Optimizing Time and Power Allocation for Cooperation Diversity in a Decode-and-Forward Three-Node Relay Channel

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***Abstract*— Providing space diversity at mobile devices such as handsets, personal digital assistants (PDA), etc is costly and problematic, due to their strict space limitation. Recent studies, however, have revealed that extraordinary diversity gain, which results in the remarkable benefits of an increased achievable transmission code-rate and a reduced information outage probability, can be obtained from a relay node in the proximity, which forwards the decoded information from the source node to the intended destination node via a diversity path. While previous works had shown the impressive gains from simple fixed-relaying schemes over non-relaying, this paper takes an information-theoretic viewpoint to study the optimal decode-and-forward (DF) single-hop relay strategy for maximizing the mutual information between the source and destination nodes exploiting the instantaneous channel state information at the nodes (CSIN), and for minimizing the outage probability if only the statistical channel information is known at the nodes (SCIN). In particular, our aim is to optimize the time and power distribution between the direct transmission and relaying phases, for a cooperative three-node relay fading channel.**

***Index Terms*— cooperation diversity, MANET, mutual information, power control, relay, outage probability**

I. INTRODUCTION

Multipath fading introduces instability and randomness of a wireless channel which presents a fundamental physical challenge of achieving high-speed reliable communications over air (e.g., [1]). The subject of providing diversity in reception to remedy the channel impairement has long been investigated for decades. Diversity techniques work under the same general principle, which reduces the risk of being in a deep fade from a number of independent copies of reception. Nonetheless, their performances may vary considerably depending on the system constraints.

For instance, high-speed communication tends to have a slow-varying channel, and time diversity becomes infeasible unless delay can be tolerated. On the other hand, frequency diversity is in general very expensive because it takes up more spectra. For this reason, space diversity has emerged as one attractive means to alleviate the channel impairments without bandwidth expansion and increase in transmit power [2]–[4]. Traditionally, space diversity is obtained by employing multiple receive antennas for independent receptions. Recent advanced multi-antenna technologies such as multiple-input multiple-output (MIMO) antennas have also been widely acknowledged (e.g., [5]–[8]). Difficulty arises, nevertheless, if a mobile station has to be compact and employing multiple antennas may not be viable. This problem is more pronounced in a mobile ad hoc network (MANET) which is self-organized and formed by a number of mobile terminals without relying on any pre-existed infrastructure.

In a MANET, every terminal is regarded as a node that can transmit or receive information to or from neighboring nodes. Range is potentially a critical issue, due to power limitation, and channel fading will further deteriorate the link quality. A promising alternative to mitigate fading in MANET is *cooperation diversity* in which several nodes cooperate together (through signaling) to form a virtual MIMO system for the space diversity benefits [9]–[11].

Cooperation diversity is accomplished by having a node acting as a relay to forward the received information from the source to the intended destination node (see Figure 1). The fact that the channel responses from the source and the relay nodes to the destination node are independent, is exploited to obtain space diversity. In recent years, much attention has been received on the use of user cooperation diversity in wireless networks (e.g., [12]–[15]).

In [12], [13], Sendonaris *et al.* presented an extensive set of simulation results demonstrating the great potential of cooperation diversity and discussed some implementation issues. Most recently in [14], Hunter *et al.* looked into coded cooperation in which cooperation operates through channel coding in the spatial domain. Instead of repeating the received bits [in decode-and-forward (DF) relaying], the cooperating node sends an incremental redundancy for its partner. [14] also derived the outage probability for the coded cooperation. In [15], Laneman *et al.* developed and analyzed low-complexity cooperation diversity protocols for delay-constrained or non-ergodic wireless channels in which the fading effect cannot be averaged out by the coding design. Incremental relaying with limited feedback was proposed to decrease the outage probability.

In this paper, we consider a three-node wireless network with a source, relay and destination, as in [15] where transmission time is divided into two periods: 1) τ_d units of time for *direct transmission* from the source node to

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both the relay and destination nodes, and 2) τ_r units of time for *forwarded transmission (or relaying)* from the relay to the destination node. Moreover, we assume that the relaying strategy is operated in a DF fashion so that the received information at the relay is first decoded, then forwarded to the destination node. In particular, our aim is to optimize the time-division (τ_d, τ_r) and the power allocation for the two phases. The work presented in this paper is an extension of [15] where $\tau_d = \tau_r = 0.5$ was considered for simplicity. The problems of interest are:

1) To maximize the mutual information of the relayed channel if the channel state information is available at all the participating nodes (CSIN); and
2) To minimize the outage probability of the relayed channel if only the statistical channel information is available at the nodes (SCIN).

The rest of the paper is organized as follows. In Section II, the channel model is described and some fundamental information-theoretic results for a relayed channel will be derived. In Section III, we analyze the mutual information maximization problem and present the optimal relaying scheme. The outage probability minimization will be dealt with in Section IV. Section V extends the results for a user cooperation network. Numerical results will be provided in Section VI and finally, Section VII concludes the paper.

II. The Three-Node Relay Channel

A. Channel Model

Consider a three-node relay channel as shown in Figure 1 where the source node intends to send a message w to the destination node by a codeword of N symbols with power P_d during the direct transmission phase. Assuming that N is large (ideally infinite) and a Gaussian codebook is used, the rate achievable at the destination node directly from the source node is given by

$$\mathcal{I}_{\mathsf{SD}} = \tau_d \log_2(1 + \rho_d g_{\mathsf{SD}}), \tag{1}$$

where τ_d denotes the units of time for the transmission, $\rho_d \triangleq \frac{P_d}{N_0}$ with N_0 being the noise power density denotes the signal-to-noise ratio (SNR) at the destination node, and g_{SD} is the instantaneous channel power gain between the source and the destination nodes.

The relay node in the proximity with the channel power gain g_{SR} from the source also listens to the transmission and can therefore support up to rate

$$\mathcal{I}_{\mathsf{SR}} = \tau_d \log_2(1 + \rho_d g_{\mathsf{SR}}). \tag{2}$$

If the transmission code-rate, R_0, that the source node is transmitting is below the channel capacity between the source and the relay nodes, i.e., $R_0 \le \mathcal{I}_{\mathsf{SR}}$, then the relay is able to decode the data reliably and forward the re-encoded message to the destination node with τ_r units of time. Note that for a proper design, if it happens that $R_0 > \mathcal{I}_{\mathsf{SR}}$, then no relaying should take place and essentially $\tau_r = 0$. Therefore, as a summary, we have

$$\begin{cases} \tau_r \ge 0 & \text{if } R_0 \le \mathcal{I}_{\mathsf{SR}}, \\ \tau_r = 0 & \text{if } R_0 > \mathcal{I}_{\mathsf{SR}}. \end{cases} \tag{3}$$

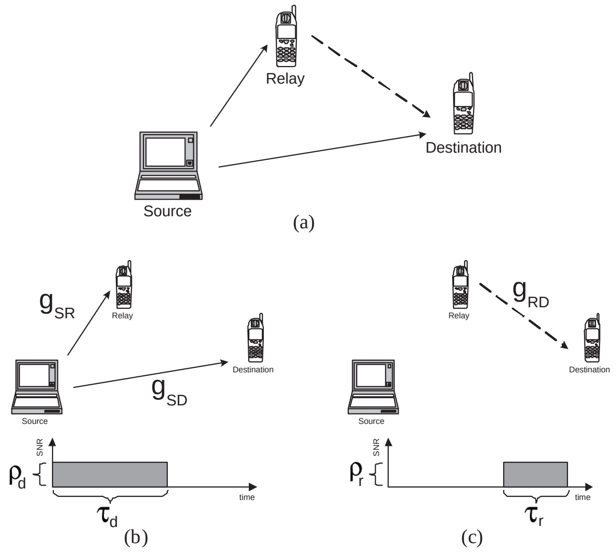

Figure 1. (a) A three-node relay channel with (b) showing the direct transmission phase and (c) showing the relaying phase.

