Capacity Enhancement in Hybrid Wireless Relay Network with Network Coding

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***Abstract*— Network coding technique increases wireless network communication efficiency. Wireless multihop relay network has been shown to achieve capacity gain over conventional single-hop wireless networks. Hybrid wireless relay networks integrate multihop ad hoc relay and infrastructure base stations to achieve better wireless network performance. Applying the promising network coding technique to hybrid wireless relay networks could increase wireless network capacity. Capacity of multihop cellular network with network coding is derived. To enhance network throughput, data forwarding strategy is proposed to determine if a packet should be transmitted through intra-cell ad hoc relay, inter-cell ad hoc relay, or infrastructure-assisted inter-cell transmission.**

***Index Terms*— network coding, hybrid wireless relay networks, inter-cell load balancing, network capacity, multihop cellular networks**

I. Introduction

Wireless ad hoc network architecture apply multihop forwarding scheme to deliver end-to-end user data traffic. Wireless network capacity of ad hoc relay network is derived in the seminal work by Gupta and Kumar [1]. In wireless ad hoc networks, data packets transmitted from source to destination are forwarded in a multi-hop fashion, which may be time-consuming and energy-wasting when the relay hop count is large. In contrast, in hybrid wireless relay network, this problem is solved by placing infrastructure base stations (BSs) sparsely. These BSs are assumed to be connected with high-speed wired backhaul network. Information can be transmitted through wireless ad hoc relay or through infrastructure BSs, as shown in Fig. I.

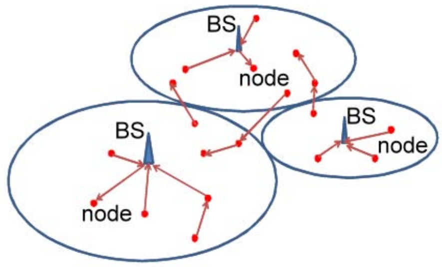

Figure 1. Hybrid wireless relay network architecture

The network capacity of hybrid wireless network has been studied recently [2]. This research work focus on the infrastructure-assisted hybrid network with n wireless nodes and m BSs uniformly and independently located in the network. The results show that the throughput capacity depends on the ratio of BSs and nodes. If m grows asymptotically slower than n, adding BSs cannot improve the throughput capacity greatly. However, if m grows faster than n ,the throughput capacity can increase linearly with the amount of BSs. The main differences between this paper and [2] are (1) we investigate the network capacity with network coding and (2) we consider both infra-structure assisted inter-cell transmission and intra-cell ad hoc relay transmission, while inter-cell communications in [2] is limited to infra-structure assisted transmission only.

The concept of network coding introduced by Ahlswede, Cai, Li and Yeung [5] may ameliorate the throughput capacity. The network coding technology has been intensively researched since then. Many researchers strive to achieve better network performance with the concept of network coding [6] [7] [8]. In wireless network, the application of network coding can really improve the throughput [6] [7]. Additionally, in [8], the researchers applied network coding into practical deployment and found out the throughput with network coding is significantly greater than throughput without network coding.

Furthermore, recent research gave the upper bound of the throughput capacity gain in ad hoc network with network coding [3] [4]. They find that in 2D ad hoc network, however, the benefit of network coding is of $c(1)$. In other word, the network coding can only improve the capacity in constant scale. They use the concept that the information transmitted will be more than the information needed. Base on this, they set a sparse cut line to restrain the throughput across the cut line. They analyze the maximum simultaneous transmission traffic across the cut and obtain the upper bound of throughput capacity. In our paper, we extend the cut capacity concept to model the hybrid wireless relay network with network coding.

In this paper, we apply network coding into hybrid network to seek network performance improvement. Due to interference limitation, transmissions to the same destination cannot occur simultaneously, so the traffic near BS may be over-loaded. To achieve higher throughput capacity, we assume that transmission to another cell need

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not across its BS in some conditions. We use a circular cut in a cell to model the capacity limitation across the cut. Then we calculate the average traffic across the cut and get the bound of throughput capacity. In addition to network capacity gain, hybrid wireless relay network architecture could be applied to achieve load balancing between neighboring cells [9]. Since network region close to the base station is usually the network bottleneck, the inter-cell transmission should not go through BS if possible . We derive an inter-cell traffic forwarding strategy to achieve maximum throughput.

We could summarize the multihop relay network capacity related research work as follows:

- ad hoc relay, without network coding [1]
- hybrid relay, without network coding [2]
- ad hoc relay, with network coding [4]
- hybrid relay, with network coding (this paper)

II. Model Formulation

A. Overview

In this section, we will describe a hybrid wireless relay network model which consists of two communication components. The first component is wireless ad hoc network. The network capacity model is investigated by Gupta and Kumar [1]. In the network capacity model n wireless nodes are randomly placed, i.e., independently and uniformly distributed in a region of fixed area. The second communication component is a sparse network of infrastructure base stations(BS). There are m BSs uniformly located in this area with a regular pattern. Furthermore, these n wireless nodes are homogeneous and assumed to employ the same transmission range r_0. A finite bandwidth limit of W bits per second is given for transmission between wireless nodes. Each BS is connected by a wired backhaul network. The base stations can transmit data with unlimited rate to any other BS. Data source nodes and destination nodes are selected randomly from wireless nodes.

