Performance Analysis of Dual-Hop Systems with Decode-and-Forward Relays over Generalized Fading Channels

Mohammed S. Aloqlah[1] and Osamah S. Badarneh[2]
[1] Yarmouk University, Irbid 21163, Jordan
[2] University of Tabuk, Tabuk, KSA
Email: mohamads@yu.edu.jo; obadarneh@ut.edu.sa

Abstract—Dual-hop transmission systems employing decode-and-forward relay are studied in this paper. The examination has been done for independent, but not necessarily identical Extended Generalized-K fading channels. New, exact, closed-form expressions are derived for the outage probability, the nth moments of the end-to-end signal-to-noise ratio (SNR), and the ergodic capacity. Moreover, the average symbol error probability (ASEP) for coherent and non-coherent modulation schemes is derived. Analytical results along with simulation results are obtained with excellent agreement.

Index Terms—decode-and-forward relays, Extended Generalized-K fading channels, outage probability, end-to-end SNR, ergodic capacity, average symbol error probability

I. Introduction

The emergence of a demand for high data rates, increasing connectivity, and capacity in current and future wireless communication systems, has made multi-hop transmission attract the attention of researchers[1]-[4]. In cooperative transmission, a destination terminal staying or moving in the radio range of the source terminal may receive signals due to the existence of two distinct paths: i) a direct transmission path originating from the source terminal; and ii) an indirect path, arriving through the intermediate relaying (neighboring) terminal. However, if the line-of-sight (i.e., direct) path is completely blocked, the resulting received signal will be the received signal of the relaying path only. In a wireless relay system, the complexity depends on the number of relay terminals between the source and the destination, i.e. when a single relay terminal exists, multi hop transmission can be simplified to it is special case of which the dual-hop transmission, this type of transmission was encountered originally in bent-pipe satellites [5]-[7].

With three general classifications of cooperative strategies, i.e., amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF), it is uncertain which is more practically better [8]. All are deployed in practical systems to facilitate secure communication, but each class has its own advantages and disadvantages, compared to each other. The advantage of using DF relaying is that the relay node forwards a clean copy of the decoded source message to the destination node, by contrast to AF relaying, in which the relay node scales and forwards the received signal to the destination node. The noise amplification while repeating the received signal in AF relaying makes the amplification of both the actual desired signal and the noise happens simultaneously. On the other hand, there are several advantages of using AF relaying starting with the low implementation complexity and simplicity. The other advantages are the high utilization of the available resources and the increased performance gains, by contrast to DF relaying in which the relay node facilitates data transmission only if it can reliably decode the source message. In addition to the aforementioned advantages, AF ends up with much less delay along with much less power consumption as no decoding or quantizing operation is performed at the relay side.

Recently, an extensive amount of research has been accomplished on the performance of dual hop transmission over fading channels. In the work presented in [9], Hasna et al. (2004) have analyzed the end-to-end outage probability of DF relaying system over Rayleigh fading channels. In [10], Suraweera *et al.* have studied the outage probability of DF relaying system over Nakagami-m fading channels. Following the work investigated in [11], Wei-Guang Li and Ming Chen have presented useful performance, approximated by finite series, for the outage capacity of dual-hop wireless communication systems with DF relays operating over η-μ and κ-μ fading channels.

More recently, the Extended Generalized-K (EGK) fading model has been proposed [12]. EGK can represent a wide range of realistic fading conditions and can also be used to perfectly fit experimental data, simplifying the way how to model the source-relay and relay-destination channels. However, concerning DF relaying systems, to the best of the authors' knowledge, there are no analytical results for the outage probability, the nth moment of end-to-end SNR and the ergodic capacity in the technical literature, even for the upper bound. Thus, the aim of this paper is to analyze the performance of dual-hop DF relaying system in terms of outage probability, ergodic capacity, and ASEP for different coherent and non-coherent modulations. The received signals from the two hops are subjected to independent, not necessarily identical EGK fading. New, closed-form expressions for

Manuscript received April 1st, 2013; revised July 12th, 2013.
Corresponding author email: mohamads@yu.edu.jo.
doi:10.12720/jcm.8.7.407-413

the outage probability, the ergodic capacity, and the ASEP in terms of H-Fox and bivariate H-Fox functions [13] are derived. These derived results are applicable for both integer-order as well as non-integer order fading. Our results depict the effects of the power imbalance between the two hops. We showed and discussed how these effects can be positive or negative on the overall system performance.

The remainder of this paper is structured as follows. Section II introduces the considered system and channel models. Section III evaluates the end-end performance of wireless communication systems with DF relays. These results are applied and discussed in Section IV. Finally, Section V concludes the paper.

II. System and Channel Models

Consider a dual-hop DF relaying system. In the system, a single relay node, which is equipped with one antenna, decodes and forwards the received signal to the destination node. Employing an efficient relay communication, we assume that the source node communicates with the destination node via the relay node with the relay node is employed in half-duplex mode to reduce the relay self-interference. For the considered system, the end-to-end SNR can be approximated by its upper bound, γ_{eq} [14] as

$$\gamma_{\mathrm{eq}} = \min(\gamma_1, \gamma_2) \tag{1}$$

The received signal power from the i[th] (i=1, 2) hop α_i can be modeled as the product of a short-term fading X_i and a long-term shadowing S_i. Hence, we use EGK distribution to model multipath fading due to its perfect fit to experimental data. Then the probability density function (PDF) of γ_i can be written as [12]

$$f_{\gamma_i}(\gamma_i) = \frac{\beta_i}{2\Gamma(m_i)\Gamma(m_{si})}\left(\frac{b_{si}b_i}{\bar{\gamma}_i}\right)^{\frac{m_i\beta_i}{2}} \gamma_i^{\frac{m_i\beta_i}{2}-1} \times \Gamma\left(m_{si} - m_i\frac{\beta_i}{\beta_{si}}, 0, \left(\frac{b_{si}b_i}{\bar{\gamma}_i}\right)^{\frac{m_i\beta_i}{2}} \gamma_i^{\frac{\beta_i}{2}}, \frac{\beta_i}{\beta_{si}}\right) \tag{2}$$

