A Flexible Wavelength Converter Placement Scheme for Guaranteed Wavelength Usage

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***Abstract*— Wavelength is one of the most important resources in Wavelength Division Multiplexing(WDM) networks. In optical routing, we are given a set of communication paths (or ligthpaths) in a WDM network and we must assign a wavelength to each path so that paths sharing a link must be assigned with different wavelengths. By properly choosing a set of nodes that are equipped with wavelength converters, the number of wavelengths which is required to support all lights paths can be reduced. In this paper, we study the problem of placing the minimum number of wavelength converters in a network to ensure that the number of wavelengths needed will not exceed a given bound αL, where L is the maximum link load in the network and α is a parameter defined by the network designer to reflect the overall availability of wavelength resources. This problem, however, is proved to be NP-hard. Hence we develop an efficient heuristic algorithm for the problem and extensive theoretical analysis and experimental studies are carried out to verify the effectiveness and performance of the algorithm.**

***Index Terms*— wavelength converter, optical network, placement, optimization, wavelength**

I. INTRODUCTION

Wavelength division multiplexing (WDM) [1] [2] divides the bandwidth of an optical fibre into multiple wavelength channels so that multiple users can transmit data at distinct wavelengths through the same fibre concurrently. Since all-optical WDM networks can provide communication service with huge bandwidth and low latency, such networks are considered as candidates for the next generation wide-area networks which are required to meet the increasing traffic demand in the foreseeable future.

A *lightpath* is an optical communication path between a pair of source and destination nodes which may span multiple hops. In WDM networks, any pair of lightpaths(traffic demand) must be assigned with different wavelengths if they share the same link in any hops. Hence it is easy to see that the number of wavelengths required in a network is at least equal to the *natural congestion bound* or *maximum link load*, defined to be the maximum number of paths passing through any one link in the network.

Wavelength converter is an essential device in multihop WDM networks that enhances the scalability of the network. In WDM networks without any wavelength conversion, the same wavelength must be assigned to all links in a lightpath (this is often referred to as the *wavelength continuity constraint*). If a node is equipped with a wavelength converter, any lightpath that passes through this node may change its wavelength. Clearly wavelength assignments in networks with wavelength converters can be more efficient (uses less wavelengths) than wavelength assignments for the same set of paths where no wavelength converter is available. However, wavelength converters are expensive devices and it has been anticipated that they will continue to be so in the foreseeable future [3]. In addition, densely placed converters may cause the signal distortion [4]. Hence, it is not practical to equip every node with a wavelength converter.

Several wavelength converter placement schemes [5] [6] [7] have been proposed in the literature to reduce the overall wavelength requirements of a given network by employing a minimal set of converters nodes. However, we note that existing converter placement schemes do not take into account the availability of resources, such as the number of wavelengths and converters that are available for utilization, in a given network; hence they are not able to adapt to the availability of resources of different networks.

In this paper, we aim to take above-mentioned issues into consideration in the design of efficient wavelength converter placement schemes for WDM networks. Furthermore, we aim to design a scheme that is able to provide a flexible trade-off between the number of wavelength converter nodes and the number of wavelengths required to support the communications of all lightpaths in a given network. In particular, the problem that we interested in is as follows: given the traffic demand (a set of lightpaths) in a network, determine the placement of the minimum number of wavelength converters in the network so that the number of required wavelengths does not exceed a given upper bound αL, where L is the maximum link load in the network and α is a parameter that can be defined by the network designer to reflect the overall availability of wavelength resources.

The rest of this paper is organized as follows: Section II reviews the related work. Section III presents the problem assumptions, problem formulation and the methodology used in this work. Section IV addresses the problem of determining the wavelength requirements for the networks with special topologies. The results we obtained in Section IV are applied in Section V and a two-step algorithm is proposed and analyzed. Results from our empirical studies are discussed in Section VI. Section VII concludes the paper.

II. RELATED WORK

In general, given a network and a set of lightpaths, if no wavelength converter is placed, the number of wavelength required to support all lightpaths can be determined by solving the *routing and wavelength assignment problem*(RWA). This problem, however, has been proved to be NP-complete [8]. In [9], Baroni *et.al* studied the relationship between wavelength requirement and network topology. In particular, they evaluated the number of required wavelengths as a function of the physical connectivity, which is a parameter that can be calculated from the network topology.

Some studies focus on determining the wavelength requirements of the networks with special topologies. These studies usually consider two types of communicationa channels: *duplex* and *unidirectional*. In duplex channels, data can be transmitted in both directions in one fiber; in unidirectional channels, data are transmitted in one direction from the source to the destination. Wilfong *et.al* [10] proved that $2L-1$ wavelengths are sufficient and necessary for a ring network with unidirectional channels. Here "*sufficient*" means the routing and wavelength assignment can be realized with at most $2L-1$ wavelengths, without wavelength conversion and "*necessary*" means that for any integer L there is an instance in which routing and wavelength assignment requires $2L-1$ wavelengths. For the network with tree topology, Erlebach *et.al* [11] proved that $\frac{5}{3}L$ wavelengths are sufficient and $\frac{5}{4}L$ wavelengths are necessary in unidirectional channels; Raghavan *et.al* [12] proved that $\frac{3}{2}L$ wavelengths are sufficient and necessary in duplex channels.

In [10], Wilfong *et.al* defined a set S of nodes in a network to be *sufficient* if placing converters at the nodes in S can ensure that the number of wavelengths required by any set of lightpaths is equal to its maximum linkload L. They showed that the problem of finding a sufficient set of minimum size for an arbitrary WDM network with unidirectional channels, referred to as the *minimum sufficient set problem*, is NP-complete. In addition, they showed that for networks with unidirectional channels, (i) the empty set (i.e. $S=\emptyset$) is sufficient if and only if the topology of network is a spider (i.e. a tree with at most one vertex of degree greater than two) and (ii) for any ring networks, the size of a minimum sufficient set S is equal to 1.