B. Interference Model

All nodes and BSs share the same wireless channel in our model. We assume they transmit data over the wireless channel with a time-division multiplexing access scheme(TDMA). That is, time is divided into slots of fixed duration, and in each time slot, one node is scheduled to transmit data in the channel. Every other nearby node cannot transmit and receive data at the same time.

For the protocol model introduced in [1], a transmission from node X_i to X_j is successful if and only if

1. $|X_i - X_j| \leq r_0$: The transmission node pair X_i and X_j has to be within the transmission range.

2. $|X_j - X_k| > \Delta r_0$, $\forall k \neq i, j$: To avoid interference, any other simultaneously transmission from node k has to be far enough from X_j.

The interference range factor Δ depends on the property of the wireless medium. Fig. 2 illustrates two different cases of transmission that represent successful transmission and unsuccessful transmission respectively.

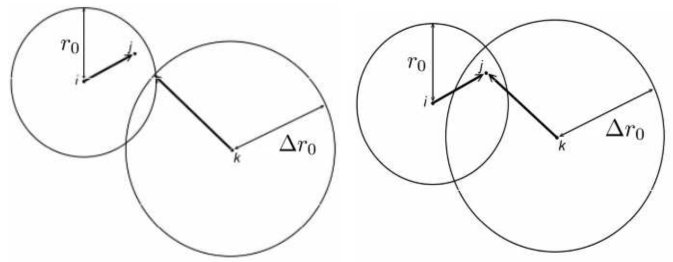


Figure 2. Successful transmission (left) & Unsuccessful transmission (right)

C. Inter-Cell Communications

In the network model, there are n wireless nodes in m cells. In order to ensure connectivity, the transmission radius of a node of the protocol model needs to be at least $r_0 = \sqrt{\frac{\log n}{n}}$ [1]. Each node i in the network is a data source that needs to route its data packets through multi hop wireless communication to another node which is a randomly chosen destination. The data transmission can either through pure ad hoc relay or passing through infrastructure BS. We denote the nearest BS, which is the base station of the serving cell, to a node X_i as $B(X_i)$.

With infrastructure-assisted relay, the source node X_i transmits data packets through base stations to the destination node Y_i. First, X_i would transmit data to $B(X_i)$, then forward to $B(Y_i)$ through the wired backbone network, and finally to its destination Y_i by relay. On the other hand, in the pure ad hoc relay mode, X_i can transmit data packets directly to Y_i by wireless multihop relay without going through base stations.

To investigate the inter-cell ad hoc relay to neighboring cells, we introduce the concept of neighbor region N_X for a wireless node X. As shown in Fig 3, the base station locates at the origin O of the cell C. For any given wireless node X, the line segment $\overline{XO}$ is the distance between wireless node X and the base station. A perpendicular line to segment $\overline{XO}$ creates an half plane S_X that does not include O. For any given node X, it may forward data to the destination node in its neighbor region through inter-cell ad hoc relay, since N_X covers the neighboring cell region that is close to X.

Definition 1: The neighbor region of node X, N_X include a set of nodes that are located in the intersection of S_X and the neighboring cells.

D. Network Traffic Characteristics

In this paper, we investigate the network capacity with a generalized traffic model, which considers the possibility of non-uniformly distributed intra-cell and inter-cell traffic. Notice that the uniformly distributed traffic scenario

