A Two-Stage Power and Rate Allocation Strategy for Secondary Users in Cognitive Radio Networks

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Abstract— **In this paper, we attempt to evaluate the optimal power and rate distribution choices for secondary users in order to maintain their quality of service (QoS) in a multi-channel cognitive radio network (CRN). We assume that multiple secondary users share a single channel and multiple channels are simultaneously used by a single secondary user (SU) to satisfy their rate requirements. Our measures for QoS include signal to interference plus noise ratio (SINR)/bit error rate (BER) and minimum rate requirement. We propose a two-stage optimization framework in order to solve for the optimal resource management strategy. In the first stage, we formulate a convex optimization problem to determine the minimum transmit power that SUs should employ in order to maintain a certain SINR. At first, the convex optimization problem is solved in a centralized manner and then we employ dual decomposition theory to derive three different distributed solutions. In the second stage, we formulate the rate distribution problem as a maximum flow problem in graph theory. We also develop a heuristic approach to determine the rate distribution. Simulation results demonstrate that optimal transmit power follows "reverse water filling" process and rate allocation follows SINR. We also observe that the distributed solution converges to the centralized solution and rate distribution based on our proposed heuristic is close to the graph theoretic optimal solution.**

Index Terms— **power allocation, distributed approach, rate allocation, quality of service**

I. Introduction

A CRN is built on the following principle: a network of secondary users (users without a license) continuously sense the use of a spectrum band by primary users and opportunistically utilize the band when primary users are absent. Any SU in a CRN performs three main functions - (1) sensing spectrum to identify absence or presence of primary user (PU); (2) start transmitting if PU is absent; (3) maintain QoS through the transmission duration by adaptively seeking out best transmission strategies (channel, rate, transmit power etc.).

Researchers have focused on a wide range of issues related to resource allocation in CRNs. The authors in [1] consider a CRN model with one PU and one SU coexisting in the same channel and develop a cognitive radio game to find optimal transmit power for the SU with the goal of minimizing total transmit power. However, QoS is not guaranteed for the SUs. Additionally, the optimality of the presented solution is not assured as the formulation is non-convex. In [2]- [3], the authors consider a CRN model with one PU and multiple SUs coexisting in the same channel. Here, the authors propose both centralized and distributed power allocation strategies without considering the interference temperature threshold in the underlying optimization problems. The authors in [4] consider a system model where multiple SUs coexist in a channel. They formulate an optimization problem to find optimal transmit powers with the objective of maximizing the summation of utilities (function of SINR) of all users. A lower bound on SINR is used as a QoS constraint for secondary users. A distributed suboptimal joint coordination and power control mechanism to allocate transmit powers to secondary users is also provided in [4]. However [1]- [4] fail to consider a practical scenario where (1) multiple channels may be available for opportunistic use, and (2) rate requirement for SUs may not be satisfied by employing a single channel per SU. In [5]- [6], joint allocation of channel and transmit power has been studied. However, in [5] coexistence of multiple secondary users in a channel has not been considered. Also, in both [5] and [6], the QoS requirement of SUs has been ignored. In [7]- [8], though the authors have considered multiple channels to start with, only one secondary user is eventually assigned to a channel. This assumption may not be fair to all SUs due to three reasons. Firstly, channels may be of different quality. Therefore, the SUs assigned to higher quality channels may hold an advantage over SUs assigned to the poorer channels. Secondly, when a PU enters a channel, the corresponding SU has to cease transmission completely. If a single SU is assigned to multiple channels, transmission can be continued even if one or a few channels become unavailable due to PU arrival. Thirdly, rate requirement of some secondary users may not be satisfied by assigning one channel to a user.

To summarize, while there has been extensive research on resource management in CRNs, there is still a lack of comprehensive robust framework that considers QoS for

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SUs in a fair manner. Our objective in this paper is to take a step towards such a solution. We distinguish ourselves from the previous treatments in a number of ways. First, we assume that multiple channels each with different quality is available for opportunistic use by multiple SUs. To maintain fairness, coexistence of multiple SUs in a channel is assumed and each SU can use multiple channels to satisfy their rate requirements. Secondly, our measures for QoS include SINR or BER and minimum rate requirement. Finally, our overall objective is to determine the optimal transmit power and rate (modulation order) that competing SUs need to employ in each channel. Unlike many of the prior efforts, we maintain convexity of our formulation to guarantee optimality of resulting solution.

In this paper, we propose a two-stage optimization framework to allocate power and rate to secondary users in a CRN. Our objective is to determine the optimal distribution of power and rate that a secondary user has to employ across the channels that it uses in order to (1) minimize total power consumption; (2) maximize rate, and (3) maintain QoS. In the first stage, we design a convex optimization problem to determine optimal choice for transmit power that SUs should employ in order to maintain a certain SINR. Then, with the help of dual decomposition theory [9], we also develop user-based distributed approach to solve the proposed optimization problem. Using power/SINR result from stage 1, the optimal distribution of bits/channel is determined in the second stage. At first, the rate distribution is formulated as a maximum flow problem in graph theory. Then, we propose a simple heuristic to perform the rate distribution. Simulation results illustrate that optimal transmit power follows "reverse water filling" process and allocation of rate follows SINR. We also observe that the solution from distributed implementation of stage 1 optimization problem converges to the centralized solution. We show that the rate distribution across users provided from our proposed heuristic is close to the optimal graph theoretic solution.

The rest of the paper is organized as follows. In Section II, along with the system model, we state all of our assumptions and notations used in the rest of the paper. Section III describes the proposed two-stage optimization framework in detail along with the distributed implementation of the first stage. Numerical results are presented in Section IV.