In [5], Kleinberg *et.al* extended the splitting technique to arbitrary directed graphs and proposed a 2-approximation algorithm[1] for finding the minimum sufficient set for an arbitrary directed (unidirectional channel) network. They also showed that the minimum sufficient set problem is as hard as the minimum vertex cover problem, which is believed to be unlikely to have an approximation algorithm with performance ratio less than 2 [13].

[1]An algorithm A is said to be an ω-approximation algorithm, if the ratio of the solution generated by A and an optimal solution, is bounded by ω.

Erlebach *el.al* [14] considered the case in which all lightpaths will be routed by the shortest path algorithm and they gave the complete characterization of duplex networks for which $S=\emptyset$ holds. They also noted that the restriction to shortest-path routing could reduce the converter requirements of a given network significantly.

The problem of finding a minimum sufficient set for a given network has also been addressed by Jia *et.al* [6]. In particular, Jia *el.al* [6] considered the problem of placing a minimum number of wavelength converters in a network such that the wavelength needed by the network does not exceed the maximum link load L. They refer to this feature of wavelength converter placement and wavelength assignment as $L-assignability$. Jia *et.al* proved that the problem of finding a minimum sufficient set for a network with duplex channel is optimally solvable in polynomial time. In addition, they proved that the problem of finding minimum sufficient set for a network with unidirectional channel is NP-complete (also proven by Kleinberg *et.al* [5]) and they proposed a 2-approximation algorithm for this problem.

By noticing that achieving L-assignability usually requires a large number of converters in some typical network topologies, another work of Jia *et.al* [7] aims at striking a trade-off between the number of wavelengths and the number of wavelength converters. In [7], Jia *et.al* introduced the notion of αL-assignability, which means that the number of required wavelengths will not exceed the maximum linkload by a factor of α. They showed that the problem of placing a minimum set of converters to achieve αL-assignability is NP-complete when α is fixed at $\frac{3}{2}$ (for duplex channel) and $\frac{5}{3}$(for unidirectional channel). Jia *et.al* also proposed a 2-approximation for both duplex and unidirectional cases.

Our work differ from the works in [5] [6] [7] [10] in two aspects:

1) The given set of lightpaths are taken into consideration as a design input. We note that doing so will help to reduce the redundant deployment of wavelength converters in a given network. For example, consider the scheme proposed in [6](for duplex channels) whereby converters are placed at each node whose degree is larger than two. In applying this scheme for a star network, a converter will be placed at the central node. Let's consider a case whereby there are only two lightpaths that pass through this network (as shown in Figure.1). It is easy to see that it is not necessary to place any converter in this case. Thus a redundant wavelength converter will be placed if the scheme proposed in [6] is adopted.
2) The schemes proposed by existing work only provide fixed upper bounds for the wavelength usage. Although Jia *et.al* introduced the notion of αL-assignability, where $1<\alpha<2$, they only address the case where $\alpha=\frac{3}{2}$ (for duplex channels) [7]. By noticing that there is a gap between L and $\frac{3}{2}L$, we adopt a variable upper bound αL, whereby

α is a variable which ranges between 1 to $\frac{3}{2}$. By considering the availability of converters and wavelengths, we can tighten this bound(use smaller α) if the wavelength is scarce and converter is comparably abundant and vice versa. It is easy to see that this feature provides additional flexibility for the network designers in the overall network design process.

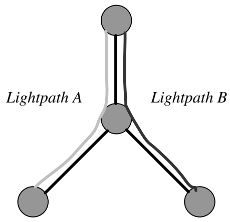

Figure 1. A case that converter is not necessary

III. Theoretical Preliminaries

A. Network Model

We model the network as an undirected simple graph G(V, E), where V is the vertex set and E is the edge set. The traffic demand is represented by a set of lightpaths $D = \{l_1, l_2, l_3 \dots l_k\}$. In this paper, we consider the case of static routing where all connections (lightpaths) are known in advance and stay for an infinite period of time in the network. The number of wavelengths needed to support all lightpaths in D is denoted by $W(G, D)$.

We assume that all communications support *duplex communication channels*, whereby data can transmit in both directions in the same fibre. The set of lightpaths that occupy the same link must be assigned with different wavelengths on this link regardless of their transmitting direction.

In this paper, we assume all converters have *full conversion capability* [15] [16]. This means the converter can translate an incoming wavelength into any outgoing wavelength. We adopt the *shared by node model* [16] which allows converters placed at a node to be shared by any lightpaths that pass through this node. In addition, we assume the capacity of each converters is large enough to support all lightpath pass though it.

Next we formally define the problem addressed in this paper as follows: given the network G and a set of traffic demand $D = \{l_1, l_2, l_3 \dots l_k\}$, locate a minimum set of nodes $S \subseteq V$ so that if we place wavelength converters at each node in S, the number of required wavelengths will not exceed the given bound αL, where L is the maximum linkload in the network and α is a parameter that can be defined by the network designer in the range of $[1, \frac{3}{2}]$. We refer this problem as *Optimal Wavelength Converter Placement with Bounded Wavelength Usage Problem*(*OPWB*).

B. The Computational Intractability of OPWB

Theorem 3.1 *OPWB* is NP-hard.

Proof. See Appendix.

