Outage Analysis of Cognitive Two-Way Relaying Network with Physical-Layer Network Coding in Nakagami-m Fading Channels

Jia Liu[1], Guixia Kang[2], Ying Zhu[1,3], and Ping Xie[1]
[1] Key Laboratory of Universal Wireless Communications, Ministry of Education
[2] Wireless Technology Innovation (WTI) Institute
Beijing University of Posts and Telecommunications, Beijing 100876, P.R.China
[3] Department of Teaching and Practicing, Guilin University of Electronic Technology, Guilin 541004, P.R.China
Email: liujia0406@126.com; gxkang@bupt.edu.cn; stefaniezhuy@163.com; xieping@bupt.edu.cn

***Abstract*—** In spectrum sharing systems, a secondary user (SU) is permitted to share frequency bands with primary user (PU) as long as its transmission does not interfere with the PU's communication. In this paper, the outage probability is investigated for the physical-layer network coding (PNC) based cognitive two-way relay network over Nakagami-m fading channel. By applying the interference temperature (IT) constraints at the transceiver nodes and relay nodes in secondary systems, we analyze the outage performance in underlay spectrum sharing with the best relay selection criterion. The cumulative distribution function (CDF) of the signal to noise ratio (SNR) for each link is derived to obtain the tight upper bound on the outage probability of the secondary relay system. Simulations results demonstrate the validity and accuracy of the theoretical analysis.

***Index Terms*—**Cognitive two-way relay network, physical layer network coding, outage probability, Nakagami-m fading channel.

I. Introduction

In cognitive radio network, unlicensed users (secondary users) are permitted to use the licensed band so long as they protect the data transmission of the licensed user (primary user) using spectrum underlay, overlay and interweave approaches [1]. Recently, underlay spectrum-sharing protocols have drawn increasing interest [2], [3]. The underlay paradigm allows cognitive (secondary) users to utilize the licensed spectrum if the interference caused to primary users is below a given interference threshold. Owing to the constraint on the transmit power, the performance of cognitive underlay protocols is severely degraded in fading environments.

One efficient method to improve the performance of the secondary network is to use cooperative communication with multiple relays [4]-[6]. As we know, cooperative communication is able to improve the channel capacity and achieve higher diversity gains in fading environments. Besides, owning to the diversity techniques, the cooperative cognitive relay is able to mitigate the signal fading arising from multipath propagation and improve the outage performance of wireless networks.

Due to bidirectional nature of communication networks, a promising relay technique, two-way relaying, has attracted much attention. Two-way relaying has higher spectral efficiency than the traditional one-way relaying. It is thus natural to incorporate two-way relaying into cognitive networks to further enhance the spectrum utilization [7]-[9]. Authors in [7], [8] introduced analog network coding into the cognitive relay network where two secondary user (SU) transceivers exchange their information with the assistance of a relay under IT constraint. The physical-layer network coding (PNC) based cognitive two-way relay network (CTRN) is discussed in [9] where two secondary transceiver nodes who are located on two different primary user (PU) coverage areas, exchange their information with the assistance of a relay in underlay spectrum sharing environment.

As we all know, the outage probability is an important performance indicator for cognitive relay systems. However, previous works mainly focus on the traditional one way relaying [10]-[12], there are few works on the outage performance of cognitive two-way relay network. As an initial attempt to this work, the authors in [13] analyzed the outage performance of a cognitive two-way relaying system under opportunistic relay selection strategy over flat and block Rayleigh fading channels. Though the two-way relaying scheme was proposed for three time slots, it had verified the two-way relaying scheme has better outage performance than one way relaying. In this paper, we build upon the work of [9] and analyze the outage performance for the PNC based CTRN with IT constraint to PUs. Meanwhile, in order to give an explicit relation between the practical channel fading and the outage probability of cognitive relay

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Corresponding author email: liujia0406@126.com.
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network, the outage performance under Nakagami-m fading channels is analyzed, since this channel model has been extensively studied in various wireless communication systems and can capture the physical channel phenomena more accurately than Rayleigh and Rician models. Eventually, we derive the tight upper bound on the outage probability of the PNC based CTRN and then investigate the impact of various key system parameters, such as interference temperature and channel fading severity on the outage probability of the system.

The rest of this paper is organized as follows. In Section II, the system model is described. In Section III, the outage probability of the spectrum sharing relay system over the Nakagami-m channel is mathematically analyzed. Simulation results are provided in Section IV and conclusions are offered in Section V.

II. System Model

Fig. 1 presents the system model for the PNC based cognitive two-way relaying under interference constraint. In this figure, there are two secondary sources S1 and S2 attempting to transmit their packet to each other. We assume that there is no direct link between the sources and hence the data transmission between these sources is realized with the help of M potential relays. It is assumed that each node is equipped with a single antenna and operates in half-duplex mode. For ease of analysis, we assume the distances between each source and all relays are same.