Denoting the power transmitted from the relay as P_r, the maximum rate attainable at the destination node from the relay node is given by

$$\mathcal{I}_{\mathsf{RD}} = \tau_r \log_2(1 + \rho_r g_{\mathsf{RD}}), \tag{4}$$

where $\rho_r \triangleq \frac{P_r}{N_0}$.

Our following consideration will assume that the channels are invariant during the two phases of transmission. However, depending on the type of channel information known to the nodes, the optimization regarding the power and time allocation $(\rho_d, \rho_r, \tau_d, \tau_r)$ may choose to adapt to the variation of the instantaneous channels or in the statistical sense (see Sections III & IV).

B. Mutual Information of the DF Relay Channel

To analyze the three-node relay channel described, it is essential to determine the mutual information between the source and the destination nodes with relay. We find the following lemmas useful for this purpose.

Lemma 1 Mutual Information for Relayed Communication when $\tau_d = \tau_r = \tau$—With the system model in Section II-A, the mutual information between the source and the destination nodes with an equal-time allocation (i.e., $\tau_d = \tau_r = \tau$) is given by

$$\mathcal{I}_{\mathsf{E}}(\tau) = \min\{\mathcal{I}_0(\tau), \mathcal{I}_{\mathsf{SR}}(\tau)\} \tag{5}$$

where

$$\mathcal{I}_0(\tau) = \tau \log_2 (1 + \rho_d g_{\mathsf{SD}} + \rho_r g_{\mathsf{RD}}). \tag{6}$$

Proof: In this relay channel, relaying is preset at τ. The definition of the mutual information already requires that the relay node reliably decodes the information from the source node so that a "proper" relaying can be done. As such, the mutual information of the relayed channel is upper-bounded by

$$\mathcal{I}_{\mathsf{E}}(\tau) \le \mathcal{I}_{\mathsf{SR}}(\tau). \tag{7}$$

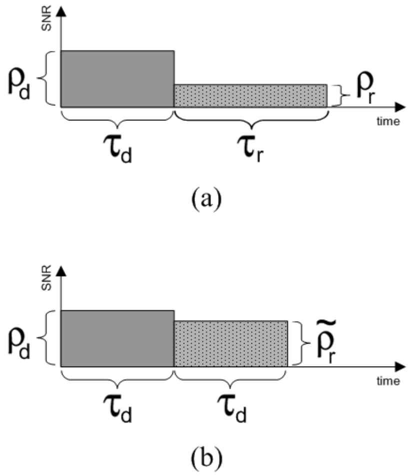

Figure 2. (a) A general relayed channel model and (b) the equivalent model of (a).

With relaying, the destination node will have two independent copies of the same message w transmitted with the same bandwidth τ, one from the source and one from the relay. The optimal maximum-likelihood (ML) receiver at the destination node can then be realized simply by maximal-ratio combining the two signals. Effectively, the resultant channel is equivalent to the maximally combined channel with SNR, $\rho_d g_{\sf SD} + \rho_r g_{\sf RD}$. In other words, the rate achievable between the source and the destination nodes permits the expression $\mathcal{I}_0(\tau)$ in (6) if it does not exceed $\mathcal{I}_{\sf SR}(\tau)$, which makes sure that the relay can decode the information reliably from the source. This has completed the proof. □

Lemma 2 Mutual Information Invariant to SNR and Time Interchange—There is a fundamental tradeoff between power (or more accurately SNR) and the amount of time on which the communication is taken place. In particular, for a transmission with SNR, ρ, and time of α, the mutual information is given by

$$\mathcal{I} = \alpha \log_2 (1 + \rho g) \tag{8}$$

where g denotes the gain channeling the communication. The same mutual information is achievable by a different time duration, β, and a different SNR given by

$$\tilde{\rho} = \frac{(1+\rho g)^{\frac{\alpha}{\beta}} - 1}{g}. \tag{9}$$

It is important to note that however in practice, both α and β should be large enough to allow a long-enough codeword for averaging out the effect of noise so that the Shannon's capacity formula is still valid and achievable.

Proof: Simply setting

$$\alpha \log_2(1+\rho g) = \beta \log_2(1+\tilde{\rho} g), \tag{10}$$

it is easily seen that $\tilde{\rho}$ is given by (9). □

The main novelty of this paper is that we address the optimization in the cases where τ_d and τ_r are generally not equal. In order to express the mutual information of this relay channel, the following lemma is needed.

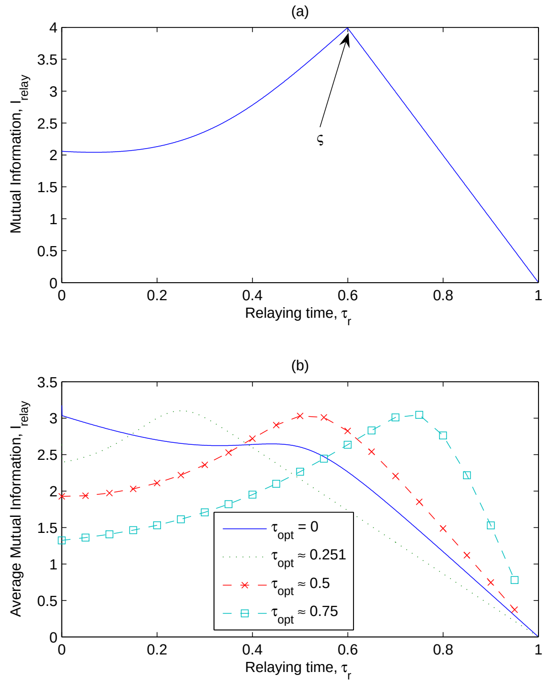

Figure 3. (a) The instantaneous mutual information for a particular relay channel realization as a function of the relaying time, τ_r. (b) The average mutual information of a relayed channel for different channel settings.

