The Impact of Combined Equalization on the Performance of MC-CDMA Systems

Barbara M. Masini

***Abstract*— In this work the performance of a combined equalization for multi-carrier code division multiple access (MC-CDMA) systems is analytically evaluated. Combined equalization consists in performing both pre-equalization at the transmitter and post-equalization at the receiver in order to counteract fading and multi user interference. In this paper we consider partial combining (PC) at the transmitter and threshold orthogonality restoring combining (TORC) at the receiver in the downlink of a MC-CDMA system affected by fading. The analytical framework here proposed allows the derivation of the bit error probability (BEP) and the bit error outage (BEO) and their dependence on the number of subcarriers, the number of active users, the mean signal-to-noise ratio (SNR) averaged over small-scale fading, the partial combining parameter and the threshold of the detector. Moreover, the dependence of the TORC threshold on the other system parameters is derived, providing a thereshold adaptive variation tracking slow processes fluctuations.**

***Index Terms*— Multi-carrier CDMA, partial combining, TORC detection, performance evaluation, fading channel.**

I. Introduction

The spreading of wireless systems and the enhancement of multimedia mobile communications have evidenced the advantages and applicability of orthogonal frequency division multiplexing (OFDM) due to its spectral efficiency, fading counteracting and robust data transmission. In order to favor the development of future broadband wireless systems and let many users share the same resources, the combination of OFDM with code division multiple access (CDMA) is gaining an increasing interest especially for its capability in counteracting multi user interference and supporting high data rate in mobile transmission. The basis principle of multi carrier (MC)-CDMA is to spread each data symbol over several (or all) subcarriers in order to exploit frequency diversity of the transmission channel [1], [2]. In order to improve the performance of MC-CDMA systems, many combining techniques are known in the literature, trying to exploit diversity and minimize the effect of fading and, as a consequence, of multiuser interference. In fact, even considering the downlink and assuming the adoption of orthogonal codes to differentiate users, the effect of different fading on each subcarriers corrupts the users' orthogonality, drastically deteriorating the final performance. For this reason, the choice of the combining technique becomes critical. Since in this work we consider the downlink, and the combination is performed also at the mobile receiver, we focus on linear combining techniques.

Within the family of linear combining techniques, different schemes based on channel state information (CSI) are known in the literature (see, e.g., [3]), where signals coming from different subcarriers are weighted by suitable coefficients to improve the system performance. Among these techniques, maximal ratio combining (MRC), equal gain combining (EGC) and orthogonality restoring combining are only some of the most known and frequently adopted when the CSI is available only at the transmitter or the receiver.

In this paper we analyze a scenario in which CSI is simultaneously available at both the transmitter and the receiver in order to evaluate if a combined equalization (i.e., combination performed at both the transmitter and the receiver side) could improve the system performance in terms of bit error probability (BEP) and bit error outage (BEO). In particular, we analytically derive the BEP and the BEO for the downlink of a MC-CDMA system when partial combining is adopted at the transmitter and threshold ORC (TORC) at the receiver [4]. We follow the scheme presented in [1], [5] modified for what concerns the combining technique of signals coming from different subcarriers. In this scheme the spreading is performed in the frequency-domain, with spreading factor equal to the number of subcarriers, and Walsh-Hadamard (W-H) codes are adopted to differentiate users. A combined equalization scheme was already presented in [6], but performance was analytically derived for a single user. In this work, we consider a general multiuser scenario and we evaluate if the combined equalization introduces some benefits with respect to classical single-side combining techniques.

The paper is organized as follows: in Section II an overview on the most common linear combining techniques is provided and the system model introduced; in Section III the channel model is presented and in Section IV the received signal and the decision variable are derived. The performance in terms of BEP and BEO is evaluated in Sections V and VI, respectively and numerical results are presented in Section VII. Finally our conclusion are drawn in Section VIII.

II. System Model

In this work we consider the MC-CDMA architecture with spreading done in the frequency-domain and W-H codes with spreading factor equal to the number of subcarriers, as presented in [1], [5] and shown in Figure 1.

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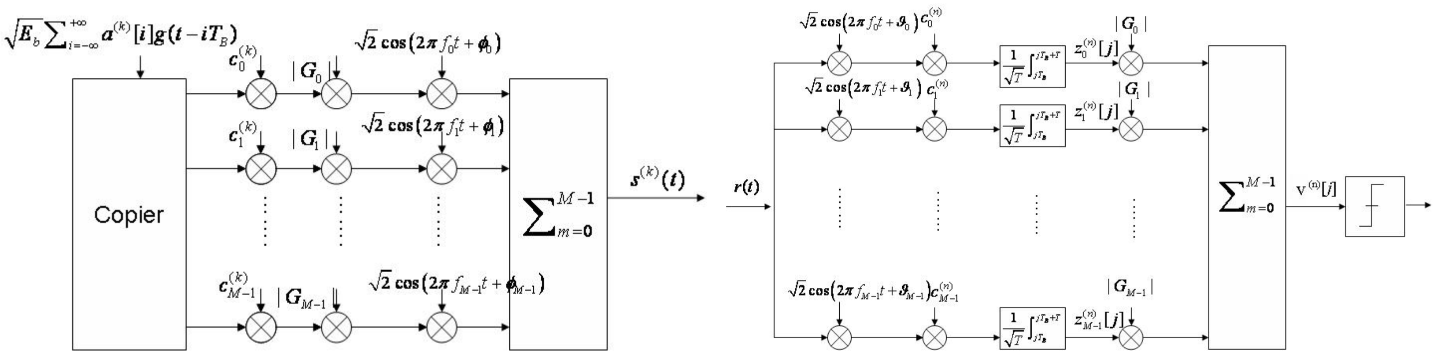

(a) Transmitter block scheme. (b) Receiver block scheme.

Figure 1. Transmitter and receiver block schemes for the $n^{\underline{\text{th}}}$ user.

Even if it can be assumed that at the receiver side of the downlink the information associated to all users experiences the same channel and the system remains always synchronous, the orthogonality between the sequences of different users is lost, in spite of the use of W-H codes, due to the different fading in each subchannel. Therefore, the choice of the combining technique becomes critical.

Several combining techniques with different complexities have been studied in the literature (see, e.g., [7]–[9]); in this work we address low complexity combining schemes such as linear combining, since, in the downlink, the detection is performed also at the user terminal.