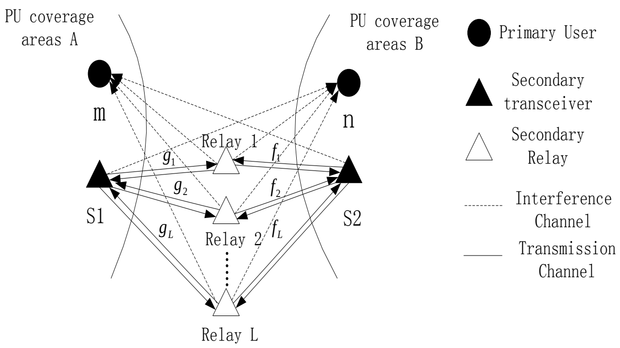


Figure 1. System model for PNC based cognitive two-way relay networks

As shown in Fig. 1, two PU coverage areas are considered, users in coverage areas use orthogonal frequency sets I_a and I_b. Assuming that m and n are the most affected PUs due to SU communication. The SU employ PNC based cognitive two way relaying as [9]. The transmission protocol consists of two orthogonal time slots. During the first timeslot, we consider the following. S1 transmits signal to the relay with the frequency set I_b, S2 transmits signal to the relay with the frequency set I_a. During the second timeslot, the relay firstly converts the received signal to a PNC modulated signal, then uses both I_a and I_b spectrum sets to transmit the PNC-modulated signal to S1 and S2 separately. The whole process has been discussed detailedly in [9]. The mutual information I_1^{PNC}, I_2^{PNC} and I_{sum}^{PNC} are given as [14]:

$$I_1^{PNC} = \min\left\{\frac{1}{2}\log_2\left(1+\gamma_{1,i}\right), \frac{1}{2}\log_2\left(1+\gamma_{i,2}\right)\right\} \quad (1)$$

$$I_2^{PNC} = \min\left\{\frac{1}{2}\log_2\left(1+\gamma_{2,i}\right), \frac{1}{2}\log_2\left(1+\gamma_{i,1}\right)\right\} \quad (2)$$

$$I_{sum}^{PNC} = \frac{1}{2}\log_2\left(1+\gamma_{1,i}+\gamma_{2,i}\right) \quad (3)$$

where $\gamma_{1,i}$, $\gamma_{2,i}$ are SNR from S1 at the *t*th relay, SNR from S2 at the *ith* relay. $\gamma_{i,1}, \gamma_{i,2}$ are SNR from *t*th relay at the S1, SNR from *t*th relay at the S2. Constant value $1/2$ denotes the information exchange takes two time-slots. To protect PNC's transmission, the following inequalities should be satisfied [9]:

$$\gamma_{1,i} \le \gamma_{i,2} \quad ; \quad \gamma_{2,i} \le \gamma_{i,1} \quad (4)$$

Assuming all the channels are reciprocal, as shown in Fig. 1, we define g_i, f_i as the channel between S1 and *t*th relay, the channel between S2 and *t*th relay respectively. Then the inequalities in (4) can be rewritten as

$$P_{1,i}\left|g_i\right|^2 \le P_{r,i}\left|f_i\right|^2, \quad P_{2,i}\left|f_i\right|^2 \le P_{r,i}\left|g_i\right|^2 \quad (5)$$

where $P_{1,i}$; $P_{2,i}$ and $P_{r,i}$ are the transmit powers of S1; S2 and relay node when *t*th relay is selected. And during the two timeslots, the system should fulfill the following IT constraints to provide stable PU communication [9]:

$$P_{1,i}\left|h_{1,n}\right|^2 \le Q_n, \quad P_{2,i}\left|h_{2,m}\right|^2 \le Q_m \quad (6)$$

$$P_{r,i}\left|h_{r,n}^i\right|^2 \le Q_n, \quad P_{r,i}\left|h_{r,m}^i\right|^2 \le Q_m \quad (7)$$

where Q_m and Q_n are the IT threshold of primary user m and primary user n respectively. $\left|h_{1,n}\right|$ and $\left|h_{2,m}\right|$ denote the channel coefficients of the $S1 \to n$ link and $S2 \to m$ link respectively. $\left|h_{r,n}^i\right|$ and $\left|h_{r,m}^i\right|$ are the channel coefficients of the *i*th relay $\to n$ link and *i*th relay $\to m$ link respectively. It is assumed that all channels are independent but not necessarily identically distributed.

$$\begin{aligned} P_{out} &= \Pr\left(I_1^{PNC} < \frac{R_{th}}{2} \text{ or } I_2^{PNC} < \frac{R_{th}}{2} \text{ or } I_{sum}^{PNC} < R_{th}\right) \\ &= \Pr\Big(\gamma_{1,r} < 2^{R_{th}} - 1 \text{ or } \gamma_{2,r} < 2^{R_{th}} - 1 \\ &\quad \text{ or } \left(\gamma_{1,r}+\gamma_{2,r}\right) < 2^{2R_{th}} - 1\Big) \end{aligned} \quad (8)$$

Similar to [14], Eq. (15)], we set the target data rate for each end-source as $R_{th}/2$ assuming the target data rate for the whole network is R_{th}. The system is in outage when the rate falls out of the capacity region. Therefore, the outage probability for PNC based cognitive relay system is given by