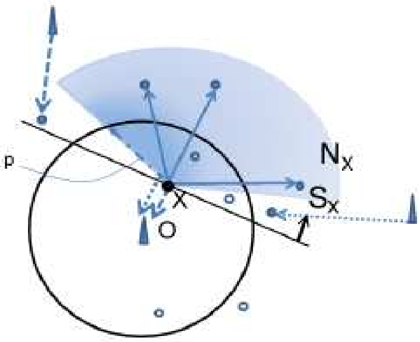


Figure 3. Neighbor N_X

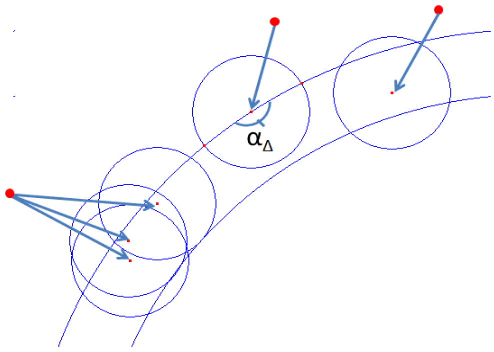


Figure 4. Cut capacity under protocol model

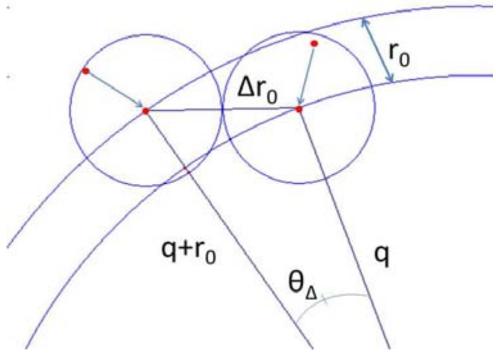


Figure 5. Cut Capacity $\Delta \geq 2$ case

that is studied in previous literatures is a special case of this generalized traffic model. We introduce parameters f and g to characterize the intra-cell (destination and source are in the same cell) and inter-cell traffic (destination and source are in different cells).

In our model, g is the ratio of intra-cell traffic to the total traffic, which includes both intra-cell and inter-cell traffic. The parameter f is the ratio of traffic to adjacent neighboring cells to the total inter-cell traffic. The parameter f characterized the locality of inter-cell traffic.

For example, there are m cells, m base stations(BS) and n nodes in the whole network. According to the model mentioned above, there are $\frac{n}{m}$ nodes in a cell. There are $\frac{n}{m}g$ nodes to transmit data to another node in the same cell. There are $\frac{n}{m}(1-g)$ nodes to transmit data to another node located in another cell. Among those inter-cell communications, $\frac{n}{m}(1-g)f$ nodes want to transmit data to another node located in one of the neighbor cells. Among that, $\frac{n}{m}(1-g)f \cdot \varphi$ nodes want to transmit data to another node located in one of the neighbor cells through the border of the cell, where φ is an integrated value of $n(r)$ which we would discuss later. For infrastructure-assisted inter-cell transmission to neighboring cell, $\frac{n}{m}(1-g)f \cdot (1-\varphi)$ nodes want to transmit data to another node located in one of the neighbor cells through the infrastructure BS. In addition, we normalize the traffic by dividing them by $\frac{n}{m}$, such that $(1-g)$ of it transmits outside the cell, and $(1-g) \cdot f$ of it transmits to neighbor cells.

III. CIRCULAR CUT CAPACITY

Capacity of wireless network with network coding could be modeled by the maximum information flow through a cut [4]. Extending the 2D linear cut capacity concept in [4], we introduce the concept of circular cut to model the network capacity for hybrid wireless relay network. In this section, we will compute the capability of a network transmitting network coded data flows based on the circular cut capacity. A circular cut is a circle with center located at the BS of a cell. The cut capacity depends on the maximum number of simultaneously success transmission across the cut, which in our model is determined by the circular cut radius q and the radio transmission range r_0.

With the protocol model, simultaneous transmissions across the cut form disjoint disks of radius $\frac{\Delta r_0}{2}$. It is obvious to see that all of the direct receivers of transmissions across the cut lie within the circular strip inside a circle of radius $q+r_0$ and outside a circle of radius q, with an area of $2qr_0 + r_0^2$. Note that disks with radius $\frac{r_0}{2}$ centered at receivers are disjoint from disks centered at the receivers of another transmission [3].We have:

Theorem 1: The capacity of a circular cut Γ with radius q has an upper bound of order $\Theta(\frac{qW}{r_0})$.

Proof: When $\Delta \leq 2$, each transmission across the circular cut at least blocks an area with the size of

$$\alpha_\Delta(\frac{\Delta r_0}{2})^2 \tag{1}$$

where $\alpha_\Delta = 2cos^{-1}(\frac{\Delta r_0}{4(q+r_0)})$.

This minimum blocking area appears when the receiver lies on the outer circle. Thus the cut capacity is upper-bounded by the area of circular strip divided by this minimum area, which is

$$\frac{(2qr_0 + r_0^2)W}{\alpha_\Delta(\frac{\Delta r_0}{2})^2} = \frac{4W}{\alpha_\Delta \Delta^2}(1 + \frac{2q}{r_0}) \tag{2}$$

, for $\frac{q}{r_0}$ large enough, α_Δ approximates constant π. When $\Delta \geq 2$, as shown in Fig. 5, receivers of different transmission result in a minimum central angle with the

