The Study of Zero-Forcing Beamforming Gain for Multiuser MIMO with Limited Feedback

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***Abstract*—**This paper discusses the finite-rate feedback-based ZF for multiuser MIMO in case of small number of candidate users to be selected and small number of quantization bits to be fed back. In this case, the multiplexing gain from ZF is limited. Under the assumption of small number of candidate users, then the paper further analyzes throughput of ZF with different number of co-scheduled users. From the analysis, the throughput of ZF is limited by the loss of target signal power and the residual interference from the co-scheduled users. The loss of target signal power comes from the steered precoder by ZF operation. From the simulation for theoretical throughput and throughput from Monte Carlo methods, we have some conclusions, for finite-rate feedback-base ZF multiuser MIMO with the small number of quantization bits, the ZF beamforming gain that is significant for perfect CSIT based ZF disappears, and the throughput for small value of co-scheduled users may be larger than the throughput for large value of co-scheduled users.

***Index Terms*—**Multiuser MIMO, limited feedback, finite-rate feedback, Zero Forcing, multiplexing gain

I. Introduction

Multiuser Multiple input multiple output (MIMO) is widely studied in 3GPP Long Term Evolution (LTE) and LTE-Advanced system. Multiuser MIMO can achieve the potential full multiplexing gain. For single-user MIMO, the multiplexing gain is limited by the number of antennas for single user. However, for multiuser MIMO, even in the extreme case that each user only has one receive antenna, multiuser MIMO can still achieve the multiplexing gain up to the number of transmit antennas. In practical, the number of receive antenna is limited by the size of user equipment and the cost of receiver. Therefore, multiuser MIMO gains many interests of engineers and academics[1-5].

However, with the limited feedback (finite-rate feedback), the throughput of Zero Forcing (ZF) method in multiuser MIMO is limited by quantization of channel, since compared with perfect Common and Internal Spanning Tree(CIST) ZF the interference part can not be eliminated any more [6]. In fact, for the limited feedback, interference and signal power both scale linearly with the Signal Noise Ratio (SNR). In Ref. [6], it is proposed that the number of quantization (feedback) bits should be scaled with the SNR, since the rate gap between perfect channel state information at transmitter(CSIT, CSI at Transmitter) based ZF and finite-rate feedback-based ZF is proportional to the SNR. In Ref. [7], the limited feedback based ZF is also studied in the similar way, and pilot for demodulation with consideration of ZF beamforming, i.e. the dedicated training, is proposed. In general view, the number of quantization bits should reach a certain level to achieve the sufficient multiuser MIMO [8].

But it is not practical to increase the number of quantization bits under the current standard framework for some reasons, such as the burden of uplink etc. In LTE standard release-10, the number of quantization bits for the 4 transmit antennas is four, and the number of quantization bits for the 8 transmit antennas is eight. In the standard, the number of quantization bits is not scaled with SNR, and is small indeed.

The one of reasons to use the small number of quantization bits is that the multiuser diversity gain can make up the absence of interference nulling gain in case of the small number of quantization bits. From this perspective, the main benefit comes from the varying of channel magnitude for large number of users, i.e. in the sense of statistics there are always users with good enough channel magnitude to achieve the high rate regardless the interference from multiuser MIMO. In Ref. [9], [10], this problem has been discussed. The scenario with the very large number of candidate users is analyzed in those papers, with help of the channel magnitude or Channel Quality Indicator (CQI) feedback, through the optimized user grouping method, the multiuser diversity gain can be achieved. However, in some scenarios, e.g. femto-cell, home access etc., the transmitter only serves a small number of candidate users. It is worth studying the ZF beamforming in multiuser MIMO. Thus, similar with Ref. [6], this paper mainly discusses the case of the small number of candidate users, or more definitely the case

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that no multiuser diversity gain is exploited. At the same time, this paper analyzes and simulates the throughput of ZF beamforming with different number of co-scheduled users.