Lemma 3 Mutual Information for Relayed Communication for (τ_d, τ_r)—The mutual information for the relayed channel is given by

$$\mathcal{I}_{\sf relay}(\tau_d,\tau_r) = \begin{cases} \tilde{\mathcal{I}}_0(\tau_d,\tau_r) & \text{if } \tau_r = 0,\\ \min\left\{\tilde{\mathcal{I}}_0(\tau_d,\tau_r), \mathcal{I}_{\sf SR}(\tau_d)\right\} & \text{if } \tau_r > 0, \end{cases} \tag{11}$$

where

$$\tilde{\mathcal{I}}_0(\tau_d,\tau_r) = \tau_d \log_2 \left[\rho_d g_{\sf SD} + (1+\rho_r g_{\sf RD})^{\frac{\tau_r}{\tau_d}}\right]. \tag{12}$$

Proof: To start with, it is understood from Lemma 2 that a duration of $\tau_r (> 0)$ with SNR of ρ_r is equivalent to a duration of τ_d with SNR of

$$\tilde{\rho}_r = \frac{(1+\rho_r g_{\sf RD})^{\frac{\tau_r}{\tau_d}} - 1}{g_{\sf RD}}. \tag{13}$$

This power-and-time interchange is illustrated in Figure 2. Then, the result of this lemma can be directly obtained by applying Lemma 1. Note that for $\tau_r = 0$, no relaying occurs and the mutual information is no longer bounded by $\mathcal{I}_{\sf SR}$. This completes the proof. □

In Fig. 3(a), the mutual information for a relay channel is plotted as a function of τ_r for some arbitrary channel settings. Note that there is a discontinuity of $\mathcal{I}_{\sf relay}$ at $\tau_r = 0$ and/or $\tau_r = \varsigma$ where ς will be given in (17) in the next section. Fig. 3(b) further demonstrates the average mutual information performance of a relayed channel for various channel settings. Results indicate that a judicious choice of τ_r could greatly boost the average capacity between the source and the destination nodes.

III. Maximizing Mutual Information with CSIN

In this section, we investigate the optimization problem of the relayed channel provided CSIN is available and we are interested in determining the optimal time and power allocation for the direct transmission and relaying phases for maximizing the mutual information. Mathematically, this is written as

$$\mathbb{P} \mapsto \begin{cases} \max\limits_{\tau_d,\tau_r,\rho_d,\rho_r} & \mathcal{I}_{\mathsf{relay}} \\ \quad\text{s.t.} & 0 \le \rho_d \le \Gamma_d, \\ & 0 \le \rho_r \le \Gamma_r, \\ & 0 \le \tau_d + \tau_r \le 1, \end{cases} \tag{14}$$

where Γ_d and Γ_r are the average power (or SNR) constraints imposed for, respectively, the source and the relay nodes.

To solve (14), first, we note that $\mathcal{I}_{\mathsf{relay}}$ is an increasing function of ρ_d and ρ_r. As a result, $(\rho_d)_{\mathsf{opt}} = \Gamma_d$ and $(\rho_r)_{\mathsf{opt}} = \Gamma_r$, and therefore, $\mathbb{P}$ can be reduced to

$$\mathbb{P} \mapsto \begin{cases} \max\limits_{\tau_d,\tau_r\ge 0} & \mathcal{I}_{\mathsf{relay}} \\ \quad\text{s.t.} & \tau_d + \tau_r \le 1. \end{cases} \tag{15}$$

Intuitively, the mutual information is also increasing with the bandwidth or the total units of time for transmission. As such, the maximum of $\mathcal{I}_{\mathsf{relay}}$ occurs when the constraint becomes active or $\tau_d + \tau_r = 1$. We therefore can write $\tau_r = \tau$ and $\tau_d = 1 - \tau$, and $\mathbb{P}$ becomes simply

$$\mathbb{P} : \max_{0\le\tau<1} \mathcal{I}_{\mathsf{relay}}(1-\tau,\tau). \tag{16}$$

To proceed further, the following useful facts are noted:

- Both the functions $\tilde{\mathcal{I}}_0(1-\tau,\tau)$ and $\mathcal{I}_{\mathsf{SR}}(1-\tau)$ are convex over τ and hence, their maxima are located at the endpoints, i.e., when $\tau = 0$ or $\tau = 1$. The convexity of the functions is proved in Appendix I.
- The intersection point (if any) between $\tilde{\mathcal{I}}_0(1-\tau,\tau)$ and $\mathcal{I}_{\mathsf{SR}}(1-\tau)$ occurs at $\tau = \varsigma$ which is given by

$$\varsigma = \frac{\log_2(1+\Gamma_d g_{\mathsf{SR}} - \Gamma_d g_{\mathsf{SD}})}{\log_2(1+\Gamma_d g_{\mathsf{SR}} - \Gamma_d g_{\mathsf{SD}}) + \log_2(1+\Gamma_r g_{\mathsf{RD}})}. \tag{17}$$

  Interestingly, note that

$$\varsigma = \begin{cases} 0 & \text{if } g_{\mathsf{SD}} = g_{\mathsf{SR}}, \\ 1 & \text{if } g_{\mathsf{RD}} = 0, \\ \varsigma & \text{otherwise}. \end{cases} \tag{18}$$

  If $g_{\mathsf{SD}} < g_{\mathsf{SR}}$, then there exists an intersection point which appears at $\tau = \varsigma$; otherwise, it is not defined (see the derivation in Appendix II).
- Following the second point, we can write

$$\mathcal{I}_{\mathsf{relay}}(\tau) = \begin{cases} \tilde{\mathcal{I}}_0(1-\tau,\tau) & \text{for } 0 \le \tau \le \varsigma, \\ \mathcal{I}_{\mathsf{SR}}(1-\tau) & \text{for } \varsigma < \tau \le 1, \end{cases} \tag{19}$$

  if ς is well defined or $g_{\mathsf{SR}} > g_{\mathsf{SD}}$.

To find the optimal choice of τ, it suffices to consider the following two cases:

C1) **No Intersection**—This occurs when $g_{\mathsf{SD}} \ge g_{\mathsf{SR}}$. In this case,

$$\mathcal{I}_{\mathsf{relay}}(\tau) = \begin{cases} \tilde{\mathcal{I}}_0(1,0) = \mathcal{I}_{\mathsf{SD}}(1), & \tau = 0, \\ \mathcal{I}_{\mathsf{SR}}(1-\tau), & 0 < \tau < 1, \end{cases} \tag{20}$$

because $\tilde{\mathcal{I}}_0(1-\tau,\tau) > \mathcal{I}_{\mathsf{SR}}(1-\tau)$. Obviously,

$$\begin{aligned} \max_\tau \mathcal{I}_{\mathsf{relay}}(\tau) &= \mathcal{I}_{\mathsf{SD}}(1) \\ &= \log_2(1+\Gamma_d g_{\mathsf{SD}}), \end{aligned} \tag{21}$$

which occurs at $\tau = 0$, and the best strategy is to allocate all the available time for direct transmission from the source and not to do relaying.