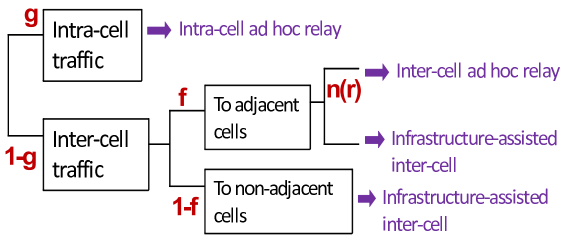


Figure 6. Classifications of user data traffic transmission

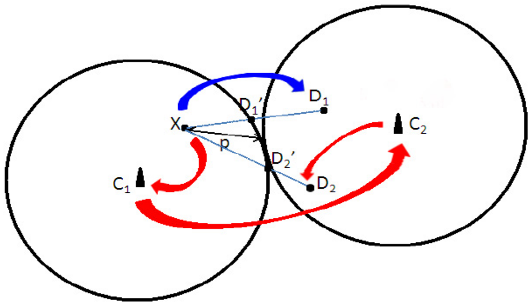


Figure 7. Traffic forwarding strategies: (a)inter-cell forwarding through cell boundary (b)infrastructure assisted inter-cell forwarding through base stations C_1 and C_2.

BS , by the law of cosine, we have

$$\theta_\triangle = cos^{-1}(1 - \frac{(\Delta^2-1)r_0^2}{2q(q+r_0)}) \tag{3}$$

. In this case, by inequality $cos^{-1}(1-x) \geq \sqrt{x}$, the circular cut capacity is upper bounded by:

$$\frac{2\pi W}{\theta_\triangle} \leq \frac{2\sqrt{2}\pi W}{\sqrt{\Delta^2-1}}\sqrt{\frac{q}{r_0}(\frac{q}{r_0}+1)} \tag{4}$$

, which is of the order $\Theta(\frac{qW}{r_0})$. ■

IV. Data Forwarding Strategies and Traffic Load

A. Data traffic forwarding strategies

As shown in Fig. 6, data traffic could be categorized as inter-cell and intra-cell. Traffic parameters (g, f) categorize the types of traffic flows in the network system. Inter-cell communication, in which destination node is located in other cells, may transmit with or without infrastructure base stations and wired backhaul. We introduce a traffic forwarding strategy to make decision from two possible methods: (1) *Infrastructure-assisted inter-cell transmission*: Data transmission uses ad hoc relay to forward packets toward the nearest BS first. Then data packets are transmitted to the base station in the destination cell. At last, data packets are relayed from the destination BS to the destination node. (2) *Inter-cell ad hoc relay*: Data transmission uses ad hoc relay to transmit directly across the border of cells to the destination located in a neighboring cell.

B. Determine Inter-Cell Traffic Forwarding Strategy

To ease BS's loading, we introduce a traffic forwarding strategy to decide how the traffic from a wireless node goes toward destination in its neighboring cells. For example, as shown in Fig. 7, node X may choose to relay data packets through ad hoc route via D_1'. For a node X and its destination $D \in N_X$, let D' be the intersection of line segment $\overline{XD}$ and the cell boundary C, as shown in Fig. 7. The traffic passes directly to D through ad-hoc network only when $\overline{XD'} \leq p$. Otherwise, the traffic from X would choose the route which goes to the infrastructure base stations C_1 and C_2. Here, we introduce p to determine whether inter-cell traffic should be forwarded via infrastructure or ad hoc relay. When $\overline{XD'} \leq p$, we denote $P_X = \{D|D \in N_X, \overline{XD'} \leq p\}$, and $P_X \subseteq N_X$. Denote r to be the distance between any given node X and the cell center O. For fixed value of p, no traffic will go through the boundary of the cell when $r + p \leq 1$. Notice that we consider normalized cellular topology with cell radius equals to 1. By the law of the cosine, we have

$$\theta_r = 2cos^{-1}(\frac{1-r^2-p^2}{2pr}) \tag{5}$$

, here θ_r is the angle such that all possible traffic should apply inter-cell ad hoc relay to route directly to neighboring cells. When $\theta_r \geq \pi$, no traffic to neighboring cells should go through infrastructure BS. Denote $n(r)$ as the ratio of inter-cell ad hoc relay traffic to the overall inter-cell traffic to neighbors, which is generated from nodes located r away from the BS. We have,

$$n(r) = \begin{cases} 0 & r \leq 1-p \\ \frac{\theta_r}{\pi} & 1-p < r \leq \sqrt{1-p^2} \\ 1 & \sqrt{1-p^2} < r \leq 1 \end{cases} \tag{6}$$

We will use $n(r)$ to determine the ratio of traffic that is transmitted through inter-cell ad hoc relay without going through BS among all inter-cell traffic to neighboring cells. The optimization of forwarding parameter p and the corresponding $n(r)$ will be discussed in Section V.