II. System Model

In this paper, we only consider the case that each user has only one antenna and assume that the channel is block Rayleigh fading. Each MISO channel for multiuser MIMO can be modeled as:

$$\mathbf{y}_i = \mathbf{h}_i^+ x + \mathbf{n}_i\,,\ i = 1,...,K \tag{1}$$

where $\mathbf{h}_i \in C^{M\times 1}$ is the channel vector for the *i*-th user (due to Rayleigh fading the mean of channel magnitude is M, i.e. $E\left[\left\|\mathbf{h}_i\right\|\right] = M$); $\mathbf{x} \in C^{M\times 1}$ is the transmit signal vector for the i-th user after beamforming; $\mathbf{n}_i$ is the Additive Gaussian White Noise (AGWN) with zero mean and unit variance; K is the number of co-scheduled users in the multiuser MIMO transmit. The total transmit power constraint is P. Here the large scale properties, such as path loss and large scale fading etc., are not involved, which has been discussed in Ref. [9-10] and approved to contribute to the multiuser diversity gain.

The channel direction $\tilde{\mathbf{h}}_i = \mathbf{h}_i / \left\|\mathbf{h}_i\right\|$ is quantized by each user with finite rate and reported in the assumed error-free uplink. In practical, the quantization is based on a fixed codebook which is known both at transmitter and receiver. If the number of feedback bits is B, the vectors number in the codebook is $N = 2^B$. The user finds the vector in the codebook which is closest to the channel direction as the quantized channel direction $\hat{\mathbf{h}}_i$, and feeds back the corresponding index of the vector. The analysis in the paper uses the theory of Random Vector Quantization (RVQ) in the similar way with Ref. [6]. The notations in the paper also can refer to Ref. [6].

With the beamforming procedure, the channel can be modeled as:

$$\mathbf{y}_i = \mathbf{h}_i^+ \textstyle\sum_{j=1}^{K} \mathbf{v}_j \mathbf{s}_j + \mathbf{n}_i\,,\ i = 1,...,K \tag{2}$$

where $\mathbf{s}_i$ denotes the symbol for the *i*-th user, and $\mathbf{v}_i$ is the beamforming vector for the *i*-th user. Due to the total transmit power constraint, $\mathbf{v}_i$ should be normalized in general, i.e. $\left\|\mathbf{v}_i\right\| = 1$, which is also called the constant module property in RVQ. The ZF beamformer $\mathbf{v}_i$ can be obtained by pseudo inverse [6] or block diagonalization [11-12] method.

The SINR for the *i*-th user can be written as:

$$SINR_i = \frac{\frac{P}{K}\left\|\mathbf{h}_i\right\|^2 \left|\tilde{\mathbf{h}}_i^+ \mathbf{v}_i\right|^2}{1 + \sum_{j=1, j\neq i}^{j=K} \frac{P}{K}\left\|\mathbf{h}_i\right\|^2 \left|\tilde{\mathbf{h}}_i^+ \mathbf{v}_j\right|^2} \tag{3}$$

where K is the number of co-scheduled users. In the paper, the equal power allocation for each user is assumed.

In case of perfect CSIT, the interference can be completely cancelled. However, in case of finite-rate feedback, the interference cannot be cancelled. As discussed in Ref. [6], the residual interference will largely limit the performance of ZF, especially in the high SNR area. In the sequel, we will analyze the throughput in case of finite-rate feedback.

III. Throughput Analysis

Since the finite-rate feedback-based ZF is interference limited, the suppression of the interference can obviously raise the throughput. The direct method to suppress the interference is to reduce the number of interference sources, i.e. the number of co-scheduled users. However, when the number of co-scheduled users is reduced, the multiplexing gain will drop as well. Thus, it is a tradeoff between the reduction of the number of co-scheduled users and the increase of the multiplexing gain.

In the next, we will analyze the theoretical throughput for finite-rate feedback-based ZF. At first, let us see the general case that the number of candidate users is larger than the number of transmit antennas, i.e. M.

As a special case, we will analyze the throughput when the number of candidate users is the same as the number of transmit antennas. In this case we The next theorem will give the clearer sight for the theoretical ergodic mean of throughput.