II. System Model

We consider a CRN with M secondary users, each equipped with spectrum sensing capability device. L free channels are detected for use. We assume that multiple secondary users may coexist in a channel and a single secondary user can use several channels at the same time. We consider (1) SINR or BER and minimum rate requirement as measures to indicate QoS of SUs, (2) M-ary QAM modulation scheme with an adaptable modulation order for all SUs, (3) simple path loss model for channel; and (4) a rate constraint for each channel. In order to enable mathematical tractability of the optimization framework, we assume that there exists a central cognitive network controller that has the knowledge about QoS requirements, power and rate limitations; Channel State Information (CSI) of all active SUs in the network. We enforce an interference temperature threshold to protect possible primary user transmission on any channel. Though Federal communications committee has removed the interference temperature limit as the quantifying metric for characterizing interference [10] but we feel that there is value to having this limit in place especially during the transition stages. That is, the interference threshold will ensure that when a PU enters a channel used by SUs, there is a certain limit to the interference that it experiences before the SUs vacate the channel. Finally, at each time instant for optimization, an estimate of the number of secondary users that may be demanding access to each of the channels denoted by $\widetilde{N}_s(k)$ where $k = 1, 2, \cdots, L$ is assumed to be known using forecasting tool developed by the authors and presented in [11].

Under this system model, we propose a two-stage optimization framework where we find transmit power and rate separately. It is important to note that we use the terms rate and bits/channel use interchangeably throughout the paper. One can also visualize the bits/channel use measure to indicate the modulation order employed by the SU in a channel. Table I defines most of the relevant terms used throughout the paper.

III. Proposed Optimization Framework

In this section, we present a detailed discussion of the optimization framework that we use to determine the best strategies for SUs from a resource utilization and QoS standpoint. We assume that the central controller has prior knowledge (based on traffic models and our proposed forecasting method in [11]) on which users are currently occupying each of the available channels. We decompose the minimization of total transmit power and maximization of total rate of SUs into two separate stages.

A. Stage 1: Optimal Transmit Power

In first stage, our objective is to determine the best choice for SU transmit powers in each channel such that the overall power consumed is minimized and QoS (defined in terms of SINR) is maintained. The mathematical description of first stage corresponds to:

$$\begin{aligned}
&\text{Determine} && \mathbf{p} \\
&\text{To Minimize} && \sum_{k=1}^{L} \sum_{i=1}^{\tilde{N}_s(k)} p_i(k) \\
&\text{subject to} && C1: \ 0 \le p_i(k) \le p_i^{max}(k), \ \forall \ i, \ k \\
& && C2: \ \sum_{i=1}^{\tilde{N}_s(k)} p_i(k) h_{i,m}(k) \le I_{th}(k), \ \forall \ k \\
& && C3: \ \gamma_i(k) \ge \gamma_i^{th}(k), \ \forall \ i, \ k; \qquad (1)
\end{aligned}$$

TABLE I.
NOTATIONS

$\widetilde{N}_s(k)$	Predicted number of users for k-th channel
$\sigma^2(k)$	Noise variance in k-th channel
$\rho_{j,i}$	Orthogonality factor between users j and i
$h_{i,i}(k)$	Power gain from i-th transmitter to i-th receiver in k-th channel
$h_{i,m}(k)$	Power gain from i-th transmitter at location m in k-th channel
$p_i(k)$	Transmit power per bit of i-th user in k-th channel
$p_i^{max}(k)$	Maximum transmit power per bit of i-th user in k-th channel
$I_{th}(k)$	Interference temperature constraint in k-th channel
$b_i(k)$	Rate of i-th user in k-th channel
$b_i^{max}(k)$	Maximum rate of i-th user in k-th channel
$R_{ch}^u(k)$	Maximum rate supported by k-th channel
R_i^l	Minimum required rate for i-th user
$p_{e,i}(k)$	BER for i-th user in k-th channel
$p_{e,i}^{th}$	BER threshold at receiver for i-th user in any channel
$\gamma_i(k)$	SINR per bit for i-th user in k-th channel
$\gamma_i^{th}(k)$	SINR per bit threshold at receiver for i-th user in k-th channel

where

$$\gamma_i(k) = \frac{p_i(k)h_{i,i}(k)}{\sum_{j=1,\ j\neq i}^{\widetilde{N}_s(k)} p_j(k)h_{j,i}(k){\rho_{j,i}}^2 + \sigma^2(k)},\ \forall\ i,\ k. \quad (2)$$

Here, $\mathbf{p} = [p_1(1), \cdots,\ p_{\widetilde{N}_s(1)}(1), \cdots,\ p_{\widetilde{N}_s(L)}(L)]^T$; $C1$ indicates limit on transmit power; $C2$ indicates the interference temperature constraint, and $C3$ is SINR constraint required to guarantee desired QoS. This is a convex optimization (linear programming) problem. The convexity of QoS/SINR constraint ($C3$) is discussed in Theorem 1 in Appendix A. Solution to this optimization problem provides optimal transmit powers that every secondary user needs to use in the channels that they are operating in.

In the following subsection, we derive the user-based distributed approach to solve the above proposed optimization problem.

Distributed Implementation of Stage 1: The centralized solution of the first stage requires a central controller and information about all users and channels. That is, centralized power allocation demands extensive control signalling and is difficult to implement in practice. Hence, we develop a distributed user-based approach to solve stage 1 of our proposed optimization framework. We use the dual decomposition of the optimization problem in order to derive the user-based power allocation algorithm.

