encoding. The destination decodes the received signals with iterative decoding. A characteristic of this system is that the decoding scheme at the destination node jointly operates over all the transmitted blocks. We expand this cooperative transmission with iterative processing from full-duplex scenario to half-duplex scenario using superposition modulation.

The rest of the paper is organized as follows. Section II presents several transmission schemes including the superposition modulated cooperative transmission and proposes to apply the iterative detection to it. In section III, we analyze each system by outage probability under the assumption that they can ideally perform dirty paper coding. In section IV, we propose to apply a constellation rotation technique to the superposition modulated system and discuss the optimum rotation angle and superposition ratio by minimum-distance analysis. In section V, we evaluate the system with computer simulations and outage

This paper is based on “Cooperative Transmit Diversity Utilizing Superposition Modulation,” by K. Ishii, which appeared in the Proceedings of the IEEE Radio and Wireless Symposium, California, USA, January 2007. © 2007 IEEE.
analyses. In section VI, we draw some conclusions.

II. COOPERATIVE TRANSMISSION WITH SUPERPOSITION MODULATION

In this section, we describe a non-cooperative transmission, conventional cooperative transmission and superposition modulated cooperative transmission.

Consider the case that the transmit-nodes A and B cooperatively transmit to the destination node D (Fig.1). Here, \(h\) stands for channel gain. The subscripts “A”, “B”, “D” indicate the nodes where the signals are transmitted or received. Channel estimation and synchronization at the receiver are assumed to be perfect.

A. Non-Cooperative Transmission

In a non-cooperative transmission, at the first time-slot, node B transmits its own packet \(S_{B1}\) to node D as shown in the top of Fig. 2. At the second time-slot, node A transmits its own packet \(S_{A2}\) to node D. Nodes A and B transmit to node D, alternately.

B. Conventional Cooperative Transmission

In a conventional cooperative transmission, at the first time-slot, node B transmits its own packet \(S_{B1}\) as shown in the middle of Fig. 2. At the same time, node A decodes the signal from node B. If the decoding task was performed successfully, node A re-encodes and transmits node B’s information \(S_{B1}'\). At the second time-slot, node A transmits its own packet \(S_{A2}\). At the same time, node B decodes the signal from node A. If the decoding was successful, node B re-encodes and transmits node A’s information. Nodes A and B alternate this operation.

C. Superposition Modulated Cooperative Transmission

In a superposition modulated cooperative transmission, at the first time-slot, node B encodes its own data and the other node’s data which is estimated at previous time-slot, as shown Fig. 3. After interleaving of the two encoded data, the each interleaved data is superposed by superposition modulation and transmitted, as shown the bottom of Fig. 2. Node B transmits its own packet with power \(1 - \gamma^2\) and the packet from A with power \(\gamma^2\). The total emitted power is normalized to 1. \(\gamma\) shows superposition ratio. Then, the received signals at nodes D and A are, respectively,

\[
y_{D1} = h_{BD} \sqrt{1 - \gamma^2} S_{B1} + \gamma S_{A0} + \epsilon_{D1}\]
\[
y_{A1} = h_{BA} \sqrt{1 - \gamma^2} S_{B1} + \gamma S_{A0} + \epsilon_{A1}\]

The additional subscript \((\cdot)_1\) stands for 1 time-slot “1”. \(\epsilon_{D1}\) and \(\epsilon_{A1}\) are noise at node D and A at time-slot 1. Node A decodes \(S_{B1}\) from the received signal \(y_{A1}\). Note that re-encoded signal \(S_{A0}\) is known at node A because the signal \(S_{A0}\) is organized from A’s signal which was transmitted two time-slot before. Therefore, node A can directly subtract \(S_{A0}\) from the received signal and estimate the signal from node B without the interference factor. In the next time-slot, node A transmits its own signal \(S_{A2}\) and the re-encoded signal \(S_{B1}'\). Assuming that the decoding of \(S_{B1}\) at node A was successful, node A transmits its own packet \(S_{A2}\) with power \(1 - \gamma^2\) and the B’s packet \(S_{B1}'\) with power \(\gamma^2\). The received signals at D and B are, respectively,

\[
y_{D2} = h_{AD} \sqrt{1 - \gamma^2} S_{A2} + \gamma S_{B1}' + \epsilon_{D2}\]
\[
y_{B2} = h_{AB} \sqrt{1 - \gamma^2} S_{A2} + \gamma S_{B1}' + \epsilon_{B2}\]

At the destination node D, the receiver decodes the two received signals ((1) and (3)). Fig. 5 shows the decoder structure at the destination node, which assumes that it knows whether the relay node could successfully decode the signal from the source node or not. In this paper, we do not consider ARQ techniques [9] [10]. Even if the destination node could decode successfully at the first time-slot, the relay node transmits the superposed signal at the second time-slot (cooperative transmission is always used). We assume that the distance between nodes, their transmitting power and transmission-rate are the same. Thus, we do not consider these factors in the system’s analysis [11].