Within the family of linear combining techniques, different schemes based on CSI are known in the literature (see, e.g., [10]–[12]), for which signals coming from different subcarriers are weighted by suitable coefficients G_m (m being the subcarrier index).

Among these techniques, EGC consists in equally weighting each subcarrier contribution and compensating only the phases as:

$$G_m = \frac{H_m^\star}{|H_m|}\,, \tag{1}$$

where H_m is the $m^{\underline{\text{th}}}$ channel coefficient (notation * stands for complex conjugation). If the system is noise-limited, (i.e., the number of active users is negligible with respect to the number of subcarriers), MRC represents the best choice with

$$G_m = H_m^\star\,. \tag{2}$$

On the other hand, this choice totally destroys the orthogonality between the codes. For this reason, when the system is interference-limited, a good choice is given by restoring the orthogonality between the sequences with ORC, for which

$$G_m = \frac{1}{H_m}\,. \tag{3}$$

On one hand ORC implies a total cancellation of the multiuser interference, but, on the other hand, it emphasizes the noise. For this reason a correction is introduced with the threshold orthogonality restoring combining (TORC), where a threshold is introduced to cancel the contributions of those subchannels highly corrupted by the noise, allowing low complexity with sufficiently good performance (see [4], [13]–[15]):

$$G_m = \frac{1}{H_m} u(|H_m| - \rho_{TH}), \tag{4}$$

where $u(\cdot)$ is the unitary-step function and the threshold ρ_{TH} is introduced to cancel the contributions of subchannels highly corrupted by the noise.

In [16], [17] a partial combining (PC) technique was introduce, with coefficient G_m function of a PC parameter, β, as given by:

$$G_m = \frac{H_m^*}{|H_m|^{1+\beta}}\,. \tag{5}$$

Note that (1), (2), (3) can be viewed as particular cases of (5) for which the parameter β assumes the values 0 (EGC), -1 (MRC) and 1 (ORC), respectively. Since MRC and ORC are optimum in the extreme cases of noise-limited and interference-limited systems, respectively, for each intermediate situation PC can represent a better solution.

In [6], a parametric combined equalization when CSI is available at both transmitter and receiver is considered and benefits are shown with respect to single side equalization, especially in single user scenarios.

In this work we consider combined equalization made up of PC pre-equalization at the transmitter and TORC post-equalization at the receiver in multiuser scenarios and we evaluate if the introduction of PC at the transmitter when already adopting TORC at the receiver gives potential advantages in terms of BEP and BEO or if TORC alone is already sufficient in a multiuser scenario. The performance of a MC-CDMA system with TORC detection at the receiver have been derived in [4].

Following the MC-CDMA architecture presented in [2], the number of subcarriers, M, is equal to the spreading factor. Each data-symbol is copied over all subcarriers, and multiplied by the chip assigned to each particular subcarrier. Consequently, the spreading is performed in the frequency-domain.

We consider W-H orthogonal code sequences for the multiple access and binary phase shift keying (BPSK) modulation; thus, the transmitted signal referred to the

$k^{\underline{\text{th}}}$ user, can be written as:

$$s^{(k)}(t) = \sqrt{\frac{2E_b}{M}} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} c_m^{(k)} a^{(k)}[i] G_{m,pre} g(t - iT_b) \times \cos(2\pi f_m t + \phi_m), \quad (6)$$

where E_b is the energy per bit, i denotes the data index, m is the subcarrier index, c_m is the $m^{\underline{\text{th}}}$ chip, $a^{(k)}[i]$ is the data-symbol transmitted during the $i^{\underline{\text{th}}}$ symbol time, $G_{m,pre}$ is the pre-equalization coefficient, $g(t)$ is a rectangular pulse waveform, with duration $[0, T]$ and unitary energy, T_b is the bit-time, $f_m = f_0 + m \cdot \Delta f$ is the subcarrier frequency (with $\Delta f \cdot T$ and $f_0 T$ integers that implies orthogonal frequencies) and ϕ_m is the random phase uniformly distributed within $[-\pi, \pi]$. In particular, $T_b = T + T_g$ is the total OFDM symbol duration, increased with respect to T of a time-guard T_g (inserted between consecutive multi-carrier symbols to eliminate the residual inter symbol interference, ISI, due to the channel delay spread). Pre-equalization is performed under the assumption that transmit power is the same as in the case without pre-equalization:

$$\sum_{m=0}^{M-1} |G_{m,pre}|^2 = M\,, \quad (7)$$

that is satisfied if

$$G_{m,pre} = G_m \sqrt{\frac{M}{\sum_{i=0}^{M-1} |G_i|^2}}\,, \quad (8)$$

where G_m is the pre-equalization coefficient without power constraint given by (5). In particular:

$$|G_{m,pre}| = \alpha_m^{-\beta} \sqrt{\frac{M}{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}}\,. \quad (9)$$

Considering that, exploiting the orthogonality of the code, all the users adopt the same carriers, the total transmitted signal results in:

$$s(t) = \sum_{k=0}^{N_u-1} s^{(k)}(t) = \sqrt{\frac{2E_b}{M}} \sum_{k=0}^{N_u-1} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} c_m^{(k)} \times G_{m,pre} a^{(k)}[i] g(t - iT_b) \cos(2\pi f_m t + \phi_m) \quad (10)$$

where N_u is the number of active users and, because of the use of orthogonal codes, $N_u \leq M$.

III. Channel Model

Since we are considering the downlink, we assume that, focusing on the $n^{\underline{\text{th}}}$ receiver, the information associated to different users experiments the same fading. Due to the CDMA structure of the system, each user receives the information of all the users and select only its own data through the spreading sequence. We assume the impulse response of the channel, $h(t)$, as time-invariant during many symbol intervals. We consider a frequency domain channel model with transfer function, $H(f)$, given by:

$$H(f) \simeq H(f_m) = \alpha_m e^{j\psi_m} \text{ for } |f - f_m| < \frac{W_s}{2}\,, \forall\, m, \quad (11)$$

where α_m and ψ_m are the $m^{\underline{\text{th}}}$ amplitude and phase coefficients, respectively, and W_s is the transmission bandwidth of each subcarrier. Hence, we assume the response $g'(t)$ to $g(t)$ as a pulse with unitary energy and duration $T' \triangleq T + T_d$, being $T_d \leq T_g$ the time delay. We assume that each $H(f_m)$ is independent identically distributed (i.i.d.) complex zero-mean Gaussian random variable (r.v.) with variance, σ_H^2, such that $\mathbb{E}\left\{\alpha^2\right\} = 2\sigma_H^2$.