For ease in presentation, we assume that there are equal number of users in each of the channels. The discussion below can be easily extended to the case when there are different number of users in each channel. Stage 1 of the proposed optimization problem (Eq. (1)) has one coupled constraint ($C2$) and one cross power term ($\sum_{j=1,\ j\neq i}^{\widetilde{N}_s(k)} p_j(k)h_{j,i}(k){\rho_{j,i}}^2$) in constraint $C3$. Introducing an auxiliary variable $in_i(k)$ (representing the interference power that user i experiences in k-th channel) for the cross power term, the optimization problem can be restated as

$$\begin{aligned}
&\text{Determine} && [\mathbf{p}^T\ \mathbf{in}^T]^T \\
&\text{To Minimize} && \sum_{k=1}^{L}\sum_{i=1}^{M} p_i(k) \\
&\text{subject to} && C1:\ 0{\leq}p_i(k){\leq}p_i^{max}(k),\ \forall\ i,\ k \\
& && C2:\ \sum_{i=1}^{M} p_i(k)h_{i,m}(k){\leq}I_{th}(k),\ \forall\ k \\
& && C3:\ p_i(k)h_{i,i}(k){\geq}(in_i(k) + \sigma^2(k)) \\
& && \times\gamma_i^{th}(k), \forall\ i,\ k; \\
& && C4:\ in_i(k){\geq}C_i(k),\ \forall\ i,\ k; \qquad (3)
\end{aligned}$$

where, $\mathbf{in} = [in_1(1), \cdots,\ in_M(1), \cdots,\ in_M(L)]^T$ and $C_i(k)$ equal to $\sum_{j=1,\ j\neq i}^{M} p_j(k)h_{j,i}(k){\rho_{j,i}}^2$ is the lower bound for $in_i(k)$. From (3), the Lagrangian of stage 1 optimization problem can be written as

$$\begin{aligned}
&\text{Determine} && [\mathbf{p}^T\ \mathbf{in}^T]^T \\
&\text{To Minimize:} && \mathcal{L}_{ts} = \sum_{k=1}^{L}\sum_{i=1}^{M} p_i(k) \\
& && + \sum_{k=1}^{L}\lambda(k)\left(\sum_{i=1}^{M} p_i(k)h_{i,m}(k) - I_{th}(k)\right) \\
& && + \sum_{k=1}^{L}\sum_{i=1}^{M}\nu_i(k)(in_i(k) - C_i(k)) \\
&\text{subject to} && CD1:\ 0{\leq}p_i(k){\leq}p_i^{max}(k),\ \forall\ i,\ k \\
& && CD2:\ p_i(k)h_{i,i}(k){\geq}(in_i(k) + \sigma^2(k)) \\
& && \times\gamma_i^{th}(k), \forall\ i,\ k; \\
& && CD3:\ in_i(k){\geq}C_i(k),\ \forall\ i,\ k; \qquad (4)
\end{aligned}$$

Here, $\lambda(k)$ and $\nu_i(k)$ are dual variables. Rearranging (4) results in

$$\begin{aligned}
&\text{Determine} && [\mathbf{p}^T\ \mathbf{in}^T]^T \\
&\text{To Minimize:} && \\
& && \mathcal{L}_{ts} = \sum_{i=1}^{M}\sum_{k=1}^{L}(p_i(k) + \lambda(k)p_i(k)h_{i,m}(k) \\
& && +\nu_i(k)in_i(k)) - \sum_{k=1}^{L}\lambda(k)I_{th}(k) \\
& && -\sum_{k=1}^{L}\sum_{i=1}^{M}\nu_i(k)C_i(k) \\
&\text{subject to} && \\
& && CD1,\ CD2,\ CD3. \qquad (5)
\end{aligned}$$

Now, we can easily decompose the optimization problem (5) into M subproblems. Based on how we model the impact of $in_i(k)$ in each of the subproblems, three formulations for decomposed problem from Lagrangian (5) can be derived.

CASE 1: For the scenario when $in_i(k)$ is assumed constant but measurable, each of the subproblems can be written as

$$\begin{aligned}
&\text{Determine} && \mathbf{p}_i \\
&\text{To Minimize:} && g_i(\mathbf{p}_i,\ \boldsymbol{\lambda}) = \sum_{k=1}^{L} p_i(k)\,(1 + \lambda(k)h_{i,m}(k)) \\
&\text{subject to} && \\
& && CDL1:\ 0 \le p_i(k) \le p_i^{max}(k),\ \forall\ k \\
& && CDL2:\ p_i(k)h_{i,i}(k) \ge (in_i(k) + \sigma^2(k)) \\
& && \times\gamma_i^{th}(k), \forall\ k, \qquad (6)
\end{aligned}$$

where,

$$\mathbf{p}_i = [p_i(1),\ p_i(2), \cdots,\ p_i(L)]^T, \qquad (7)$$

$$\boldsymbol{\lambda} = [\lambda(1),\ \lambda(2), \cdots,\ \lambda(L)]^T, \qquad (8)$$

$$in_i(k) = \sum_{j=1,\ j\neq i}^{M} p_j(k)h_{j,i}(k){\rho_{j,i}}^2, \qquad (9)$$

$\mathbf{p}_i$ and $g_i(\mathbf{p}_i,\ \boldsymbol{\lambda})$ are the transmit powers across different channels and the Lagrangian function for user i, respectively. The corresponding master dual problem is

$$\begin{aligned}
&\text{Determine} && \boldsymbol{\lambda} \\
&\text{To Minimize:} && \sum_{i=1}^{M} g_i(\mathbf{p}_i,\ \boldsymbol{\lambda}) - \sum_{k=1}^{L}\lambda(k)I_{th}(k) \\
&\text{subject to} && \\
& && \boldsymbol{\lambda} \ge \mathbf{0}. \qquad (10)
\end{aligned}$$

The user-based distributed power allocation algorithm can be summarized as follows. Dual variables $\boldsymbol{\lambda}$ are initialized. $\mathbf{in}_i$ are measured. Each user executes one optimization problem to compute transmit power for each of its intended channels. At regular intervals, each user measures $\mathbf{in}_i$ and updates the dual variables. Each user continues to do the same until it achieves desired SINR along with satisfying system constraint ($C2$). The pseudo code for the algorithm is shown in Algorithm 1. In Algorithm 1, t is the iteration counter, α is a sufficiently small positive step-size.