III. Outage Analysis

Here, we investigate the outage probability of the previously described cognitive relay system. We define the relaying set $R(n)$ to be the set of successful potential relays which correctly decode the message from the transceivers. The ith relay selected into $R(n)$ needs to satisfy $(1/2)\log_2(1+\gamma_{1,i}) \geq R_{th}$ and $(1/2)\log_2(1+\gamma_{2,i}) \geq R_{th}$. Therefore, the probability that the ith relay is selected into $R(n)$ is given as

$$\begin{aligned}\Pr(i \in R(n)) &= \Pr(\gamma_{1,i} \geq 2^{R_{th}} - 1)\Pr(\gamma_{2,i} \geq 2^{R_{th}} - 1) \\ &= \left(1 - F_{\gamma_{1,i}}\left(2^{R_{th}} - 1\right)\right)\left(1 - F_{\gamma_{2,i}}\left(2^{R_{th}} - 1\right)\right)\end{aligned} \tag{9}$$

where $F_{\gamma_{1,i}}, F_{\gamma_{2,i}}$ are the CDF of $\gamma_{1,i}, \gamma_{2,i}$ respectively. Assuming $\gamma_{1,i}$, $\gamma_{2,i}$ are independent random variables, we then have

$$\begin{aligned}\Pr(R(s)) = &\prod_{i\in R(s)}\left(1 - F_{\gamma_{1,i}}\left(2^{2R_{th}} - 1\right)\right)\left(1 - F_{\gamma_{2,i}}\left(2^{2R_{th}} - 1\right)\right)\times \\ &\prod_{i\notin R(s)} 1 - \left(1 - F_{\gamma_{1,i}}\left(2^{2R_{th}} - 1\right)\right)\left(1 - F_{\gamma_{2,i}}\left(2^{2R_{th}} - 1\right)\right)\end{aligned} \tag{10}$$

The relay selection criterion is given as

$$d = \arg\max_{i\in R(n)}\left(\gamma_{1,i} + \gamma_{2,i}\right) \tag{11}$$

where d is the "best" relay node. Consider the secondary nodes use their maximum allowable power while satisfying the interference temperature requirement perceived at the PUs. Then based on (5), (6) and (7), the transmit powers of S1, S2 and relay can be developed as following

$$P_{1,i} = \min\left\{\frac{Q_n}{\left|h_{1,n}\right|^2}, \frac{P_{r,i}\left|f_i\right|^2}{\left|g_i\right|^2}\right\} \tag{12}$$

$$P_{2,i} = \min\left\{\frac{Q_m}{\left|h_{2,m}\right|^2}, \frac{P_{r,i}\left|g_i\right|^2}{\left|f_i\right|^2}\right\} \tag{13}$$

$$P_{r,i} = \min\left\{Q_n\left|h_{r,n}^i\right|^{-2}, Q_m\left|h_{r,m}^i\right|^{-2}\right\} \tag{14}$$

Also we assume that the noise variance δ^2 as 1 without loss of generality. Therefore, the received SNR at ith relay can be obtained as

$$\gamma_{1,i} = \min\left\{\frac{Q_n\left|g_i\right|^2}{\left|h_{1,n}\right|^2}, P_{r,i}\left|f_i\right|^2\right\} \tag{15}$$

$$\gamma_{2,i} = \min\left\{\frac{Q_m\left|f_i\right|^2}{\left|h_{2,m}\right|^2}, P_{r,i}\left|g_i\right|^2\right\} \tag{16}$$

For notational convenience, we define

$$U_1 = Q_n\left|g_i\right|^2\left|h_{1,n}\right|^{-2},$$
$$V_1 = \min\left\{Q_n\left|h_{r,n}^i\right|^{-2}, Q_m\left|h_{r,m}^i\right|^{-2}\right\}\left|f_i\right|^2.$$

Starting from (15) and knowing the independence of random variables U_1 and V_1, then the CDF of SNR $\gamma_{1,i}$ can be computed as

$$\begin{aligned}F_{\gamma_{1,i}}(x) &= \Pr(\min\left(U_1, V_1\right) < x) \\ &= 1 - \Pr(\min\left(U_1, V_1\right) > x) \\ &= 1 - \Pr(U_1 > x, V_1 > x) \\ &= 1 - \left[\left(1 - F_{U_1}(x)\right)\left(1 - F_{V_1}(x)\right)\right] \\ &= F_{U_1}(x) + F_{V_1}(x) - F_{U_1}(x)F_{V_1}(x)\end{aligned} \tag{17}$$

Similarly, when we define

$$U_2 = Q_m\left|f_i\right|^2\left|h_{2,m}\right|^{-2}$$
$$V_2 = \min\left\{Q_n\left|h_{r,n}^i\right|^{-2}, Q_m\left|h_{r,m}^i\right|^{-2}\right\}\left|g_i\right|^2$$

the CDF of SNR $\gamma_{2,i}$ can be obtained as

$$F_{\gamma_{2,i}}(x) = F_{U_2}(x) + F_{V_2}(x) - F_{U_2}(x)F_{V_2}(x) \tag{18}$$

According to (8) and (10), we know that there is only one outage probability value for one given $R(n)$. Thus, the outage probability can be written in the form of the total probability law as follows:

$$\begin{aligned}P_{out} = &\Pr\left\{R(n) = \phi\right\}\times \\ &\Pr\left\{I_1 < \frac{R_{th}}{2} \text{ or } I_2 < \frac{R_{th}}{2} \text{ or } I_{sum} < \frac{R_{th}}{R(n)} = \phi\right\} + \\ &\Pr\left\{R(n)\right\}\times \\ &\sum_{R(n)\neq\phi}\Pr\left\{I_1 < \frac{R_{th}}{2} \text{ or } I_2 < \frac{R_{th}}{2} \text{ or } I_{sum} < \frac{R_{th}}{R(n)}\right\} \\ = &\, p_{o1} + p_{o2}\end{aligned} \tag{19}$$

