Power Allocation for Amplify-and-Forward Cooperative Transmission Over Rayleigh-Fading Channels

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Abstract— **Cooperative transmission has been used to achieve diversity gain through partner to create multiple independent fading channels. It has attracted lots of research attention in recent years. An important question in cooperative communication is power allocation. Given the same total power, how much should be allocated to self-information transmission and how much to relaying-information transmission. In this paper, we investigate power allocation problem using amplify-and-forward (AF) protocol considering bit error rate (BER) performance of both users and the work region of two cooperative users. We solve cooperative ratio, which is defined as the ratio of power used for cooperative transmission over the total power. We first consider both users applying the same ratio, and then extend to two-dimensional cooperative ratio problem. Our results show that it is sufficient to allocate both users the same ratio. This value is specified by the user with a weaker channel gain to the destination, *i.e.*, bottleneck link. With appropriate selection of the cooperative ratio, the BER performance of both users can be improved significantly. User fairness is also considered in the analysis.**

Index Terms— **cooperative communication, amplify-and-forward, power allocation, cooperative ratio, work region, BER.**

I. Introduction

Diversity technique is an important approach to combat fading. In mobile cellular communication, diversity has been implemented for different kind of realizations, for example, time diversity, space diversity, polarity diversity, *etc*. Recently, there is great research attention on multi-path/multiuser diversity, which can be realized by exploring cooperative transmission (see the tutorial paper [1] and the references cited in).

The basic idea of cooperative transmission is to create multiple channels for each user, while each user is equipped with only one transmit antenna, to deliver his/her information to the destination. As a result, spacial diversity gain can be achieved in the presence of channel fading. The transmission quality, measured as bit error rate (BER) at the destination can be improved since statistically speaking, it is less likely that several channels are fading low simultaneously. There are mainly two different kind of operations at the partners (or relays) and the cooperation is distinguished as amplify-and-forward (AF) and decode-and-forward (DF). AF is sometimes also named as nonregenerative, where the partners amplify and forward the received signal without any decoding. For DF, on the other hand, also known as regenerative relaying, the partners decode the signal, recode and forward to the next terminal. In this paper, our focus is on the first cooperative protocol, *i.e.*, amplify-and-forward.

One major category of the frameworks in the open literature has been based on information-theoretic point of view to investigate the performance gains, for example, the achievable system throughput, the achievable rate region, and the outage probability by using cooperative transmission. In [2], a CDMA-based two-user cooperative modulation scheme has been proposed. The main idea is to allow each user to retransmit the estimates of their partner's information such that each user's information transmission rate could be increased and the outage probability could be decreased. In the work of [3], the outage and the ergodic capacity behavior of various cooperative protocols are analyzed for a three-user case under quasi-static fading channels. In [4], optimal power allocation is investigated to maximize the set of ergodic rates achievable by block Markov superposition coding using sub-gradient methods. There also exists a large body of research on coded cooperations (for example [5]). Our focus is on uncoded cooperations, which can be regarded as repetition coding for relay transmission.

Power control/power allocation has long been playing an important role in wireless networks to dynamically combat channel fluctuations and control the co-channel interference [6]. A natural question risen in user cooperation is that how much power should be allocated for self-information transmission and how much for cooperative transmission. Recently, this question has attracted great research attention. In [7], efficient power allocation strategy is investigated in an orthogonal AF network to satisfy the target SNR requirement. In [8], an optimal power allocation scheme is proposed by optimizing the derived approximate symbol error rate subject to fixed transmission rate and total transmit power constraints for AF protocol. In [9], optimal power and bandwidth allocation algorithm is solved to maximize the overall

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system capacity for FDMA-based DF multihop links. In [10]–[12], power allocation is solved to minimize the outage probability subject to total power constraint. In [10], upper bound of outage is derived based on equal power with channel selection, *i.e.*, equally allocate power to the eligible relay channels with channel gain above a threshold. In [11], iteration is required to obtain the optimal power allocation for DF system with diversity. While for AF with diversity, the authors suggest to use the same solution as that of DF due to difficulty/complexity in formulation. In [12], outage probability is analyzed in high SNR regime and optimal power allocation is solved to minimize the outage probability of mutual information. In [13], the optimal power allocation algorithm is derived based on the general closed-form symbol error rate bound derived in [14] for AF cooperation systems. In [15], the authors present theoretic analysis for a class of multi-node cooperative protocols and provide optimal power allocation for the multi-node relay problem based on the obtained approximate expression for the symbol error rate (SER). The optimal power allocation reported in the literature is mainly based on the approximated SER or outage performance bounds. The actual optimum power allocation for cooperative diversity in fading channels with knowledge of channel statistics is still an open problem. In recent years, the problem of partner selection and power allocation is investigated jointly. In [16], optimal power allocation is solved to minimize the total energy consumption satisfying the BER target of the cooperating pair. In [17], the achievable information capacity is analyzed to obtain optimal power allocation and partner selection with the total power constraints for different cooperation/relaying schemes. In [18], effective user relaying algorithms to jointly optimize relay node selection and power allocation is investigated for wireless relay networks. The problem is formulated to maximize network capacity in terms of achievable mutual information.

In this paper, we apply the cooperative transmission framework proposed in [2], and divide the transmission into two phases. Phase I is for user self-information transmission and Phase II is for cooperative-information transmission. The suggested efficient power allocation scheme for AF is obtained from theoretical BER bound analysis, work region analysis, and simulations. One difference between our work and most of the previous work on AF cooperation (for example [7], [8], [13], [14]) is the noise component treatment method. Most of the previous research has been essentially focused on single-user transmission. Therefore, in phase II, noise components from the partner and the target user are isolated at the terminal. For a multi-user environment, as for FDMA (frequency division multiple access) and CDMA (code division multiple access), due to wideband nature of the background noise, this kind of isolation is difficult, if not impossible. For TDMA (time division multiple access), noise isolation can be achieved with the price that the number of time slots required is doubled in order to transmit self-information and cooperative-information for the target user and the partner, leading to extra requirements in radio resource. In this paper, two spreading codes are applied for two mobile users to create orthogonal channels, thus no extra radio resource is required to implement cooperative transmission. We explicitly consider the noise enhancement introduced by the desired user in cooperation when forwarding the partner's information to the destination.