Notations:

- r_0: radio transmission radius.
- g: ratio of traffic to destination inside the cell.
- f: ratio of traffic to neighboring cell and traffic to destination in all outside cells.
- N_X: set of neighbors of particular wireless node X.
- P_X: set of neighbors of particular wireless node X to which data is allowed to send through boarder directly.
- $n(r)$: the ratio of traffic that does not go through BS toward destination in neighboring cells.
- Δ: the interference range is Δ times the distance between the sender and receiver.
- Γ: a circular cut with center at O
- $C(X_i)$: the cell which node X_i located in
- $B(X_i)$: the base station which is nearest to node X_i

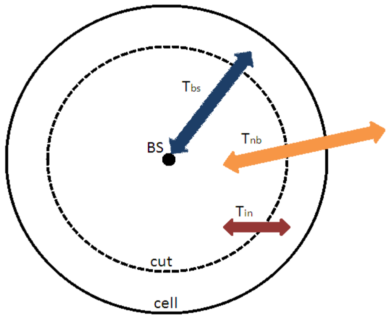


Figure 8. Definitions of $T_{bs}(r)$, $T_{nb}(r)$, and T_{in}

C. Traffic Across The Cut

We define T_Γ to be the traffic transmitted from one side to the other side of the cut Γ in both directions. T_Γ depends on several factors: the traffic pattern parameters f and g and, the most critically, the length of the circular cut's radius r. When $r \to 0$ the traffic across the cut is approximately equal to the traffic going to BS. On the contrary, while as $r \to 1$, the traffic approximates the total traffic transmit to neighbors of mobile nodes inside the cell. As shown in Fig.8, we categorize the traffic across a circular cut with radius r into three traffic flow types.

- infrastructure-assisted inter-cell transmission (T_{bs})
- inter-cell ad hoc relay (T_{nb})
- intra-cell ad hoc relay (T_{in})

Definition 2: $T_{bs}(r)$ is the amount of traffic contributed by nodes located outside the cut $\Gamma(r)$ transmitted to and received from the BS.

Definition 3: $T_{nb}(r)$ is the amount of traffic contribute by source nodes X_i locate within the cut $\Gamma(r)$ transmit to N_{X_i} and source nodes outside the cell transmit into the cut.

Definition 4: $T_{in}(r)$ is the amount of intra-cell ad hoc network traffic contributed by source-destination node pairs located within the same cell. The source node and the destination node are in different sides of the circular cut.

These three kinds of traffic contribute to the overall traffic across a circular cut. As illustrated in Fig. 8, we can get:

$$T_\Gamma(r) = T_{bs}(r) + T_{nb}(r) + T_{in}(r). \qquad (7)$$

The total amount of traffic and the portions of those three traffic types vary with the radius of cut Γ. As we assume the node density is uniform across the network, the number of nodes in certain region is directly proportional to the size of the region. Thus, with given parameters f and g, we can derive the expected value of these three types of traffic shown below.

Proposition 1: The inter-cell traffic flow through base station infrastructure is

$$T_{bs}(r) = 2(1-g)(1-f\cdot 2\pi\int_r^1 n(t)tdt) \qquad (8)$$

Proof: Similar to other network capacity papers, we also assume that nodes are uniformly distributed in cell C. Consider T_{bs} as the set of traffic connection source destination pairs (X,Y) which X and Y lie in different cells and $Y \notin N_X$, $X \notin N_Y$. Consider two subsets $T_\Gamma^1 \subset T$ and $T_\Gamma^2 \subset T$ that $T_\Gamma^1 = \{(X,Y)|X \text{ lies in } C \text{ but outside } \Gamma\}$ and $T_\Gamma^2 = \{(X,Y)|Y \text{ lies in } C \text{ but outside } \Gamma\}$. By symmetry, we have $|T_\Gamma^1| = |T_\Gamma^2|$, thus T_{bs} is two times to the traffic in one direction, i.e. transmitted from nodes outside Γ to BS. By definition of g, $(1-g)$ of the traffic generated from cell C is toward the cells other than C. Among the outgoing traffic, $f\cdot\int_r^1 n(t)dArea(\Gamma(t))$ of it will use inter-cell ad hoc relay through the border of C without going through any base station. Thus, we have $T_{bs}(r) = 2(1-g)(1-f\cdot 2\pi\int_r^1 n(t)tdt)$, by substituting $dArea(\Gamma(t)) = 2\pi tdt$. ■

Proposition 2: The inter-cell traffic flow through ad hoc relay is

$$T_{nb}(r) = 2(1-g)f\cdot 2\pi\int_0^r n(t)tdt \qquad (9)$$

.

Proof: The arguments are similar to the above proof. Now, we will consider the traffic in one direction, i.e. from nodes lie inside the cut Γ relaying their traffic through the border of C. We could derive the result based on the same argument. ■

Proposition 3: The intra-cell traffic flow through ad hoc relay is

$$T_{in}(r) = 2gr^2(1-r^2) \qquad (10)$$

.