Theorem 1: The compact upper bound of throughput for finite-rate feedback-based ZF is:

$$R_{FB}(P) \le K \log_2\left(1 + \frac{\frac{P}{K}\cdot(M-K+1)}{1 + \frac{P}{K} M(K-1) E\left[\sin^2(\angle(\tilde{\mathbf{h}}_i, \hat{\mathbf{h}}_i))\right]\frac{1}{M-1}}\right) . \tag{4}$$

Proof: The theoretical ergodic mean of throughput can be written as:

$$R_{FB}(P) = K \cdot E\left[\log_2\left(1 + \frac{\frac{P}{K}\left\|\mathbf{h}_i\right\|^2 \left|\tilde{\mathbf{h}}_i^+ \mathbf{v}_i\right|^2}{1 + \sum_{j=1, j\neq i}^{j=K} \frac{P}{K}\left\|\mathbf{h}_i\right\|^2 \left|\tilde{\mathbf{h}}_i^+ \mathbf{v}_j\right|^2}\right)\right] \tag{5}$$

Assuming that $\tilde{\mathbf{v}}_i$ is derived from the unquantized version $\tilde{\mathbf{h}}_i$ ($i = 1,...,K$) instead of the quantized version $\hat{\mathbf{h}}_i$ ($i = 1,...,K$), with this assumption the mean of $|\tilde{\mathbf{h}}_i^+ \tilde{\mathbf{v}}_i|^2$ can be regarded as a compact upper bound of the mean of $|\tilde{\mathbf{h}}_i^+ \tilde{\mathbf{v}}_i|^2$.

$$E\left[\left|\hat{\mathbf{h}}_i^+ \mathbf{v}_i\right|^2\right] \le E\left[\left|\tilde{\mathbf{h}}_i^+ \tilde{\mathbf{v}}_i\right|^2\right]$$

Furthermore, $\tilde{\mathbf{v}}_i$ has $M-K+1$ degrees of freedom to satisfy $\left|\tilde{\mathbf{h}}_j^+ \tilde{\mathbf{v}}_i\right|^2 = 0$ ($j \neq i$). In other words, $\tilde{\mathbf{v}}_i$ still has

$M-K+1$ degrees of freedom to match to the target channel $\tilde{\mathbf{h}}_i$. Thus,

$$E\left[\left|\hat{\mathbf{h}}_i^{+}\mathbf{v}_i\right|^2\right] \le E\left[\left|\tilde{\mathbf{h}}_i^{+}\tilde{\mathbf{v}}_i\right|^2\right] = \frac{M-K+1}{M}$$

In other words, if the number of co-scheduled users is smaller, the more dependence $\tilde{\mathbf{v}}_i$ to $\tilde{\mathbf{h}}_i$ can be chosen to obtain the residual array gain.

From the proof of theorem 2 in Ref. [1], we have:

$$\begin{aligned} E\left[\left|\tilde{\mathbf{h}}_i^{+}\mathbf{v}_j\right|^2\right] &= E\left[\sin^2(\angle(\tilde{\mathbf{h}}_i,\hat{\mathbf{h}}_i)\right]\cdot E_Y[Y] \\ &= E\left[\sin^2(\angle(\tilde{\mathbf{h}}_i,\hat{\mathbf{h}}_i)\right]\frac{1}{M-1} \end{aligned}$$

where Y is a beta (1, M-2) random variable.

Finally, the upper bound of throughput can be written as Equ. (4).

Similar with [1], $E\left[\sin^2(\angle(\tilde{\mathbf{h}}_i,\hat{\mathbf{h}}_i)\right]$ can be evaluated as $2^B\cdot\beta\left(2^B,\frac{M}{M-1}\right)$ from the RVQ theory.

It should be noted that:

1) The perfect CSIT-based ZF has the throughput mean as:

$$R_{FB}(P) \le K\cdot\log_2\left(1+\frac{P}{K}\cdot\frac{1}{K}\right) \tag{6}$$

2) When $K=1$, the multiuser MIMO system can be regarded as single-user MIMO system, i.e. the so-called point-to-point MIMO system.

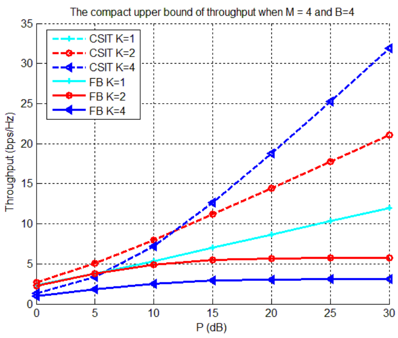
It can be observed from the equations derived above that for finite-rate feedback-based ZF the value of K will largely impact the upper bound of throughput.

IV. Throughput Analysis

To verify the upper bound of throughput, we compared them with the simulated throughput by Monte Carlo method with consideration of Eq. (3). In Monte Carlo method, the channels are realized randomly as the block Rayleigh fading. We choose the number of transmit antennas as 4 and 8. The numbers of quantization bits are selected as 4 and 8 respectively, since they are the number of feedback bits for 4 and 8 transmit antennas in current LTE and LTE-Advanced standard.