It is important to note that the distributed approach does not fully avoid central control. This is due to the requirement of updating dual variables $\boldsymbol{\lambda}$. The dual variables capture information regarding how well the interference temperature threshold constraint is being satisfied. If the interference temperature threshold constraint is violated, then the corresponding dual variable increases in magnitude. This increase forces the objective function in our optimization problem to increase. To counter this effect, the optimization variables (power of users) are reduced which in turn improves the ability of satisfying the interference temperature threshold constraint.

CASE 2: Consider the case when $in_i(k)$ is assumed a variable. However, in each iteration of distributed approach, a lower bound for this interference is measurable. In this case, each of the subproblems can be written as,

$$\begin{aligned}
&\text{Determine} && [{\mathbf{p}_i}^T\ {\mathbf{in}_i}^T]^T \\
&\text{To Minimize:} && g_i(\mathbf{p}_i,\ \mathbf{in}_i,\ \boldsymbol{\lambda}) = \sum_{k=1}^{L} p_i(k) \\
& && \times(1 + \lambda(k)h_{i,m}(k)) \\
&\text{subject to} && \\
& && CDL1,\ CDL2 \\
& && CDL3:\ in_i(k) \ge C_i(k),\ \forall\ k, \qquad (11)
\end{aligned}$$

where, $C_i(k)$ is the lower bound for $in_i(k)$ and equal to $\sum_{j=1,\ j\neq i}^{M} p_j(k)h_{j,i}(k){\rho_{j,i}}^2$. The corresponding master dual problem is

$$\begin{aligned}
&\text{Determine} && \boldsymbol{\lambda} \\
&\text{To Minimize:} && \sum_{i=1}^{M} g_i(\mathbf{p}_i,\ \mathbf{in}_i,\ \boldsymbol{\lambda}) - \sum_{k=1}^{L}\lambda(k)I_{th}(k) \\
&\text{subject to} && \\
& && \boldsymbol{\lambda} \ge \mathbf{0}. \qquad (12)
\end{aligned}$$

The pseudo code for the corresponding algorithm is shown in Algorithm 2. In Algorithm 2, $\mathbf{C}_i = [C_i(1), \cdots, C_i(L)]^T$ and α is as defined before.

CASE 3: An alternative formulation can be created by absorbing the constraint $CD3$ in the objective function. The subproblems for this case can be formulated as

$$\begin{aligned}
&\text{Determine} && [{\mathbf{p}_i}^T\ {\mathbf{in}_i}^T]^T \\
&\text{To Minimize:} && g_i(\mathbf{p}_i,\ \mathbf{in}_i,\ \boldsymbol{\lambda},\ \boldsymbol{\nu}_i) = \sum_{k=1}^{L}(p_i(k) \\
& && \times(1 + \lambda(k)h_{i,m}(k)) + \nu_i(k)in_i(k)) \\
&\text{subject to} && \\
& && CDL1,\ CDL2,\ CDL3. \qquad (13)
\end{aligned}$$

The corresponding master dual problem is

$$\begin{aligned}
&\text{Determine} && [\boldsymbol{\lambda}^T\ \boldsymbol{\nu}_1^T\ \cdots\ \boldsymbol{\nu}_i^T\ \cdots\ \boldsymbol{\nu}_M^T]^T \\
&\text{To Minimize:} && \sum_{i=1}^{M} g_i(\mathbf{p}_i,\ \mathbf{in}_i,\ \boldsymbol{\lambda},\ \boldsymbol{\nu}_i) - \sum_{k=1}^{L}\lambda(k)I_{th}(k) \\
& && -\sum_{k=1}^{L}\sum_{i=1}^{M}\nu_i(k)C_i(k) \\
&\text{subject to} && \\
& && \boldsymbol{\lambda} \ge \mathbf{0} \\
& && \boldsymbol{\nu}_i \ge \mathbf{0}; \qquad (14)
\end{aligned}$$

where,

$$\boldsymbol{\nu}_i = [\nu_i(1),\ \nu_i(2), \cdots, \nu_i(L)]^T. \qquad (15)$$

The pseudo code for the corresponding algorithm is shown in Algorithm 3. In Algorithm 3, β is also a sufficiently small positive step-size, α is as defined before and $[\cdot]^+$ denotes the projection onto nonnegative orthant.

Algorithm 1 Dual Algorithm to solve (5) based on **CASE 1**

```
Initialization: λ(0);
while termination criterion is not true do
  {% Execute subproblems}
  for i = 1, 2, ··· , M do
    Measure in_i;
    Solve optimization subproblem (6);
  end for
  {% Update λ}
  for k = 1, 2, ··· , L do
    if (∑_{i=1}^{M} p_i(k)h_{i,m}(k) > I_th(k)) then
      λ^{t+1}(k) = [λ^t(k) − α(−∑_{i=1}^{M} p_i(k)h_{i,m}(k) + I_th(k))];
    else
      λ^{t+1}(k) = λ^t(k);
    end if
  end for
end while
```

Algorithm 2 Dual Algorithm to solve (5) based on **CASE 2**

```
Initialization: λ(0);
while termination criterion is not true do
  {% Execute subproblems}
  for i = 1, 2, ··· , M do
    Measure C_i;
    Solve optimization subproblem (11);
  end for
  {% Update λ}
  for k = 1, 2, ··· , L do
    if (∑_{i=1}^{M} p_i(k)h_{i,m}(k) > I_th(k)) then
      λ^{t+1}(k) = [λ^t(k) − α(−∑_{i=1}^{M} p_i(k)h_{i,m}(k) + I_th(k))];
    else
      λ^{t+1}(k) = λ^t(k);
    end if
  end for
end while
```

In summary, based on a priori information or ability to measure interference power, we can formulate the different versions of distributed implementation of stage 1. It is also important to note that initializing dual variables and choice of step sizes are critical in convergence and speed of the distributed solution [9].