C. Graph Decomposition

Consider the case whereby node v_i is equipped with wavelength converters. All lightpaths that pass though v_i can convert their wavelength at v_i. The set of lightpaths which shared these converters are thus split into two parts, one from source node to the converter node v_i while another one from v_i to the destination node. The wavelength assignments for these two parts are independent from each other; thus placing a set of wavelength converters at a set of nodes S will result in the splitting of lightpaths that pass through the nodes in S into shorter lightpaths. This feature can be described by the splitting operation which is defined as follows:

Given a graph $G(V, E)$ and subset $S \subseteq V$, let $G_S(V', E')$ be a new graph derived from G by splitting each node $x \in S$ into $deg(x)$ nodes in V', where $deg(x)$ denote the degree of node x in G. Each edge (x, y) in $G(V, E)$ becomes edge (x^*, y) in $G_S(V', E')$, where x^* is a new node that is generated by splitting x in $G(V, E)$. Let $W_x \subseteq V'$ denote the set of vertices in G_S which are derived from node x in G. Note that each node in W_x is of degree one. The process of decomposing node x in G into a new set of nodes W_x in G_S is referred to as the *splitting operation*(as in [5] [6] [7] [10]). Figure. 2 illustrates the decomposition of a given graph G into a new graph G_S by splitting nodes in the set S, where $S = \{3, 4\}$. Given a graph $G(V, E)$ and a set $S \subseteq V$, the process of decomposing a graph $G(V, E)$ by splitting the nodes in S can be expressed as follows: $G_S(V', E) = split[G(V, E), S]$

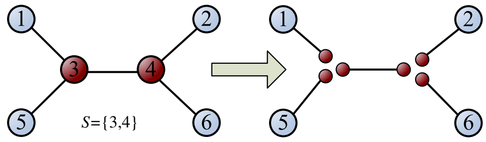

Figure 2. Derive a new graph by splitting operation

Since the task of wavelength assignment in networks with special topologies, such as paths, stars and trees, can be done more easily than in network with arbitrary topologies, we adopt the approach of decomposing a given network into edge-disjoint subgraphs with special topologies which include paths, stars and trees. The decomposition process is carried out by using the splitting operation described above. We note that such an approach has also been used in [5] [6] [7] [10]. However the objectives of our approach differ from those in [5] [6] [7] [10] as follow: the objectives of the work in [5] [6] [7] [10] are to select a set of converters for placement to satisfy L-assignability or $\frac{3}{2}L$-assignability, i.e. fixed bounds on wavelength usage; on the other hand, the objectives of our approach is to place a minimal set of wavelength converters to satisfy αL-assignability, where α is a parameter that may be specified by the user. Hence the problem addressed in this work is a generalization of those addressed in [5] [6] [7] [10].

IV. Networks with Special Topologies

A. Network with Path Topology

Theorem 4.1 [6] Given a network with path topology (which is often referred to as *linear network*), denoted by G_{path}, then $W(G_{path}, D) = L$ holds for arbitrary D, where L is the maximum linkload of G_{path}.

Theorem 4.2 [7] Given a network G, if every connected component of G is a path, then $W(G_{path}, D) = L$ holds for arbitrary D, where L is the maximum linkload of G.

It follows from Theorem 4.2 that if we split an arbitrary network into a set of linear networks, then L-assignability can always be achieved for the network. However, a major drawback of this approach is that a large number of nodes will have to be split in the process, thus resulting in high usage of wavelength converters.

B. Network with Star Topology

A *star* $G_{star}(V, E)$ is a graph whereby each vertex in G_{star} is of degree one except for one vertex whose degree is at least three. The vertex whose degree is three or above is referred to as the *center node* and all edges that adjacent to this vertex are called *legs*. We note that each lightpath in $G_{star}(V, E)$ has at most 2 hops. We will show that the wavelength assignment problem for a network with star topology can be transformed into an edge coloring problem which can be defined as follows.

Definition 4.1 Let G be a graph without loops, A $k-edge\ coloring$ of G is an assignment of k colors to the edges of G in such a way that any two edges meeting at a common vertex are assigned with different colors. If G has a k-edge coloring, then G is said to be $k-edge\ colorable$. The chromatic index of G, denoted by $\chi'(G)$, is the smallest value of k for which G is k-edge colorable. The problem of finding a k-edge coloring of G whereby $k = \chi'(G)$ is called the *edge coloring problem*.

Given a star network $G_{star}(V, E)$, we can construct a new graph $H^*(V^*, E^*)$ which we refer to as the *edge compatibility graph*, as follows:

Edge compatibility graph construction scheme(EGCS)

Input: A star network $G_{star}(V, E)$, $V = \{v_1, v_2, \dots v_n\}$, $E = \{e_1, e_2, \dots e_m\}$ and traffic demand $D = \{l_1, l_2, l_3 \dots l_k\}$.

Output: Edge compatibility graph $H^*(V^* \cup W^*, E^*)$.

1) $V^* = \emptyset; W^* = \emptyset; E^* = \emptyset;$
2) For each edge $e_i \in E$, create a vertex $v_i^* \in V^*$(shown as Figure.3-i);
3) For each lightpath $l_i \in D$ we do the following:

$Case(i)$: l_i is a 2-hop lightpath.
In this case, l_i will occupy two edges, say e_x and e_y in G_{star}. Insert an edge $e_i^* \in E^*$ in H^* that connects the two vertices v_x^* and v_y^* in H^* that correspond to the edges e_x and e_y(shown as Figure.3-ii).

$Case(ii)$: l_i is a 1-hop lightpath.
In this case, l_i will occupy an edge, say e_x in G_{star}. Insert a new vertex $w_i^* \in W^*$ in H^* and insert an edge $e_i^* \in E^*$ in H^* that will connect the pair of vertices $v_x^* \in V^*$ and $w_i^* \in W^*$in H^*(shown as Figure.3-iii).