The results are plotted in Fig. 1 and Fig. 2 as follows.

From Fig. 1 and Fig. 2, we have the conclusions:



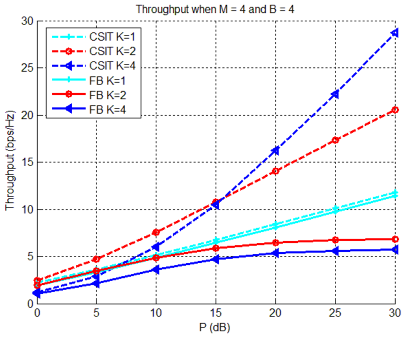


Figure. 1. The compact upper bound of throughput and the simulated throughput when M=4 and B=4

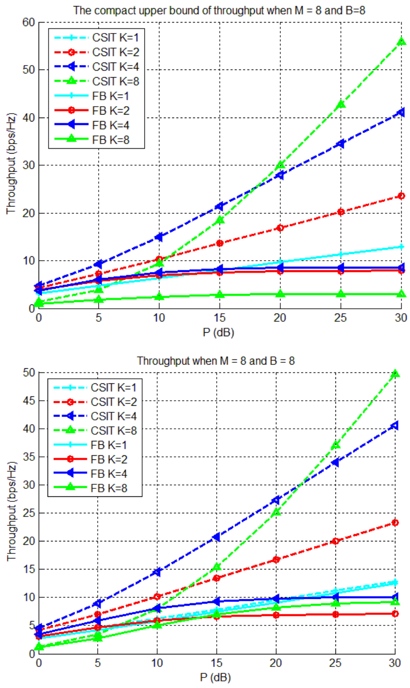


Figure. 2. The compact upper bound of throughput and the simulated throughput when M=8 and B=8

1) The curves show the similar trend for the compact upper bound of throughput and the simulated throughput;
2) For finite-rate feedback-base ZF with the small number of quantization bits, e.g. 4 and 8 bits in our simulation, the ZF beamforming gain is not obvious;
3) For finite-rate feedback-base ZF with the small number of quantization bits, the throughput for small value of K may be larger than the throughput for large value of K;
4) Even for perfect CSIT-based ZF, in low SNR area, the throughput for small value of K may be larger than the throughput for large value of K.

The conclusions above seem too pessimistic for multiuser MIMO. It is noted that any conclusion in the paper is based on the assumption of no multiuser diversity gain exploited. If the multiuser diversity gain is exploited, i.e. the number of candidate users is large enough, the expectation of $\left|\tilde{\mathbf{h}}_i^{+}\mathbf{v}_i\right|^2$ is not that small as

Theorem 1 when K is large, so the multiplexing gain will be obvious since the each user has the larger power of target signal. Specifically, when the number of candidate users is large enough, we will choose the group of users who have the semi-orthogonal or orthogonal channel direction, so the ZF beamformer will approach to the channel direction, and thereby the power of target signal will approach to that in single-user MIMO, i.e. $\left|\tilde{\mathbf{h}}_i^{+}\mathbf{v}_i\right|^2$ will approach the optimal value. In other words, the array gain is kept when the number of candidate users is large enough. However, in this case, the ZF beamforming gain is not shown yet.

To verify this point, we will show the other extreme case, i.e. the number of candidate tends to infinity (of course much larger than the number of transmit antennas).

Theorem 2: When the number of candidate users tends to infinity, the upper bound of throughput for finite-rate feedback-based ZF is:

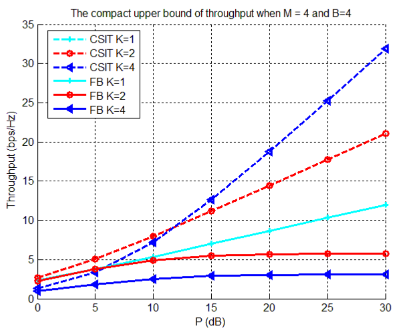
$$R_{FB}(P) \le K\log_2\left(1+\frac{\frac{P}{K}\cdot M}{1+\frac{P}{K}M(K-1)E\left[\sin^2(\angle(\tilde{\mathbf{h}}_i,\hat{\mathbf{h}}_i))\right]\frac{1}{M-1}}\right). \quad (7)$$