IV. Decision Variable

The received signal can be written as:

$$r(t) = s(t) * h(t) + n(t), \quad (12)$$

where $n(t)$ is the wide sense stationary additive white Gaussian noise with two-side power spectral density (PSD) $N_0/2$. Hence, we can write:

$$r(t) = \sqrt{\frac{2E_b}{M}} \sum_{k=0}^{N_u-1} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} \alpha_m c_m^{(k)} a^{(k)}[i] g'(t - iT_b) \times G_{m,pre} \cos(2\pi f_m t + \overbrace{\phi_m + \psi_m}^{\vartheta_m}) + n(t), \quad (13)$$

where $\vartheta_m \triangleq \phi_m + \psi_m$. Note that, since ϑ_m can be considered uniformly distributed in $[-\pi, \pi]$, we can consider the argument of $H(f_m)$ distributed as ϑ_m.
Focusing, without loss of generality, to the $l^{\underline{\text{th}}}$ subcarrier of user n, the receiver performs the correlation at the $j^{\underline{\text{th}}}$ instant (perfect synchronization and phase tracking are assumed) of the received signal with $c_l^{(n)} \sqrt{2} \cos(2\pi f_l t + \vartheta_l)$, as:

$$z_l^{(n)}[j] = \frac{1}{\sqrt{T}} \int_{jT_b}^{jT_b+T} r(t)\; c_l^{(n)} \sqrt{2} \cos(2\pi f_l t + \vartheta_l) dt\,. \quad (14)$$

Following the analytical procedure adopted in [17] and after some algebra, we obtain:

$$z_l^{(n)}[j] = \sqrt{\frac{E_b \delta_d}{M}} \alpha_l^{1-\beta} \sqrt{\frac{M}{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}} a^{(n)}[j] \quad (15)$$
$$+ \sqrt{\frac{E_b \delta_d}{M}} c_l^{(n)} \alpha_l^{1-\beta} \sum_{k=0, k\neq n}^{N_u-1} c_l^{(k)} \sqrt{\frac{M}{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}} \times a^{(k)}[j] + n_l[j]\,,$$

where $\delta_d \triangleq 1/(1 + T_d/T)$ represents the loss of energy caused by the time-spreading of the impulse and $n_l[j]$ is the noise contribution.

The decision variable, $v^{(n)}[j]$, is obtained by linearly combining the weighted signals from each subcarrier as:[1]

$$v^{(n)} = \sum_{l=0}^{M-1} |G_{l,post}| z_l^{(n)}\,, \quad (16)$$

where the post-equalization coefficient $G_{l,post}$ is referred to a TORC detector; it has to counteract not only the

[1] For the sake of conciseness in our notation, since ISI is avoided, we will neglect the time-index j in the following.

fading channel, but also additional distortions caused by pre-equalization, thus it can be written as:

$$G_{l,post} = \frac{1}{H_l G_{l,pre}} u(|H_l G_{l,pre}| - \rho_{\mathrm{TH}}) \,, \tag{17}$$

and

$$\begin{aligned} |G_{l,post}| &= \frac{1}{|H_l G_{l,pre}|} u(|H_l G_{l,pre}| - \rho_{\mathrm{TH}}) \quad (18) \\ &= \alpha_l^{\beta-1} \sqrt{\frac{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}{M}} \\ &\times\; u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right) . \end{aligned}$$

By substituting (15) and (18) in (16), the decision variable can be re-written as:

$$\begin{aligned} v^{(n)} &= \overbrace{\sqrt{\frac{E_b\delta_d}{M}} \sum_{l=0}^{M-1} u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right)}^{U} \\ &\times\; a^{(n)} + \overbrace{\sqrt{\frac{E_b\delta_d}{M}} \sum_{l=0}^{M-1} \sum_{k=0,k\neq n}^{N_u-1} c_l^{(n)} c_l^{(k)}}^{I} \\ &\times\; \overbrace{u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right) a^{(k)}}^{I} \\ &+\; \overbrace{\sum_{l=0}^{M-1} \alpha_l^{\beta-1}\sqrt{\frac{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}{M}}}^{N} \\ &\times\; \overbrace{u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right) n_l}^{N} . \end{aligned} \tag{19}$$

Now, the BEP evaluation can be obtained by studying the statistic distributions of the useful term, U, the noise term, N and the interference term, I. In particular, it is reasonable, for sufficiently high number of subcarriers, as for practical systems (such as digital video broadcasting, WiMax, etc.), to adopt the law of large number (LLN) and the central limit theorem (CLT). The reliability of the approximation on the BEP obtained through these assumptions will be verified by simulation.

A. Useful term

By applying the CLT to the useful term, we obtain the following statistical distribution:

$$U \sim \mathcal{N}\left(\mu_U, \sigma_U^2\right) \tag{20}$$

where notation $\mathcal{N}(\mu, \sigma^2)$ stands for Gaussian distribution with mean μ and variance σ^2, $\mathbb{E}\{\cdot\}$ denoting the statistical expectation operation. In particular:

$$\begin{aligned} \mu_U &= \sqrt{E_b\delta_d M}\,\mathbb{E}\left\{u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right)\right\} \\ \sigma_U^2 &= E_b\delta_d\zeta(\alpha) . \end{aligned}$$

It can be derived that:

$$\mathbb{E}\left\{u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right)\right\} = e^{-\frac{\rho_{\mathrm{TH}}^2}{2\sigma_{\mathrm{H}}^2}} \tag{21}$$

and

$$\begin{aligned} \zeta(\alpha) &\triangleq \mathbb{E}\left\{\left[u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right)\right]^2\right\} \\ &- \left[\mathbb{E}\left\{u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right)\right\}\right]^2 \\ &= e^{-\frac{\rho_{\mathrm{TH}}^2}{2\sigma_{\mathrm{H}}^2}} - e^{-\frac{\rho_{\mathrm{TH}}^2}{\sigma_{\mathrm{H}}^2}} . \end{aligned} \tag{22}$$