Based on the construction scheme described above, it is easy to see that the *edge compatibility graph* H^* of a star network G_{star} satisfies the following properties:

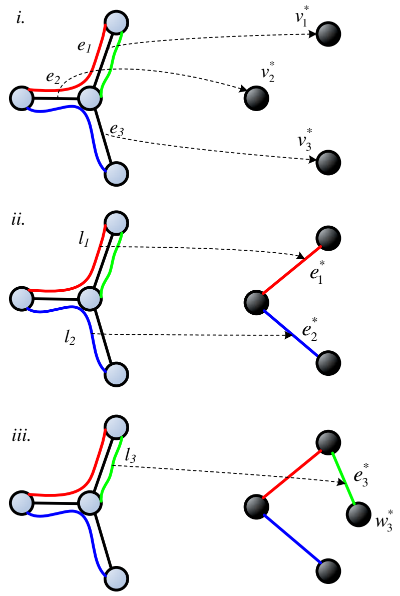

Figure 3. Constructing the edge compatibility graph for a star network

- Each vertex $v_i^* \in V^*$ in H^* corresponds to an edge $e_i \in E$ in G_{star};
- Each edge $e_i^* \in E^*$ in H^* corresponds to a lightpath $l_i \in D$ in G_{star};
- Any two edges in H^* are adjacent if and only if their corresponding lightpaths occupy the same edge in G_{star}.

Next we note that if G_{star} is a part of a larger network G, i.e. G_{star} is a subgraph of G, then there may exist more than one lightpaths traversing through the same pair of links in G_{star}. This in turn implies that there may exist more than one edges connecting the same pair of vertices in H^*, i.e.H^* is a multi-graph. Hence for the rest of the discussion in this paper, we shall assume that H^* is a multi-graph.

Since each pair of lightpaths in G_{star} must be assigned with different wavelengths if they occupy the same link, it is easy to see the task of assigning wavelengths to lightpaths in G_{star} is equivalent to that of assigning colors to the edges in H^* such that any two adjacent edges are assigned with different colors, i.e. solving the edge coloring problem on H^*. The edge coloring problem is known to be NP-hard [18] [19] and various results have been proposed in the literature to provide upper bounds on the chromatic index of a given graph. Some of these results are listed as follow.

Bounds on the chromatic index:

König's Theorem [20] If G is a bipartite multi-graph whose maximum vertex degree is d, then its chromatic index $\chi'(G) = d$.

Shannon's Theorem [21] If G is a multi-graph whose maximum vertex degree is d, then $d \leq \chi'(G) \leq \frac{3}{2}d$.

Vizing's Theorem (extended version) [22] If G is a multi-graph whose maximum vertex degree is d, and if h is the maximum number of edges joining a pair of vertices, then $d \leq \chi'(G) \leq d + h$.

Bounds on the wavelength requirement of a given network:

Theorem 4.3 Given a star network G_{star} with traffic demand D and it's maximum link load is denoted by L, let H^* be its edge compatibility graph constructed using *EGCS*. If H^* is a bipartite graph, then $W(G_{star}, D) = L$.

Proof. We note the maximum link load L of G_{star} is equal to the maximum degree d of H^*. Thus it follows from König's theorem the chromatic index of H^* is equal to L. This in turn implies that the wavelength requirement of G_{star} is L.

Theorem 4.4 Given a star network G_{star} with traffic demand D, let h denote the maximum number of lightpaths occupying the same pair of edges(links) in G_{star}, and let L denote the maximum linkload of G_{star}. Then $W(G_{star}, D) \leq Min(\frac{3}{2}L, L + h)$.

Proof. Let H^* be the edge compatibility graph of G_{star} constructed by *EGCS*. The maximum linkload L of G_{star} is equal to the maximum degree d of H^*. The maximum number of edges joining a pair of vertices in H^* is equal to the maximum number of lightpaths traversing the same pair of edges in G_{star}, i.e. h. Hence it follows from Shannon's Theorem and Vizing's Theorem that the chromatic index of H^* is bounded from above by $\frac{3}{2}L$ and $L + h$, respectively. This in turn implies that the wavelength requirement of G_{star} is bounded by $Min(\frac{3}{2}L, L + h)$.

C. Network with Bridges

Definition 4.2. Given a network $G(V, E)$, an edge e is called a *bridge* if $G - e$ is disconnected. Let C_1 and C_2 denote the two connected components of $G - e$, let $G_1 = C_1 \cup e$ and $G_2 = C_2 \cup e$, Then we say the two networks G_1 and G_2 are *singly connected* by bridge e.

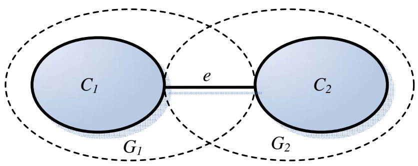

Figure 4. Two singly connected networks:G_1 and G_2

Theorem 4.5. Given two networks G_1 and G_2, if G_1 and G_2 are singly connected by bridge e, then $W(G, D) = max[W(G_1, D_1), W(G_2, D_2)]$, where $G = G_1 \cup G_2, D = D_1 \cup D_2$; D_1 and D_2 are the set of lightpaths that traverses G_1 and G_2, respectively.

Proof. Without lost the generality, we assume that $W(G_1, D_1) \geq W(G_2, D_2)$. We note that e is the only common edge of G_1 and G_2. Let T denote the set of lightpaths over e and $T = \{l_1, l_2, \ldots l_k\}$,$|T| = k$. Consider the case whereby wavelengths have been assigned to all lightpaths in G_1 and G_2 using their respective assignment schemes, which we refer to as $Scheme\,1$ and $Scheme\,2$.

We note that the wavelengths that have been assigned to G_1 and G_2 will form a valid assignment for G if the two schemes assign the same set of wavelengths to each lightpaths in T. The overall wavelength requirement of G in this case is $W(G, D) = W(G_1, D_1) = max[W(G_1, D_1), W(G_2, D_2)]$.

Next consider the case whereby Schemes 1 and Scheme 2 assign different set of wavelengths to the lightpaths in T. In this case conflict will arise between Scheme 1 and Scheme 2 in the assignment of wavelengths to the common lightpaths in T. Without loss of generality, we can resolve this conflict by keeping Scheme 1 unchanged while reassigning the wavelengths in Scheme 2 as described below:

Wavelength reassignment for scheme 2:

Let the set of wavelengths assigned to G_1 and G_2 be denoted by A and B, respectively. For each lightpath $l_i \in T$, let $x_i \in A$, $y_i \in B$ denote the wavelengths that have been assigned to l_i using Scheme 1 and Scheme 2, respectively.