In (2), $\Gamma(.)$ denotes the Gamma function [15] and $\Gamma(.,.,.,.)$ is the extended incomplete Gamma function defined as $\Gamma(\alpha, x, b, \beta) = \int_x^\infty r^{\alpha-1}e^{-(r+br^{-\beta})}dr$, where $\alpha, \beta, b \in \mathbb{C}$ and $x \in \mathbb{R}^+$ [15]. Furthermore, m and β represent the severity and shaping factor of the fading, respectively; while m_s and β_s represent the severity and shaping factor of the shadowing, respectively. Moreover, $b=\Gamma(m+2/\beta)/\Gamma(m)$, $b_s = \Gamma(ms+2/\beta s)/\Gamma(ms)$, and $\bar{\gamma}_i$ is the average power.

III. Performance Analysis

A. Cumulative Distribution Function (CDF)

As mentioned above, γ_i is an EGK random variable with PDF given by (2). With the reasonable assumption that the γ_1 and γ_2 are statically independent due to large spacing between the source, relay and destination nodes, then their joint PDF is $f_{\gamma_1,\gamma_2}(\gamma_1,\gamma_2) = f_{\gamma_1}(\gamma_1)f_{\gamma_2}(\gamma_2)$. Accordingly, the CDF of γ_{eq} can be derived as

$$\begin{aligned} F_{\gamma_{eq}}(\gamma) &= P\{\gamma_{eq} \le \gamma\} \\ &= 1 - P\{\gamma_1 > \gamma\}\,P\{\gamma_1 > \gamma\} \end{aligned} \tag{3}$$

where P(.) denotes the probability operator.

The $P\{\gamma_1 > \gamma\}$ in (3) can be written as

$$\begin{aligned} P\{\gamma_1 > \gamma\} &= 1 - P\{\gamma_1 \le \gamma\} \\ &= 1 - \int_0^\gamma f_{\gamma_1}(\gamma_1)d\gamma_1 \end{aligned} \tag{4}$$

A closed-form expression for the previously defined integral can be obtained after performing some algebraic manipulations involving H-Fox function as

$$P\{\gamma_1 > \gamma\} = 1 - \frac{1}{\Gamma(m_1)\Gamma(m_{s1})} \times H_{2,3}^{2,2}\left[\frac{b_{s1}b_1\gamma}{\bar{\gamma_1}}\middle| \begin{matrix} (1,1),(0,0) \\ \left(m_1,\frac{2}{\beta_1}\right),\left(m_{s1},\frac{2}{\beta_{s1}}\right),(0,1) \end{matrix}\right] \tag{5}$$

Similarly,

$$P\{\gamma_2 > \gamma\} = 1 - \frac{1}{\Gamma(m_2)\Gamma(m_{s2})} \times H_{2,3}^{2,2}\left[\frac{b_{s2}b_2\gamma}{\bar{\gamma_2}}\middle| \begin{matrix} (1,1),(0,0) \\ \left(m_2,\frac{2}{\beta_2}\right),\left(m_{s2},\frac{2}{\beta_{s2}}\right),(0,1) \end{matrix}\right] \tag{6}$$

B. Outage Probability Analysis

The outage probability (OP), in regenerative systems, is defined as the probability that γ_{eq}, described in (1), goes below a predefined threshold, γ_{th}. Significantly, knowing the outage probability provides an important indication of the QoS level.

For the prescribed system model, the outage probability can be found as

$$P_{out}(\gamma_{th}) = F_{\gamma_{eq}}(\gamma_{th}) \tag{7}$$

A closed-form expression for the outage probability can be obtained after performing some mathematical manipulations as (8).

$$P_{out}(\gamma_{th}) = 1 - \left\{1 - \frac{1}{\Gamma(m_1)\Gamma(m_{s1})} H_{2,3}^{2,2}\left[\frac{b_{s1}b_1\gamma_{th}}{\bar{\gamma_1}}\middle| \begin{matrix} (1,1),(0,0) \\ \left(m_1,\frac{2}{\beta_1}\right),\left(m_{s1},\frac{2}{\beta_{s1}}\right),(0.1) \end{matrix}\right]\right\} \times \left\{1 - \frac{1}{\Gamma(m_2)\Gamma(m_{s2})} H_{2,3}^{2,2}\left[\frac{b_{s2}b_2\gamma_{th}}{\bar{\gamma_2}}\middle| \begin{matrix} (1,1),(0,0) \\ \left(m_2,\frac{2}{\beta_2}\right),\left(m_{s2},\frac{2}{\beta_{s2}}\right),(0,1) \end{matrix}\right]\right\} \tag{8}$$

It can be observed that the outage probability can be accurately and efficiently expressed by using the H-Fox function. To the best of our knowledge, (8) is novel. Furthermore, it should be noted that the H-Fox function is not a standard built-in function in any of the mathematical software packages such as MATLAB, MAPLE or MATHEMATICA. However, in this paper we used a similar approach as in [16] for its numerical evaluation.