Proof: By the same symmetry argument, we consider the half amount of intra-cell traffic with source node outside Γ. Since nodes are distributed uniformly in each cell, the traffic across Γ of radius r is proportional to $r^2(1-r^2)$. Thus, we have $T_{in} = 2gr^2(1-r^2)$. ■

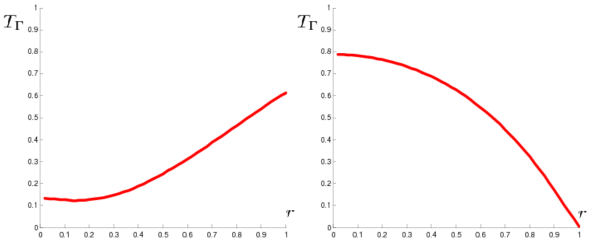
The overall information traffic flow is $T_\Gamma = T_{bs} + T_{nb} + T_{in}$. By reorganizing equations (8),(9),(10), we could derive

$$\begin{aligned} T_\Gamma(r) &= gr^2(1-r^2) + (1-g)\cdot \\ (1-f\cdot 2\pi\int_r^1 n(t)tdt &+ f\cdot 2\pi\int_0^r n(t)tdt) \end{aligned} \qquad (11)$$

For given traffic loading parameters (p,f,g), we can plot T_Γ, the traffic across the cut Γ, versus the cut radius r. Note that r and T_Γ are normalized values that the total amount of traffic between $r=0$ and $r=1$ is 1. The offered traffic load through a circular cut $T_\Gamma(r)$ depends on the traffic characteristics. The traffic characteristics of flow through circular cut at different distance r from BS varies quite differently, as shown in two cases given in Fig.9.

V. System Throughput Capacity

Denote λ as the system throughput. The system throughput value is determined by the cut capacity and the offered traffic flow. Notice that by changing the value of



$p = 0.9,\ g = 0.2,\ f = 0.85$ $\quad$ $p = 0.1,\ g = 0.2,\ f = 0.1$

Figure 9. Examples of T_Γ versus r

parameters f and g, the offered traffic pattern at different cuts can alter significantly. Notice that, by changing the value of f and p, the traffic can alter significantly. To find the maximum throughput, we have,

Theorem 2: λ is upper bounded by the minimum value of $\frac{C_\Gamma}{T_\Gamma}$ for every cut Γ.

Proof: For arbitrary coding scheme under which the messages are transmitted in this hybrid network, for messages transmitted across a cut to be decoded successfully, the system throughput λ is constrained by Shannon's data compression theorem. To find $\lambda_{max} = \max_{\forall r} \lambda$. For any achievable system throughput, the λ value must satisfy $\lambda T_\Gamma \leq C_\Gamma$. The cut capacity is defined as $C_\Gamma(r)$ for a circular cut with radius r. For any circular cut within a normalized cell (with radius of 1),

$$\lambda(r)T_\Gamma(r) \leq C_\Gamma(r),\ \forall r\ 0 \leq r \leq 1 \tag{12}$$

That is to say,

$$\lambda \leq \min_{0\leq r\leq 1} \frac{C_{\Gamma(r)}}{T_{\Gamma(r)}} \tag{13}$$

Thus, we have the system throughput

$$\lambda_{max} = \min_{0\leq r\leq 1} \frac{C_{\Gamma(r)}}{T_{\Gamma(r)}} \tag{14}$$

$\blacksquare$

From Theorem 1, network coding capacity of a circular cut is

$$C_{\Gamma(r)} = \frac{2\pi W}{cos^{-1}(1 - \frac{(\Delta^2-1)r_0^2}{2r(r+r_0)})} \tag{15}$$

By substituting the equations (11) and (15) to (14), the achievable system throughput could be derived. As a result, to optimize the performance, we should find the optimized p value for the best relay strategy. For source node, which is p away from the base station, the data packets are forwarded through pure ad hoc relay to neighboring cells when the destination node is located in neighboring cell. The optimized inter-cell forwarding strategy is

$$p^* = \underset{0\leq p\leq 1}{argmax}\,\{\min_{0\leq r\leq 1}\{\frac{C_{\Gamma(r)}}{T_{\Gamma(r)}}\}\} \tag{16}$$

Accordingly, the achievable system throughput under optimized inter-cell forwarding scheme is

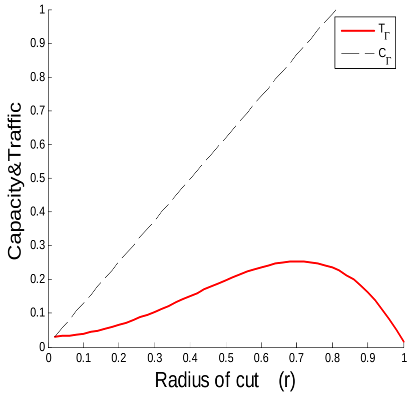


Figure 10. System Capacity Analysis: capacity bottleneck at BS (k=2.0399e+003 , $p = 0.6$, $g = 0.95$, $f = 0.5$)

$$\lambda^*_{max} = \max_{0\leq p\leq 1}\{\min_{0\leq r\leq 1}\{\frac{C_{\Gamma(r)}}{T_{\Gamma(r)}}\}\} \tag{17}$$

The method of analyze and optimize the performance is described in Appendix.