Power allocation problem is first investigated by assuming that both users apply the same cooperative ratio. We analyze the optimal cooperative ratio in the sense to minimize BER based on the BER lower bound. The obtained optimal ratio is only related to the channel gain of the inter-user channel and the relay channel. A natural question is that this ratio will be different from each user's point of view. As a second step of the research, we investigate the two dimensional situation, *i.e.*, each user is allowed to select different cooperative ratio, subject to the constraint that the total power keeps fixed. Through work region and simulation results analysis, we conclude that it is efficient to apply the same ratio to both users. This common ratio is selected based on the bottleneck user, *i.e.*, the user with weaker channel gain to the destination.

In the remaining of the paper, system model is described in Section II. Analysis and numerical results for one-dimensional power allocation problem are presented in Section III. Two-dimensional cooperative ratio assignment are discussed in Section IV. Conclusions are summarized in Section V.

II. System Model

A cooperative transmission scheme, which consists of two phases in a wireless network is considered. In phase I, each mobile user sends its data to the destination, and the data is also received by its partner. This phase is referred to as "self-information transmission phase". In phase II, each mobile user helps its partner to forward the data received in phase I to the destination. Therefore, this phase is also called "cooperative-information transmission phase" or "relaying phase". The orthogonality of the different user information can be achieved by using multiple access technologies, for example, TDMA, FDMA or CDMA, *etc*. In this work, CDMA method as proposed in [2] is applied. Data modulation applies Binary Phase Shift Keying (BPSK) scheme. The generic model is shown in Fig.1. The transmit power levels for each user in each phase are listed in Table 1. The powers, P_1^t and P_2^t, are the allocated powers to transmit user's self-information directly to the destination; while P_1^c and P_2^c are the powers for cooperative transmission in phase II. Assuming that the total power is fixed for each user, *i.e.*, $P_i^t + P_i^c = P, i = [1, 2]$, where P is the total power for each user.

In phase I, the transmitted signals for mobile user 1

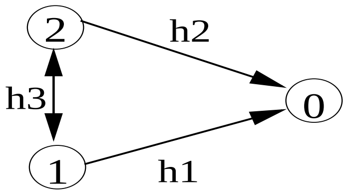

Figure 1. Model for user cooperation.

TABLE I.
TRANSMIT POWERS IN PHASE I AND PHASE II.

	Phase I	Phase II
User 1	P_1^t	P_1^c
User 2	P_2^t	P_2^c

and 2 are respectively given as:

$$X_{11} = \sqrt{P_1^t}\, b_1\, \mathbf{c}_1 \quad (1)$$

$$X_{21} = \sqrt{P_2^t}\, b_2\, \mathbf{c}_2 \quad (2)$$

where the first and the second subscripts of the signal distinguish the indexes for the nodes and the phases respectively; b_1 denotes user 1's information bit and $\mathbf{c}_1$ denotes user 1's spreading sequence. The spreading sequence is selected to be unit energy and be orthogonal to each other, *i.e.*, $\mathbf{c}_i^T\mathbf{c}_j = \delta(i-j), i,j = [1,2]$, where $\mathbf{c}_i^T$ denotes the transpose of column vector $\mathbf{c}_i$.

We assume that all receivers have channel state information and use coherent detection. Therefore, in the analysis, we only need to consider the fading coefficient magnitudes. The received signals at mobile user 1, mobile user 2 and the destination in phase I are given as

$$Y_{11} = \sqrt{h_3}X_{21} + n_1 = \sqrt{h_3P_2^t}b_2\mathbf{c}_2 + n_1 \quad (3)$$

$$Y_{21} = \sqrt{h_3}X_{11} + n_2 = \sqrt{h_3P_1^t}b_1\mathbf{c}_1 + n_2 \quad (4)$$

$$\begin{aligned} Y_{01} &= \sqrt{h_1}X_{11} + \sqrt{h_2}X_{21} + n_0 \\ &= \sqrt{h_1P_1^t}b_1\mathbf{c}_1 + \sqrt{h_2P_2^t}b_2\mathbf{c}_2 + n_0 \\ &= \alpha_1 b_1\mathbf{c}_1 + \alpha_2 b_2\mathbf{c}_2 + n_0 \end{aligned} \quad (5)$$

where $\alpha_1 = \sqrt{h_1P_1^t}$ and $\alpha_2 = \sqrt{h_2P_2^t}$. The terms, h_1, h_2, h_3, are the fading power gain for the channels of mobile user 1 to the destination, mobile user 2 to the destination, and the inter-user channel, respectively, where a reciprocal inter-user channel fading power gain has been assumed. This assumption makes the two cooperating users see identical fading coefficients between them, as in [5]. Each h_i follows exponential distribution with mean $H_i, (i = 1,2,3)$. Therefore, the amplitude gain $\sqrt{h_i}, (i = 1,2,3)$ follows Rayleigh distribution. We further assume a flat fading channel that in one cooperation interval, the fading gain doesn't change and is independent with each other and independent for each cooperation interval. Additive white Gaussian noise terms are represented in $n_i, i = 0,1,2$, seen at the destination, user 1 and user 2, respectively. They are modeled as zero-mean, complex Gaussian random variables with variance σ_N^2.

The main difference for the two methods: *Amplify-and-Forward* and *Decode-and-Forward* is depicted in phase II. In phase II, AF algorithm doesn't attempt to decode the partner's information, but simply forward what has been received in phase I to the destination. Therefore, the transmitted signals are:

$$X_{12} = \frac{\sqrt{P_1^c}}{\sqrt{h_3P_2^t + \sigma_N^2}}Y_{11} \quad (6)$$

$$X_{22} = \frac{\sqrt{P_2^c}}{\sqrt{h_3P_1^t + \sigma_N^2}}Y_{21} \quad (7)$$

The denominator term ensures the transmit power at phase II is P_1^c and P_2^c respectively for user 1 and user 2. At the destination, a combined signal is received as

$$\begin{aligned} Y_{02} &= \sqrt{h_1}X_{12} + \sqrt{h_2}X_{22} + n_0 \\ &= \frac{\sqrt{h_1P_1^c}}{\sqrt{h_3P_2^t + \sigma_N^2}}\left(\sqrt{h_3P_2^t}b_2\mathbf{c}_2 + n_1\right) \\ &\quad + \frac{\sqrt{h_2P_2^c}}{\sqrt{h_3P_1^t + \sigma_N^2}}\left(\sqrt{h_3P_1^t}b_1\mathbf{c}_1 + n_2\right) + n_0 \\ &= \alpha_3 b_1\mathbf{c}_1 + \alpha_4 b_2\mathbf{c}_2 + \alpha_5 n_1 + \alpha_6 n_2 + n_0 \end{aligned} \quad (8)$$