When $R(n)$ is empty, there is no relay will be selected, and the outage probability is p_{o1}. When $R(n)$ is not empty, the outage probability is p_{o2}. Then we have

$$\Pr\left\{I_1^{PNC} < \frac{R_{th}}{2} \text{ or } I_2^{PNC} < \frac{R_{th}}{2} \text{ or } I_{sum}^{PNC} < R_{th}\left|R(n) = \phi\right.\right\} = 1 \tag{20}$$

$$\begin{aligned}&\Pr(I_1^{PNC} < \frac{R_{th}}{2} \text{ or } I_2^{PNC} < \frac{R_{th}}{2} \text{ or } I_{sum}^{PNC} < R_{th}\left|R(n)\right.) \\ &= \Pr(\gamma_{1,d} + \gamma_{2,d} < 2^{2R_{th}} - 1) \\ &= \prod_{i\in R(n)}\Pr(\gamma_{1,i} + \gamma_{2,i} < 2^{2R_{th}} - 1) \\ &\leq \prod_{i\in R(n)}\Pr(\gamma_{1,i} < 2^{2R_{th}} - 1)\Pr(\gamma_{2,i} < 2^{2R_{th}} - 1) \\ &= \prod_{i\in R(n)}F_{\gamma_{1,i}}\left(2^{2R_{th}} - 1\right)F_{\gamma_{2,i}}\left(2^{2R_{th}} - 1\right)\end{aligned} \tag{21}$$

Then, we can get the closed form upper bound of total outage probability within the spectrum sharing relay system as

$$\begin{aligned}P_{out} &= \prod_{i=1}^{M}\left(1-\left(1-F_{\gamma_{1,i}}\left(2^{R_{th}}-1\right)\right)\left(1-F_{\gamma_{2,i}}\left(2^{R_{th}}-1\right)\right)\right)+\\ &\sum_{R(n)\neq\phi}\left(\left(\prod_{i\in R(n)}\left(1-F_{\gamma_{1,i}}\left(2^{R_{th}}-1\right)\right)\left(1-F_{\gamma_{2,i}}\left(2^{R_{th}}-1\right)\right)\times\right.\right.\\ &\left.\prod_{i\notin R(n)}\left(1-\left(1-F_{\gamma_{1,i}}\left(2^{R_{th}}-1\right)\right)\left(1-F_{\gamma_{2,i}}\left(2^{R_{th}}-1\right)\right)\right)\right)\times\\ &\left.\prod_{i\notin R(n)}F_{\gamma_{1,i}}\left(2^{R_{th}}-1\right)F_{\gamma_{2,i}}\left(2^{R_{th}}-1\right)\right)\end{aligned} \quad (22)$$

where $F_{\gamma_{1,i}}(x)$, $F_{\gamma_{2,i}}(x)$ are given in (17) and (18). $F_{U_1}(x)$, $F_{V_1}(x)$, $F_{U_2}(x)$ and $F_{V_2}(x)$ are derived in Appendix.

IV. Numerical Results

Based on the analysis above, the factors which affect the outage performance are: a. interference temperature; b. the quality of data and interference channels, i.e. the fading exponent m in Nakagami-m channel model; c. the number of SUs which can be relay. In this section, Monte-Carlo simulations are implemented and the impacts on outage performance from these factors are also analyzed. We assume that the coordinates of S1, S2, relay, m and n are (0, 0); (1, 0); (x1, 0); (0, y1) and (1, y1) respectively, where $0 < x1 < 1, y1 > 0$. Therefore, the relay position is measured as $d_1 = x1$.

These simulations parameters are as follows: the wireless channels are modeled as Nakagami-m channel, and the fading exponents of interference channels are respectively $m_{1,n}$, $m_{2,m}$, $m_{r,n}$ and $m_{r,m}$. The fading exponents of data channels are respectively m_{g_i} and m_{f_i}. The threshold of interference temperature of PUs is set as $Q_m = Q_n = Q$. The number of potential relay nodes is M. The rate threshold of cognitive relay networks is set as $R_{th} = 1 bit/s/Hz$. Further, we set $\Omega_{g_i} = \Omega_{f_i} = \Omega_{1,n} = \Omega_{2,m} = \Omega_{r,n} = \Omega_{r,m} = 1$.

Fig. 2 shows the impact of fading parameters on the outage probability of the cognitive relay system with interference temperature constraints. We set $m_{1,n} = m_{2,m} = m_{s,p}$, $M = 3$, $m_{r,n} = m_{r,m} = m_{r,p}$. The simulation results validate the theoretical results. Furthermore, the outage probability will decrease when the interference temperatures Q increases. It can be seen that the better the channel is , the better the outage performance is. The interference channel will mainly affect the interference to primary users from secondary users. If the data channels are perfect, the better interference channel can also improve the outage performance.

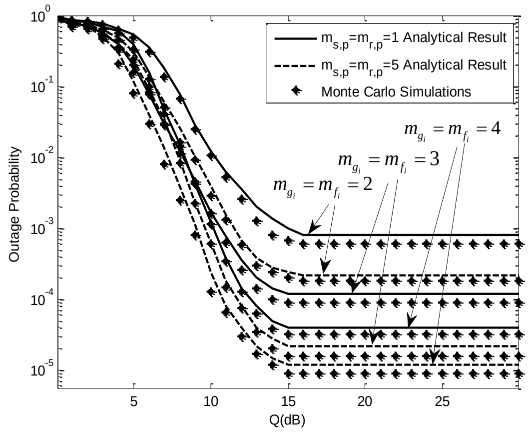


Figure 2. Impact of fading parameters on the outage probability of the PNC based CTRN with interference temperature constraints.