1) Construct a bipartite graph $G_R(V, E)$, $V = A \cup B$, $E = \emptyset$;
2) Insert the edge (x_i, y_i) into E for $i = 1, 2, \ldots, k$;
3) For each vertex $b_j \in B - \{y_1, y_2, \ldots, y_k\}$, insert the edge (a_j, b_j) into E, where a_j is a vertex in $A - \{x_1, x_2, \ldots, x_k\}$ which is not adjacent to any vertex in B, i.e. $deg(a_j) = 0$. Since $|A| > |B|$, it is always possible to find such a vertex a_j;
4) Reassign the wavelength in Scheme 2 as follows: Let l_p be a lightpath in D_2 which has been assigned with wavelength p, i.e. $p \in B$. Let the vertex which is adjacent to p in the graph G_R be denoted by q, i.e. $(p, q) \in E$. In this case the lightpath l_p will be reassigned with wavelength q.

From the process described above, we can ensure that the reassignment of Scheme 2 satisfy the following two properties:

- After the reassignment, the set of lightpaths in T will be assigned with the same set of wavelengths by Scheme 1 and Scheme 2. This property is guaranteed by step 2. We can also note since all lightpaths in T shared a common edge, each lightpath will be assigned with a distinct wavelength in both Scheme 1 and Scheme 2. Thus each edge $(x_i, y_i) \in G_R$ inserted in step 2 will not adjacent to any other edges.
- This reassignment will not destroy the validity of Scheme 2. In the step 2 and step 3 described above, we always choose a vertex whose degree is 0 to insert a new edge, hence every vertex in A is of degree 1 or degree 0, this implies that in the reassignment of step 4, each wavelength in set B will be replaced by a distinct wavelength, any pair of lightpaths that are assigned with different wavelengths in Scheme 2 before reassignment will still be assigned with different wavelengths when the reassignment is finished.

Following the reassignment of wavelengths in G_2, the overall wavelength requirements of network G is again bounded by the $W(G_1, D_1) = max[W(G_1, D_1), W(G_2, D_2)]$.

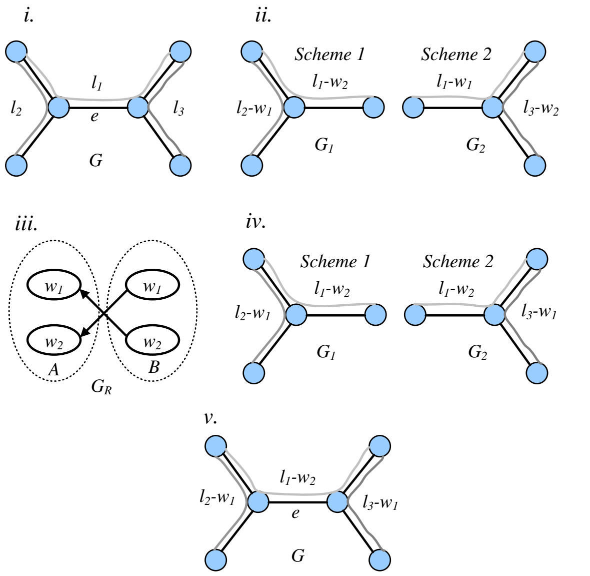

Figure 5. Reassignment of wavelengths in Scheme 2

In Figure. 5, we illustrate this process by a small example. As shown in Figure. 5-i, network G is composed by two singly connected stars G_1 and G_2, three lightpaths l_1, l_2, l_3 are traversing over G. l_1 is the common lightpath shared by G_1 and G_2. Scheme 1 and Scheme 2 assigned different wavelengths(shown in Figure. 5-ii) to l_1, thus we need to reassign the wavelengths in Scheme 2. We construct the bipartite graph G_R shown in Figure. 5-iii, $w_1 \in B$ is connected to $w_2 \in A$ because w_1 is assigned to l_1 in Scheme 2 and w_2 is assigned to l_1 in Scheme 1. After the reassignment for Scheme 2, two schemes assigned same wavelength to their common lightpath l_1(Figure. 5-iv), thus they can form a valid assignment for G(Figure. 5-v).

Theorem 4.6. A tree $G_{tree}(V, E)$ can be constructed by taking a union of a series components $C_1, C_2, \ldots, C_r$, whereby the following conditions hold:

i). $G_{tree} = \bigcup_{i=1}^{r} C_i$;
ii). C_i is either a path or a star, for $i = 1, 2, \ldots, r$;
iii). Given two components: $C_a = \bigcup_{i=1}^{m} C_i, C_b = C_{m+1}$, C_a and C_b are singly connected for $m = 1, 2, 3, \ldots, r-1$.

Proof. We prove this theorem by proposing a tree-construction scheme described as follow:

Tree construction scheme(*TCS*):

Input: A tree $G_{tree}(V, E)$.

Output: An ordered list $C = \{C_1, C_2, \ldots, C_r\}$ that satisfy conditions (i) to (iii) above.

1) Create an empty set C^*;
2) For each vertex $v_i \in V$, if $deg(v_i) > 2$, then v_i is the center vertex of a star. Thus we insert a star $S_i(V_i, E_i)$ into C^*, where V_i is composed of v_i(center node) and all its neighboring vertices(terminal nodes) in $G_{tree}(V, E)$, E_i is composed by the edges that adjacent to v_i in $G_{tree}(V, E)$.
3) The rest of $G_{tree}(V, E)$, i.e. the linear sub-networks between two star centers or between one star center and one 1-degree node are also inserted into C^* as a set of paths.
4) Now we have a set C^* whereby each member of C^* is either a star or a path. We randomly choose a member in C^* and insert this member into C as C_1. Then for $m = 1$ to $m = r - 1$, we do following: $C_a = \bigcup_{i=1}^{m} C_i$, choose a member which is singly connected to C_a in C^* and insert it into C as C_{m+1}. This process will finish when all members from C^* are inserted into C.