where

$$\alpha_3 = \sqrt{h_2h_3P_1^tP_2^c}/\sqrt{h_3P_1^t + \sigma_N^2} \quad (9)$$

$$\alpha_4 = \sqrt{h_1h_3P_1^cP_2^t}/\sqrt{h_3P_2^t + \sigma_N^2} \quad (10)$$

$$\alpha_5 = \sqrt{h_1P_1^c}/\sqrt{h_3P_2^t + \sigma_N^2} \quad (11)$$

$$\alpha_6 = \sqrt{h_2P_2^c}/\sqrt{h_3P_1^t + \sigma_N^2} \quad (12)$$

It can be observed that in phase II at the destination, noise component is enhanced and accumulated by using AF protocol.

At the receiver, in order to recover the transmitted symbols, despreading, maximal ratio combining and decision procedures are carried out and outlined in next Section.

III. ONE DIMENSION POWER ALLOCATION

We define a parameter, β $(0 \leq \beta \leq 1)$, as the cooperative ratio, which implies the ratio of the power used for cooperative transmission over the total power. In this section, we will present our earlier result by using the same β for both user 1 and user 2 [19]. Let P denote the total power for each user. The corresponding power allocation is listed in Table II. The problem is to find the desirable value of β .

TABLE II.
ONE DIMENSION POWER ALLOCATION PROBLEM.

	Phase I	Phase II
User 1	$P_1^t = (1-\beta)P$	$P_1^c = \beta P$
User 2	$P_2^t = (1-\beta)P$	$P_2^c = \beta P$

From user 1's point of view, it is expected to have P_1^t and P_2^c as large as possible; while for user 2, it is expected to have P_2^t and P_1^c as large as possible.

A. BER Lower Bound Analysis

Considering the retrieval of user 1's information. The first stage at the receiver is de-spreading. Multiplying the received composite signal by the spreading sequence for user 1, $\mathbf{c}_1$, leads to

$$\mathbf{Y} = \mathbf{c}_1^T \begin{bmatrix} Y_{01} \\ Y_{02} \end{bmatrix} \quad (13)$$

$$= \begin{bmatrix} \alpha_1 b_1 + n_0' \\ \alpha_3 b_1 + \alpha_5 n_1' + \alpha_6 n_2' + n_0' \end{bmatrix} \quad (14)$$

The noise component $n_i', (i = 0, 1, 2)$ has zero mean and variance of σ_N^2 since the spreading sequence has unit energy. By applying MRC (maximal ratio combining), the signal-to-noise ratio (SNR) after combining is the sum of the SNR from each phase,

$$\Gamma_M = \left(\alpha_1^2 + \frac{\alpha_3^2}{1 + \alpha_5^2 + \alpha_6^2}\right) \frac{1}{\sigma_N^2}. \quad (15)$$

The corresponding instantaneous bit error rate (BER) for BPSK modulation is given as

$$P_b = \mathbf{E}_{\{h_i\}_{i=1}^3} \left\{ Q\left(\sqrt{2\Gamma_M}\right) \right\} \quad (16)$$

where $\mathbf{E}_{h_i}\{\cdot\}$ denotes the expectation with respect to the random fading channel gains h_i; $Q(x)$ is complementary cumulative distribution function of standard Gaussian distribution. An efficient power allocation strategy is to find the value of β to obtain a low BER value for both users. Unfortunately, we cannot have a closed-form solution for the BER expression given in (16). Motivated by the work in [14], the SNR after MRC combining in (15) can also be written as

$$\Gamma_M = \gamma_1 + \frac{\gamma_2 \gamma_3}{1 + \gamma_2 + \gamma_3 + \gamma_4} \quad (17)$$

where $\gamma_1 = h_1 P_1^t / \sigma_N^2 = h_1(1 - \beta)P/\sigma_N^2$, $\gamma_2 = h_2 P_2^c / \sigma_N^2$, $\gamma_3 = h_3 P_1^t / \sigma_N^2$, and $\gamma_4 = h_1 P_1^c / \sigma_N^2$. Let $\gamma = P/\sigma_N^2$, and it is referred as SNR without fading in sequel. Compared with [14], (17) has an extra term γ_4 at the denominator of the second term. The term γ_4 reflects the noise seen at the terminal and collected at user 1 when user 1 forwards user 2's signal in Phase II. In addition, the closed-form result in [14] has ignored the constant 1 in the denominator for large SNR. Therefore, the BER result derived in [14] serves as a BER lower bound, which is given as

$$P_b \geq \frac{3}{4k^2} \left(\frac{1}{\bar{\gamma}_2} + \frac{1}{\bar{\gamma}_3} \right) \frac{1}{\bar{\gamma}_1} \quad (18)$$

where k is specified by the modulation method and for BPSK, $k = 2$. $\bar{\gamma}_i$ is the average branch SNR, obtained by replacing h_i with H_i in γ_i. Taking the first derivative of P_b lower bound with respect to β and equating to zero, we can solve the optimal β value to minimize the lower bound of P_b and it is given as [19]

$$\beta_{LB}^* = \frac{3H_3 - \sqrt{H_3^2 + 8H_2H_3}}{4(H_3 - H_2)} \quad (19)$$

which is illustrated in Fig. 2 as a function of H_2 when H_3 is fixed.

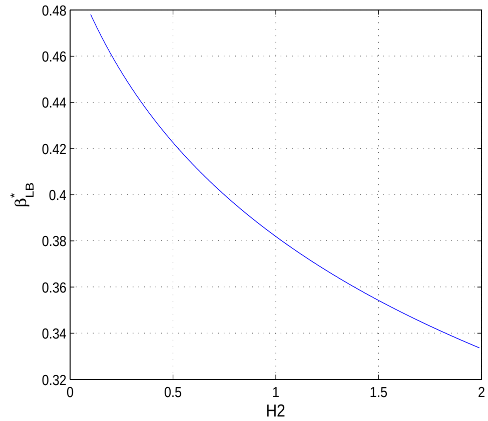

Figure 2. Solution for β_{LB}^* as a function of H_2, H_3 is fixed at 2.