Fig. 3 shows the impact on the outage performance from the numbers of potential relay nodes. We set $m_{s,p} = m_{r,p} = 1$. It is indicated that the outage probability can be decreased by increasing M. Meanwhile, due to the threshold of the interference temperature, the floor of outage probability upper bound exists.

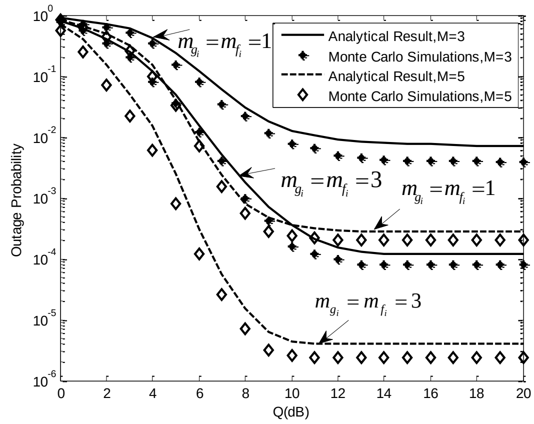


Figure 3. Impact of numbers of potential relay on the outage probability of the PNC based CTRN.

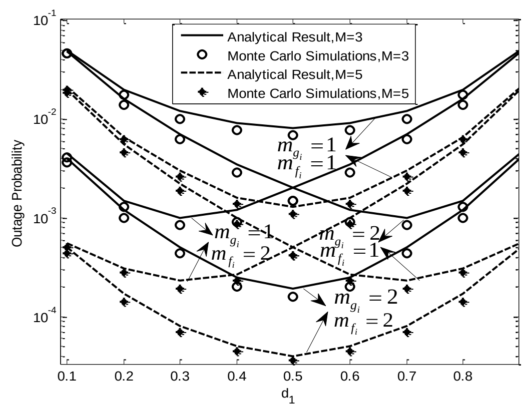


Figure 4. Impact of the relay position on the outage probability of the PNC based CTRN.

Fig. 4 shows the effect of the relay position on the outage performance. We set $Q = 15dB$, $m_{s,p} = m_{r,p} = 1$.

As we can see, if $m_{g_i} = m_{f_i}$, the system has the optimal outage performance when the relay is located at the center of S1 and S2, whereas the optimal relay position is inclined to near the transceiver node with bad channel condition when $m_{g_i} \neq m_{f_i}$.

V. CONCLUSION

In this paper, the outage performance of PNC based cognitive two-way relay network over the Nakagami-m channel is investigated. The impact of fading exponent in the wireless channel model on outage performance is analyzed. Accordingly, the PDF and CDF of SNR are derived at secondary receiver nodes over the Nakagami-m channel, and the tight upper bound of outage probability is obtained. The simulation results validate the accuracy of theoretical analysis. Furthermore, the interference temperature of PUs will affect the actual transmission power, and it will cause the floor of outage probability. Moreover, the better the channel is and the more relay nodes are, the better the outage performance is. As expected, the optimal relay position exits at the center of two SU transceivers when the channel coefficients from two SU transceivers to the relay are equal. Therefore, the outage performance can be improved by modifying the interference temperature of PUs and optimizing the relay position for the cognitive network.

VI. APPENDIX

This appendix provides the derivation of cumulative density function (CDF) of U1, V1, U2 and V2 in section III. We first present the probability density function (PDF) and CDF of a gamma random variable g with parameters m and Ω as follows:

$$f_g(x) = \frac{\beta^m}{\Gamma(m)} x^{m-1} e^{-\beta x} \tag{23}$$

$$F_g(x) = \frac{\gamma(m, \beta x)}{\Gamma(m)} \tag{24}$$

where $\beta = (m/\Omega)$, and $\gamma(a, x)$ is the incomplete gamma function [15], which is defined as

$$\gamma(n,x) = \Gamma(n)\left(1 - e^{-x}\sum_{i=0}^{n-1}\frac{x^i}{i!}\right) \tag{25}$$

Note that $U_1 = Q_n |g_i|^2 |h_{1,n}|^{-2}$, where $|g_i|$ and $|h_{1,n}|$ are independent but nonidentically distributed Nakagami-m random variables with parameters m_{g_i}, Ω_{g_i} and $m_{1,n}, \Omega_{1,n}$, respectively. Then, the CDF of random variable U_1 is given by

$$F_{U_1}(x) = \Pr\left\{\frac{Q_n |g_i|^2}{|h_{1,n}|^2} \le x\right\} = \int_0^\infty f_{h_{1,n}}(y) \int_0^{\frac{xy}{Q_n}} f_{g_i}(z)\,dz\,dy$$

$$= \frac{\beta_{1,n}^{m_{1,n}}}{\Gamma(m_{g_i})\Gamma(m_{1,n})}\int_0^\infty \gamma\left(m_{g_i}, \frac{\beta_{g_i}xy}{Q_n}\right)\frac{y^{m_{1,n}-1}}{e^{\beta_{1,n}y}}dy$$

$$= \frac{\beta_{1,n}^{m_{1,n}}}{\Gamma(m_{1,n})}\int_0^\infty \left(1-\sum_{i=0}^{m_{g_i}-1}\frac{(\beta_{g_i}xy)^i e^{\frac{-\beta_{g_i}xy}{Q_n}}}{(Q_n)^i\, i!}\right)\frac{y^{m_{1,n}-1}}{e^{\beta_{1,n}y}}dy$$