Figure. 6 gives an illustration for how a tree can be constructed by singly connecting a set of stars and paths. It is easy to verify that the ordered list of components derived by *TCS* satisfies conditions i-iii, so Theorem 4.6 holds.

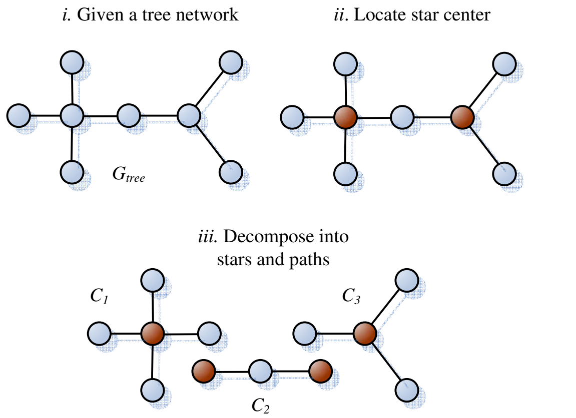

Figure 6. Constructing a tree by singly connecting a set of stars and paths

Theorem 4.7. For a tree network G_{tree}, $W(G_{tree}, D) \leq \alpha L$ if and only if for each star sub-network $C_i \subseteq G_{tree}$, $W(C_i, D_i) \leq \alpha L$, where D_i is the set of lightpaths that traverses C_i.

Proof. If: From Theorem 4.5 and Theorem 4.6 we have: $W(G_{tree}, D) = Max[W(C_1, D_1), W(C_2, D_2), \ldots, W(C_r, D_r)]$, where $C_1, C_2, \ldots, C_r$ is a set of star networks or linear networks that satisfy conditions i-iii stated in Theorem 4.6 and $D_1, D_2, \ldots, D_r$ are the lightpath sets traversing over $C_1, C_2, \ldots, C_r$, respectively. For each linear network C_j, Theorem 4.2 states that $W(C_j, D_j) \leq L \leq \alpha L$. Thus $W(G_{tree}, D) \leq \alpha L$ holds if the wavelength usage of each star networks, $W(C_i, D_i)$, is bounded by αL.

Only if: Since C_i is a sub-network of G_{tree}, $W(C_i, D_i) \leq W(G_{tree}, D)$. Thus if $W(C_i, D_i) > \alpha L$, then we will have: $W(G_{tree}, D) \geq W(C_i, D_i) > \alpha L$. Hence $W(G_{tree}, D) \leq \alpha L$ holds only when $W(C_i, D_i) \leq \alpha L$.

V. Proposed Algorithm

A. Algorithm for OPWB

As proven in [17], the problem of determining the wavelength usage of a network is NP-hard. In fact, to the best of our knowledge, no efficient upper bound has been proposed for the wavelength usage of a network with arbitrary topology. In [7] Jia *et.al* showed that even for a network with simple topology(a 4-nodes graph), its wavelength requirement may exceed $\frac{3}{2}L$. Furthermore, in [10] Wilfong *et.al* showed that for a single ring network, its wavelength requirement may also exceed $\frac{3}{2}L$.

Fortunately, if G is a tree network then its wavelength usage can be bounded by $\frac{3}{2}L$ regardless of the traffic demand [12]. Based on this fact, in the first step of our proposed algorithm, we aim to determine the minimum set S_1 so that $G_{S1}(V', E) = split[G(V, E), S_1]$ will be a tree or a forest(a set of disconnected trees). This problem is often referred to as the minimum feedback set problem and is known to be NP-complete [19]. However, as a well-studied problem, there exist some approximation algorithms with good performance guarantee. For example, in [23], a 2-approximate algorithm is proposed for this problem. Thus we can determine the vertex set S_1 which will be equipped with converters by applying these approximation algorithms.

After the converters are placed at each node in S_1, the wavelength usage of $G_{S1}(V', E)$ is bounded by $\frac{3}{2}L$. We can further tighten this bound by applying step 2. In this step, for each star sub-network, we examine the upper bound for its wavelength requirement which is determined using Theorem 4.3 and Theorem 4.4. For those star sub-networks whose upper bounds on wavelength usage exceed αL, we will include their center nodes into set S_2. Converters will be placed at each node in S_2. After all these converters are placed, some stars are split into paths and we let the resultant network be denoted as G_S. We can guarantee that for all remaining stars $C_i \subseteq G_S$, $W(C_i, D_i) \leq \alpha L$. Thus from Theorem 4.7 we have $W(G_S, D) \leq \alpha L$, which implies that the total wavelength usage is bounded by αL and the total number of converter nodes is $|S_1| + |S_2|$. Our algorithm can be described by the pseudo code:

Input: Network $G(V, E)$, $V = \{v_1, v_2, \ldots, v_n\}$, $E = \{e_1, e_2, \ldots, e_m\}$ with traffic demand set $D = \{l_1, l_2, \ldots, l_k\}$, the upper bound for the wavelength usage αL.

Output: Vertex set S.

1) Step one(place converters at the feedback set nodes):
   $S = \emptyset, S_1 = \emptyset$
   find the minimum feedback set S_1 for G
   $S = S \cup S_1$
   $G_S(V, E) = split[G(V, E), S]$
2) Step two(place converters at centers of star networks):
   $S_2 = \emptyset$
   for $i = 1$ to n **do**: /* for each vertex v_i
     if $deg(v_i) \leq 2$ /*v_i is not a star center
       $i{+}{+}$
     else
     build the edge-compatibility graph H_i^* of the star with center v_i by the *EGCS* scheme
     check whether H_i^* is a bipartite graph
     check the value of l and h, which denotes the maximum linkload and the maximum number of edges joining a pair of vertices, separately
     if H_i^* is not a bipartite graph **and** $Min(\frac{3}{2}l, l + h > \alpha L)$
       $S_2 = S_2 \cup v_i$, $i{+}{+}$
   $S = S \cup S_2$
   end and **output** vertex set S

B. Performance Analysis

1) Computational complexity

Theorem 5.1. The computational complexity of proposed two-step algorithm is $O(|E||V| + |D||V|)$.