B. Interference Term

By exploiting the properties of orthogonal codes as in [5], after some algebra the interference term can be rewritten as:

$$\begin{aligned} I &= \sqrt{\frac{E_b\delta_d}{M}} \sum_{k=0,k\neq n}^{N_u-1} a^{(k)} \\ &\times \left[\overbrace{\sum_{h=1}^{\frac{M}{2}} u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right)}^{A_1}\right. \\ &\left. - \overbrace{\sum_{h=1}^{\frac{M}{2}} u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right)}^{A_2}\right] \end{aligned} \tag{23}$$

where indexes x_h and y_h define the following partition:

$$\begin{aligned} &c^{(n)}[x_h]c^{(k)}[x_h] = 1 \\ &c^{(n)}[y_h]c^{(k)}[y_h] = -1 \\ &\{x_h\} \cup \{y_h\} = 0, 1, 2, ..., M-1 . \end{aligned} \tag{24}$$

For large values of M, we can exploit the CLT, by obtaining that terms A_1 and A_2 in (23) are distributed as:

$$\mathcal{N}\left(\sqrt{\frac{M}{2}}\mathbb{E}\left\{u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\mathrm{TH}}\right)\right\}, \frac{M}{2}\zeta(\alpha)\right) .$$

Hence, by exploiting the symmetry of the Gaussian p.d.f. and the property of the sum of uncorrelated Gaussian r.v.'s, we have:

$$a^{(k)}\left(A_1 - A_2\right) \sim \mathcal{N}\left(0, M\zeta(\alpha)\right) .$$

Hence, it can be derived that the general interference term is distributed as:

$$I \sim \mathcal{N}\left(0, \sigma_I^2\right) , \tag{25}$$

where $\sigma_I^2 \triangleq E_b\delta_d(N_u - 1)\zeta(\alpha)$.

C. Noise Term

$$
\begin{aligned}
N &= \sum_{l=0}^{M-1} \alpha_l^{\beta-1} \sqrt{\frac{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}{M}} \\
&\times \ u\left(\alpha_l^{1-\beta} \sqrt{\frac{M}{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}} - \rho_{\text{TH}}\right) n_l \\
&= \sqrt{\frac{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}{M}} \sum_{l=0}^{M-1} \alpha_l^{\beta-1} \\
&\times \ u\left(\alpha_l^{1-\beta} \sqrt{\frac{M}{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}} - \rho_{\text{TH}}\right) n_l \\
&\simeq \sqrt{\frac{M\mathbb{E}\left\{\alpha_i^{-2\beta}\right\}}{M}} \sum_{l=0}^{M-1} \alpha_l^{\beta-1} \\
&\times \ u\left(\alpha_l^{1-\beta} \sqrt{\frac{M}{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}} - \rho_{\text{TH}}\right) n_l \\
&= \sqrt{(2\sigma_H)^{-\beta}\Gamma[1-\beta]} \sum_{l=0}^{M-1} \alpha_l^{\beta-1} \\
&\times \ u\left(\alpha_l^{1-\beta} \sqrt{\frac{M}{\sum_{i=0}^{M-1} \alpha_i^{-2\beta}}} - \rho_{\text{TH}}\right) n_l \qquad (26)
\end{aligned}
$$

Considering that the terms α_l and n_l are independent and n_l is zero mean, the noise term N results distributed as:

$$
N \sim \mathcal{N}\left(0, \sigma_N^2\right) , \qquad (27)
$$

where

$$
\begin{aligned}
\sigma_N^2 &= M\frac{N_0}{2}(2\sigma_H^2)^{-\beta}\Gamma[1-\beta] \\
&\times \ \mathbb{E}\left\{\left[\alpha_l^{\beta-1} u\left(\alpha_l^{1-\beta}\sqrt{\frac{M}{\sum_{i=0}^{M-1}\alpha_i^{-2\beta}}} - \rho_{\text{TH}}\right)\right]^2\right\} \\
&= M\frac{N_0}{2}(2\sigma_H^2)^{-\beta}\Gamma[1-\beta](2\sigma_H^2)^{\beta-1}\Gamma\left[\beta, \frac{\rho_{\text{TH}}^2}{2\sigma_H^2}\right] \\
&= M\frac{N_0}{2}(2\sigma_H^2)^{-1}\Gamma[1-\beta]\Gamma[\beta, \frac{\rho_{\text{TH}}^2}{2\sigma_H^2}] \qquad (28)
\end{aligned}
$$

D. Independence between terms

By noting that $a^{(k)}$ is zero mean and statistically independent on α_l, $(A_1 - A_2)$ and n_l, it follows that $\mathbb{E}\{I\ N\} = \mathbb{E}\{I\ U\} = 0$. Since n_l and α_l are statistically independent, then $\mathbb{E}\{N\ U\} = 0$. Since I, N and U are uncorrelated Gaussian r.v.'s, they are also statistically independent.

V. Bit Error Probability Evaluation

From (20), (25) and (27), we obtain:

$$
P_b \simeq \frac{1}{2}\text{erfc}\sqrt{\frac{\mu_U^2}{2(\sigma_N^2+\sigma_I^2)}} = \frac{1}{2}\text{erfc}\sqrt{SNIR},
$$

where $SNIR$ is given by:

$$
SNIR = \frac{\overline{\gamma} e^{-\frac{\rho_{\text{TH}}^2}{\sigma_{\text{H}}^2}}}{\Gamma[1-\beta]\Gamma[\beta, \frac{\rho_{\text{TH}}^2}{2\sigma_H^2}] + 2\overline{\gamma}\frac{(N_u-1)}{M}\left[e^{-\frac{\rho_{\text{TH}}^2}{2\sigma_{\text{H}}^2}} - e^{-\frac{\rho_{\text{TH}}^2}{\sigma_{\text{H}}^2}}\right]} \qquad (29)
$$

and $\overline{\gamma}$ represents the mean SNR averaged over small-scale fading defined as:

$$
\overline{\gamma} \triangleq \frac{E_b\delta_d}{N_0}2\sigma_H^2 . \qquad (30)
$$

Note that when $\beta = 0$ (i.e., we do not perform PC at the transmitter side), we obtain the BEP expression presented in [4], confirming the validity of our analysis. By comparing (29) with the SNIR given in [4], it is also worth noting that the presence of PC pre-equalization only affects the noise term at the denominator of (29); the useful and interference contribution are the same also not considering any pre-equalization.