B. Stage 2: Optimal Rate

In the second stage, we attempt to satisfy the rate requirement for each secondary user. Our goal in this stage is to determine how each SU distributes its information across the multiple channels in a way that the overall rate is maximized and the individual rate requirement is

Algorithm 3 Dual Algorithm to solve (5) based on **CASE 3**

```
Initialization: λ(0);
Initialization: ν_1(0), ··· , ν_i(0), ··· , ν_M(0);
Measure C_1, ··· , C_i, ··· , C_M;
while termination criterion is not true do
  {% Execute subproblems}
  for i = 1, 2, ··· , M do
    Solve optimization subproblem (13);
  end for
  {% Update λ}
  for k = 1, 2, ··· , L do
    if (∑_{i=1}^{M} p_i(k)h_{i,m}(k) > I_th(k)) then
      λ^{t+1}(k) = [λ^t(k) − α(−∑_{i=1}^{M} p_i(k)h_{i,m}(k) + I_th(k))];
    else
      λ^{t+1}(k) = λ^t(k);
    end if
  end for
  {% Update ν_1, ··· , ν_i, ··· , ν_M}
  for i = 1, 2, ··· , M do
    Measure C_i;
    for k = 1, 2, ··· , L do
      ν_i^{t+1}(k) = [ν_i^t(k) − β(−in_i(k) + C_i(k))]^+;
    end for
  end for
end while
```

met. Employing the optimal transmit powers and SINRs from first stage, the following rate allocation problem is proposed:

$$
\begin{aligned}
&\text{Determine} && \mathbf{b} \\
&\text{To Maximize} && \\
& && \sum_{k=1}^{L}\sum_{i=1}^{\tilde{N}_s(k)} b_i(k) \\
&\text{subject to} && \\
& && C5:\ b_i(k)\in[1,\cdots,\ b_i^{max}(k)],\ \forall\, i,\ k \\
& && C6:\ \sum_{i=1}^{\tilde{N}_s(k)} b_i(k)\le R_{ch}^{u}(k),\ \forall\, k \\
& && C7:\ \sum_{k=1}^{L} b_i(k)\ge R_i^{l}\ \forall\, i, \\
& && C8:\ p_{e,i}(k)\le p_{e,i}^{th},\ \forall\, i,\ k; \qquad (16)
\end{aligned}
$$

where,

$$
p_{e,i}(k) = \frac{4}{b_i(k)}\left(1-2^{-\frac{b_i(k)}{2}}\right)Q\left(\sqrt{\frac{3b_i(k)\gamma_i(k)}{(2^{b_i(k)}-1)}}\right), \quad \forall\, i,\ k,\ \text{even } b_i(k); \qquad (17)
$$

$$
p_{e,i}(k)\le\frac{4}{b_i(k)}Q\left(\sqrt{\frac{3b_i(k)\gamma_i(k)}{(2^{b_i(k)}-1)}}\right),\ \forall\, i,\ k,\ \text{odd } b_i(k). \qquad (18)
$$

Here, $\mathbf{b} = [b_1(1),\cdots,\ b_{\tilde{N}_s(1)}(1),\cdots,\ b_{\tilde{N}_s(L)}(L)]^T$; $C5$ indicates limit on rate; $C6$ indicates total rate that a channel can support, $C7$ captures the rate requirements for each SU, and $C8$ is BER requirement for every SU.

It is to be noted that $Q(x)$ is defined as $\int_x^\infty e^{-\zeta^2/2} d\zeta$. It is very easy to show that to achieve a certain BER, constraint $C8$ is equivalent to the following constraint

$$C9: \ -\gamma_i(k) \leq -C_{qarg}(2^{b_i(k)}-1), \ \forall\ i,\ k; \qquad (19)$$

where, C_{qarg} is a constant and can be determined using (1) minimum rate, $b_i^{min}(k) = 1$ (in our system); (2) $b_i^{max}(k)$, and (3) value of $p_{e,i}^{th}$ from $C8$. As an example, with $b_i^{min}(k) = 1$ to achieve $p_{e,i}^{th} = 10^{-3}$, Eq. (18) suggests that $\gamma_i(k)/(2^{b_i(k)}-1)$ has to be greater than 4.08 and with $b_i^{max}(k) = 6$, Eq. (17) suggests that $\gamma_i(k)/(2^{b_i(k)}-1)$ has to be greater than 0.50. From this, we can conclude that by setting $C_{qarg} = 4.08$, we can guarantee a BER that is less than or equal to 10^{-3} for the feasible values of $b_i(k)$. For ease in presentation, we define $Q_{qarg,i}(k) = -\gamma_i(k)/(2^{b_i(k)}-1)$; so that constraint $C9$ can be rewritten as

$$C9: \ Q_{qarg,i}(k) \leq -C_{qarg}. \qquad (20)$$

1) Rate Distribution by Graph Theoretic Analysis: The rate allocation problem (Eq. (16)) can be formulated as a maximum flow problem in a directed network in graph theory. A directed network is expressed as $G = (N,\ A)$ defined by a set N of n nodes and a set A of m directed links [12]. Each link $(i,\ j) \in A$ is associated with a capacity u_{ij} that denotes the maximum amount that can flow on the link and a lower bound l_{ij} that denotes the minimum amount that must flow on the link. The maximum flow problem seeks a feasible solution that sends the maximum amount of flow from a specified source node s to another specified sink node d in such a directed network. The rate measure in our problem of interest takes the role of flow in the maximum flow problem formulation. Therefore, the equivalent graph formulation of Eq. (16) corresponds to Fig. 1.