The value for β_{LB}^* has a few interesting features. Firstly, it is not related to the link gain of the direct link (H_1), but only related to the inter-user link (H_3) and the relayed link (H_2). Secondly, it is a monotonic decreasing function with respect to H_2. Thirdly when $H_3 > H_2$, which is usually a reasonable assumption, the range of β_{LB}^* is [1/3, 1/2]. Next, the proof of the third feature is presented below.

Proof. First we prove that (19) is greater than zero. If $H_3 > H_2$, multiplying this condition by $8H_3$ to both sides and adding H_3^2, we have $9H_3^2 > H_3^2 + 8H_2H_3$. Square root both sides yields $3H_3 - \sqrt{H_3^2 + 8H_2H_3} > 0$. Both the numerator and denominator of (19) are greater than zero, therefore the ratio is greater than zero. If $H_3 < H_2$, similarly we can show that both the numerator and denominator are less than zero and therefore, the ratio is greater than zero.

Next we show if $H_3 > H_2$, then $\beta_{LB}^* < 0.5$; otherwise, if $H_2 > H_3$, then $\beta_{LB}^* > 0.5$.

If $H_2 < H_3$, multiplying both sides of this condition by $4H_2$, and adding $H_3^2 + 4H_2H_3$, we have

$$H_3^2 + 4H_2H_3 + 4H_2^2 < H_3^2 + 8H_2H_3.$$

Taking square root to both sides leads to

$$2H_2 + H_3 < \sqrt{H_3^2 + 8H_2H_3}.$$

Adding $2H_3$ to both sides, and manipulating, we have

$$3H_3 - \sqrt{H_3^2 + 8H_2H_3} < 0.5 \times 4(H_3 - H_2),$$

which readily shows (19) is less than 0.5.

If $H_2 > H_3$, similarly we can prove (19) is greater than 0.5. Furthermore, by applying L'Hôpital law, the limiting value of $\beta_{LB}^* = 1/3$ when H_2 approaches H_3. This limit value can be observed from Fig. 2.

Since this optimal value is obtained when ignoring some noise component, it is expected that this value will be an upper bound of the solution from simulation. Furthermore, in this section, β_{LB}^* is solved to minimize the BER of user 1. Except symmetric situation, β_{LB}^*

calculated from user 2's point of view will be different. This motivates us to investigate two dimensional optimal power allocation problem in Section IV.

B. Simulation Analysis

In the simulation, we generate 5,000,000 fading realizations and capture the average performance. From these simulation results, the numerical solution to minimize BER is denoted as β^*_{sim}. More insights of the effects of proper power allocation can be gained through simulation analysis.

Fig. 3 shows average BER for user 1 and user 2 as a function of β. From the top to the bottom, the solid curves represent the BER results for user 1 when $P/\sigma_N^2 = 10, 12, 14, 16$ dB respectively and the dashed curves represent the corresponding BER for user 2. In this simulation, the mean fading gains are $H_1 = 0.5$, $H_2 = 1$, $H_3 = 2$. It is clear that selecting different value of β leads to different value of BER. By introducing cooperative transmission ($\beta > 0$), the BER curves for user 1 and user 2 exhibit a certain level of parallelism.

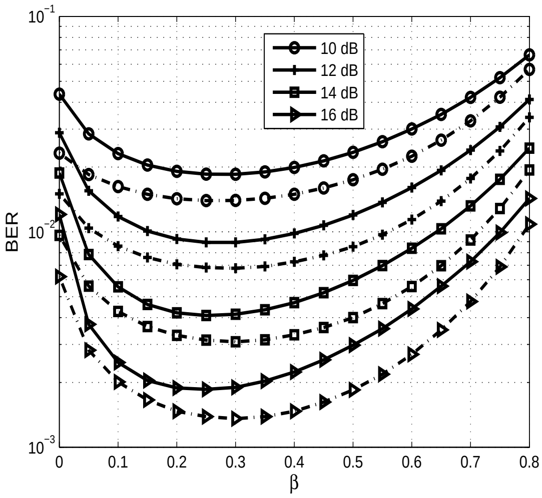

Figure 3. BER for user 1 and user 2 ($P/\sigma_N^2 = [10, 12, 14, 16]$ dB, $H_1 = 0.5$, $H_2 = 1$, $H_3 = 2$).

Fig. 4 shows average BER vs. β with different average inter-user channel fading gain. From the top to the bottom, H_3 changes from 2, 3, 4 to 5. It shows that with the increasing of H_3, BER decreases, and the preferred β^*_{sim} has a trend to increase, *i.e.*, to allocate more power for cooperative-information transmission. When H_3 changes from 2 to 5, the desired β^*_{sim} vales are simulated as 0.25, 0.3, 0.35, 0.35, while the closed form β^*_{LB} values are calculated as 0.3820, 0.4069, 0.4226, 0.4336 respectively.

Table III lists the BER values for non-cooperative transmission ($\beta = 0$) and the corresponding minimal BER values with cooperative transmission and the desired β^*_{sim} when SNR values changing from 10 dB to 16 dB. The first two rows compare our simulation results when no cooperation ($\beta = 0$) with the theoretical results of the BER over flat Rayleigh fading channel. It can be observed that our simulation results match very well with the theoretical results. The third and the fourth rows list the simulated minimal BER values with user cooperation and the corresponding percentage reduction of the BER value over that of non-cooperation. It shows that with the increasing of the SNR values, the BER improvement is more significant by using cooperation. User 1 obtained more benefit by introducing cooperation since user 1's direct channel (H_1) is the weakest channel. Therefore, the diversity gain obtained from the better relay channel improves the BER performance greatly. For user 2, even the average channel quality for cooperative transmission is worse than that of direct channel, user 2 also benefits from the diversity gain by using statistically independent fading channels. The last row lists the cooperative ratios achieving the minimal simulated BER values. It shows that the desirable value of β^*_{sim} is between 0.25 to 0.30. From Fig. 3, it can be observed that the BER value isn't sensitive to the minor changing of the β^*_{sim}. Therefore, $\beta^*_{sim} = 0.30$ will be used as the desirable cooperative ratio in this range of SNR values. From equation (19), the corresponding optimal value is calculated as $\beta^*_{LB} = 0.3820$. As analyzed before, since some noise components were ignored at the denominator of the combined SNR expression, the β^*_{LB} is an upper bound for the simulation results, where noise terms shown in (5) and (8) are all included.