$$= \frac{\beta_{1,n}^{m_{1,n}}}{\Gamma(m_{1,n})}\left[\int_0^\infty y^{m_{1,n}-1}e^{-\beta_{1,n}y}dy\right] - \frac{\beta_{1,n}^{m_{1,n}}}{\Gamma(m_{1,n})}\sum_{i=0}^{m_{g_i}-1}\frac{(\beta_{g_i}x)^i}{(Q_n)^i\, i!}\int_0^\infty e^{\frac{-\beta_{g_i}xy}{Q_n}-\beta_{1,n}y}\,y^{m_{1,n}+i-1}dy$$

$$= 1-\frac{\beta_{1,n}^{m_{1,n}}}{\Gamma(m_{1,n})}\sum_{i=0}^{m_{g_i}-1}\frac{(\beta_{g_i}x)^i\,\Gamma(m_{1,n}+i)}{\left(\frac{\beta_{g_i}x}{Q_n}+\beta_{1,n}\right)^{m_{1,n}+i}(Q_n)^i\, i!} \tag{26}$$

Meanwhile, note that the variable $V_1 = \min\left\{Q_n|h_2|^2|h_{r,n}|^{-2}, Q_m|h_2|^2|h_{r,m}|^{-2}\right\}$, where $|h_{r,n}|, |h_{r,m}|$ and $|h_2|$ are independent but nonidentically distributed Nakagami-m random variables with parameters $m_{r,n}, \beta_{r,n}, m_{r,m}, \beta_{r,m}$ *and* m_{h_2}, β_{h_2} respectively. Then, the CDF of random variable V_1 is given by

$$F_{V_1}(x) = \Pr\left(\min\left\{\frac{Q_n|f_i|^2}{|h_{r,n}|^2}, \frac{Q_m|f_i|^2}{|h_{r,m}|^2}\right\} < x\right)$$

$$= \Pr\left(|h_{r,n}|^2 > \max\left(\frac{Q_n}{Q_m}|h_{r,m}|^2, \frac{Q_n|f_i|^2}{x}\right)\right) + \Pr\left(|h_{r,m}|^2 > \max\left(\frac{Q_m}{Q_n}|h_{r,n}|^2, \frac{Q_m|f_i|^2}{x}\right)\right)$$

$$= \int_0^\infty f_{h_{r,n}}(t)\Pr\left[\max\left(\frac{Q_n}{Q_m}|h_{r,m}|^2, \frac{Q_n|f_i|^2}{x}\right) < t\right]dt + \int_0^\infty f_{h_{r,m}}(t)\Pr\left[\max\left(\frac{Q_m}{Q_n}|h_{r,n}|^2, \frac{Q_m|f_i|^2}{x}\right) < t\right]dt$$

$$= \int_0^\infty f_{h_{r,n}}(t)F_{h_{r,m}}\left(\frac{tQ_m}{Q_n}\right)F_{f_i}\left(\frac{tx}{Q_n}\right)dt + \int_0^\infty f_{h_{r,m}}(t)F_{h_{r,n}}\left(\frac{tQ_n}{Q_m}\right)F_{f_i}\left(\frac{tx}{Q_m}\right)dt$$

$$= \underbrace{\int_0^\infty \frac{\beta_{r,n}^{m_{r,n}}\gamma\left(m_{r,m}, \frac{tQ_m\beta_{r,m}}{Q_n}\right)\gamma\left(m_{f_i}, \frac{tx\beta_{f_i}}{Q_n}\right)}{\Gamma(m_{r,n})\Gamma(m_{r,m})\Gamma(m_{f_i})t^{1-m_{r,n}}e^{\beta_{r,n}t}}dt}_{J1(x)} + \underbrace{\int_0^\infty \frac{\beta_{r,m}^{m_{r,m}}\gamma\left(m_{r,n}, \frac{tQ_n\beta_{r,n}}{Q_m}\right)\gamma\left(m_{f_i}, \frac{tx\beta_{f_i}}{Q_m}\right)}{\Gamma(m_{r,m})\Gamma(m_{r,n})\Gamma(m_{f_i})t^{1-m_{r,m}}e^{\beta_{r,m}t}}dt}_{J2(x)} \tag{27}$$

Then, utilizing the expansion for an incomplete gamma function defined in (25) , we can evaluate $J1(x)$ as

$$J1(x)=\int_0^\infty \frac{\beta_{r,n}^{m_{r,n}} t^{m_{r,n}-1} e^{-\beta_{r,n}t}}{\Gamma(m_{r,n})}dt-\frac{\beta_{r,n}^{m_{r,n}}}{\Gamma(m_{r,n})}\times$$

$$\sum_{i=0}^{m_{r,m}-1}\frac{\left(Q_m\beta_{r,m}\right)^i}{\left(Q_n\right)^i i!}\int_0^\infty t^{m_{r,n}+i-1}e^{-\left(\beta_{r,n}+\frac{Q_m\beta_{r,m}}{Q_n}\right)t}dt-$$

$$\frac{\beta_{r,n}^{m_{r,n}}}{\Gamma(m_{r,n})}\sum_{j=0}^{m_{f_i}-1}\frac{\left(x\beta_{f_i}\right)^j}{\left(Q_n\right)^j j!}\int_0^\infty t^{m_{r,n}+j-1}e^{-\left(\beta_{r,n}+\frac{x\beta_{f_i}}{Q_n}\right)t}dt+$$