Proof. In the first step, finding the minimum feedback set by using the approximation algorithm proposed in [23] can be done in $O(|E||V|)$ time; splitting operation can be done in $O(|E|)$ time in the worst case. In the second step, each legs of the stars should be checked to determine the maximum linkload of stars, we note each edge can be included in two stars at most so the complexity of checking linkload is $O(|E|)$. Next we note the number of lightpaths after Step 1 is $|D||V|$ at most, thus building edge compatibility graph by *EGCS* can be done in $O(|E| + |D||V|)$; checking whether the edge compatibility graph is bipartite for all stars can be done in $O(|E|+|D||V|)$. Step 2 will cost $O(|E|+|D||V|)$ and the two-step algorithm we proposed will cost $O(|E||V|+|D||V|)$ in the worst case.

2) The setting of α

As mentioned in Section I, the size of converter nodes set S is determined by network topology, traffic demand and given bound for the wavelength usage. In this section we will study the relationship between $|S|$ and the value of α:

1) $\alpha = 1$: In this case the wavelength usage is the minimum possible. Thus the size of S would be large. In the worst case, every center node of stars will be equipped with converters so the network will be split into a set of linear networks by node set S; this is the case that studied by [6].
2) $\alpha = \frac{3}{2}$: It is proved in [12] and [7] that for the network with tree topology, this upper bound can always be met for arbitrary traffic demand. Thus we do not need to place any converter in the second step. We can also note that in this case the *OPWB* is equivalent to the minimum feedback set problem. The wavelength converter placement problem under this case is studied by [7].
3) $1 < \alpha < \frac{3}{2}$: This is the general case that we are addressing in this paper, as shown our algorithm will generate a vertex set S with the size between case 1) and case 2).

VI. Experimental Study

In this section, we adopt experimental approach to study the relationship between the size of S and the value of α . The converter set S is constructed by the proposed algorithm. Three typical networks were studied which include NSFnet network, USA long haul network and mesh network. We also varied the size of mesh network from 4×4 to 7×7 to evaluate the effects of the network size. Some statistics of these networks are listed in Table I.

Traffic demand are generated for each pair of vertices with probability p, where p is a parameter controlling the total traffic load of the network. In this study we defined three types of traffic load condition:

i). Low traffic load, where p is set to 0.2;
ii). Moderate traffic load, where p is set to 0.5;
iii). High traffic load, where p is set to 0.8;

All traffic demands are routed by the shortest path algorithm. For each traffic load condition, we repeat the

TABLE I.
STATISTICS OF SOME TYPICAL NETWORKS

Topology	Number of vertices	Feedback set size	Number of vertices whose degree larger than two
NSFnet	14	3	10
USA long haul	28	8	21
4×4 mesh	16	4	12
7×7 mesh	49	13	45

experiment by ten times to get the mean value of $|S|$, which denotes the size of wavelength converter nodes set. The results are shown in Figures 7 to 10.

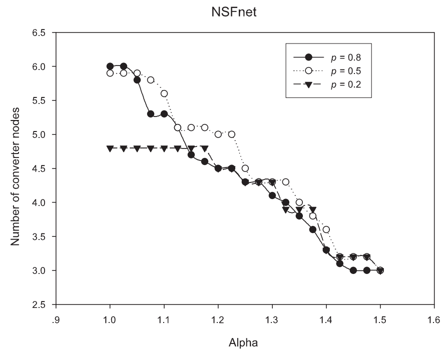

Figure 7. Number of converter nodes in NSFnet

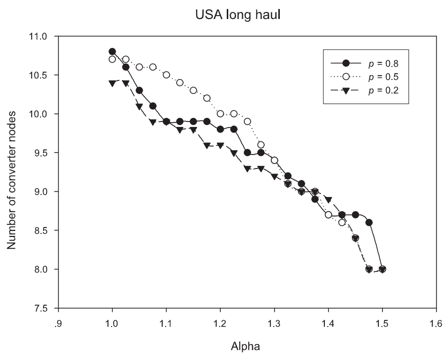

Figure 8. Number of converter nodes in USA long haul network

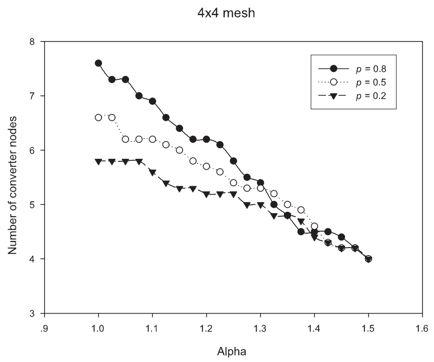

Figure 9. Number of converter nodes in 4×4 mesh network

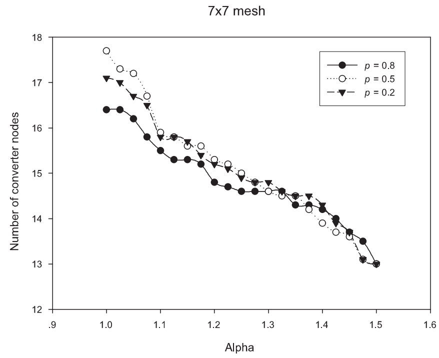

Figure 10. Number of converter nodes in 7×7 mesh network

The results show that the size of S decreases as α increases from 1 to $\frac{3}{2}$. This phenomenon is expected and it indeed verify that correctness of the implementation of our proposed algorithm. With these $|S|$ vs α curves, the network designer can easily estimate the upper bound of wavelength usage when given the number of wavelength converters or estimate the number of required converters when given the upper bound for the wavelength usage.