VI. Numerical Results

In this section, we investigate the network capacity and system bottleneck in different network scenarios. As shown in Fig. 10 and 11, we could find the cut capacity and traffic loading in different radius. The point where cut capacity curve and traffic loading curve intersect is the bottleneck of the system.

The capacity is C_Γ approximately a linear function of r, since the cut capacity is almost proportional to the diameter of the cut. The slope of the C_Γ line is inversely proportional to r_0. In practical scenarios, taking WLAN or WPAN for example, wireless devices might have a transmission range between 10 meters to 100 meters. The cell size can vary from 1 to 10 kilometers. We could set parameters $r_0 \in [\frac{1}{100}, \frac{1}{10}]$, with cut capacity plot versus r with slope range in $[10, 100]$. To determine the maximum throughput, we need to find the minimum value of $\frac{C_\Gamma}{T_\Gamma}$. It is equivalent to fit T_Γ by scaling C_Γ as close to T_Γ as possible. That is to say, we will divide C_Γ by a scaling factor k such that C_Γ is equal or greater than T_Γ and is tangent to T_Γ. The scaling value k is the achievable throughput when given parameters (g, f, p). In scenarios like Fig. 10, tangential point of the two curves locate at $r = 0$, reflecting the fact that the cut near BS is the capacity bottleneck, since this cut of radius 0 has the smallest capacity.

In Fig.11, the bottleneck is at the cut of radius $r = 1$, in this case $g = 0.2$ with relatively large f and p, indicating traffic flow mainly through the cell border. Notice that the system capacity in the $k = 3417.5$ case is greater than that of the $k = 2039.9$ case. Comparison between these two cases reveals the fact that a better throughput λ can be reached when the traffic distribution is higher at the outer part of the cell. Furthermore, we plot the maximum throughput k, which is equivalent to λ^*_{max}, versus f, g in

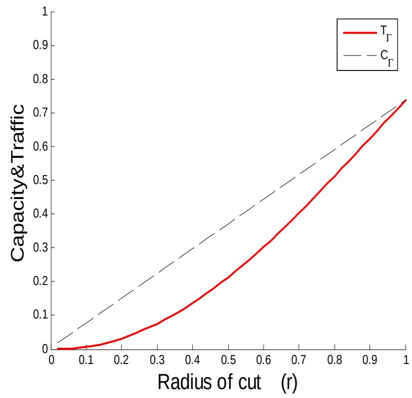


Figure 11. System Capacity Analysis: capacity bottleneck at cell boundary (k=3.4175e+003 , $p = 0.99$, $g = 0.2$, $f = 0.98$)

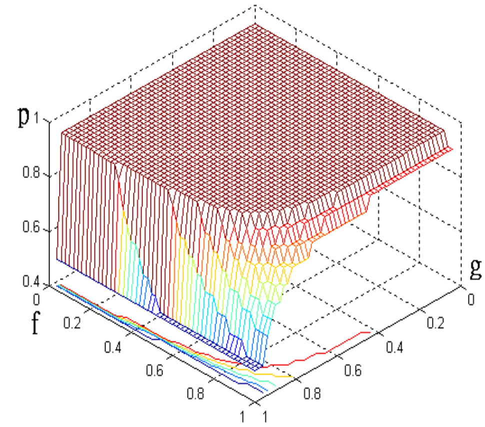


Figure 13. Optimal forwarding strategy p^* versus (g, f), $r_0 = 0.1$

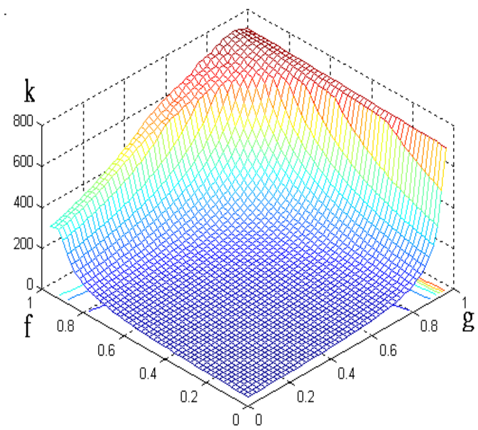


Figure 12. k versus (g, f), $r_0 = 0.1$

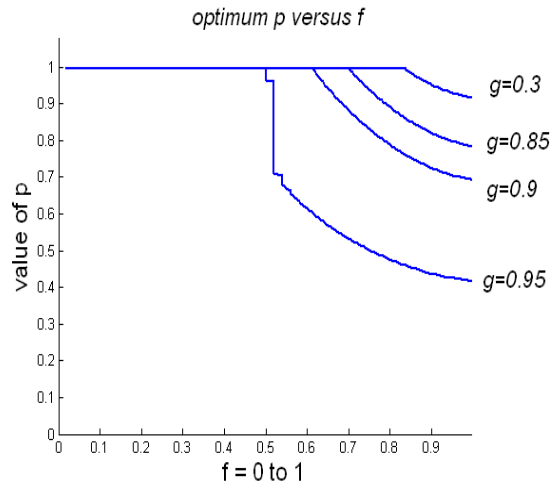


Figure 14. Optimal p^* in different traffic conditions

Fig.12. We also plot the value of p^* versus parameters f, g in Fig.13, to determine whether traffic bound for N_X is transmitted to the cell border directly with inter-cell ad hoc relay.