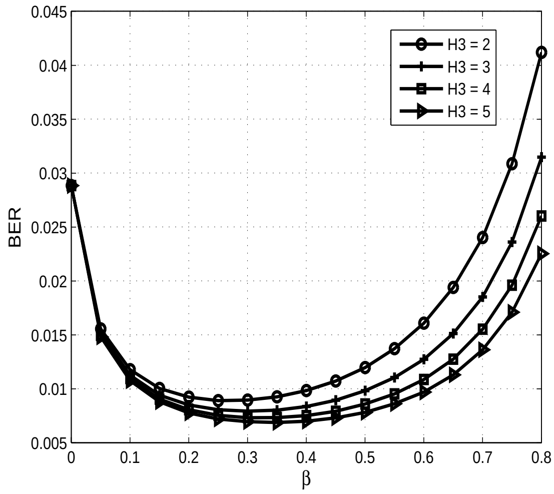

Figure 4. BER for user 1 ($P/\sigma_N^2 = 12$ dB, $H_1 = 0.5$, $H_2 = 1$, $H_3 = [2, 3, 4, 5]$).

Fig. 5 shows the BER performance as a function of SNR. The leftmost curve is the result for AWGN channel. The circle-marked and square-marked solid curves are the results for Rayleigh fading channel with second-order full diversity and no diversity respectively. These two curves set lower and upper bounds for any cooperative transmission scheme. The diamond-marked and right-triangle-marked dashed curves are the simulated BER results for AF transmission with cooperative ratio 0.3 and 0.5 re-

TABLE III.
BER VALUES FOR NON-COOPERATION TRANSMISSION, NUMERICAL MINIMAL BER FOR COOPERATIVE TRANSMISSION, PERCENTAGE GAIN AND β^*_{sim}, $H_1 = 0.5$, $H_2 = 1$, $H_3 = 2$.

	User 1				User 2			
SNR (dB)	10	12	14	16	10	12	14	16
Non-cooperation	0.0436	0.0289	0.0188	0.0121	0.0232	0.0150	0.0096	0.0062
Theoretical value	0.0436	0.0288	0.0188	0.0121	0.0233	0.0151	0.0097	0.0062
Min. BER using cooperation	0.0185	0.0089	0.0041	0.0019	0.0140	0.0068	0.0031	0.0014
BER reduc. %	57.48 %	69.02%	78.24%	84.64%	39.98%	54.97%	68.03%	77.98%
β^*_{sim}	0.3	0.25	0.25	0.25	0.25	0.3	0.3	0.3

spectively. The solid curve without any marker is the BER lower bound calculated from equation (18) with $\beta^*_{LB} = 0.3820$. As expected, the BER curves for cooperative AF systems fall between the curves of the second-order full diversity and no diversity over Rayleigh fading channels. It can be observed that $\beta = 0.3$ is a better choice over the equal power allocation $\beta = 0.5$, which makes the curve down closer to the second-order full diversity curve and achieves a 2 dB SNR gain at BER equating 10^{-3}. The gap between the dashed curves (cooperative AF transmission) and the second-order full diversity curve comes from the fact that in an amplify-and-forward mechanism, the noise component collected at phase I in each user is also amplified and forwarded together with the signal, leading to a performance degradation compared with the second-order full diversity scheme. It also shows that the validity of the BER closed-form lower bound approximation in (18) [14] requires the SNR values to be at least greater than 10 dB.

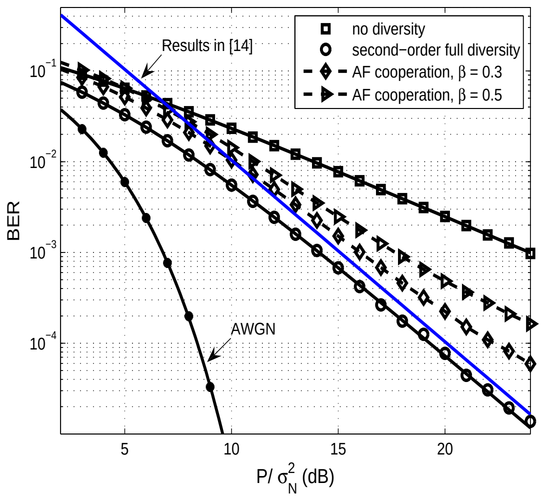

Figure 5. BER as a function of SNR for different systems ($H_1 = H_2 = 1$, $H_3 = 2$).

IV. TWO DIMENSIONAL POWER ALLOCATION

In this section, we assume that users 1 and 2 can select different cooperation ratio, denoted as β_1 and β_2, respectively, subject to the same total power restriction. Power allocation problem is thus shown in Table IV. In this section, we will analyze the feasible work region for both users to find the desirable power allocation strategies.

TABLE IV.
TWO DIMENSION POWER ALLOCATION PROBLEM.

	Phase I	Phase II
User 1	$P_1^t = (1-\beta_1)P$	$P_1^c = \beta_1 P$
User 2	$P_2^t = (1-\beta_2)P$	$P_2^c = \beta_2 P$

Let P_{b1} and P_{b2} denote the BER for user 1 and 2 respectively. From (18), we have

$$P_{b1} \geq \frac{3}{16}\left[\left(\frac{P_1^t}{\sigma_N^2}H_3\right)^{-1} + \left(\frac{P_2^c}{\sigma_N^2}H_2\right)^{-1}\right]\cdot\left(\frac{P_1^t}{\sigma_N^2}H_1\right)^{-1}$$

$$P_{b2} \geq \frac{3}{16}\left[\left(\frac{P_2^t}{\sigma_N^2}H_3\right)^{-1} + \left(\frac{P_1^c}{\sigma_N^2}H_1\right)^{-1}\right]\cdot\left(\frac{P_2^t}{\sigma_N^2}H_2\right)^{-1}$$