C2) **Intersection at** $\tau = \varsigma$—This occurs when $g_{\mathsf{SD}} < g_{\mathsf{SR}}$. Then, we have

$$\max_\tau \mathcal{I}_{\mathsf{relay}}(\tau) = \max\left\{\tilde{\mathcal{I}}_0(1,0), \tilde{\mathcal{I}}_0(1-\varsigma,\varsigma)\right\} \tag{22}$$

where ς is given by (17), and

$$\begin{cases} \tilde{\mathcal{I}}_0(1,0) = \log_2(1+\Gamma_d g_{\mathsf{SD}}), \\ \tilde{\mathcal{I}}_0(1-\varsigma,\varsigma) = (1-\varsigma)\log_2(1+\Gamma_d g_{\mathsf{SR}}). \end{cases} \tag{23}$$

Apparently, if

$$g_{\mathsf{SD}} < \frac{(1+\Gamma_d g_{\mathsf{SR}})^{1-\varsigma} - 1}{\Gamma_d} < g_{\mathsf{SR}}, \tag{24}$$

then

$$\begin{aligned} \mathcal{I}_{\mathsf{relay}}(\varsigma) &= \max_\tau \mathcal{I}_{\mathsf{relay}}(\tau) \\ &= \tilde{\mathcal{I}}_0(1-\varsigma,\varsigma) > \tilde{\mathcal{I}}_0(1,0), \end{aligned} \tag{25}$$

and $\tau_{\mathsf{opt}} = \varsigma$. On the other hand, if

$$\frac{(1+\Gamma_d g_{\mathsf{SR}})^{1-\varsigma} - 1}{\Gamma_d} < g_{\mathsf{SD}} < g_{\mathsf{SR}}, \tag{26}$$

then $\max_\tau \mathcal{I}_{\mathsf{relay}}(\tau) = \tilde{\mathcal{I}}_0(1,0)$ with $\tau_{\mathsf{opt}} = 0$.

Now, summarizing the above results, the optimal value of τ can be found as

$$\tau_{\mathsf{opt}} = \begin{cases} \varsigma & \text{if } g_{\mathsf{SD}} < \frac{(1+\Gamma_d g_{\mathsf{SR}})^{1-\varsigma}-1}{\Gamma_d}, \\ 0 & \text{if } g_{\mathsf{SD}} \ge \frac{(1+\Gamma_d g_{\mathsf{SR}})^{1-\varsigma}-1}{\Gamma_d}, \\ 0 & \text{if } \varsigma \text{ is not defined or } g_{\mathsf{SD}} > g_{\mathsf{SR}}. \end{cases} \tag{27}$$

The corresponding maximum mutual information of the relayed channel can be obtained by

$$\begin{aligned} &\max_\tau \mathcal{I}_{\mathsf{relay}}(\tau) = \\ &\max\left\{\min\{\tilde{\mathcal{I}}_0(1-\tau,\tau), \mathcal{I}_{\mathsf{SR}}(1-\tau)\}, \mathcal{I}_{\mathsf{SD}}(1)\right\}\Big|_{\tau=\tau_{\mathsf{opt}}}, \end{aligned} \tag{28}$$

which can be simplified to (29) (see top of the next page) where we have also

$$\begin{aligned} &(1-\varsigma)\log_2(1+\Gamma_d g_{\mathsf{SR}}) = \\ &\qquad \frac{\log_2(1+\Gamma_d g_{\mathsf{SR}})\log_2(1+\Gamma_r g_{\mathsf{RD}})}{\log_2(1+\Gamma_d g_{\mathsf{SR}} - \Gamma_d g_{\mathsf{SD}})(1+\Gamma_r g_{\mathsf{RD}})}. \end{aligned} \tag{30}$$

$$\mathcal{I}_{\mathsf{relay}}(\tau_{\mathsf{opt}}) = \begin{cases} (1-\varsigma)\log_2(1+\Gamma_d g_{\mathsf{SR}}) & \text{if } g_{\mathsf{SD}} < \frac{(1+\Gamma_d g_{\mathsf{SR}})^{1-\varsigma}-1}{\Gamma_d}, \\ \log_2(1+\Gamma_d g_{\mathsf{SD}}) & \text{if } g_{\mathsf{SD}} \geq \frac{(1+\Gamma_d g_{\mathsf{SR}})^{1-\varsigma}-1}{\Gamma_d} \text{ or } \varsigma \text{ is not defined,} \end{cases} \tag{29}$$

IV. Minimizing Outage Probability with SCIN

When CSIN is absent, it is more preferred to minimize the outage probability with a specific constant transmission code-rate R_0, i.e.,

$$\mathbb{Q}: \min_{0\leq\tau<1} \mathcal{P}\left(\{\mathcal{I}_{\mathsf{relay}}(\tau) < R_0\}\right), \tag{31}$$

where again $\rho_d = \Gamma_d$, $\rho_r = \Gamma_r$, $\tau_r = \tau$ and $\tau_d = 1-\tau$ are used. To obtain the outage probability, we shall consider that the channel power gains between the nodes are all independent and exponential distributed (so the channels are in Rayleigh fading) to have the following probability density function (pdf)

$$\mathcal{F}(g) = \begin{cases} \frac{1}{\mathtt{E}[g]}e^{-\frac{g}{\mathtt{E}[g]}} & \text{if } g \geq 0, \\ 0 & \text{if } g < 0, \end{cases} \tag{32}$$

with their respective mean channel power gains $\mathtt{E}[g_{\mathsf{SD}}] = G_{\mathsf{SD}}$, $\mathtt{E}[g_{\mathsf{SR}}] = G_{\mathsf{SR}}$ and $\mathtt{E}[g_{\mathsf{RD}}] = G_{\mathsf{RD}}$.

A. Exact Outage Probability

To derive the outage probability expression, we find the following lemma useful.

Lemma 4 Cumulative Distribution Function (cdf) of the Minimum of Two Independent Random Variables—Given two independent random variables X and Y, the cdf of $Z = \min\{X,Y\}$ is given by

$$\mathcal{P}(\{Z \leq z\}) = 1-(1-\mathcal{P}(\{X \leq z\}))(1-\mathcal{P}(\{Y \leq z\})). \tag{33}$$

Proof: See [16]. □

Knowing (11) and Lemma 4, calculation of the outage probability can be done by obtaining the two probabilities

$$\begin{cases} p_1 = \mathcal{P}\left(\{\mathcal{I}_{\mathsf{SR}}(1-\tau) < R_0\}\right), \\ p_2 = \mathcal{P}(\{\tilde{\mathcal{I}}_0(1-\tau,\tau) < R_0\}). \end{cases} \tag{34}$$

To find them, we note that for a pair of random variables X and Y related by $Y = a\log_2(b+X)$ for $a,b>0$, we have

$$\mathcal{P}(\{Y \leq y\}) = \mathcal{P}\left(\left\{X \leq 2^{\frac{y}{a}} - b\right\}\right), \tag{35}$$

and if X is exponential distributed, then

$$\mathcal{P}(\{Y \leq y\}) = \begin{cases} 1 - e^{-\frac{2^{\frac{y}{a}}-b}{\mathtt{E}[X]}}, & \text{for } y \geq a\log_2 b, \\ 0, & \text{for } y < a\log_2 b. \end{cases} \tag{36}$$

Using this neat result, p_1 can be expressed as

$$p_1 = 1 - e^{-\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\mathsf{SR}}}} \tag{37}$$

and we can also get

$$\mathcal{P}(\{\tilde{\mathcal{I}}_0(1-\tau,\tau) < R_0|g_{\mathsf{RD}}\}) = \begin{cases} 1 - e^{-\frac{2^{\frac{R_0}{1-\tau}}-(1+\Gamma_r g_{\mathsf{RD}})^{\frac{\tau}{1-\tau}}}{\Gamma_d G_{\mathsf{SD}}}}, & \text{for } g_{\mathsf{RD}} < \frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r}, \\ 0, & \text{for } g_{\mathsf{RD}} \geq \frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r}. \end{cases} \tag{38}$$