At the destination node D, the joint MAP detector described in Fig. 5 calculates the log-likelihood ratio for
an information bit $b_k$ from the received signals $y_{D1}$ and $y_{D2}$. The log-likelihood ratio is given by

$$LLR(b_k|y_D) = \log \frac{P(b_k = 1|y_D)}{P(b_k = 0|y_D)}$$

(5)

where $y_D$ is the received sample, $\{S_X, S_Y\}$ are the two symbols to be jointly demodulated, and $\sigma^2$ is the noise variance per dimension. Also, $P(S_X), P(S_Y)$ are the a-priori probabilities of $S_X, S_Y$. In this system, the receiver does not use these a-priori probabilities. The calculated LLRs are de-interleaved. If the relay node could decode successfully, each interleaved LLRs are maximum-ratio combined and become the input of the decoder. If the relay could not decode successfully, the only LLR calculated at the former time-slot becomes the input. We assume that the destination node knows whether the relay node could decode successfully or not.

D. Superposition Modulated Cooperative Transmission with Iterative Detection

In this section, we propose to apply an iterative detection to the superposition modulated cooperative transmission. The transmitter structure at the relay and source nodes is the same, but the receiver structure at the destination node is not. In the system with iterative detection, the results of decoder are interleaved and go back to the detectors as a-priori probabilities of (5). The aim of this study is to realize “dirty paper coding” theory. From the viewpoint of node A’s data, the node B’s data can be considered as the interference factor which is known at the transmitter, and vice versa. Thus, in order to realize it, it is very important for the destination node to calculate each LLR from the superposing signal. Due to use of iterative detection at the destination node, the ability of detection can be improved.

III. OUTAGE ANALYSIS

In this subsection, we analyze the performances by means of outage probability. We assume that the error-correcting code can achieve Shannon limit, in other words, the performance depends on only modulation scheme. In the same fashion of sec. II, we investigate three transmission schemes.

A. Non-cooperative Transmission

The received signal from node A to D is given by

$$y_{non-coop} = h_AD S_A + e.$$  

(6)

Assuming that the average signal to noise ratio is $\text{SNR}(= S^2/\sigma_n^2); S^2$ is signal power, $\sigma_n^2$ is the variance of $e$. We define the minimum SNR which can achieve the transmission-rate $R$ bits/symbol using $M$-PAM signal as $\text{SNR}_R(M)$. Then, the outage probability is expressed as

$$\Pr[C(h_{AD}) < R] = \Pr |h_{AD}|^2SNR < \text{SNR}_R(M).$$  

(7)

Since $h_{AD}$ is a circularly symmetric Gaussian random variable with zero mean and unit variance, $|h_{AD}|^2$ follows the exponential distribution. Therefore, the outage
probability, averaged over all realizations of $h_{AD}$, is given by

$$
\Pr[C(h_{AD}) < R] = \Pr \left[ \frac{|h_{AD}|^2}{SNR_R(M)} < SNR_R(M) \right] = \int_0^{SNR_R(M)/SNR} \exp{-x} \, dx = 1 - \exp{\frac{SNR_R(M)}{SNR}}. \quad (8)
$$

Moreover, we consider the diversity order in the high SNR region, formally defined in [12] as

$$
d \triangleq \lim_{SNR \to \infty} \frac{\log P_{e}(SNR)}{\log(SNR)} \quad (9)
$$

where $P_{e}(SNR)$ is the outage probability. Therefore, the diversity order is $d = 1$.