C. Moments of End-to-End SNR

Assuming the overall end-to-end SNR at the destination in a dual-hop transmission model with DF relay as in (1). The n^{th} moments of the end-end SNR can be calculated from $\mu_n = E[\gamma_{eq}^n]$, where $E[.]$ denotes the expectation operator.

In this paper, in order to obtain a closed expression of μ_n, the PDF of γeq is derived by taking the derivative of (8) with respect to γ_{th} as

$$f_{\gamma_{eq}}(\gamma) = \left.\frac{dP_{out}(\gamma_{th})}{d\gamma_{th}}\right|_{\gamma_{th}=\gamma} \tag{9}$$

Having derived a closed form expression of $f_{\gamma_{eq}}(\gamma)$ as (10), finally we can get μ_n as

$$f_{\gamma_{eq}}(\gamma) = \left\{1 - \frac{1}{\Gamma(m_1)\Gamma(m_{s1})} H_{2,3}^{2,2}\left[\frac{b_{s1}b_1\gamma}{\overline{\gamma_1}}\middle|\begin{matrix}(1,1),(0,0)\\ \left(\frac{m_{1,2}}{\beta_1}\right),\left(\frac{m_{s1,2}}{\beta_{s1}}\right),(0.1)\end{matrix}\right]\right\}\left\{H_{3,4}^{2,3}\left[\frac{b_{s2}b_2\gamma}{\overline{\gamma_2}}\middle|\begin{matrix}(0,1),(1,1),(0,0)\\ \left(\frac{m_{2,2}}{\beta_2}\right),\left(\frac{m_{s2,2}}{\beta_{s2}}\right),(0,1),(1,1)\end{matrix}\right]\right\}$$
$$\frac{1}{\Gamma(m_2)\Gamma(m_{s2})\gamma} + \frac{1}{\Gamma(m_1)\Gamma(m_{s1})\gamma}\times$$
$$\left\{1 - \frac{1}{\Gamma(m_2)\Gamma(m_{s2})} H_{2,3}^{2,2}\left[\frac{b_{s2}b_2\gamma}{\overline{\gamma_2}}\middle|\begin{matrix}(1,1),(0,0)\\ \left(\frac{m_{2,2}}{\beta_2}\right),\left(\frac{m_{s2,2}}{\beta_{s2}}\right),(0,1)\end{matrix}\right]\right\}\left\{H_{3,4}^{2,3}\left[\frac{b_{s1}b_1\gamma}{\overline{\gamma_1}}\middle|\begin{matrix}(0,1),(1,1),(0,0)\\ \left(\frac{m_{1,2}}{\beta_1}\right),\left(\frac{m_{s1,2}}{\beta_{s1}}\right),(0,1),(1,1)\end{matrix}\right]\right\} \tag{10}$$

$$\mu_n = \int_0^\infty \gamma_{eq}^n f_{\gamma_{eq}}(\gamma_{eq})d\gamma_{eq} \tag{11}$$

$$\mu_n = \frac{1}{\Gamma(m_2)\Gamma(m_{s2})}\left(\frac{b_{s2}b_2\gamma}{\overline{\gamma_2}}\right)^{-n}\Gamma\left[\begin{matrix}m_2+\frac{2n}{\beta_2}, m_{s2}+\frac{2n}{\beta_{s2}}, 1-n, -n, 1\\ 1-n, -n\end{matrix}\right] + \frac{1}{\Gamma(m_1)\Gamma(m_{s1})}\left(\frac{b_{s1}b_1\gamma}{\overline{\gamma_1}}\right)^{-n}\times$$
$$\Gamma\left[\begin{matrix}m_1+\frac{2n}{\beta_1}, m_{s1}+\frac{2n}{\beta_{s1}}, 1-n, -n, 1\\ 1-n, -n\end{matrix}\right] - \frac{1}{\Gamma(m_1)\Gamma(m_{s1})\Gamma(m_2)\Gamma(m_{s2})}\times$$
$$\left\{H_{6,6}^{5,4}\left[\frac{b_{s2}b_2\gamma}{\overline{\gamma_2}}\middle|\begin{matrix}(1,1),(0,0),\left(1-m_2-\frac{2n}{\beta_2},\frac{2}{\beta_2}\right),\left(1-m_{s2}-\frac{2n}{\beta_{s2}},\frac{2}{\beta_{s2}}\right),(1-n,1),(-n,1)\\ \left(m_1,\frac{2}{\beta_1}\right),\left(m_{s1},\frac{2}{\beta_{s1}}\right),(1-n.1),(-n,1),(1,0),(0,1)\end{matrix}\right]\left(\frac{b_{s2}b_2\gamma}{\overline{\gamma_2}}\right)^{-n}+\right.$$
$$\left.\left(\frac{b_{s1}b_1\gamma}{\overline{\gamma_1}}\right)^{-n}H_{6,6}^{5,4}\left[\frac{b_{s1}b_1\gamma}{\overline{\gamma_1}}\middle|\begin{matrix}(1,1),(0,0),\left(1-m_1-\frac{2n}{\beta_1},\frac{2}{\beta_1}\right),\left(1-m_{s1}-\frac{2n}{\beta_{s1}},\frac{2}{\beta_{s1}}\right),(1-n,1),(-n,1)\\ \left(m_2,\frac{2}{\beta_2}\right),\left(m_{s2},\frac{2}{\beta_{s2}}\right),(1-n.1),(-n,1),(1,0),(0,1)\end{matrix}\right]\right\} \tag{12}$$