Then, p_2 can be evaluated by

$$\begin{aligned} p_2 &= \mathtt{E}_{g_{\mathsf{RD}}}\left[\mathcal{P}(\{\tilde{\mathcal{I}}_0(1-\tau,\tau) < R_0|g_{\mathsf{RD}}\})\right] \\ &= \int_0^{\frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r}} \left(1 - e^{-\frac{2^{\frac{R_0}{1-\tau}}-(1+\Gamma_r x)^{\frac{\tau}{1-\tau}}}{\Gamma_d G_{\mathsf{SD}}}}\right) \frac{e^{-\frac{x}{G_{\mathsf{RD}}}}}{G_{\mathsf{RD}}} dx \\ &= 1 - e^{-\frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r G_{\mathsf{RD}}}} \\ &\quad - \frac{1}{G_{\mathsf{RD}}} \int_0^{\frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r}} e^{-\frac{2^{\frac{R_0}{1-\tau}}-(1+\Gamma_r x)^{\frac{\tau}{1-\tau}}}{\Gamma_d G_{\mathsf{SD}}}} e^{-\frac{x}{G_{\mathsf{RD}}}} dx. \end{aligned} \tag{39}$$

Finally, for $\tau > 0$, the outage probability can be found as (40) (see top of the next page) while when $\tau = 0$, the outage probability is simply

$$\begin{aligned} \mathcal{P}(\{\mathcal{I}_{\mathsf{relay}}(0) < R_0\}) &= \mathcal{P}(\{\mathcal{I}_{\mathsf{SD}}(1) < R_0\}) \\ &= 1 - e^{-\frac{2^{R_0}-1}{\Gamma_d G_{\mathsf{SD}}}}. \end{aligned} \tag{41}$$

B. Upper Bounding Outage Probability

The difficulty of (40) is that it is too complicated to be used for optimizing τ because of the integration involved (even though it is a finite integral). Here, we shall upper-bound the outage probability and derive some closed form expressions from the bounds for ease of optimization of τ. This is achieved by considering the lower bound of the integral of the form

$$\mathcal{S} \triangleq \int_0^{A^{\frac{1}{t}}-1} e^{\frac{(1+y)^t}{\Gamma_1}} e^{-\frac{y}{\Gamma_2}} dy, \quad \text{for } A>1, t, \Gamma_1, \Gamma_2 > 0. \tag{42}$$

It is easily shown that $e^{\frac{(1+y)^t}{\Gamma_1}}$ is an increasing function of y. Therefore,

$$\begin{aligned} \mathcal{S} &> e^{\frac{1}{\Gamma_1}} \int_0^{A^{\frac{1}{t}}-1} e^{-\frac{y}{\Gamma_2}} dy \\ &= \Gamma_2 e^{\frac{1}{\Gamma_1}}\left(1 - e^{-\frac{A^{\frac{1}{t}}-1}{\Gamma_2}}\right) \equiv \mathcal{L}_1. \end{aligned} \tag{43}$$

On the other hand, we know that $e^{-\frac{y}{\Gamma_2}}$ is a decreasing function of y. Hence, we have also

$$\mathcal{S} > e^{-\frac{A^{\frac{1}{t}}-1}{\Gamma_2}} \int_0^{A^{\frac{1}{t}}-1} e^{\frac{(1+y)^t}{\Gamma_1}} dy, \tag{44}$$

$$
\begin{aligned}
\mathcal{P}(\{\mathcal{I}_{\text{relay}}(\tau) < R_0\}) &= 1-(1-p_1)(1-p_2) \\
&= 1-\left(e^{-\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\text{SR}}}}\right)\left(e^{-\frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r G_{\text{RD}}}}+\frac{1}{G_{\text{RD}}}\int_0^{\frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r}} e^{-\frac{2^{\frac{R_0}{1-\tau}}-(1+\Gamma_r x)^{\frac{\tau}{1-\tau}}}{\Gamma_d G_{\text{SD}}}} e^{-\frac{x}{G_{\text{RD}}}}\,dx\right) \\
&= 1-e^{-\left(\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\text{SR}}}+\frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r G_{\text{RD}}}\right)}-\frac{e^{-\left(\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\text{SR}}}+\frac{2^{\frac{R_0}{1-\tau}}}{\Gamma_d G_{\text{SD}}}\right)}}{G_{\text{RD}}}\int_0^{\frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r}} e^{\frac{(1+\Gamma_r x)^{\frac{\tau}{1-\tau}}}{\Gamma_d G_{\text{SD}}}} e^{-\frac{x}{G_{\text{RD}}}}\,dx
\end{aligned}
\tag{40}
$$

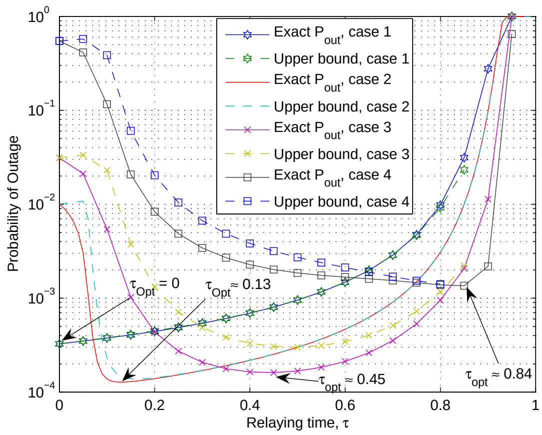

Figure 4. Outage probability of a relay channel as a function of relaying time for various channel settings.

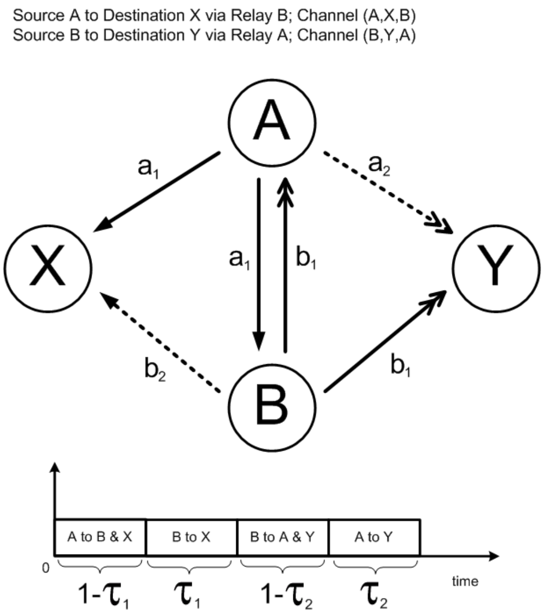

Figure 5. A cooperative network with four nodes.

which can be evaluated as (see Appendix III for details)

$$
\mathcal{S} > e^{-\frac{A^{\frac{1}{t}}-1}{\Gamma_2}}\left[A^{\frac{1}{t}}\,{}_1F_1\left(\frac{1}{t};1+\frac{1}{t};\frac{A}{\Gamma_1}\right)\right.
$$
$$
\left.-\,{}_1F_1\left(\frac{1}{t};1+\frac{1}{t};\frac{1}{\Gamma_1}\right)\right] \triangleq \mathcal{L}_2 \tag{45}
$$

where ${}_pF_q$ is the generalized hypergeometric function.