Using the power levels in Table IV and $\gamma = P/\sigma_N^2$, we can have

$$P_{b1} \geq \frac{3}{16\gamma^2}\left[\frac{1}{(1-\beta_1)H_3} + \frac{1}{\beta_2 H_2}\right]\frac{1}{(1-\beta_1)H_1} \quad (20)$$

$$P_{b2} \geq \frac{3}{16\gamma^2}\left[\frac{1}{(1-\beta_2)H_3} + \frac{1}{\beta_1 H_1}\right]\frac{1}{(1-\beta_2)H_2} \quad (21)$$

Then for user 1, from (20), we can obtain the restriction for β_2 as

$$\beta_2 \geq \frac{1}{H_2}\left[\frac{16\gamma^2 P_{b1}}{3}(1-\beta_1)H_1 - \frac{1}{(1-\beta_1)H_3}\right]^{-1} \quad (22)$$

Similarly, for user 2, we have following restriction to β_1 derived from (21):

$$\beta_1 \geq \frac{1}{H_1}\left[\frac{16\gamma^2 P_{b2}}{3}(1-\beta_2)H_2 - \frac{1}{(1-\beta_2)H_3}\right]^{-1} \quad (23)$$

In the remaining of this section, we first consider symmetrical channels, *i.e.*, $H_1 = H_2$, then generalize to unsymmetrical channels, *i.e.*, $H_1 \neq H_2$.

A. Symmetrical Channels

For symmetrical channel, $H_1 = H_2 = H$, $P_{b1} = P_{b2} = P_b$. We further assume good inter-user channel, *i.e.*, $H_3 > H_1$. The optimal value for β should be less than 0.5 from our analysis in previous Section.

Fig. 6 plots the work region of the users as the shadowed area illustrated in Fig. 6(a). The plus-marker marked curve and dashed curve are obtained from (22)

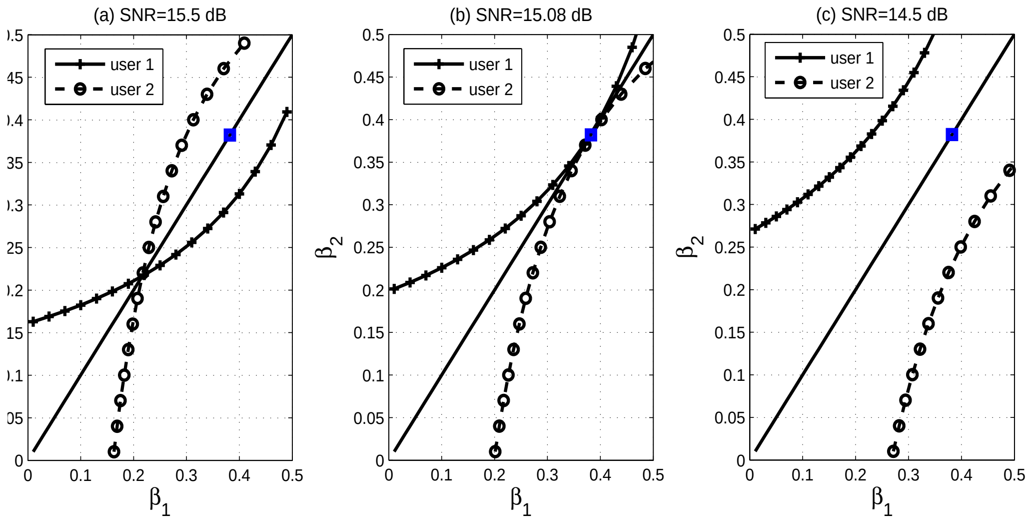

Figure 6. Work region for user 1 and user 2. plus-marker curve: user 1; dashed-curve: user 2; straight line: $\beta_1 = \beta_2$. From left to right, SNR values: [15.5 15.08 14.5]dB; $H_1 = H_2 = 1, H_3 = 2$, $P_b = 1e-3$; square: solution from (19).

and (23) respectively with equal sign instead of inequality. For user 1, the region above plus-marker curve is the work region; while for user 2, the work region is the area below the dashed curve. Therefore, the shadowed area is the region to meet the requirements for both users.

When SNR is high, the work regions for users have overlapped area (shadowed area in Fig. 6(a)). The cooperative ratios, β_1 and β_2, can be selected any values in this area to meet the BER target of both users. With the decreasing of the SNR, the overlap area reduces. In Fig. 6(b), the two curves have only one point to overlap. In Fig. 6(c), the two curves are further separated. In this case, no matter how we select the values of β_1 and β_2, we cannot meet the BER requirement for both users since the provided SNR is not high enough. Therefore, when SNR changes from high to low, the selection of the acceptable cooperative ratios changes from loose to stringent to no solution.

For user 1, it is preferable to select working point further above the plus-marked curve. User 2 prefers the working point further down the dashed curve. When we consider user fairness, the work point should be selected with equal distance to the two curves. Studying Fig. 6(a), we find the two BER restriction curves are symmetric with respect to the line $\beta_1 = \beta_2$. Therefore, the working point should be on this line. Following is a proof of this symmetrical relation.

Substituting the conditions $H_1 = H_2 = H$, $P_{b1} = P_{b2} = P_b$ in the β restrictions (22) and (23) and using equal sign, we have

$$\beta_2 = \frac{1}{H}\left[K(1-\beta_1) - \frac{1}{(1-\beta_1)H_3}\right]^{-1} \quad (24)$$

$$\beta_1 = \frac{1}{H}\left[K(1-\beta_2) - \frac{1}{(1-\beta_2)H_3}\right]^{-1} \quad (25)$$

where $K = 16\gamma^2 P_b H/3$. Let $\beta_2 = f(\beta_1)$ denote the function defined by (24). Point A is on the curve defined by (24) with coordinates $(\beta_0, f(\beta_0))$, with β_0 being the horizontal coordinate of an arbitrary point on the curve. Point B, whose coordinates are obtained by swapping the coordinates of point A as $(f(\beta_0), \beta_0)$, is on the curve defined by (25). The middle point of the line AB should have coordinates $((\beta_0 + f(\beta_0)/2, \beta_0 + f(\beta_0)/2))$, which is on the line $\beta_1 = \beta_2$. As a result, the two curves are symmetric with respect to the line $\beta_1 = \beta_2$.