$$\overline{\gamma_{eq}} = \frac{1}{\Gamma(m_2)\Gamma(m_{s2})}\left(\frac{b_{s2}b_2\gamma}{\overline{\gamma_2}}\right)^{-1}\Gamma\left[\begin{matrix}m_2+\frac{2}{\beta_2}, m_{s2}+\frac{2}{\beta_{s2}}, 0, -1, 1\\ 0, -1\end{matrix}\right] + \frac{1}{\Gamma(m_1)\Gamma(m_{s1})}\left(\frac{b_{s1}b_1\gamma}{\overline{\gamma_1}}\right)^{-1}\times$$
$$\Gamma\left[\begin{matrix}m_1+\frac{2}{\beta_1}, m_{s1}+\frac{2}{\beta_{s1}}, 0, -1, 1\\ 0, -1\end{matrix}\right] - \frac{1}{\Gamma(m_1)\Gamma(m_{s1})\Gamma(m_2)\Gamma(m_{s2})}\times$$
$$\left\{H_{6,6}^{5,4}\left[\frac{b_{s2}b_2\gamma}{\overline{\gamma_2}}\middle|\begin{matrix}(1,1),(0,0),\left(1-m_2-\frac{2}{\beta_2},\frac{2}{\beta_2}\right),\left(1-m_{s2}-\frac{2}{\beta_{s2}},\frac{2}{\beta_{s2}}\right),(0,1),(-1,1)\\ \left(m_1,\frac{2}{\beta_1}\right),\left(m_{s1},\frac{2}{\beta_{s1}}\right),(0.1),(-1,1),(1,0),(0,1)\end{matrix}\right]\left(\frac{b_{s2}b_2\gamma}{\overline{\gamma_2}}\right)^{-1}+\right.$$
$$\left.\left(\frac{b_{s1}b_1\gamma}{\overline{\gamma_1}}\right)^{-1}H_{6,6}^{5,4}\left[\frac{b_{s1}b_1\gamma}{\overline{\gamma_1}}\middle|\begin{matrix}(1,1),(0,0),\left(1-m_1-\frac{2}{\beta_1},\frac{2}{\beta_1}\right),\left(1-m_{s1}-\frac{2}{\beta_{s1}},\frac{2}{\beta_{s1}}\right),(0,1),(-1,1)\\ \left(m_2,\frac{2}{\beta_2}\right),\left(m_{s2},\frac{2}{\beta_{s2}}\right),(0,1),(-1,1),(1,0),(0,1)\end{matrix}\right]\right\} \tag{13}$$

After performing integrals involving H-Fox functions we can obtain (12). A special case of μ_n, when n= 1, is the average end-to-end SNR, $\overline{\gamma_{eq}}$, expressed as (13).

D. Ergodic Capacity

The ergodic capacity is analyzed in this paper owing to its good insight of the bound on the data rate for reliable communication over fading channels. For a dual–hop relay, the ergodic capacity can be obtained as [17]

$$C = \frac{BW}{2} E[\log_2(1+\gamma_{overall})] \tag{14}$$

After some manipulations involving Mellin Barnes integral we can obtain (15). It can be observed that the ergodic capacity can be expressed by using the bivariate H-Fox function. To the best of our knowledge, (15) is novel. Furthermore, it should be noted that the bivariate H-Fox function is not a standard built-in function in any of the mathematical software packages such as MATLAB, MAPLE or MATHEMATICA. However, in this paper we used a similar approach as in [16] for its numerical evaluation.

$$C = \frac{BW}{2}\Bigg\{\frac{1}{\Gamma(m_2)\Gamma(m_{s2})} H^{4,4}_{6,5}\left[\left(\frac{b_{s2}b_2}{\overline{\gamma_2}}\right)^{-1}\middle|\begin{matrix}(1,1),(1,1),\left(1-m_2,\frac{2}{\beta_2}\right),\left(1-m_{s2},\frac{2}{\beta_{s2}}\right),(1,1),(0,1)\\(1,1),(1,1),(0.1),(1,0),(0,1)\end{matrix}\right] + \frac{1}{\Gamma(m_1)\Gamma(m_{s1})}\times$$
$$H^{4,4}_{6,5}\left[\left(\frac{b_{s1}b_1}{\overline{\gamma_1}}\right)^{-1}\middle|\begin{matrix}(1,1),(1,1),\left(1-m_1,\frac{2}{\beta_1}\right),\left(1-m_{s1},\frac{2}{\beta_{s1}}\right),(1,1),(0,1)\\(1,1),(1,1),(0.1),(1,0),(0,1)\end{matrix}\right] - \frac{1}{\Gamma(m_1)\Gamma(m_{s1})\Gamma(m_2)\Gamma(m_{s2})}\times$$
$$\Bigg\{H^{0,3:2,2;2,1}_{0,1:3,3;1,3}\left[\begin{matrix}\frac{b_{s2}b_2}{\overline{\gamma_2}}\\\frac{b_{s1}b_1}{\overline{\gamma_1}}\end{matrix}\middle|\begin{matrix}(0;1,1),(1;-1,-1),(1;-1,-1):(0,1),(1,1),(0,-1);(1,1)\\(0;-1,-1):\left(m_2,\frac{2}{\beta_2}\right),\left(m_{s2},\frac{2}{\beta_{s2}}\right),(0,1);\left(m_1,\frac{2}{\beta_1}\right),\left(m_{s1},\frac{2}{\beta_{s1}}\right),(0,1)\end{matrix}\right]+$$
$$H^{0,3:2,2;2,1}_{0,1:3,3;1,3}\left[\begin{matrix}\frac{b_{s1}b_1}{\overline{\gamma_1}}\\\frac{b_{s2}b_2}{\overline{\gamma_2}}\end{matrix}\middle|\begin{matrix}(0;1,1),(1;-1,-1),(1;-1,-1):(0,1),(1,1),(0,-1);(1,1)\\(0;-1,-1):\left(m_1,\frac{2}{\beta_1}\right),\left(m_{s1},\frac{2}{\beta_{s1}}\right),(0,1);\left(m_2,\frac{2}{\beta_2}\right),\left(m_{s2},\frac{2}{\beta_{s2}}\right),(0,1)\end{matrix}\right]\Bigg\}\Bigg\} \tag{15}$$