As a result of $\mathcal{L}_1$ and $\mathcal{L}_2$ by substituting $\Gamma_1 = \Gamma_d G_{\text{SD}}$, $\Gamma_2 = \Gamma_r G_{\text{RD}}$, $t = \frac{\tau}{1-\tau}$ and $A = 2^{\frac{R_0}{1-\tau}}$, we can have the upper bound outage probability written in closed form as

$$
\mathcal{P}(\{\mathcal{I}_{\text{relay}}(\tau) < R_0\}) \le \min\{\mathcal{P}_1(\tau), \mathcal{P}_2(\tau)\} \tag{46}
$$

where $\mathcal{P}_1$ and $\mathcal{P}_2$ are defined in (47) (see top of the next page) and they are functions of τ.

In Fig. (4), results are provided to reveal the tightness of the probability upper bound for various relay channels. As can be seen, the proposed bound is generally very tight at the optimal relaying point (i.e., at $\tau = \tau_{\text{opt}}$), which makes it useful in optimizing the relaying time.

C. Optimizing τ Using DIRECT

Provided the outage probability in (40), we can solve $\mathbb{Q}$, or (31) numerically since it is a minimization problem over a single bounded variable. To perform this optimally and efficiently, we propose to apply the DIviding RECTangle (DIRECT) algorithm, originally proposed by Jones *et al.* [17], which samples points in the domain, and uses the information it has obtained to decide where to search next. It is well understood that DIRECT will converge to the global minimal value of the objective function if the number of function evaluations is sufficient, regardless of the convexity of the problem. Evaluating (40), however, requires to compute the numerical integration for each sample τ and is therefore very complex. A more efficient, yet suboptimal, approach is to solve

$$
\tilde{\mathbb{Q}}:\ \min_{0\le\tau<1}\ \min\{\mathcal{P}_1(\tau), \mathcal{P}_2(\tau)\}. \tag{48}
$$

V. USER COOPERATION NETWORK

Thus far, we have considered a scenario which a (relay) node is spending its resources (e.g., bandwidth and power) to assist another node selflessly. A more reasonable and likely situation would be that two nodes cooperate each other and compromise their resources in such a way that the performance is improved overall and individually. In this section, we extend the use of our results to address a cooperative network with four nodes: two source nodes and two destination nodes, which is basically constructed by two 3-node relay channels (see Figure 5). Here, the nodes X and Y have been assumed to be too far from each other to communicate.

$$\begin{cases} \mathcal{P}_1(\tau) = 1 - e^{-\left(\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\mathsf{SR}}} + \frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r G_{\mathsf{RD}}}\right)} - e^{-\left(\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\mathsf{SR}}} + \frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\mathsf{SD}}}\right)} + e^{-\left(\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\mathsf{SR}}} + \frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\mathsf{SD}}} + \frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r G_{\mathsf{RD}}}\right)}, \\ \mathcal{P}_2(\tau) = 1 - e^{-\left(\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\mathsf{SR}}} + \frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r G_{\mathsf{RD}}}\right)} \\ \qquad - \frac{e^{-\left(\frac{2^{\frac{R_0}{1-\tau}}-1}{\Gamma_d G_{\mathsf{SR}}} + \frac{2^{\frac{R_0}{1-\tau}}}{\Gamma_d G_{\mathsf{SD}}} + \frac{2^{\frac{R_0}{\tau}}-1}{\Gamma_r G_{\mathsf{RD}}}\right)}}{\Gamma_r G_{\mathsf{RD}}} \left[2^{\frac{R_0}{\tau}} {}_1F_1\left(\frac{1}{\tau}-1; \frac{1}{\tau}; \frac{2^{\frac{R_0}{1-\tau}}}{\Gamma_d G_{\mathsf{SD}}}\right) - {}_1F_1\left(\frac{1}{\tau}-1; \frac{1}{\tau}; \frac{1}{\Gamma_d G_{\mathsf{SD}}}\right)\right] \end{cases} \tag{47}$$

In this network, we regulate the power consumption of the transmitter nodes A and B as follows:

$$\begin{cases} \frac{a_1(1-\tau_1)+a_2\tau_2}{1-\tau_1+\tau_2} \le \Gamma_{\mathsf{A}}, \\ \frac{b_1(1-\tau_2)+b_2\tau_1}{1-\tau_2+\tau_1} \le \Gamma_{\mathsf{B}}. \end{cases} \tag{49}$$

Also, note that the same bandwidths in time are allocated to achieve the communications at the destination nodes. An optimal relaying strategy requires to find $\boldsymbol{a} = (a_1, a_2)$, $\boldsymbol{b} = (b_1, b_2)$ and $\boldsymbol{\tau} = (\tau_1, \tau_2)$ jointly that maximizes the performance metric under the given constraints.

A. Mutual Information Fair Cooperation

Denoting the mutual information of the relayed channel (A, X, B) as $\mathcal{I}_{\mathsf{AXB}}$ [similarly for the channel (B, Y, A)], given the CSIN, we consider

$$\begin{aligned} \max_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{\tau}} \quad & \min\left\{\mathcal{I}_{\mathsf{AXB}}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{\tau}), \mathcal{I}_{\mathsf{BYA}}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{\tau})\right\} \\ \text{s.t.} \quad & \frac{\boldsymbol{a}^T\left(\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\boldsymbol{\tau} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)}{1+[-1\ 1]\boldsymbol{\tau}} \le \Gamma_{\mathsf{A}}, \\ & \frac{\boldsymbol{b}^T\left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\boldsymbol{\tau} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)}{1+[1\ -1]\boldsymbol{\tau}} \le \Gamma_{\mathsf{B}}, \\ & \boldsymbol{0} \le \boldsymbol{\tau} < \boldsymbol{1}, \end{aligned} \tag{50}$$

where $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{\tau}$ are treated as column vectors and the superscript T denotes vector transposition. By maximizing the minimum of the information rates at the destination nodes, some "fairness" is built in.