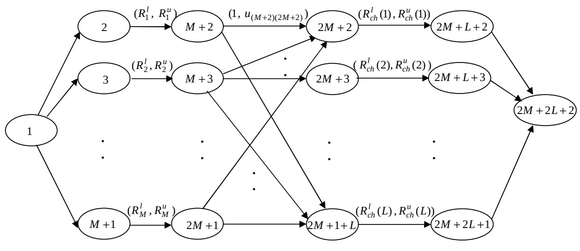

Figure 1. Rate distribution problem as a maximum flow problem in graph theory.

In the graph shown in Fig. 1, nodes 1 and $2M + 2L + 2$ are the source and sink nodes, respectively. Nodes $(2,\ 3, \cdots,\ M+1,\ M+2, \cdots,\ 2M+1)$ and $(2M+2,\ 2M+3, \cdots,\ 2M+1+L,\ 2M+L+2, \cdots,\ 2M+2L+1)$ represent users and channels, respectively. The lower bounds on link capacities between nodes $(2,\ 3, \cdots,\ M+1)$ and $(M+2, \cdots,\ 2M+1)$ can be obtained from the minimum rate requirement of the users (constraint $C7$). The upper bounds on link capacities for these links can be set to some reasonable high values. As for example, the upper bound can be set to the value obtained by multiplying the maximum capacity of the links between nodes $(M+2,\ M+3, \cdots,\ 2M+1)$ and $(2M+2,\ 2M+3, \cdots,\ 2M+1+L)$ by the number of total available channels. The upper bounds on link capacities between nodes $(2M+2,\ 2M+3, \cdots,\ 2M+1+L)$ and $(2M+L+2,\ 2M+L+3, \cdots,\ 2M+2L+1)$ can be obtained from the maximum rate supporting capabilities of the channels (constraint $C6$). The lower bounds on link capacities for these links i.e., $R_{ch}^l(k)$'s can be set to 0. The lower and upper bounds on link capacities between nodes $(M+2,\ M+3, \cdots,\ 2M+1)$ and $(2M+2,\ 2M+3, \cdots,\ 2M+1+L)$ can be computed from the constraints $C5$ and $C9$ with obtained SINR in first stage. The upper bounds on the link capacities between nodes 1 to $(2,\ 3, \cdots,\ M+1)$ and $(2M+L+2,\ 2M+L+3, \cdots,\ 2M+2L+1)$ to $2M+2L+2$ can be set to the total capacity of the links between nodes between $(M+2,\ M+3, \cdots,\ 2M+1)$ to $(2M+2,\ 2M+3, \cdots,\ 2M+1+L)$.

There are several algorithms to solve maximum flow problem such as labeling algorithm, capacity scaling algorithm, successive shortest path algorithm. The running time of the labeling algorithm, capacity scaling algorithm and successive shortest path algorithm are $O(nmU)$, $O(nmlogU)$ and $O(n^2m)$, respectively [12]. Here, U is the maximum capacity of the links in the network. The running time may increase with a high number of nodes (n) or links (m) or maximum capacity (U) of the link in the network. The running time of the above mentioned algorithms for finding maximum flow in a network corresponding to Fig. 1 is shown as $O(M^2LU)$, $O(M^2LlogU)$ and $O(M^2L)$, respectively. In the following, we develop an heuristic to solve problem (16).

2) Rate Distribution by Proposed Heuristic: The rate allocation algorithm that can be employed to solve rate allocation problem (16) as follows. First, we allocate the maximum feasible rate (i.e., maximum $b_i(k)$ that satisfies Eq. (20)) to all users across channels. Based on this allocated rate, the average $Q_{qarg,i}(k)$ is calculated for all users and compared with $-C_{qarg}$ in the next step. For a specific user, if average $Q_{qarg,i}(k)$ does not satisfy constraint $C9$, the maximum rate allocated to a channel for that user is reduced by 1. This process is repeated until constraint $C9$ is satisfied or a maximum number of iterations (l_1^{max}) are completed. In the latter case, the average $Q_{qarg,i}(k)$ (though not satisfactory) is the achievable $Q_{qarg,i}(k)$ for that user. In the following step, the algorithm checks if the total rate limits that are set for all channels are violated. If the rate constraint per channel (constraint $C6$) is not met, the maximum rate allocated to a user in that channel is reduced by 1. This process is repeated until constraint $C6$ is satisfied or a maximum number of iterations (l_2^{max}) are completed.

The running time of our developed heuristic is shown as $O(M(max\{L,\ l_1^{max},\ l_2^{max}\}))$ which is less than that of the optimal graph theoretic algorithms.

In summary, the two-stage optimization framework decomposes the power calculation and rate allocation into two stages. In the first stage, the transmit power for every

TABLE II.
USAGE PATTERN ACROSS CHANNELS

Channel, k	1	2	3	4	5	6	7	8	9	10	11
User, 1	1	0	1	0	0	1	1	0	0	0	1
User, 2	1	1	1	0	0	1	1	0	1	0	1
User, 3	0	1	1	0	1	1	0	0	1	0	1
User, 4	0	1	1	0	1	1	1	0	1	1	1
User, 5	0	1	1	1	1	1	1	1	1	1	1
User, 6	0	1	1	1	1	0	1	1	1	1	1
User, 7	0	1	0	1	1	0	1	1	0	1	1
User, 8	1	1	1	0	1	0	1	1	0	1	1
User, 9	1	0	1	0	1	0	1	1	0	1	1
User, 10	1	0	1	0	1	1	0	1	0	1	1

TABLE III.
CHANNEL QUALITY PARAMETERS

Channel, k	$\sigma^2(k)$ $(\times 10^{-3})$
1	5.0
2	4.0
3	3.0
4	2.0
5	2.5
6	6.0
7	4.0
8	4.0
9	5.0
10	3.5
11	4.5

SU is governed by the SINR threshold and in the second stage, we attempt to maximize the rate for each SU given the BER requirement.