E. *Error Performance Analysis*

The average symbol error probability, $\overline{P_{se}}$, is obtained in this work for both coherent and non-coherent frequency shift keying (FSK) and phase shift keying (PSK) by averaging the conditional symbol error rate for optimum detection of nonfading binary signals in Gaussian noise,$P_{se}(\gamma)$, over the distribution of the end-to-end SNR γ_{eq} as [18]

$$\overline{P_{se}} = \int_{-\infty}^{\infty} P_{se}(\gamma) f_{\gamma_{eq}}(\gamma) \tag{16}$$

In (16) the symbol error rate, $P_{se}(\gamma)$, for coherent and non-coherent detection of both PSK and FSK with optimum matched filter receiver is given by [19]

$$P_{se}(\gamma) = a\cdot e^{-b\gamma}$$

$$\begin{cases}a=0.5, b=0.5 & \text{for NCFSK}\\a=0.5, b=1.0 & \text{for DCPSK}\end{cases} \tag{17}$$

and

$$P_{se}(\gamma) = a\cdot \mathrm{erfc}\left(\sqrt{2b\gamma}\right)$$
$$\begin{cases}a=0.5, b=0.5 \text{ for CFSK}\\a=1.0, b=1.0 \text{ for CPSK}\end{cases} \tag{18}$$

where erfc(.) is the complementary error function [15].

Evaluating the integral in (16) for non-coherent FSK and PSK, then $\overline{P_{se}}$ can be expressed as (19). Similarly, the closed form of, $\overline{P_{se}}$, for coherent FSK and PSK, is obtained as (20). To the best of our knowledge, the derived error performance results reported in this paper are new.

$$\overline{P_{se}} = \frac{A}{\sqrt{\pi}\Gamma(m_2)\Gamma(m_{s2})} H^{5,2}_{5,4}\left[B\left(\frac{b_{s2}b_2}{\overline{\gamma_2}}\right)^{-1}\middle|\begin{matrix}\left(1-m_2,\frac{2}{\beta_2}\right),\left(1-m_{s2},\frac{2}{\beta_{s2}}\right),(1,1),(0,1),(1,1)\\(0,1),(0.5,1),(1,1),(0,1),(1,0)\end{matrix}\right]+$$
$$\frac{A}{\sqrt{\pi}\Gamma(m_1)\Gamma(m_{s1})} H^{5,2}_{5,4}\left[B\left(\frac{b_{s1}b_1}{\overline{\gamma_1}}\right)^{-1}\middle|\begin{matrix}\left(1-m_1,\frac{2}{\beta_1}\right),\left(1-m_{s1},\frac{2}{\beta_{s1}}\right),(1,1),(0,1),(1,1)\\(0,1),(0.5,1),(1,1),(0,1),(1,0)\end{matrix}\right]-$$
$$\frac{A}{\sqrt{\pi}\Gamma(m_1)\Gamma(m_{s1})\Gamma(m_2)\Gamma(m_{s2})}\times$$
$$\Bigg\{H^{0,1:3,2;2,2}_{2,0:4,2;2,3}\left[\begin{matrix}\left(\frac{b_{s2}b_2}{B\overline{\gamma_2}}\right)^{-1}\\\left(\frac{b_{s1}b_1}{B\overline{\gamma_1}}\right)^{-1}\end{matrix}\middle|\begin{matrix}(0.5;-1,-1),(0;1,1):(1,-1),(0,0),(1,1),(0,1);(1,-1),(0,0)\\-:\left(m_2,\frac{-2}{\beta_2}\right),\left(m_{s2},\frac{-2}{\beta_{s2}}\right),(1,1);\left(m_1,\frac{-2}{\beta_1}\right),\left(m_{s1},\frac{-2}{\beta_{s1}}\right),(0,-1)\end{matrix}\right]+$$
$$H^{0,1:3,2;2,2}_{2,0:4,2;2,3}\left[\begin{matrix}\left(\frac{b_{s1}b_1}{B\overline{\gamma_1}}\right)^{-1}\\\left(\frac{b_{s2}b_2}{B\overline{\gamma_2}}\right)^{-1}\end{matrix}\middle|\begin{matrix}(0.5;-1,-1),(0;1,1):(1,-1),(0,0),(1,1),(0,1);(1,-1),(0,0)\\-:\left(m_1,\frac{-2}{\beta_1}\right),\left(m_{s1},\frac{-2}{\beta_{s1}}\right)(1,1);\left(m_2,\frac{-2}{\beta_2}\right),\left(m_{s2},\frac{-2}{\beta_{s2}}\right),(0,-1)\end{matrix}\right]\Bigg\} \tag{19}$$

$$\overline{P_{se}} = \frac{A}{\Gamma(m_2)\Gamma(m_{s2})} H_{4,4}^{4,2}\left[B\left(\frac{b_{s2}b_2}{\overline{\gamma_2}}\right)^{-1} \middle| \begin{matrix} \left(1-m_2,\frac{2}{\beta_2}\right),\left(1-m_{s2},\frac{2}{\beta_{s2}}\right),(1,1),(0,1) \\ (0,1),(1,1),(0,1),(1,0) \end{matrix} \right] +$$