A. Performance Optimization

Keeping β fixed, we now aim at finding the optimum values of ρ_{TH} that minimizes the BEP, defined as:

$$
\rho_{\text{TH}}^{(opt)} = \arg\min_{\rho_{\text{TH}}}\{P_b\} \simeq \arg\max_{\rho_{\text{TH}}}\{SNIR\} . \qquad (31)
$$

By deriving (29) with respect to ρ_{TH}, we obtain (32) (bottom of the next page). Then, after some manipulation, the following solving equation can be found out:

$$
\begin{aligned}
\xi &= e^{\frac{\rho_{\text{TH}}^2}{2\sigma_{\text{H}}^2}}\Gamma[1-\beta] \qquad (33) \\
&\times \ \left\{-2\Gamma\left[\beta, \frac{\rho_{\text{TH}}^2}{2\sigma_H^2}\right] + \left(\frac{2\sigma_H^2}{\rho_{\text{TH}}}\right)^{1-\beta} e^{-\frac{\rho_{\text{TH}}^2}{2\sigma_{\text{H}}^2}}\right\} ,
\end{aligned}
$$

where

$$
\xi \triangleq 2\overline{\gamma}\frac{(N_u-1)}{M} . \qquad (34)
$$

Note that the parameter ξ quantifies how much the system is noise-limited (low values) or interference-limited (high values), and (33) represents the implicit solution, for the problem of finding the optimum value of ρ_{TH} for all possible values of SNR, number of subcarriers, number of users and pre-equalization combining techniques. Moreover, (33) opens the way to an important consideration. In fact, the optimum ρ_{TH} only depends, through ξ, on slowly varying processes such as the SNR (averaged over fast fading then randomly varying according to shadowing), the number of users, the number of subcarriers and the pre-equalization coefficient. This means that it could be reliable an adaptive TORC detection technique in which ρ_{TH} is slowly adapted to the optimum value for the current set of $\overline{\gamma}$, N_u, M and β.

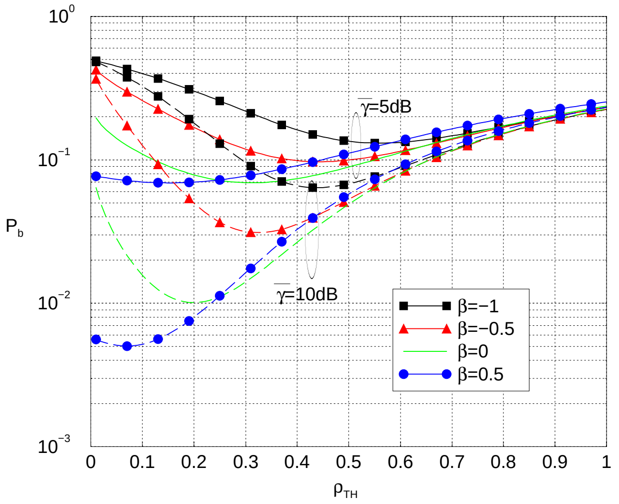

Figure 2. BEP vs. ρ_{TH} for different values of β and mean SNR $\overline{\gamma}$ in fully loaded system conditions ($M = N_U = 64$).

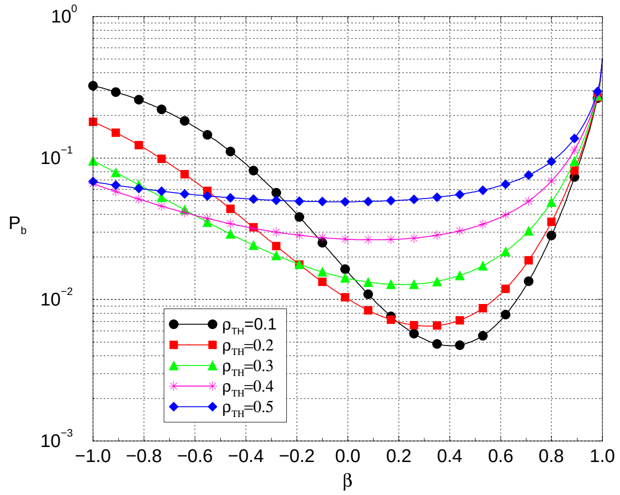

Figure 3. BEP vs. β for different values of ρ_{TH} and $\overline{\gamma} = 10$ dB in fully loaded system conditions ($M = N_U = 64$).

VI. Outage Probability Evaluation

In digital wireless communications where shadowing is superimposed to small-scale fading [18]–[20], another important performance metric is given by the BEO defined as the probability that BEP exceeds a maximum tolerable level (the target BEP, $P_b^\star$) as:

$$P_o \triangleq \mathbb{P}\{P_b > P_b^\star\} = \mathbb{P}\{\overline{\gamma}_{\mathrm{dB}} < \overline{\gamma}_{\mathrm{dB}}^\star\}\,, \tag{35}$$

where $\overline{\gamma}_{\mathrm{dB}} = 10\log_{10}\overline{\gamma}$ and $\overline{\gamma}_{\mathrm{dB}}^\star$ is the SNR (in dB) giving the BEP equal to $P_b^\star$ (i.e., $P_b(\overline{\gamma}^\star) = P_b^\star$). We consider the case of a shadowing environment in which $\overline{\gamma}$ is log-normal distributed with parameters μ_{dB} and σ_{dB}^2 (i.e., $\overline{\gamma}_{\mathrm{dB}}$ is a Gaussian r.v. with mean μ_{dB} and variance σ_{dB}^2). Hence, the BEO results in:

$$P_o = \frac{1}{2}\mathrm{erfc}\left\{\frac{\mu_{\mathrm{dB}} - \overline{\gamma}_{dB}^\star}{\sqrt{2}\sigma_{\mathrm{dB}}}\right\}. \tag{36}$$

Thus, target SNIR, SNIR*, giving $P_b = P_b^\star$ is:

$$\mathrm{SNIR}^\star = \left(\mathrm{erfc}^{-1}\{2P_b^\star\}\right)^2, \tag{37}$$

where erfc^{-1} is the inverse complementary error function. Hence, we derive the required SNR, $\overline{\gamma}^\star$, as a function of SNIR*, ρ_{TH}, M, N_u and β, which is related to the system load, as given by:

$$\overline{\gamma}^\star = \frac{\mathrm{SNIR}^\star\ \Gamma[1-\beta]\Gamma[\beta, \frac{\rho_{\mathrm{TH}}^2}{2\sigma_{\mathrm{H}}^2}]}{e^{-\frac{\rho_{\mathrm{TH}}^2}{\sigma_{\mathrm{H}}^2}} - 2\frac{N_u-1}{M}\cdot\mathrm{SNIR}^\star\cdot\left(e^{-\frac{\rho_{\mathrm{TH}}^2}{2\sigma_{\mathrm{H}}^2}} - e^{-\frac{\rho_{\mathrm{TH}}^2}{\sigma_{\mathrm{H}}^2}}\right)}. \tag{38}$$

Equation (38) enables the derivation of optimal ρ_{TH} which minimizes the required SNR for a given target $P_b^\star$, system load and pre-equalization technique. In addition, given target BEP and BEO, from (29) and (36), we can obtain the required value of μ_{dB} that is the median value of the SNR. This is useful for wireless digital communication systems design, since it is strictly related to the link budget when the path-loss law is known.