$$\frac{\beta_{r,n}^{m_{r,n}}}{\Gamma(m_{r,n})}\sum_{i=0}^{m_{r,m}-1}\sum_{j=0}^{m_{f_i}-1}\frac{\left(Q_m\beta_{r,m}\right)^i\left(x\beta_{f_i}\right)^j}{Q_n^{i+j}i!\,j!}\times$$

$$\int_0^\infty e^{\frac{-t(Q_m\beta_{r,m}+x\beta_{f_i}+\beta_{r,n}Q_n)}{Q_n}}t^{m_{r,n}+i+j-1}dt$$

$$=1-\frac{\beta_{r,n}^{m_{r,n}}}{\Gamma(m_{r,n})}\sum_{i=0}^{m_{r,m}-1}\frac{\left(Q_m\beta_{r,m}\right)^i\Gamma(m_{r,n}+i)}{\left(Q_n\right)^i i!\left(\beta_{r,n}+\frac{Q_m\beta_{r,m}}{Q_n}\right)^{m_{r,n}+i}}-$$

$$\frac{\beta_{r,n}^{m_{r,n}}}{\Gamma(m_{r,n})}\sum_{j=0}^{m_{f_i}-1}\frac{\left(x\beta_{f_i}\right)^j\Gamma(m_{r,n}+j)}{\left(Q_n\right)^j j!\left(\beta_{r,n}+\frac{x\beta_{f_i}}{Q_n}\right)^{m_{r,n}+j}}+$$

$$\frac{\beta_{r,n}^{m_{r,n}}}{\Gamma(m_{r,n})}\sum_{i=0}^{m_{r,m}-1}\sum_{j=0}^{m_{f_i}-1}\frac{\left(Q_m\beta_{r,m}\right)^i\left(x\beta_{f_i}\right)^j\Gamma(m_{r,n}+i+j)}{Q_n^{i+j}i!\,j!\left(\beta_{r,n}+\frac{Q_m\beta_{r,m}+x\beta_{f_i}}{Q_n}\right)^{m_{r,n}+i+j}} \quad (28)$$

By the same way discussed above, the last term of (27) can be derived as follows

$$J2(x)=1-\frac{\beta_{r,m}^{m_{r,m}}}{\Gamma(m_{r,m})}\sum_{i=0}^{m_{r,n}-1}\frac{\left(Q_n\beta_{r,n}\right)^i\Gamma(m_{r,m}+i)}{\left(Q_m\right)^i i!\left(\beta_{r,m}+\frac{Q_n\beta_{r,n}}{Q_m}\right)^{m_{r,m}+i}}-$$

$$\frac{\beta_{r,m}^{m_{r,m}}}{\Gamma(m_{r,m})}\sum_{j=0}^{m_{f_i}-1}\frac{\left(x\beta_{f_i}\right)^j\Gamma(m_{r,m}+j)}{\left(Q_m\right)^j j!\left(\beta_{r,m}+\frac{x\beta_{f_i}}{Q_m}\right)^{m_{r,m}+j}}+ \quad (29)$$

$$\frac{\beta_{r,m}^{m_{r,m}}}{\Gamma(m_{r,m})}\sum_{i=0}^{m_{r,n}-1}\sum_{j=0}^{m_{f_i}-1}\frac{\left(Q_n\beta_{r,n}\right)^i\left(x\beta_{f_i}\right)^j\Gamma(m_{r,m}+i+j)}{Q_m^{i+j}i!\,j!\left(\beta_{r,m}+\frac{Q_n\beta_{r,n}+x\beta_{f_i}}{Q_m}\right)^{m_{r,m}+i+j}}$$

Note that $U_2=Q_m\left|f_i\right|^2\left|h_{2,m}\right|^{-2}$ can be directly derived from the CDF of U_1 after substituting the respective parameters by their counterparts (i.e. $Q_n\to Q_m, m_{g_i}\to m_{f_i}, m_{1,n}\to m_{2,m}, \Omega_{g_i}\to\Omega_{f_i}, \Omega_{1,n}\to\Omega_{2,m}$).

Then , the CDF of random variable U_2 is given by

$$F_{U_2}(x)=1-\frac{\beta_{2,m}^{m_{2,m}}}{\Gamma(m_{2,m})}\sum_{i=0}^{m_{f_i}-1}\frac{\left(\beta_{f_i}x\right)^i\Gamma(m_{2,m}+i)}{\left(Q_m\right)^i i!\left(\frac{\beta_{f_i}x}{Q_m}+\beta_{2,m}\right)^{m_{2,m}+i}} \quad (30)$$

Similarly, consider that $V_2=\min\left\{Q_n\left|h_1\right|^2\left|h_{r,n}\right|^{-2}, Q_m\left|h_1\right|^2\left|h_{r,m}\right|^{-2}\right\}$, the CDF of V_2 can be obtained from the cdf of V_1 by substituting the following parameters ($m_{f_i}\to m_{g_i}, \Omega_{f_i}\to\Omega_{g_i}$). Therefore, we can obtain the CDF of random variable V_2 as following