$$\frac{A}{\Gamma(m_1)\Gamma(m_{s1})} H_{4,4}^{4,2}\left[B\left(\frac{b_{s1}b_1}{\overline{\gamma_1}}\right)^{-1} \middle| \begin{matrix} \left(1-m_1,\frac{2}{\beta_1}\right),\left(1-m_{s1},\frac{2}{\beta_{s1}}\right),(1,1),(0,1) \\ (0,1),(1,1),(0,1),(1,0) \end{matrix} \right] -$$

$$\frac{A}{\Gamma(m_1)\Gamma(m_{s1})\Gamma(m_2)\Gamma(m_{s2})} \times$$

$$\left\{ H_{1,0:4,2;2,2}^{0,1:2,2;2,1}\left[\begin{matrix} \left(\frac{b_{s2}b_2}{B\overline{\gamma_2}}\right)^{-1} \\ \left(\frac{b_{s1}b_1}{B\overline{\gamma_1}}\right)^{-1} \end{matrix} \middle| \begin{matrix} (1;-1,-1):(0,-1),(1,-1),(1,1),(0,1);(1,-1),(1,1) \\ -:\left(m_2,\frac{-2}{\beta_2}\right),\left(m_{s2},\frac{-2}{\beta_{s2}}\right);\left(m_1,\frac{-2}{\beta_1}\right),\left(m_{s1},\frac{-2}{\beta_{s1}}\right) \end{matrix} \right] + \right.$$

$$\left. H_{1,0:4,2;2,2}^{0,1:2,2;2,1}\left[\begin{matrix} \left(\frac{b_{s1}b_1}{B\overline{\gamma_1}}\right)^{-1} \\ \left(\frac{b_{s2}b_2}{B\overline{\gamma_2}}\right)^{-1} \end{matrix} \middle| \begin{matrix} (1;-1,-1):(0,-1),(1,-1),(1,1),(0,1);(1,-1),(1,1) \\ -:\left(m_1,\frac{-2}{\beta_1}\right),\left(m_{s1},\frac{-2}{\beta_{s1}}\right);\left(m_2,\frac{-2}{\beta_2}\right),\left(m_{s2},\frac{-2}{\beta_{s2}}\right) \end{matrix} \right] \right\} \quad (20)$$

IV. DISCUSION OF RESULTS

In this section, we present the numerical results of outage probability, moments of end-to-end SNR, ergdic capacity and ASEP for dual-hop DF relaying system. In Fig. 1, the outage probability is plotted as a function of the average SNR per hop γ_1, assuming $\gamma_{th} = 1$ dB. As it can be observed, OP improves as the fading parameter m increases from ($m_1 = m_2 = 2$) to ($m_1 = m_2 = 8$). Also, by considering the same value for $\overline{\gamma_1}$, it can be seen that the power imbalance may be either advantageous or harmful for the overall system performance. Indeed, for $\overline{\gamma_2} > \overline{\gamma_1}$ it is beneficial, otherwise, it is detrimental.

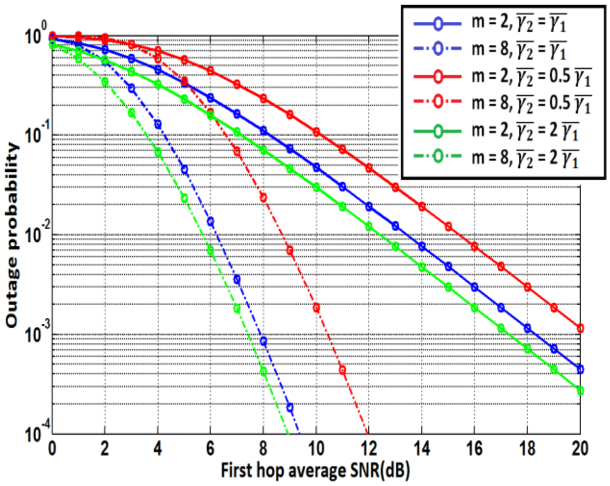

Figure. 1. Outage probability for different values of the parameter m_1=m_2=m.

The results for the average SNR from (13) over EGK fading channels with different values of the fading parameter m are shown in Fig. 2. As expected, the average SNR increases with increasing $\overline{\gamma_1}$ and also with increasing the fading parameter m. In the same figure, it can be seen that, by considering the same value for $\overline{\gamma_1}$, the overall system performance improves for $\overline{\gamma_2} > \overline{\gamma_1}$ and degrades for $\overline{\gamma_2} < \overline{\gamma_1}$.

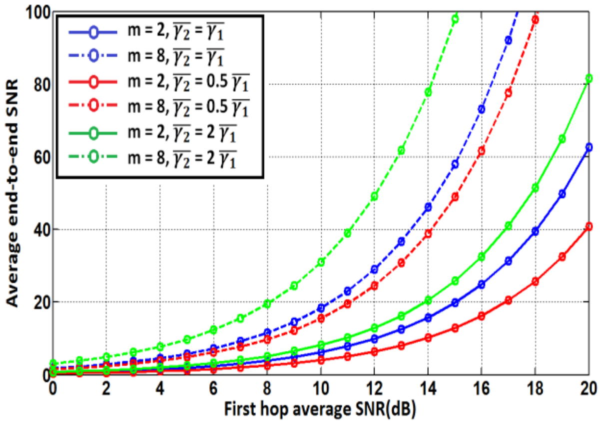

Figure. 2. Average end-to-end SNR for different values of the parameter m_1=m_2=m

In Fig. 3, the ergodic capacity is plotted versus the average SNR per hop γ1. The ergodic capacity increases with increasing $\overline{\gamma_1}$ and also with increasing the fading parameter m. Also, it can be seen that, by considering the same value for $\overline{\gamma_1}$, the power imbalance may be either advantageous or harmful for the overall system performance. Indeed, for $\overline{\gamma_2} > \overline{\gamma_1}$ it is beneficial, otherwise, it is detrimental.