Under symmetric assumption, the one dimensional β_{LB}^* presented in Section III is reasonable. This solution is marked by the squares in Fig. 6. We need to find the working point which is on the line of $\beta_2 = \beta_1$, and which is the tangent point of the two curves. Let this point is denoted as β_W^*, where the subscript W denotes work region. The corresponding β_W^* value is optimal in the sense that it is the point to satisfy the BER target for both users under the most stringent SNR condition. The following equation set should be met:

$$\frac{d\beta_2}{d\beta_1}\Big|_{\beta_1=\beta_W^*} = 1 \quad (26)$$

and

$$\beta_W^* = \frac{1}{H}\left[K(1-\beta_W^*) - \frac{1}{(1-\beta_W^*)H_3}\right]^{-1} \quad (27)$$

Equation (26) implies the tangent point of two curves is on the line $\beta_2 = \beta_1$. Equation (27) implies that (β_W^*, β_W^*) is on the curve defined by (24) or (25). Using (24), (26) can be written as

$$K + \frac{1}{H_3(1-\beta_W^*)^2} = H\left[K(1-\beta_W^*) - \frac{1}{H_3(1-\beta_W^*)}\right]^2 \quad (28)$$

Solving equation set (27)-(28), we can obtain β_W^* and K. The parameter K defines the most stringent working condition, *i.e.*, using as low SNR as possible to meet the BER target for both users. The optimal cooperation ratio, β_W^*, is solved to be the point satisfying both users' BER target under most stringent SNR condition. When SNR is higher, β_W^* is still the preferred working point considering fairness of the resource allocation between the users.

At this stage, the closed form solution for (27)-(28) is not available. Nevertheless we can use numerical method or the optimal ratio solved from (19) in Section III as β_W^*. Fig. 7 shows the numerical solution of β_W^*, where the square marker is the solution from (19). The three curves denote the decreasing of SNR from the bottom to the top. The middle curve is the solution of equation set (27)-(28).

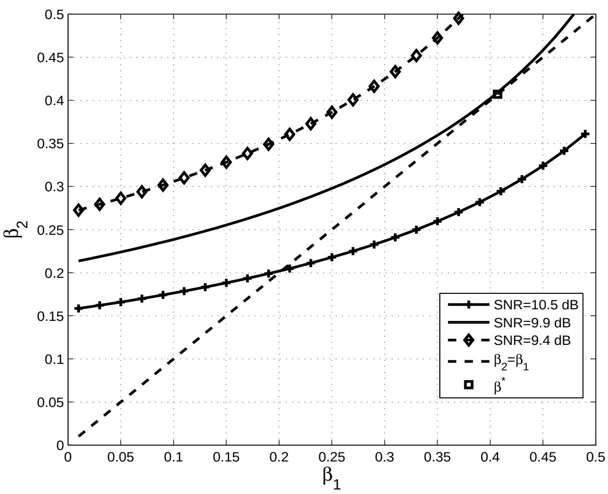

Figure 7. Solution for β^*: $H_1 = H_2 = 1, H_3 = 2$, $P_b = 1e-2$.

B. Unsymmetrical Channel

For unsymmetrical channel, $H_1 \neq H_2$. Without loss of generality, we assume that $H_1 < H_2$. Fig. 8 shows the corresponding work region for the users. The three subfigures denote SNR changing from 18, 17.5, to 17 dB from left to right, respectively. Similar to symmetric channel, when SNR is high, the work region is the shadowed area in Fig. 8(a). With the decreasing of the SNR, the work region shrinks. The desirable cooperative ratio pair selection can be determined from Fig. 8(b) when the two curves are tangent with each other. The desirable point is optimal in the sense that with this pair of β values, the target BER requirement can be met for both users with this most stringent condition (SNR value, channel gains, BER target).

Numerical solution is investigated to shed light on the target problem. From Fig. 8, we can obtain the numerical solution as $[\beta_{1W}^*, \beta_{2W}^*] = [0.38, 0.43]$. Compared with the solution of using (19), which gives solution $\beta_{LB}^* = 0.3820$ for user 1 and $\beta_{LB}^* = 0.4461$ for user 2, shown by the squares in Fig. 8, we can see that the two-dimensional cooperative ratio is close to that of the combination of the one-dimensional solution.

In order to gain more insights for unsymmetrical channel situation, we conducted simulation of BER vs. SNR as shown in Figs. 9 and 10, where $H_1 = 0.3$, $H_2 = 1$, and $H_3 = 2$. Obviously, the direct link of user 1 is the bottleneck link. Without cooperation, the BER curves for user 1 and user 2 are shown as solid-star curve and dashed-star curve respectively. A 5 dB SNR gap between these two curves can be observed due to better direct link of user 2. With application of the cooperation, this difference is reduced. We simulated three cases: (a) both users apply the same cooperative ratio ($\beta = 0.38$), which is derived from user 1's point (bottleneck user); (b) the same cooperative ratio ($\beta = 0.43$), which is derived from user 2's point; (c) two-dimensional cooperative ratios ($\beta_1 = 0.38, \beta_2 = 0.43$), each is derived based on each user's own point of view. We can have following interesting observations: (i) for three situations, user 1's BER curves form a cluster above the cluster of user 2's BER curves; (ii) both clusters of BER curves are lower than the case without cooperation. Even for user 2, where the relayed channel from user 1 is much worse than its direct channel, diversity gain still can be achieved; (iii) inside both clusters, we can observe that $\beta = 0.38$ (derived from the weak user) leads to a lower BER performance compared to $\beta = 0.43$. This is reasonable. $\beta = 0.38$ is derived in favor of user 1. For user 2, who is experiencing a better direct link, benefits from the lower value of β, which means lower power used for relay transmission; (iv) when two dimensional ratios are applied, the BER performance has very minor improvement for user 1. For user 2, the BER is almost identical with the case of one dimensional β at 0.43. As a result, it is suggested that one dimensional β is sufficient in terms of BER performance. This common cooperative ratio should be derived from the user who has a weaker direct link.

Furthermore, from Figs. 6 and 8, we can also observe that the work region is sensitive to SNR. The loss in term of work region from the selection of the one dimensional cooperative ratio can be well justified by the BER performance, complexity reduction, and fairness.