IV. NUMERICAL RESULTS

In this section, we evaluate the performance of the optimization framework. We assume a CRN with $L = 11$ available channels and a total of $M = 10$ secondary users. We assume a usage pattern as shown in Table II, where a 1 indicates that the corresponding channel is being used by the SU. Table III provides information on the channel quality for all L channels. Table IV lists the minimum rate requirement for each SU. Finally, Table V contains all other system parameters that are relevant to our optimization framework. Based on all this information, our objective is to find the optimal transmit power and rate that each of the M SUs should employ to guarantee their QoS.

As discussed in Section III, the first stage of optimization framework is a linear programming problem, and any LP solver can be used to find the solution. In this work, we use the "Linear Interior Point Solver (LIPSOL)" to solve stage 1. We use "Mixed Integer Programming (MIP)" to solve second stage for optimal rate distribution.We set the SINR threshold, $\gamma_i^{th}(k)$ to 12 dB. Based on the system

TABLE IV.
MINIMUM RATE REQUIREMENT OF USERS

User, i	1	2	3	4	5	6	7	8	9	10
R_i^l	3	8	4	12	9	7	14	5	10	8

TABLE V.
SYSTEM PARAMETERS

$p_i^{max}(k)\ \forall\ i,\ k$	5
$b_i^{max}(k)\ \forall\ i,\ k$	6
$p_{e,i}^{th}\ \forall\ i$	10^{-3}
$I_{th}(k)\ \forall\ k$	$200 \times \sigma^2(k)$
$\rho_{j,i}$	0.03125

parameters defined earlier, we can calculate C_{qarg} at 4.5. For second stage, we set $R_{ch}^u(k)$ to 20 for all channels.

Figure 2 illustrates the transmit power and rate allocation across channels for users 1 and 8. The channel noise variance and resulting SINR are also plotted for reference. Here, user 1 operates on channels 1, 3, 6, 7 and 11; user 8 operates on channels 1, 2, 3, 5, 7, 8, 10 and 11. From Fig. 2, it is evident that for both users, higher transmit powers are allocated to channels with higher noise variance. In other words, optimal transmit power allocation follows "reverse water filling" process since the goal is to satisfy the minimum SINR threshold. Figure 2 also indicates that the SINR threshold is attained in every channel. The allocated rate across channels directly follows SINR and since SINR is maintained at the threshold value, the rate allocation is also a constant across channels. The allocated power and rate for other users follow the pattern presented for users 1 and 8. Figure 3 shows the total transmit power and rate allocation across users. From this figure, we can conclude that the proposed optimization framework has been successful in meeting the rate requirement for every active SU. Figure 4 captures the effects of increasing number of users on the total transmit power and rate for user 1. It is clear that with increase in the number of users in the system (i.e., increasing the number of users in the channels based on usage pattern), user 1 is forced to use higher transmit power. This is because, with increase in number of users, interference increases and therefore more power is required to satisfy the SINR threshold. Since, SINR is maintained at the threshold level irrespective of the number of users, the total rate (that is a function of SINR) for user 1 remains unchanged as seen in Fig. 4.

Figure 5 shows us the allocation of transmit power across users obtained from three formulations of distributed approach, i.e., CASE 1, CASE 2 and CASE 3 with centralized solution. In each case, we initialize dual variable $\lambda(k)$ to 0 for all channels. The step size α is set to 0.1. For CASE 3, $\nu_i(k)$ is initialized to 0 for all users and channels and the step size β is set to 0.1. From Fig. 5, we can conclude that the solution from each of the distributed formulations converges to the centralized solution.

The distributed formulations require measurement of interference power ($\mathbf{in}_i$ for CASE 1, $\mathbf{C}_i$ for CASE 2 and 3). Figure 6 shows the number of iterations that the three formulations of distributed approach need to converge with error (overestimation) in measurements. This figure illustrates that with increase in percentage of error, the

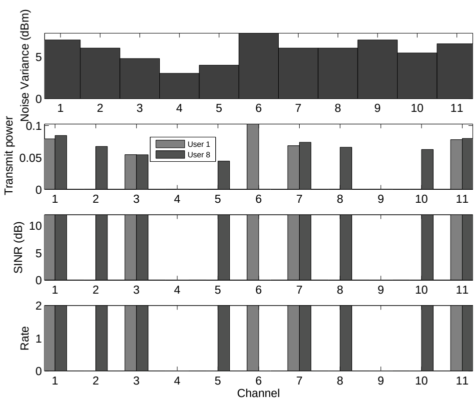

Figure 2. Allocation of transmit power and rate with channel noise variance and SINR for users 1 and 8.

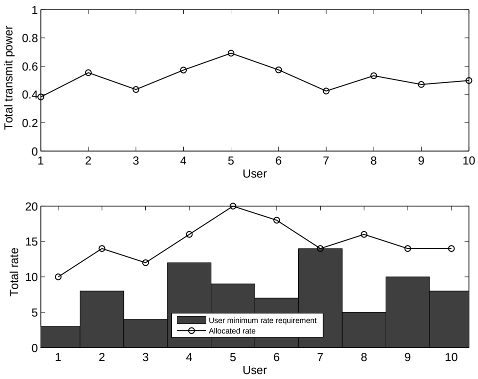

Figure 3. Allocation of total transmit power and total rate across users.

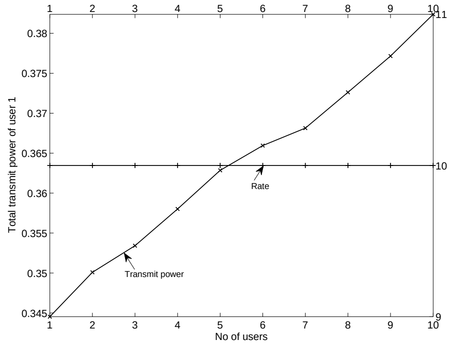

Figure 4. Total transmit power and total rate for user 1 with number of users.