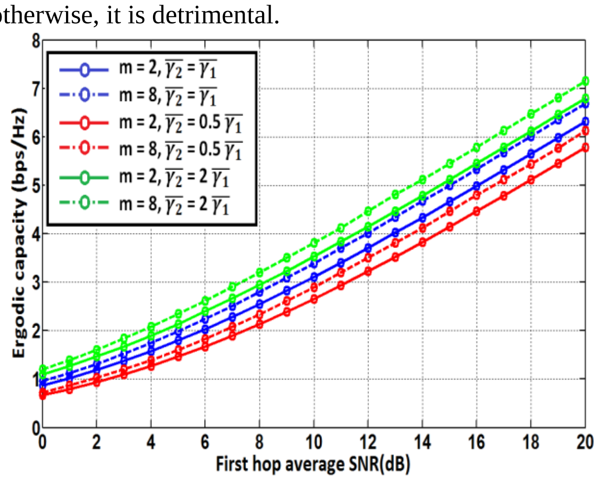

Figure. 3. Ergodic capacity for different values the parameter m_1=m_2=m.

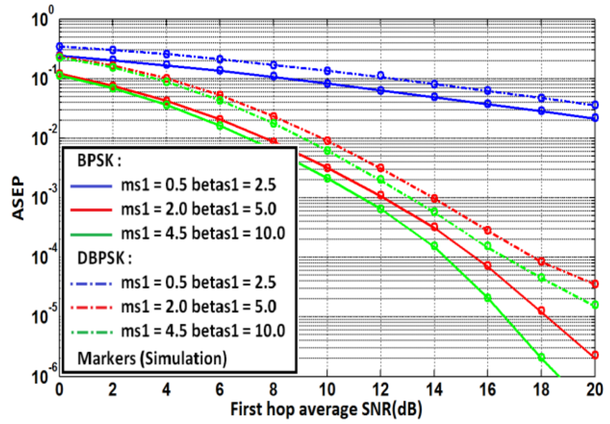

Figure. 4. Error performance of BPSK and DBPSK for different values of shadowing severity m_{s1}= m_{s2} and shadowing shaping factor β_{s1}=β_{s2}.

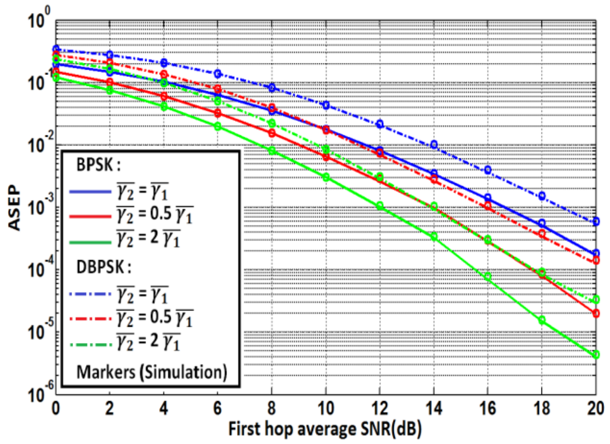

Figure. 5. Error performance of BPSK and DBPSK for balanced and imbalanced links.

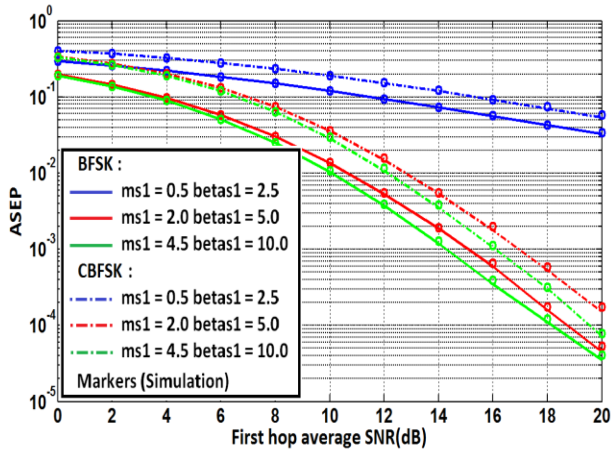

Figure. 6. Error performance of BFSK and CBFSK for different values of shadowing severity m_{s1}= m_{s2} and shadowing shaping factor β_{s1}=β_{s2}.

Finally, the closed form mathematical results for the ASEP given in (19) and (20) are presented in Fig. 4 for BPSK and DBPSK as a function of the first hop average SNR, $\overline{\gamma_1}$, for $\overline{\gamma_2} = 2\overline{\gamma_1}$, different values of shadowing severity m_{s1}= m_{s2}= m_s and different values of shadowing shaping factor β_{s1}= β_{s2}= β_s. Depending on Fig. 4, the ASEP decreases with the increase of m_s and β_s; because increasing m_s decreases the fading severity and increasing β_s skews the PDF of the fading around the average power. Moreover, in Fig. 5, the performance of the aforementioned signaling constellations is studied for both balanced and imbalanced links. This figure shows that the error performance improves as $\overline{\gamma_2}$ exceeds $\overline{\gamma_1}$. Similar performance results are derived for BFSK and CBFSK using Fig. 6 and Fig. 7. All these figures include both analytical and Monte Carlo simulation results with excellent agreement.