Fig. 10 shows the BER for both users under similar fading conditions as in Fig. 9 when both users are allocated the same cooperative ratio. Similar to Fig. 9, the two star-marker curves are the BER for two users without cooperation. The middle cluster is the BER curves for user 1 when the cooperative ratio changes from 0.38, 0.30, to 0.25, from the top to the bottom. The first value is based on BER lower bound and the later two values are based on simulation. We can see that the BER performance for $\beta = 0.30$ and $\beta = 0.25$ are very close in low-to-medium SNR values and is better than that when $\beta = 0.38$. With high SNR values, $\beta = 0.25$ outperforms other two selected ratios. Meanwhile, for user 2, the BER curves form the lower cluster. With these cooperative ratio values, the BER for user 2 has identical performance, which on the other side, verifies our analysis that the selection of the cooperative ratio should focus on the weaker link.

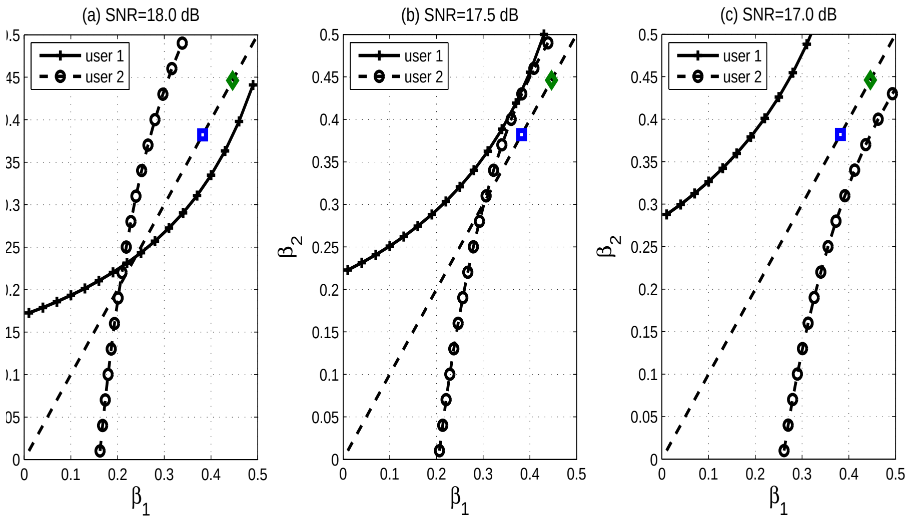

Figure 8. Work region for user 1 and user 2. Plus-marker curve: user 1; circle-marker curve: user 2; straight line: $\beta_1 = \beta_2$. From left to right, γ values: [18.0 17.5 17.0] dB; $H_1 = 0.3, H_2 = 1, H_3 = 2$, $P_b = 1e-3$; squares: solution from (19) for users 1 and 2 respectively.

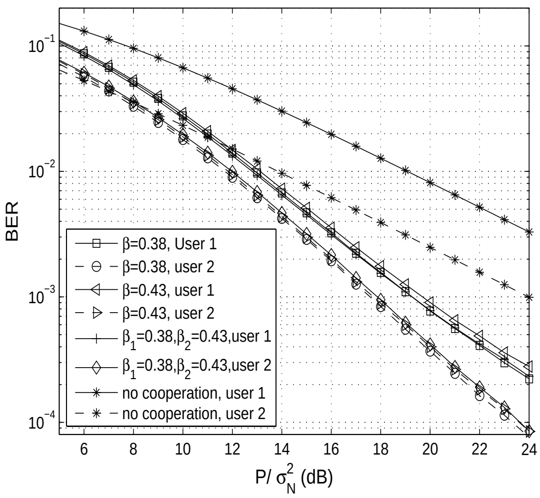

Figure 9. BER vs. SNR with one and two dimensional cooperative ratios, $H_1 = 0.3, H_2 = 1, H_3 = 2$.

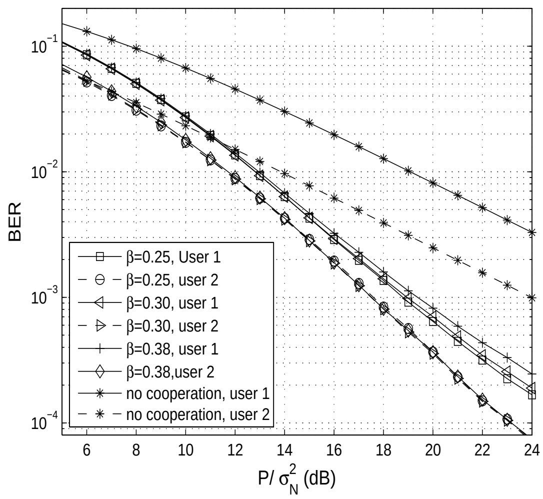

Figure 10. BER vs. SNR with the same cooperative ratio for both users, $H_1 = 0.3, H_2 = 1, H_3 = 2$.

V. CONCLUSIONS

In this paper, power allocation strategies for amplify-and-forward (AF) cooperative transmission systems are investigated. By exploiting spacial diversity in wireless channels, BER performance can be significantly improved by properly allocating powers for self-information transmission and cooperative-information transmission under the constraint of fixed total transmit power of each user.

We investigated cooperative ratio, which is defined as the ratio of the cooperative-information transmission power over total power. For one dimensional case, a closed-form expression, denoted as β_{LB}^*, is obtained to minimize the BER lower bound. This value is a function of only the average fading gain of inter-user channel and the relaying channel. Under the assumption that inter-user channel has a higher average channel gain over the relay-to-destination channel, β_{LB}^* is in the range of [1/3, 1/2]. Simulation results show that the optimal cooperative ratio is also related to SNR, besides channel fading gains. The value of β_{LB}^* is actually an upper bound of the numerical solution from the simulation analysis.

We further studied two dimensional cooperative ratio allocation problem using work region analysis. Our results show that it is sufficient to allocate the same ratio to each user. This common power allocation is based on the optimal solution of the user who has a weaker channel fading gain to the destination, *i.e.*, the bottleneck user. The BER of the user with better channel condition is less sensitive to the minor change of the cooperative ratio.

Even though we only discussed AF protocol, similar approaches can be applied to investigate DF protocol. Our future work includes further investigation the closed-form expressions or tight bounds of the BER under realistic cooperative transmission, tradeoff between performance gains and overhead of user cooperation, and adaptive power allocation strategies. In addition, in a wireless network, where there exists a large number of parallel transmissions if using CDMA or FDMA protocols, AWGN accumulation and propagation may become prohibitive for AF cooperative protocol. Our future work also includes analyzing this factor and to provide design guidelines for when to cooperate and when not to cooperate.

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