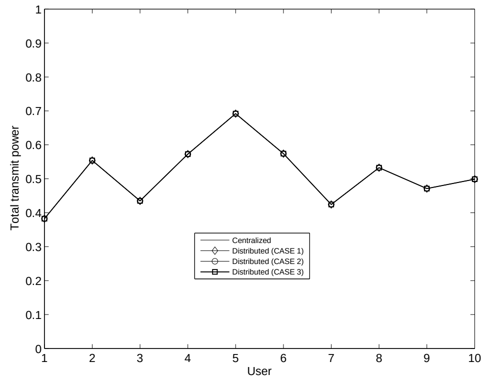

Figure 5. Allocation of total transmit power across users from different distributed approaches.

number of iteration decreases. The reason behind this behavior can be better understood by observing the impact of erroneous measurement of interference power in each of the subproblems. When $\mathbf{in}_i$ for CASE 1, $\mathbf{C}_i$ for CASE 2 and 3 are overestimated, then it causes an increase in the magnitude of optimizing variables $\mathbf{p}_i$. The increase in magnitude of optimizing variables improves the ability to satisfy the SINR constraints at a faster rate.

Figure 7 illustrates the rate distribution resulting from our proposed heuristic with optimal graph theoretic approach. We consider three example cases. In example 1, $R_{ch}^u(k)$ is set as 20 for all channels; In example 2, $R_{ch}^u(k)$ is set as 15 for all channels; In example 3, the maximum rate supporting capabilities of the channels are set as 10, 12, 14, 18, 15, 8, 11, 11, 8, 14 and 14, respectively for channels 1 through 11. The total minimum rate requirement for all users in the system is 80 in all examples. In example 1, both heuristic and graph theoretic approaches result in identical rate distribution for users. However in examples 2 and 3, the rate distribution profile from both approaches are different. In all three example cases, the total rate $\left(\sum_{k=1}^{L}\sum_{i=1}^{\widetilde{N}_s(k)} b_i^{opt}(k)\right)$ supported is found to be equal. From Fig. 7, we can conclude that our proposed heuristic performs comparable to optimal graph theoretic approach.

It is important to note that we have investigated the cases where we assume that a feasible distribution of transmit power and rate across all active SUs exists and we have shown that our proposed approach is a useful tool in determining that solution optimally. In practice, some cases may arise when the total minimum rate requirement of all active secondary users may exceed the rate supporting capability of the entire system or channel condition may be so poor that the desired SINR for all the active SUs is not attainable. In such scenarios, the only viable option is that some SUs may have to stop their transmission and retransmit when the conditions become favorable. Developing optimal scheduling policies for SUs to access the channels across time in such over-burdened systems is the focus of our future research.

V. Conclusion

In this paper, we propose a two-stage optimization framework that provides the optimal transmit power and rate distribution that each SU needs to employ while maintaining QoS in a multi-channel CRN. We assume that multiple secondary users may coexist in a single channel and a single secondary user can simultaneously employ multiple channels to meet its rate requirements. We show that optimal transmit power follows reverse water filling process and optimal rate allocation is proportional to SINR. We also observe that the dual decomposed user-

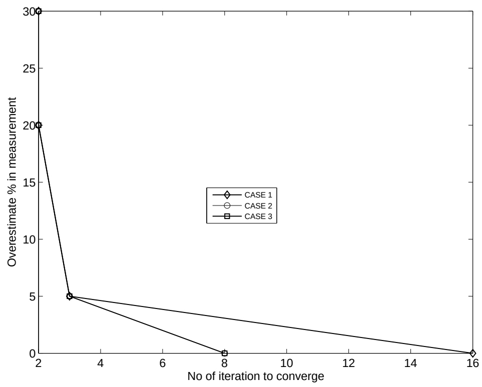

Figure 6. Convergence speed of the distributed approach with imperfect measurement of interference power of adjacent users.

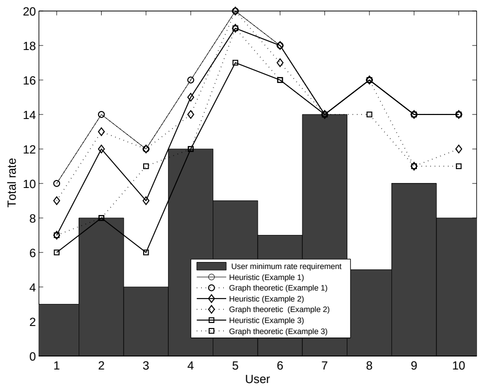

Figure 7. Total rate allocation across users from proposed heuristic and graph theoretic analysis.

based distributed solution of stage 1 converges to centralized solution and rate distribution in stage 2 based on our proposed heuristic is close to optimal graph theoretic solution.

APPENDIX A

Theorem 1: $\gamma_i(k) \geq \gamma_i^{th}(k), \ \forall \ i, \ k$ is a convex constraint.

Proof: From Eqs. (1) and (2), we can write the constraint as

$$p_i(k)h_{i,i}(k) \geq \gamma_i^{th}(k)\left(\sum_{j=1,\ j\neq i}^{\widetilde{N}_s(k)} p_j(k)h_{j,i}(k)\rho_{j,i}{}^2 + \sigma^2(k)\right). \quad (21)$$

Equation (21) is equivalent to

$$\gamma_i^{th}(k)\left(\sum_{j=1,\ j\neq i}^{\widetilde{N}_s(k)} p_j(k)h_{j,i}(k)\rho_{j,i}{}^2 + \sigma^2(k)\right) - p_i(k)h_{i,i}(k) \leq 0. \quad (22)$$

This inequality is a linear combination of the variables $p_i(k)$. Hence, the inequality is linear and can be treated as convex. ■

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