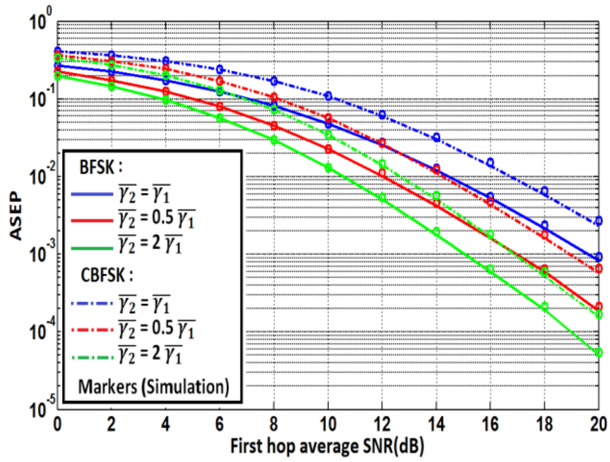

Figure. 7. Error performance of BFSK and CBFSK for balanced and imbalanced links.

V. Conclusion

In this paper, we investigated the end-to-end performance of dual-hop wireless communication systems with DF relays over EGK fading channels. The outage probability and the moments of the end-to-end SNR were extracted in closed form in terms of the H-Fox function. The results were used to study important performance criteria, such as the average end-to-end SNR. Furthermore, a closed form expressions of the ergodic capacity and ASEP were derived in terms of the bivariate H-Fox function. The results showed that the impact of the power/fading imbalance between the two hops may have gainful or harmful effects on the overall system performance.

References

[1] M. O. Hasna and M. S. Alouini, "Outage probability of multihop transmissions over Nakagami fading channels," *IEEE Commun. Lett.*, vol. 7, no. 5, pp. 216–218, 2003.

[2] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1820–1830, 2004.

[3] K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, "Bounds for multihop relayed communications in nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 18–22, 2006.

[4] G. K. Karagiannidis, "Performance bounds of multihop wireless communications with blind relays over generalized fading channels," *IEEE Trans. Wirel. Commun.*, vol. 5, no. 3, pp. 1498–1503, 2006.

[5] M. O. Hasna and M. S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wirel. Commun.*, vol. 2, no. 6, pp. 1126–1131, 2003.

[6] M. O. Hasna and M. S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wirel. Commun.*, vol. 3, no. 6, pp. 1963–1968, 2004.

[7] M. O. Hasna and M. S Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130–135, 2004.
[8] M. Yu and J. Li, "Is amplify-and-forward practically better than decode-and-forward or vice versa?" in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 3, Mar 2005, pp. 365-368.
[9] M. O. Hasna and M. S. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1999-2004, Nov 2004.
[10] H. A. Suraweera, P. J. Smith, and J. Armstrong, "Outage probability of cooperative relay networks in Nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 10, pp. 834-836, Dec 2006.
[11] W.-G. Li and M. Chen, "Outage capacity of dual-hop decode-and-forward relaying system over generalized fading channels," in *Proc. 2nd International Conference on Future Computer and Communication*, vol. 3, 21-24 May 2010, pp. 827-831.
[12] F. Yilmaz and M.-S. Alouini, "A new simple model for composite fading channels: second order statistics and channel capacity," in *Proc. International Symposium on Wireless Communication Systems*, 2010, pp. 676–680.
[13] T. Wang, A. Cano, G. B. Giannakis, and J. N. Laneman, "High-performance cooperative demodulation with decode-and-forward relays," *IEEE Transactions on Communications*, vol. 55, no. 7, pp. 1427-1438, July 2007.
[14] I. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, 6th edition. Academic Press, 2000.
[15] K. P. Peppas, "A new formula for the average bit error probability of dual-hop amplify-and-forward relaying systems over generalized shadowed fading channels," *IEEE Wireless Communications Letters*, vol. 1, no. 2, pp. 85-88, April 2012.
[16] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design," *IEEE J. Sel. Areas Commun*., vol. 22, no. 6, pp. 1099–1109, 2004.
[17] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*, 2nd ed., New York: Wiley, 2004.
[18] M. Schwartz, W. R. Bennett, and S. Stein, *Communicarion Systems and Techniques*, New York: McGraw-Hill, 1966.

Mohammed S. Aloqlah was born in Al husn, Jordan, in 1979. He received his B.S. degree in Telecommunications Engineering from Yarmouk University, Jordan, in 2002. Between 2002 and 2004, he worked as a teaching assistant at Yarmouk University where he was awarded a scholarship to pursue his graduate studies in the US. He received his M.S. degree in Electrical Engineering from Colorado State University in 2006. Between 2006 and 2010, Mohammad was a Ph.D. candidate and research assistant at Case Western Reserve University where he received his PhD degree in Electrical Engineering in August 2010. His research focuses on modeling of mobile radio channels, cooperative communication, smart antennas and MIMO systems, body sensors for healthcare and wireless sensor networks for energy efficient. Now Dr. Aloqlah is an assistant professor in the Telecommunications Engineering Department at Hijjawi Faculty for Engineering Technology, Yarmouk University, Jordan.

Osamah S. Badarneh received the Ph.D. in Electrical Engineering from University of Quebec-ETS (Canada) in 2009. He was an Assistant Professor in the Department of Telecommunication Engineering at Yarmouk University from 2010-2012. In 2012 he joined the Department of Electrical Engineering at University of Tabuk where he is currently an Assistant Professor. His research includes work in video multicast over ad hoc wireless networks, routing in Cognitive Radio Networks, Cross-layer design, MIMO systems, and performance analysis for wireless communication systems. He serves as a reviewer for prestigious international journals and conferences.