**B. Conventional Cooperative Transmission**

At the first time-slot, node A transmits to node B. $P_f$ is the error probability of the transmission from node A to B. Assuming that each transmitted power, transmission-rate and distance between nodes are the same, the error probability of transmission from A to D is also $P_f$. In the strict sense, since the relay node does not transmit in the non-cooperative case (the relay node failed to decode the signal from the source node), to average SNR, the SNRs of non-cooperative and cooperative cases should be weighted by $P_f$ and $1 - P_f$, respectively. However, for the sake of simplicity, we use not the weighted SNR but the SNR of the cooperative case as the average SNR. Then, the probabilities of cooperative ($P_{coop}$) and non-cooperative ($P_{non-coop}$) transmission are given by

$$
P_{coop} = 1 - P_f \quad \quad \quad \quad (10)
$$

$$
P_{non-coop} = P_f = 1 - \exp{\frac{SNR_R(M)}{SNR}}. \quad (11)
$$

The error probabilities at the destination node in the cases of the cooperative ($P_{div}$) and non-cooperative ($P_{single}$) transmissions are given by

$$
P_{div} = P_f^2 \quad \quad \quad \quad (12)
$$

$$
P_{single} = P_f. \quad \quad \quad \quad (13)
$$

With (10) - (13), the outage probability ($P_{conv}$) can be expressed as

$$
P_{conv} = P_{coop}P_{div} + P_{non-coop}P_{single} = 1 - \exp{\frac{SNR_R(M)}{SNR}} \times 1 + \exp{\frac{SNR_R(M)}{SNR}}. \quad (14)
$$

The diversity order can be shown to be $d = 2$.

**C. Superposition Modulated Cooperative Transmission**

Consider the same situation of sec. II. The probabilities of cooperative transmission $P_{coop}$ and non-cooperative transmission $P_{non-coop}$ are expressed using the error probability $P_f$ as

$$
P_{coop} = 1 - P_f(\gamma A) \quad \quad \quad \quad (15)
$$

$$
P_{non-coop} = P_f(\gamma A) \quad \quad \quad \quad (16)
$$

$$
P_{f}(\gamma A) = 1 - \exp{\frac{SNR_R(M)}{(1-\gamma^2)SNR}}. \quad (17)
$$

Since the relay node A knows $S_{A0}$, it can decode $S_{B1}$ without interference factor (see sec. II). In this analysis, we assume that the cooperative transmission with superposition modulation can perfectly perform the dirty paper coding. In dirty paper coding theory [4], if the transmitter knows the interference factor, the capacity does not decrease even if the receiver does not know the interference (superposing) factor. Therefore, since the relay node A knows the interference factor $S_{A0}$, the capacity for $S_{B1}$ at the destination node does not decrease. Thus, the probability of the relay to destination channel is the same as the one of the source to destination channel ($P_{f}(\gamma A) = P_{f}(\gamma A)$). Here, if both the transmitter and receiver do not know the interference factor $S_{A0}$, the SNR in (17) is defined as $(\frac{(1-\gamma^2)^2}{\frac{2\gamma^2}{(1-\gamma^2)^2}})$. Under this assumption, the transmit power for $S_{B1}$ at time-slot 1 and 2 are $(1-\gamma^2)$ and $\gamma^2$, respectively. Therefore, the error probability between the source node B and destination node D is the same as (17), and the error probability between the relay node A and destination node D is given by

$$
P_{f}(\gamma A) = 1 - \exp{\frac{SNR_R(M)}{\gamma^2SNR}}. \quad (18)
$$

The error probabilities at the destination node in the cases of the cooperative ($P_{div}$) and non-cooperative ($P_{single}$) transmissions are given by

$$
P_{div} = P_f(\gamma A)P_{f}(\gamma A) \quad \quad \quad \quad (19)
$$

$$
P_{single} = P_f(\gamma A). \quad \quad \quad \quad (20)
$$

Using (15) - (20), the outage probability for superposition modulated transmission $P_{sup}$ is given by

$$
P_{sup} = P_{coop}P_{div} + P_{non-coop}P_{single} \times 1 - \exp{\frac{1}{\gamma^2}}. \quad (21)
$$

Thus, the diversity order is 2.