B. Outage Probability Fair Cooperation

With SCIN in a cooperative network, a sensible solution can be obtained by

$$\begin{aligned} \min_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{\tau}} \quad & \max\left\{\begin{matrix} \mathcal{P}(\{\mathcal{I}_{\mathsf{AXB}}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{\tau}) < R_1\}), \\ \mathcal{P}(\{\mathcal{I}_{\mathsf{BYA}}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{\tau}) < R_2\}) \end{matrix}\right\} \\ \text{s.t.} \quad & \frac{\boldsymbol{a}^T\left(\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\boldsymbol{\tau} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)}{1+[-1\ 1]\boldsymbol{\tau}} \le \Gamma_{\mathsf{A}}, \\ & \frac{\boldsymbol{b}^T\left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\boldsymbol{\tau} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)}{1+[1\ -1]\boldsymbol{\tau}} \le \Gamma_{\mathsf{B}}, \\ & \boldsymbol{0} \le \boldsymbol{\tau} < \boldsymbol{1}, \end{aligned} \tag{51}$$

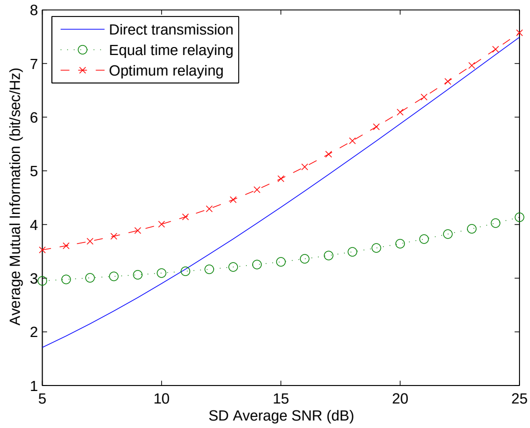

Figure 6. Simulation results showing the average mutual information performance of a relay channel with CSIN using different schemes.

where R_1 and R_2 are the respective target code-rates at the destination nodes. Using the expression derived in (40) or the bound in (47), the optimal relaying solution can be numerically solved.

VI. Numerical Results

In this section, numerical results are presented to show the benefits of optimizing relaying and user cooperation. For this purpose, comparison will be made with direct-only transmission and equal time allocation schemes. In all the simulations provided, the noise power density is assumed to be the same for all nodes and equal to unity. The channels are Rayleigh faded with coefficients $\sqrt{g_{\mathsf{XX}}}$ where X can be S, R or D. Channel coefficients account for path loss, shadowing and multipath fading.

First, a three-node network shown in Fig. 1 is considered. The average channel power gain to the noise ratio from the source to the destination is set to be -30 (dB) and from the source to the relay is -5 (dB) and from relay to destination is -10 (dB). CSIN is available at all nodes. The simulations are carried out by varying the SNR from the source to the destination node with the average mutual information results shown in Fig. 6. Results illustrate that the optimum relaying outperforms the direct transmission and the equal time relaying and significant performance improvement is possible especially at low SNR.

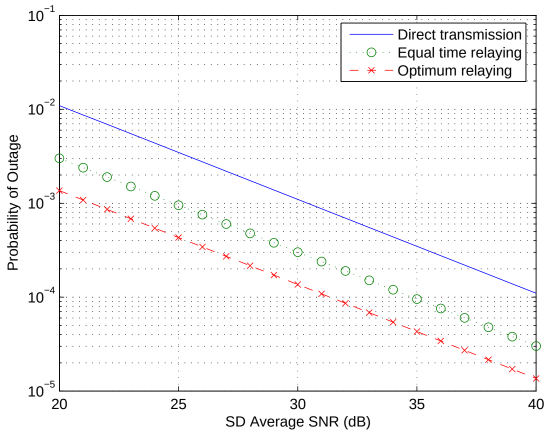

Figure 7. Simulation results showing the outage probability performance of a relay channel with SCIN using different schemes.

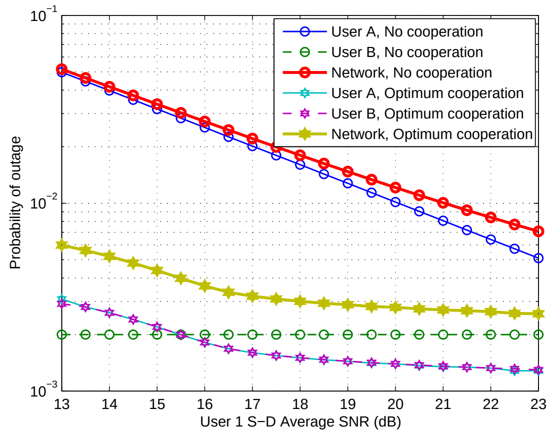

Figure 9. Outage probability performances of a four-node network with only SCIN for optimal and no cooperation protocols.

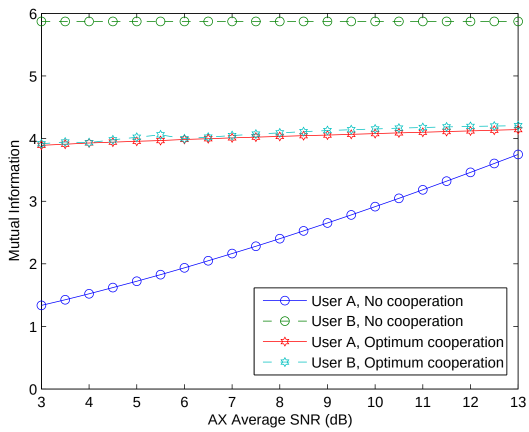

Figure 8. Average mutual information performances of a four-node network with CSIN for optimal and no cooperation protocols.

In the second example, we investigate the system where only SCIN is available at the nodes and we attempt to optimize the relaying for minimizing the outage probability. In this simulation, the channel gain to the noise power ratios from both the source to the relay and from the relay to the destination are -5 (dB) while the source to the destination channel gain to noise ratio has an value of -15 (dB). Results are provided in Fig. 7 which demonstrates a remarkable advantage of optimizing the time allocation for outage probability minimization. As we can see, the capacity advantages become more apparent at low SNR.

In the last two simulations, we study the performance of the user cooperation protocols in Section V for a 4-node network in Fig. 5. In Fig. 8, we assume that CSIN is available to all the nodes and cooperation is done to maximize the minimum of the mutual information at the destination nodes. The inter-user channels (g_{AB} and g_{BA}) are assumed to be symmetric with average channel to noise power ratio of -5 (dB) while other channel gain to noise power ratios (note that the noise power has been assumed to be unity) are as follow: $g_{\mathsf{AX}} = -52$ (dB), $g_{\mathsf{AY}} = -35$ (dB), $g_{\mathsf{BY}} = -45$ (dB), and $g_{\mathsf{BX}} = -35$ (dB). As can be seen in Fig. 8, after user cooperation, User A's mutual information is almost the same as that of User B. In addition, results also illustrate that user cooperation significantly improves the worse user's performance (User B in the direct transmission case) by balancing the users' resources through cooperation in a fair manner.

In Fig. 9, results are provided for the four-node network with only SCIN at the nodes. It is assumed that the inter-user channel power to noise ratio is -10 (dB) and other channels are: $g_{\mathsf{AX}} = -27$ (dB) and $g_{\mathsf{AY}} = g_{\mathsf{BX}} = g_{\mathsf{BY}} = -10$ (dB). In this case, the user cooperation is optimized in order to minimize the maximum outage probability of the destination nodes. Results show that a significant improvement for User A's performance is observed while User B is only slightly deteriorated. For a wide range of SNR, User's B performance is actually better after cooperation than without cooperation. Moreover, the fairness of the system is illustrated by bringing the performance of the two users very close together.