VII. Numerical Result

In this Section, numerical results related to the BEP, the optimum ρ_{TH} and the BEO in different systems conditions will be shown.

In Figure 2, the BEP is plotted as a function of ρ_{TH} for various values of β and mean SNR, $\overline{\gamma}$, in fully loaded system conditions with $M = N_u = 64$. Note that there is always an optimum value of ρ_{TH} minimizing the BEP and this value depends on β, thus on the combining technique adopted at the transmitter. Moreover, it is important to observe that the BEP is also drastically dependent on β; in particular, the curve referred to $\beta = -1$ (i.e., by performing MRC at the transmitter), is outperformed by the curve related to $\beta = 0$ (i.e., we equalize at the receiver side only); this means that, a not suitable pre-equalization technique can even deteriorate the performance with respect to post-equalization only. However, when $\beta = 0.5$, the BEP is quite improved with respect to TORC detection only. At instance, when $\rho_{\mathrm{TH}} \simeq 0.1$ and $\overline{\gamma} = 10$ dB, the BEP is about $2\cdot10^{-2}$ for $\beta = 0$, while is $5\cdot10^{-3}$ for $\beta = 0.5$. Similar trends and considerations could be drawn in different system loads conditions.

$$\left\{\Gamma[1-\beta]\Gamma[\beta, \frac{\rho_{\mathrm{TH}}^2}{2\sigma_H^2}] + 2\overline{\gamma}\frac{(N_u-1)}{M}\left[e^{-\frac{\rho_{\mathrm{TH}}^2}{2\sigma_{\mathrm{H}}^2}} - e^{-\frac{\rho_{\mathrm{TH}}^2}{\sigma_{\mathrm{H}}^2}}\right]\right\}\overline{\gamma}e^{-\frac{\rho_{\mathrm{TH}}^2}{\sigma_H^2}}\left(-\frac{2\rho_{\mathrm{TH}}}{\sigma_H^2}\right)$$
$$= \overline{\gamma}e^{-\frac{\rho_{\mathrm{TH}}^2}{\sigma_H^2}}\left\{-\Gamma[1-\beta]\left(\frac{2\sigma_H^2}{\rho_{\mathrm{TH}}}\right)^{1-\beta}\frac{\rho_{\mathrm{TH}}}{\sigma_H^2}e^{-\frac{\rho_{\mathrm{TH}}^2}{2\sigma_H^2}} + 2\overline{\gamma}\frac{(N_u-1)}{M}\left[-\frac{\rho_{\mathrm{TH}}}{\sigma_H^2}e^{-\frac{\rho_{\mathrm{TH}}^2}{2\sigma_{\mathrm{H}}^2}} + \frac{2\rho_{\mathrm{TH}}}{\sigma_H^2}e^{-\frac{\rho_{\mathrm{TH}}^2}{\sigma_{\mathrm{H}}^2}}\right]\right\}. \tag{32}$$

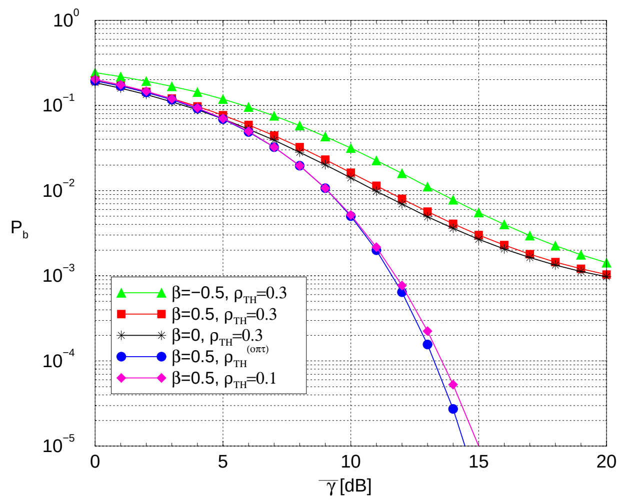

Figure 4. BEP vs. $\overline{\gamma}$ for different values of β and $\rho_{\rm TH}$ in fully loaded system conditions ($M = N_U = 64$).

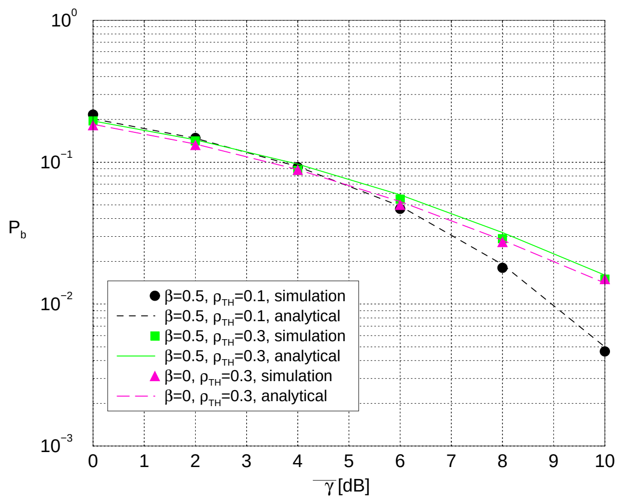

Figure 6. BEP vs. $\overline{\gamma}$ for different values of β and $\rho_{\rm TH}$ in fully loaded system conditions ($N_U = M = 64$): comparison between analysis and simulation.