**IV. Constellation Rotation**

In this section, we discuss the two-dimensional superposition modulated transmission. We extend the one-dimensional superposition modulation to the two-dimensional one, where the constellation is shown in Fig. 6(a). In order to achieve maximum coding gain, we propose to apply the constellation rotation technique [13]
rotation is given by
\[ y = \sqrt{1 - \gamma^2} S_A + \gamma S_B \] (23)
where \( S_A \) and \( S_B \) are \( M \)-PSK signals without and with constellation rotation \( \{s_A, s_B \in \{0, 1, \ldots, M - 1\}\} \) and \( \theta \) is the rotation angle. By substituting (23) by (22), the minimum distance is given by
\[ d_{\text{min}} = \min_{(s_A, s_B) \neq (s', s')} \sqrt{1 - \gamma^2 (S_A - S') + \gamma (S_B - S')} \] (24)
In Figs. 7(a) and 7(b), the minimum distance in the cases of BPSK and QPSK modulation are shown. If the modulation is \( M \)-PSK, the effective rotation angle is in the interval \( [0, \pi/M] \) from the symmetry of signals in \( M \)-PSK [14]. Then, we evaluate the minimum-distance at the region where \( \gamma^2 \) is from 0 to 0.5 and \( \theta \) is from 0 to \( \pi/M \). For BPSK modulation, the parameters \( (\gamma^2, \theta) \) at the maximum minimum-distance are \( (0.5, 1/3\pi) \) (see Fig. 7(a)). For QPSK modulation, the parameters \( (\gamma^2, \theta) \) at the maximum minimum-distance are \( (0.2, 1/12\pi) \) (see Fig. 7(b)). However, in this analysis of the minimum distance, we do not consider that the relay node has the possibility to cooperate or not depending on these parameters. In a cooperative transmission, the performance is greatly influenced by whether the system can cooperate or not. In this superposition modulated system, the higher \( \gamma \) the worse the performance between source and relay nodes becomes. The details are described in the latter section.

V. EVALUATIONS

A. Evaluations of FER

In this subsection, we evaluate the frame error rate (FER) performances of each system with computer simulations. The simulation parameters are as follows. One frame consists of 1000 symbols. The element code of turbo code is 7-5 RSC code. Each interleaver is a random interleaver. All the channel links are subject to a quasi-static frequency non-selective Rayleigh fading. Thus, the fading coefficient is constant within each frame but vary from frame to frame. We evaluate five systems, non-cooperative transmission (non-coop.), conventional cooperative transmission (conv. coop.), superposition modulated cooperative transmission without iterative detection (sup. coop. w/o ID) [3] [16], and superposition modulated cooperative transmission with iterative detection (sup. coop. w/ ID). In this subsection, we use only one-dimensional modulation. The discussion for the superposition modulated system with constellation rotation is described in following subsection. In the non-cooperative
transmission, the data is encoded by turbo encoder (coding rate is transformed into 1/2 with puncturing) which consists of two 7-5 RSC encoders and a random interleaver. The coded signals are interleaved and modulated by 2PAM. In the conventional cooperative transmission, the data is encoded by turbo encoder, interleaved and modulated by 4PAM. Since the transmission period of the conventional cooperative transmission is half (see section II), to provide equal transmission rate, it uses 4PAM. Therefore, the spectral efficiency of all transmission schemes is the same. The superposition modulated system uses the same turbo code and the superposition ratio, which is discussed below, is set to 0.15. The number of turbo decoding at all transmission schemes is 7, the number of iterative detection is 3 and assuming that the destination node knows whether the relay node could decode successfully or not. In the strict sense, since the relay node does not transmit in the non-cooperative case (the relay node failed to decode the signal from the source node), to average SNR, the SNRs of non-cooperative and cooperative cases should be weighted by the probability of non-cooperation and cooperation, respectively. However, for the sake of simplicity, we did not use the weighted SNR but the SNR of the cooperative case as the average SNR.

Fig. 8 shows the FER performances. Compared to the non-cooperative transmission, the cooperative transmissions achieve about twice diversity gain, in other words, it can obtain diversity order of 2. From the comparison between the conventional cooperative transmission and superposition modulated cooperative transmissions (“sup. coop. w/o ID” and “sup. coop. w/ ID”), it is seen that the superposition modulated systems achieve better performance. Since the fading coefficient is constant during one frame, the system cannot achieve bit-interleaved coded modulation (BICM) gain due to iterative detection, but can improve the performance. In other words, the LLRs calculated by joint-MAP detector at the destination node can be improved.