Proof: When the number of candidate users tends to infinity, the scheduler can always select the users with the orthogonal channel directions, so ZF precoding need to be operated since the channel directions for the selected users are already interference free. In other words, the precoder $\tilde{\mathbf{v}}_i$ will not be adjusted in ZF operation, so it will still match to the channel direction $\tilde{\mathbf{h}}_i$, i.e.

$$E\left[\left|\hat{\mathbf{h}}_i^{+}\mathbf{v}_i\right|^2\right] \le E\left[\left|\tilde{\mathbf{h}}_i^{+}\tilde{\mathbf{v}}_i\right|^2\right] = 1$$

The left derivation is the same with Theorem 1, and the result is reached.

From above theorem, it is observed that, when the number of candidate users tends to infinity the precoder will not be adjusted and will match to the target channel direction, so the multiplexing gain of multiuser MIMO comes back. Fig. 3 and Fig. 4 show that with the optimal user grouping the throughput for large number of candidate users is larger than that for small number of candidate users, in all SNR area for perfect CSIT case and in low SNR area for finite-rate feedback case.



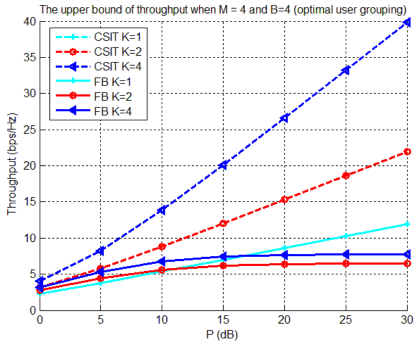


Figure. 3. The upper bound of throughput 1) without any user grouping 2) with the optimal user grouping, when M=4 and B=4

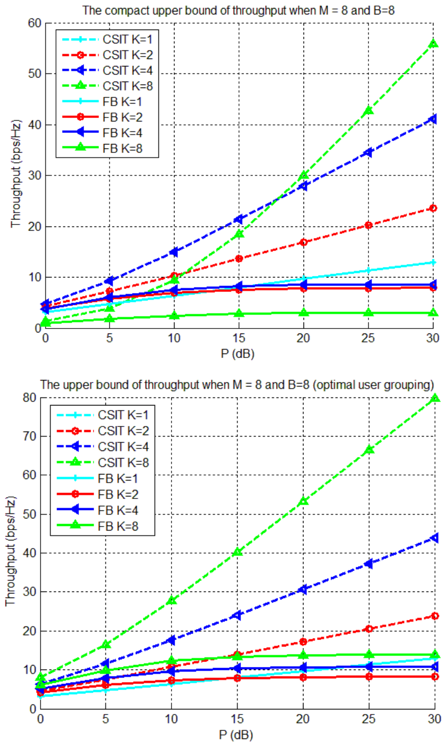


Figure. 4: The upper bound of throughput: 1) without any user grouping and 2) with the optimal user grouping, when M=8 and B=8

On the other hand, the large number of quantization bits will lead to the significant ZF beamforming gain. Nevertheless, the increase of the feedback bits will cause some issues, such as uplink payloads etc., and is not accepted in current standard. When the number of quantization bits is large enough, the throughput is sure to be close to perfect CSIT-based ZF which shows large ZF beamforming gain, but it is out of scope of this paper.

Thus, we can foresee that the potential exceptions or improvements for low ZF beamforming gain are:

1) The larger number of candidate users;
2) The larger number of quantization bits.

V. Conclusions

This paper discusses the finite-rate feedback-based ZF for multiuser MIMO in case of small number of candidate users to be selected and small number of quantization bits to be fed back. In this case, the multiplexing gain from ZF is limited.

Under the assumption of small number of candidate users, the paper then further analyzes throughput of ZF with different number of co-scheduled users. From the analysis, the throughput of ZF is limited by the loss of target signal power and the residual interference from the co-scheduled users. The loss of target signal power comes from the steered precoder by ZF operation.

From the simulation for theoretical throughput and throughput from Monte Carlo methods, we have some conclusions:, for finite-rate feedback-base ZF multiuser MIMO with the small number of quantization bits, the ZF beamforming gain that is significant for perfect CSIT based ZF is disappear, and the throughput for small value of co-scheduled users may be larger than the throughput for large value of co-scheduled users.

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