Next we note when the traffic load of network increase, both the number of lightpaths and the maximum linkload increase. This in turn results in an increase in the number of wavelengths needed. In addition, we note that our proposed algorithm is able to achieve similar trend of performance (in terms of the trade-off between the number of wavelength converters needed and the value of α) as the traffic load increases.

We could also note when $\alpha = 1$, which means the wavelength usage is the minimum possible, the size of S constructed by the proposed algorithm is much smaller than the size of converter set constructed by the algorithm proposed in [6], which is equal to the number of vertices whose degree is larger than two (listed in the last column of Table.I). For instance, the number of wavelength converters needed for the case of NSFnet when $\alpha = 1$ and $p = 0.2$ in Figure. 7 is equal to 5 while the number needed by the algorithm proposed by Jia et. al. [6] is equal to 10.

This result shows that taking the traffic demand into consideration will helps to reduce the redundant deployment of wavelength converters.

VII. Conclusion and Discussion

We have studied the problem of placing a minimal set of wavelength converters in WDM networks with arbitrary topology and the total wavelength usage is bounded. The traffic demand is also taken into consideration. In this work, the network designer can set the upper bound for wavelength usage in the range of $[L, \frac{3}{2}L]$. Thus the proposed algorithm is more flexible compared to existing work in this area. A two-step algorithm is proposed for this problem, its correctness is guaranteed by a set of theorems and its effectiveness is evaluated by both theoretical and experimental studies.

This work can benefit WDM network design and development in several aspects. Firstly, by considering the traffic status, the number of converter required can be further reduced compared to earlier works. Secondly, it can help us to understand the relationship between the number of converters and the bound on wavelength usage, thus enabling more efficient utilization of wavelength converters. Thirdly, by adopting our two-step algorithm and wavelength switching techniques, the wavelength assignment problem for a network with arbitrary topology can be reduced to a wavelength assignment problem in a set of independent stars and paths, which in turn helps in reducing the overall computational complexity.

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APPENDIX

Proof of Theorem 3.1: Let's formulate the corresponding decision problem of *OPWB*, which is referred as *OPWB'*, as follows:

Instance: A network $G(V,E)$, a lightpath set D, non-negative integer i_1 and positive real number r_1, where $i_1 = |S|$, $r_1 = \alpha L$.

Question: Is it possible to place i_1 converters in G so that the number of wavelengths required to support all lightpaths in D will not exceed r_1?

Now we consider the special case where $i_1 = 0$ and G is a star. We note it is equalent to the wavelength assignment problem in a star network:

Instance: A star network $G_{star}(V,E)$, a lightpath set D, positive real number r_1, where $r_1 = \alpha L$.

Question: When no converter is placed, is it possible establish all lightpaths in D so that the number of required wavelengths will not exceed r_1?

Lemma. The wavelength assignment problem in a star network is NP-hard.

Proof: In Section IV.B we have shown that the wavelength assignment problem for any star network can be transformed to the edge coloring problem by constructing the *edge compatibility graph*. In this proof we will show the complement is also true, i.e. an edge coloring problem can be transformed into a wavelength assignment problem of a star network. This transformation can be done by the scheme described below:

Star network construction scheme(*SNCS*)

Input: A graph $G(V, E)$, $V = \{v_1, v_2, \dots v_n\}$, $E = \{e_1, e_2, \dots e_m\}$.

Output: A star network $G_{star}(V^* \cup v_c, E^*)$, $V^* = \{v_1^*, v_2^*, \dots v_n^*\}$, $E^* = \{e_1^*, e_2^*, \dots e_n^*\}$ lightpath set $D = \{l_1, l_2, l_3 \dots l_m\}$.

1) $V^* = \emptyset; E^* = \emptyset; D^* = \emptyset;$
2) Create a vertex v_c as the center of the star network;
3) For each vertex $v_i \in V$, create a vertex $v_i^* \in V^*$ and insert an edge $e_i^* \in E^*$ by connecting v_c and v_i^*;
4) For each edge $e_i \in E$, where $e_i = (v_a, v_b), v_a \in V, v_b \in V$, we create a 2-hop lightpath $l_i \in D$ which traversing over edge $e_a^* \in E^*, e_b^* \in E^*$ that correspond to $v_a \in V, v_b \in V$.

It's easy to see that the star network G_{star} constructed by the scheme described above satisfies the following properties:

- Each vertex $e_i^* \in E^*$ in G_{star} corresponds to a vertex $v_i \in V$ in G;
- Each lightpath $l_i \in D$ in G_{star} corresponds to an edge $e_i \in E$ in G;
- Any two lightpaths in G_{star} will share an edge(link) $e_i^* \in E^*$ if and only if their corresponding edges are adjacent to the same vertex $v_i \in V$ in G.

From Definition 4.1 in Section IV.B, the task of edge coloring is to assign colors to all edges so that any pair of edges which are adjacent to the same vertex will be assigned with different colors; On the other hand, the task of wavelength assignment is to assign wavelengths to all lightpaths so that any pair of lightpaths which occupying the same link will be assigned with different wavelengths. It's easy to see the edge coloring problem in G is equivalent to the wavelength assignment problem in G_{star} when $r_1 = \chi'(G)$, i.e. the edge coloring problem of an arbitrary graph G is reducible to the wavelength assignment problem by *SNCS*, which can be done in polynomial time. Since the edge coloring problem is known to be NP-hard [18] [19], the wavelength assignment problem in a star network is also NP-hard.

As mentioned above, the wavelength assignment problem in a star network is a special case of *OPWB'*, hence *OPWB'* is NP-hard.