B. Evaluations of Outage Probabilities

Fig. 9 shows the outage probabilities in the case of superposition modulated cooperative transmission, the superposition factor $\gamma^2$ is 0.15. Recall that the non-cooperative and superposition modulated cooperative transmissions use 2PAM and the conventional cooperative transmission uses 4PAM, and assuming that the error-correcting code is Shannon code and the superposition modulated system can be ideally perform “dirty paper coding” theory. In the case of $R = 0.5$, the minimum SNR using 2PAM and 4PAM are $\text{SNR}_{R=0.5}(2\text{PAM}) = 0.2$ and $\text{SNR}_{R=1}(4\text{PAM}) = 5.1$ [dB], respectively [17]. The aim of iterative detection is to improve the LLRs calculated by joint-MAP detector, thus under this assumption, the performances of superposition modulated transmission with and without iterative detection are the same. It is seen from Fig. 9 that the best performance is the superposition modulated cooperative transmission and that the cooperative transmissions achieve a diversity order close to 2.

C. Relationship between Superposition factor and Performances

Fig. 10 shows the relationship between the superposition ratio $\gamma^2$ and FER performances in the case where SNR is 15 dB and coding rate is 1/2. Recall that the outage probabilities are calculated under the assumption that the system can ideally perform dirty paper coding theory. Obviously, it is not fair to compare the FER performances with its outage probability. But if the system can ideally perform dirty paper coding theory, we can consider that the shape of FER performances correspond to the one of its outage probability (e.g., the optimum superposition ratio is the same). It is seen from the outage probability in Fig. 10 that the optimum superposition ratio $\gamma^2$ is
Consider the performances without the constellation rotation and with/without iterative detection ($\theta = 0$, w/o ID) and ($\theta = 0$, w/ ID). The higher the superposition ratio $\gamma^2$ grows above 0.15, the worse the system performance without iterative detection becomes. The reason is that the relay channel has worse performance at the region of high superposition ratio since the power ratio of source node’s information in the received signal at the relay node is inversely proportional to the superposition ratio. Especially, at the region where $\gamma^2 = 0.5$, the performance of the superposition modulated system without iterative detection becomes the worst. Consider the case where the superposition ratio $\gamma^2$ is 0.5 and the superposing signals have inverse values (e.g., $(S_A, S_B) = (+1, -1)$). Since the received signal superposed $S_A$ and $S_B$ without noise factor becomes zero, the probabilities of $(S_A, S_B) = (+1, -1)$ and $(S_A, S_B) = (-1, +1)$ are the same. Thus, since LLRs become zero, the decoder cannot decode the signal. However, using iterative detection, the performance considerably improves at the region where $0.2 \leq \gamma^2$. As the result of applying the iterative detection, the system can get close to realization of dirty paper coding theory but the optimum superposition ratio is not the same as the one of outage analysis.

Next, we consider the system with constellation rotation. We evaluate three cases when the rotation angle is 30, 60, and 90 degree. The system with constellation rotation can achieve better performance than the one without constellation rotation. The best performance is the case when $\theta = 90$. It is seen that the FER performance almost corresponds with minimum-distance analyses but does not correspond perfectly. The reason is that for minimum-distance analyses, we do not consider that the relay node has the possibility to cooperate or not depending on these parameters. Due to use of iterative detection, the constellation rotated system can improve the performances. However, the higher the rotation angle ($0 \leq \theta \leq 90$), the less the improvement the system has. The reason is that the system with constellation rotation can get close to the theoretical limit, since the shape of performance is similar to the one of outage probability calculated under the assumption that the system can ideally perform the dirty paper coding theory.

VI. CONCLUSION

This paper has focused on the cooperative transmission with superposition modulation for a half-duplex scenario. In this work, we proposed to apply the constellation rotation technique to the superposition modulated cooperative transmission and provided the optimum rotation angle and superposition ratio. Moreover, we proposed the system with iterative detection. From the empirical and theoretical outage analysis, the cooperative transmission with superposition modulated system can achieve better performance than the conventional cooperative transmission and can get close to the realization of dirty paper coding theory.

REFERENCES


Koji Ishii was born in Aichi, Japan. He received the B.S., M.E., and Ph.D. degrees in electrical and computer engineering from Yokohama National University, Japan, in 2000, 2002 and 2005, respectively. He is currently an Assistant Professor with department of reliability-based information system engineering, Kagawa university, Japan. His current research interests include wireless and mobile communications, information theory, channel coding, turbo coding, MIMO systems and cooperative communications.