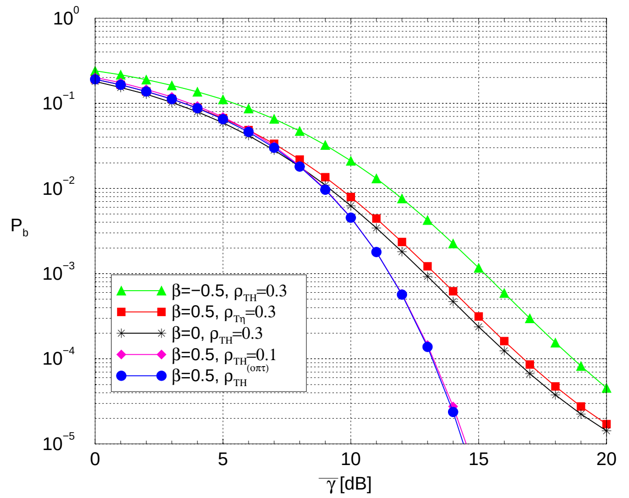

Figure 5. BEP vs. $\overline{\gamma}$ for different values of β and $\rho_{\rm TH}$ in half loaded system conditions ($N_U = M/2 = 32$).

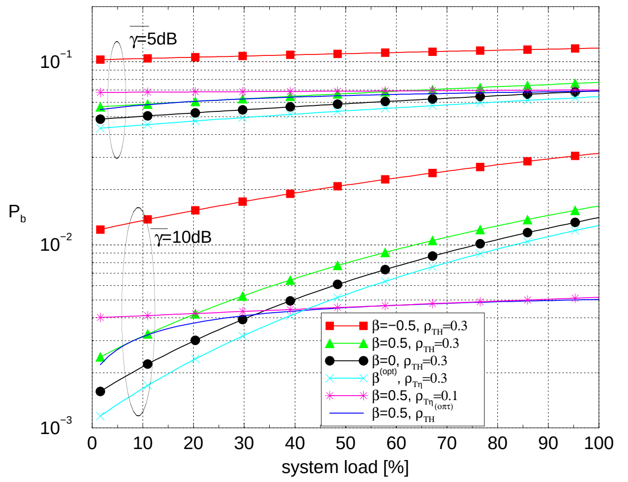

Figure 7. BEP vs. system load for different values of β, $\rho_{\rm TH}$ and $\overline{\gamma}$.

In Figure 3, the BEP as a function of β for $\overline{\gamma} = 10$ dB and different values of $\rho_{\rm TH}$ is shown in fully loaded system conditions. In particular, the BEP is minimum when $\rho_{\rm TH} = 0.1$ and $\beta = 0.5$. However, in spite of the values for $\rho_{\rm TH}$, the values of β (i.e., the kind of pre-equalization technique) granting good performance are in the range [0.1, 0.6]. Hence, the choice of pre-equalization technique is crucial, drastically improving or deteriorating the performance with respect to post-equalization only.

In Figures 4 and 5, the BEP is plotted as a function of the mean SNR, $\overline{\gamma}$, in fully and half loaded system conditions, respectively, for different couples of β and $\rho_{\rm TH}$. In particular, by fixing $\rho_{\rm TH} = 0.3$, we can observe an improvement in the performance by passing from $\beta = -0.5$ to $\beta = 0$ up to $\beta = 0.5$. The best performance can be obtained by adaptively changing $\rho_{\rm TH}$ tracking the slow variations of the mean SNR, $\overline{\gamma}$, with $\beta = 0.5$ (both in fully and half loaded system conditions). In this case, $P_b = 10^{-3}$ can be obtained with $\overline{\gamma} \simeq 12$ dB rather then 20 dB in fully loaded system conditions. Finally note that, by adopting $\beta = 0.5$ and $\rho_{\rm TH} = 0.1$ (i.e., fixed values), we obtain almost the same performance as in case of adaptive TORC detection. This means that a suitable choice of combined equalization gives optimum performance without tracking system variations. By comparing Figures 4 and 5 referred to fully and half loaded system, respectively, it is also worth noting that, when adaptively changing the threshold or by adopting $\beta = 0.5$ with $\rho_{\rm TH} = 0.1$, the system load does not affect the performance, whereas in all the other cases, the system loads highly affects the BEP.

In order to confirm the validity of our analysis, in Figure 6 a comparison between analytical and simulation results is proposed for some values of β and $\rho_{\rm TH}$ in fully loaded system conditions ($N_U = M = 64$). As can be observed analysis and simulation are in good agreement confirming the validity of the analytical framework.

In Figure 7, the BEP as a function of the system load, N_u/M, in percentage is shown for different values of $\overline{\gamma}$

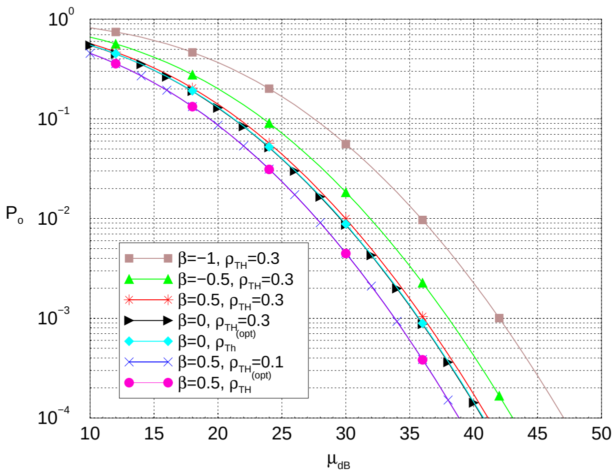

Figure 8. BEO vs. μ_{dB} giving $P_b^\star = 10^{-2}$ for different values of β and ρ_{TH} in fully loaded system conditions ($N_U = M = 64$).

and couples (β, ρ_{TH}). Note that a suitable choice of pre- and post-equalization allows to increase the supportable system load, thus serving an higher number of users, on equal terms of BEP. At instance, by fixing as target BEP, $P_b = 5 \cdot 10^{-3}$, when $\beta = 0$ the system satisfies the 40% of users, whereas when $\beta = 0.5$ and adaptively changing ρ_{TH}, the totality of users (i.e., 100% system load) can be satisfied.