$$F_{V_2}(x)=\underbrace{\int_0^\infty\frac{\beta_{r,n}^{m_{r,n}}\gamma(m_{r,m},\frac{tQ_m\beta_{r,m}}{Q_n})\gamma(m_{g_i},\frac{tx\beta_{g_i}}{Q_n})}{\Gamma(m_{r,n})\Gamma(m_{r,m})\Gamma(m_{g_i})t^{1-m_{r,n}}e^{\beta_{r,n}t}}dt}_{J3(x)}+$$
$$\underbrace{\int_0^\infty\frac{\beta_{r,m}^{m_{r,m}}\gamma(m_{r,n},\frac{tQ_n\beta_{r,n}}{Q_m})\gamma(m_{g_i},\frac{tx\beta_{g_i}}{Q_m})}{\Gamma(m_{r,m})\Gamma(m_{r,n})\Gamma(m_{g_i})t^{1-m_{r,m}}e^{\beta_{r,m}t}}dt}_{J4(x)} \quad (31)$$

where $J3(x), J4(x)$ can be easily obtained from $J1(x), J2(x)$ by substituting m_{f_i},Ω_{f_i} with m_{g_i},Ω_{g_i} respectively.

REFERENCES

[1] A. Goldsmith, S. A. Jafar, *et al.*, “Breaking spectrum gridlock with cognitive radios: An information theoretic perspective,” in *Proc. IEEE*, vol. 97, no. 5, May 2009.

[2] Z. G. Duan, “Performance enhancement with cooperation based transmission in the cognitive radio networks,” in *Proc. WASE International Conference on Information Engineering*, Taiyuan, Shanxi, vol. 1, 2009, pp. 359-362.

[3] Y. Guo, G. Kang, N. Zhang, W. Zhou, and P. Zhang, “Outage performance of relay-assisted cognitive-radio system under spectrum sharing constraints,” *Electron. Letter*, vol. 46, no. 2, pp. 182-183, 2010.

[4] J. Lee, H. Wang, J. G. Andrews, and D. Hong, “Outage probability of cognitive relay networks with interference constraints,” *IEEE Trans. Wirel. Commun*, vol. 10, no. 2, pp. 390-395, 2011.

[5] D. Li, “Outage probability of cognitive radio networks with relay selection,” *IET Commun.*, vol. 5, no. 18, pp. 2730-2735, 2011.

[6] M. Xia and S. Aissa, “Cooperative AF relaying in spectrum-sharing systems: performance analysis under average interference power constraints and nakagami-m fading,” *IEEE Trans. Commun.*, vol. 60, pp. 1523-1533, 2012.

[7] L. L. Xi, J. Wei, *et al.*, ”New optimal power allocation for bidirectional communications in cognitive relay network using analog network coding,” *China Communications*, vol. 7, no. 4, pp. 144-148, 2010.

[8] P. Ubaidulla and S. Aissa, “Optimal relay selection and power allocation for cognitive two-way relaying networks,” *IEEE Wireless Communications Letters*, vol. 1,no. 3, June 2012.

[9] L. K. S. Jayasinghe, N. Rajatheva, and M. Latva-aho, “Optimal power allocation for PNC relay based communications in cognitive radio,” in *Proc. IEEE Intnl. Conf. on Commun.*, 2011, pp. 1-5.

[10] W. Xu and J. H. Zhang, *et al.*, “Outage probability of decode-and-forward cognitive relay in presence of primary user’s interference,” *IEEE Communications Letters*, vol. 16, no. 8, Aug 2012.

[11] L. Luo, P. Zhang, G. Zhang, and J. Qin, "Outage performance for cognitive relay networks with underlay spectrum sharing," *IEEE Commun. Lett.*, vol. 15, no. 7, pp. 710-712, 2011.

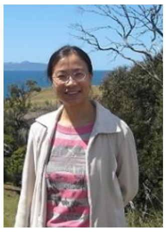
[12] Z. Yan, X. Zhang, and W. Wang, "Exact outage performance of cognitive relay networks with maximum transmit power limits," *IEEE Commun. Lett.*, vol. 15, no. 12, Dec 2011.

[13] T. T .Duy and H. Y. Kong, "Exact outage probability of cognitive two-way relaying scheme with opportunistic relay selection under interference constraint," *IET Communications*, vol. 6, no. 16, November 2012.

[14] P. Liu and I.-M. Kim, "Performance analysis of bidirectional communication protocols based on decode-and-forward relaying," *IEEE Trans. Commun*, vol. 58, pp. 2683-2696, September 2010.

[15] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. Orlando, FL: Academic, 1994.

Jia Liu received the M.S. degree in electronics and information engineering from Guilin University of Electronic Technoloy, Guilin, China, in 2008. He is currently working toward the Ph.D. degree in electrical engineering with the School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing, China. His research interests include multiple-input multiple-output, cognitive radio, relay, and information theory.



Guixia Kang received the M.S. degree from Tianjin University, Tianjin, China, and the Ph.D. degree in electrical engineering from Beijing University of Posts and Telecommunications (BUPT), Beijing. She was a research scientist in the Future Radio Concept Department of Siemens, Munich, Germany. She is currently a Professor with BUPT. Her interests include the research, development and standardization of 3G and beyond 3G (B3G) wireless communications systems as well as wireless sensor networks.

Ying Zhu received the M.S. degree in electronics and information engineering from Guilin University of Electronic Technoloy, Guilin, China, in 2009. She is currently working toward the Ph.D. degree in electrical engineering with the School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing, China. Her research interests include wireless communications and electrical contact reliability.

Ping Xie received the M.S. degree in electronics and information engineering from Hunan University, Changsha, China, in 2011. She is currently working toward the Ph.D. degree in electrical engineering with the School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing, China. Her research interests include wireless communication and signal processing.