Before we conclude this section, results in Fig. 9 also show that the user cooperation to minimize the maximum outage probability works excitingly well for minimizing the network outage probability when a network outage is declared if at least one user's code-rate is not met at the destination. This is because

$$\begin{aligned}\mathcal{P}_{\mathsf{out}}^{(\mathsf{Network})} &= 1-\left(1-\mathcal{P}_{\mathsf{out}}^{(\mathsf{A})}\right)\left(1-\mathcal{P}_{\mathsf{out}}^{(\mathsf{B})}\right)\\ &= \mathcal{P}_{\mathsf{out}}^{(\mathsf{A})}+\mathcal{P}_{\mathsf{out}}^{(\mathsf{B})}-\mathcal{P}_{\mathsf{out}}^{(\mathsf{A})}\mathcal{P}_{\mathsf{out}}^{(\mathsf{B})}\\ &\approx \mathcal{P}_{\mathsf{out}}^{(\mathsf{A})}+\mathcal{P}_{\mathsf{out}}^{(\mathsf{B})}\end{aligned} \quad (52)$$

where $\mathcal{P}_{\mathsf{out}}^{(\mathsf{Network})}$, $\mathcal{P}_{\mathsf{out}}^{(\mathsf{A})}$, and $\mathcal{P}_{\mathsf{out}}^{(\mathsf{B})}$ denote, respectively, the outage probabilities of the overall network, User A and User B. Since $\min\{\mathcal{P}_{\mathsf{out}}^{(\mathsf{A})}, \mathcal{P}_{\mathsf{out}}^{(\mathsf{B})}\}$ has a strong tendency to minimize the sum of the two probabilities, the network outage probability can be greatly reduced by the proposed user cooperation, as confirmed by the results in the figure.

VII. CONCLUSION

In this paper, we have addressed the optimal allocation for bandwidth (measured in time) in a DF relay channel. An expression for the mutual information of a three-node relay channel with arbitrary relaying-time allocation has been derived and its maximization has been examined in the presence of CSIN with the optimal solution derived in closed form. We have also investigated the minimization of the outage probability if only SCIN is available. In this case, a closed form probability upper bound is proposed, which has permitted us to efficiently obtain the optimal relaying strategy numerically using such as the DIRECT algorithm. Simulation results have indicated that the optimal relaying scheme offers tremendous performance gains over the direct-only transmission and the fixed equal-time relaying schemes. The findings have also been extended to a four-node network with two-user cooperation showing promising results.

APPENDIX I
CONVEXITY OF $\tilde{\mathcal{I}}_0(1-\tau,\tau)$ AND $\mathcal{I}_{\mathsf{SR}}(1-\tau)$

Differentiating $\tilde{\mathcal{I}}_0(1-\tau,\tau)$ in (12) with respect to τ gives

$$\frac{\partial\tilde{\mathcal{I}}_0(1-\tau,\tau)}{\partial\tau} = -\log_2\left(\Gamma_d g_{\mathsf{SD}} + (1+\Gamma_r g_{\mathsf{RD}})^{\frac{\tau}{1-\tau}}\right) + \left(\frac{1}{1-\tau}\right)\frac{(1+\Gamma_r g_{\mathsf{RD}})^{\frac{\tau}{1-\tau}}\log_2(1+\Gamma_r g_{\mathsf{RD}})}{\Gamma_d g_{\mathsf{SD}} + (1+\Gamma_r g_{\mathsf{RD}})^{\frac{\tau}{1-\tau}}}. \quad (53)$$

Differentiating it once again, we get

$$\frac{\partial^2\tilde{\mathcal{I}}_0(1-\tau,\tau)}{\partial\tau^2} = \frac{\ln 2\cdot\Gamma_d g_{\mathsf{SD}}(1+\Gamma_r g_{\mathsf{RD}})^{\frac{\tau}{1-\tau}}\log_2^2(1+\Gamma_r g_{\mathsf{RD}})}{(1-\tau)^3\left[\Gamma_d g_{\mathsf{SD}} + (1+\Gamma_r g_{\mathsf{RD}})^{\frac{\tau}{1-\tau}}\right]^2} > 0. \quad (54)$$

Therefore, $\tilde{\mathcal{I}}_0(1-\tau,\tau)$ is convex. On the other hand, as $\mathcal{I}_{\mathsf{SR}}(1-\tau)$ is a straight line, it is also convex. In addition, its maximum occurs when $\tau=0$ while the minimum is located at $\tau=0$.

APPENDIX II
INTERSECTION OF $\tilde{\mathcal{I}}_0(1-\tau,\tau)$ AND $\mathcal{I}_{\mathsf{SR}}(1-\tau)$

At the intersection ($\tau=\varsigma$), it is required that

$$\tilde{\mathcal{I}}_0(1-\varsigma,\varsigma) = \mathcal{I}_{\mathsf{SR}}(1-\varsigma), \quad (55)$$

which yields

$$\begin{aligned}(1+\Gamma_r g_{\mathsf{RD}})^{\frac{\varsigma}{1-\varsigma}} &= 1+\Gamma_d g_{\mathsf{SR}} - \Gamma_d g_{\mathsf{SD}} \\ \frac{\varsigma}{1-\varsigma} &= \frac{\log_2(1+\Gamma_d g_{\mathsf{SR}}-\Gamma_d g_{\mathsf{SD}})}{\log_2(1+\Gamma_r g_{\mathsf{RD}})}.\end{aligned} \quad (56)$$

Finally, it gives

$$\varsigma = \frac{\log_2(1+\Gamma_d g_{\mathsf{SR}}-\Gamma_d g_{\mathsf{SD}})}{\log_2(1+\Gamma_d g_{\mathsf{SR}}-\Gamma_d g_{\mathsf{SD}}) + \log_2(1+\Gamma_r g_{\mathsf{RD}})}. \quad (57)$$

Therefore, at most, one intersection point is possible, and it exists only if

$$g_{\mathsf{SR}} > g_{\mathsf{SD}}; \quad (58)$$

otherwise, it is undefined.

APPENDIX III
DERIVATION OF $\int_0^x e^{a(1+y)^t}dy$

First, rewrite the integral as

$$\int_0^x e^{a(1+y)^t}dy = a^{-\frac{1}{t}}\int_{a^{\frac{1}{t}}}^{a^{\frac{1}{t}}(1+x)} e^{y^t}dy. \quad (59)$$

Then, the integration can be computed by expanding e^{y^t} so that

$$\begin{aligned}&\int_0^x e^{a(1+y)^t}dy \\ =&a^{-\frac{1}{t}}\sum_{k=0}^{\infty}\frac{1}{k!}\int_{a^{\frac{1}{t}}}^{a^{\frac{1}{t}}(1+x)} y^{tk}dy \\ =&a^{-\frac{1}{t}}\left[\sum_{k=0}^{\infty}\frac{1}{k!}\frac{y^{tk+1}}{tk+1}\right]_{a^{\frac{1}{t}}}^{a^{\frac{1}{t}}(1+x)} \\ =&a^{-\frac{1}{t}}\left[y\,{}_1F_1\left(\frac{1}{t};1+\frac{1}{t};y^t\right)\right]_{a^{\frac{1}{t}}}^{a^{\frac{1}{t}}(1+x)} \\ =&(1+x)\,{}_1F_1\left(\frac{1}{t};1+\frac{1}{t};a(1+x)^t\right) \\ &\quad - {}_1F_1\left(\frac{1}{t};1+\frac{1}{t};a\right).\end{aligned} \quad (60)$$

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