Finally, in Figure 8, the BEO as a function of μ_{dB} is shown for different values of β and ρ_{TH} in fully loaded system conditions ($N_U = M = 64$). The target BEP is assumed 10^{-2} (a typical value for uncoded systems) and $\sigma_{dB} = 8$ dB. Also in this case, it can be observed that a suitable combination of pre- and post-equalization improves the system outage. In particular, by adopting the optimum value of ρ_{TH} and $\beta = 0.5$, we gain 2.5 dB (in terms of μ_{dB}) with respect to TORC detection only and up to 8 dB with respect to MRC at the transmitter and TORC at the receiver. This gain could be exploited to save energy or increase the coverage range of the considered system.

VIII. Conclusion

In this paper we analytically evaluated the performance of a MC-CDMA system with combined equalization at the transmitter and the receiver. In particular, we considered PC at the transmitter and TORC detection at the receiver and we derived the performance in terms of BEP and BEO in the downlink of fading channels. We found out that a suitable combination of pre- and post-equalization techniques can greatly improve the performance both in terms of BEP and BEO. However a wrong choice of pre-equalization technique (such as MRC) can even deteriorate the performance with respect to only TORC detection at the receiver. We also derived the TORC threshold expression as a function of the other system parameters, such as the number of subcarriers, the number of active users, the mean SNR averaged over small-scale fading and the pre-equalization parameter, opening the way to an adaptive variation of the threshold following slow processes fluctuations. Numerical results showed that an adaptive TORC significantly improves the performance in fast fading and log-normal shadowing conditions.

Acknowledgment

The author would like to thank Prof. Oreste Andrisano for motivating this work, Dr. Andrea Conti for helpful suggestions and comments and Flavio Zabini for helpful discussions.

References

[1] N. Yee, J.-P. Linnartz and G. Fettweis, "Multi-Carrier-CDMA in indoor wireless networks", in *Conference Proceedings PIMRC '93, Yokohama, Sept, 1993. p 109-113*

[2] S. Hara and R. Prasad, "An Overview of multicarrier CDMA", *IEEE 4th International Symposium on Spread Spectrum Techniques and Applications Proceedings, 22-25 Sept. 1996, Pages:107 - 114 vol.1*

[3] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*, New York, NY, 10158: John Wiley & Sons, Inc., first ed., 2000.

[4] B. M. Masini, "On the Performance of MC-CDMA Systems with TORC Detection and Channel Estimation Error", *Vehicular Technology Conference, 2007. VTC-2007 Fall. 2007 IEEE 66th Sept. 30 2007-Oct. 3 2007 Page(s):1117 - 1121*

[5] N. Yee and J.-P. Linnartz, "BER of multi-carrier CDMA in an indoor Rician fading channel", *Conference Record of The Twenty-Seventh Asilomar Conference on Signals, Systems and Computers, 1993., 1-3 Nov. 1993 Pages:426 - 430 vol.1*

[6] I. Cosovic, S. Kaiser, "A Unified Analysis of Diversity Exploitation in Multicarrier CDMA", IEEE Transactions on Vehicular Technology, Vol. 56, No 4, Pages: 2051 - 2062, July 2007.

[7] S. Kaiser, "On the performance of different detection techniques for OFDM-CDMA in fading channels", in *Globecom '95 pp.2059-2063 Nov. 1995*

[8] L. Sanguinetti, I. Cosovic, M. Morelli, "Channel estimation for MC-CDMA uplink transmissions with combined equalization", Selected Areas in Communications, IEEE Journal on Volume 24, Issue 6, June 2006 Page(s):1167 - 1178.

[9] I. Cosovic, M. Schnell, A. Springer, "Combined equalization for uplink MC-CDMA in Rayleigh fading channels", Communications, IEEE Transactions on Volume 53, Issue 10, Oct. 2005 Page(s):1609 - 1614.

[10] M. K. Simon and M. S. Alouini, "Digital Communication over Fading Channels: A Unified Approach to Performance Analysis", New York, NY, 10158: John Wiley & Sons, Inc., first ed., 2000.

[11] M. Z. Win and J. H. Winters, "Virtual branch analysis of symbol error probability for hybrid selection/maximal-ratio combining in Rayleigh fading", IEEE Trans. Commun., vol. 49, pp. 19261934, Nov. 2001.

[12] M. Z. Win, G. Chrisikos, and J. H. Winters, "MRC performance for M-ary modulation in arbitrarily correlated Nakagami fading channels", IEEE Commun. Lett., vol. 4, pp. 301303, Oct. 2000.

[13] Kalofonos, D.N.; Stojanovic, M.; Proakis, J.G., "Performance of adaptive MC-CDMA detectors in rapidly fading Rayleigh channels", Wireless Communications, IEEE Transactions on Volume 2, Issue 2, March 2003 Page(s):229 - 239.

[14] T. Müller, H. Rohling and R. Grünheid, "Comparison of different Detection Algorithms for OFDM-CDMA in Broadband Rayleigh Fading",Vehicular Technology Conference, 1995 IEEE 45th Volume 2, 25-28 July 1995 Page(s):835 - 838.

[15] N. Yee and J.-P. Linnartz, "Controlled equalization of multi-carrier CDMA in an indoor Rician fading channel" *Vehicular Technology Conference, 1994 IEEE 44th, 8-10 June 1994 Pages:1665 - 1669 vol.3*

[16] S.B. Slimane, "Partial equalization of multi-carrier CDMA in frequency selective fading channels", *IEEE International Conference on Communications, 2000,Volume: 1 ,18-22 June 2000 Pages:26 - 30 vol.1*

[17] A. Conti, B. Masini, F. Zabini and O. Andrisano, "On the downlink Performance of Multi-Carrier CDMA Systems with Partial Equalization", IEEE Transactions on Wireless Communications, Vol. 6, Issue 1, Jan. 2007 Page(s):230 - 239.

[18] V. Erceg, L. J. Greenstein, S. Y. Tjandra, S. R. Parkoff, A. Gupta, B. Kulic, A. A. Julius, R. Bianchi, "An empirically based path loss model for wireless channels in suburban environments", Selected Areas in Communications, IEEE Journal on Volume 17, Issue 7, July 1999 Page(s):1205 - 1211.

[19] A. Conti, M. Win, M. Chiani, "Bit Error Outage for Diversity Reception in Shadowing Environment", IEEE Communications Letters, Volume: 07 , Issue: 1 , Jan. 2003. Pages: 15 - 17.

[20] A. Conti, "MC-CDMA bit error probability and outage minimization through partial combining", Communications Letters, IEEE Volume 9, Issue 12, Dec. 2005 Page(s):1055 - 1057.

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