To evaluate the performance in both cases, we plot the optimized values of p^* and their respective maximum scaling k versus the given parameters $(g, f) \in [0, 1] \times [0, 1]$. From the two plots, we can observe that the maximum throughput λ^*_{max} given $r_0 = 1$ occurs when $f = 1$ and with g close to 1. This result is reasonable since the slope of plot of C_Γ is $O(\frac{1}{r_0})$ and $r_0 = 0.1$ in this case, it is inclined to have a better throughput with relatively higher traffic load near the border of the cell. In such a case, a higher percent of inter-cell ad hoc relay is conducted to reduce the congestion around the highly loaded base stations. In addition, notice that the maximum value of throughput occurs when g close to 1, this reflects the fact that as the portion of traffic transmitted inside the cell increases, the traffic load on BS and border can be reduced significantly. Thus, we have a greater value of λ, even though the cut capacity is the greatest at $r = 1$.

Observing the plot of p^* value for all $(g, f) \in [0, 1] \times [0, 1]$, the decision of p are invariant ($p \cong 1$) for most of the cases with relatively small g and f. Basically, for p^* approaching 1 for most of the cases, we should mostly apply inter-cell ad hoc relay when needed. As an example in Fig.14, fix the value of g to plot the optimal value of p versus the value of f, we can see that for small f the optimal throughput occurs at $p = 1$, i.e. all the portion of traffic toward neighboring cells should be forwarded through the cell border. For cases having maximum throughput occurs at $p = 1$, the performance in these cases is determined only by the given network traffic characteristics g and f. Only when $g \to 1$ and $f \to 1$, the smaller p^* setting occurs. Furthermore, for cases with larger g values, i.e. about 10% to 30% of the traffic are going outside of the cell, our traffic forwarding strategy can enhance the throughput performance significantly. To determine the optimized network forwarding strategy, given the traffic characteristics, we could use Fig.14 to enhance network operation.

VII. Conclusion

In this paper, we have investigated the hybrid wireless network model with network coding. With traffic permitted to transmit across the border of cells and through infrastructure base stations, a packet forwarding strategy to maximize system throughput is proposed. We discovery that the system throughput can be improved significantly by the proposed traffic forwarding strategy to apply inter-cell ad hoc relay or infrastructure-assisted transmission. As shown in the numerical results, the forwarding strategy selection and the hybrid wireless network throughput improvement depends on the inter-cell and intra-cell traffic flow characteristics.

APPENDIX
METHOD TO ANALYZE SYSTEM THROUGHPUT WITH TAYLOR EXPANSION

To calculate the value of p and λ, we use the Taylor Expansion for arc cosine function:

$$cos^{-1}(x) = \frac{\pi}{2} - x - \frac{1}{6}x^3 - \frac{3}{40}x^5 - \frac{5}{112}x^7 - \frac{35}{1152}x^9 ...$$

We have for $1 - p < r \leq \sqrt{1-p^2}$,

$$\int_0^r n(t)tdt = \frac{2}{\pi}\int_{1-p}^r [\frac{\pi}{2} - (\frac{1-t^2-p^2}{2pt}) - \frac{1}{6}(\frac{1-t^2-p^2}{2pt})^3 - \frac{3}{40}(\frac{1-t^2-p^2}{2pt})^5 ...]tdt \tag{18}$$

and

$$\int_r^1 n(t)tdt = \frac{p^2}{\pi} + \frac{2}{\pi}\int_r^{\sqrt{1-p^2}} [\frac{\pi}{2} - (\frac{1-t^2-p^2}{2pt}) - \frac{1}{6}(\frac{1-t^2-p^2}{2pt})^3 - \frac{3}{40}(\frac{1-t^2-p^2}{2pt})^5 ...]tdt \tag{19}$$

By Eq.(18) and Eq.(19), both $\int_r^1 n(t)tdt$ and $\int_0^r n(t)tdt$ in Eq.(7) can be expand by infinite series in which each term can be integrated directly. Except for that the Taylor Expansion of arc cosine converges rather slowly, we can approximate T_Γ by fraction of polynomials. Additionally, by using the same Taylor Expansion for arc cosine function in α of C_Γ, we can derive the right side of Eq.(14) as a fraction of two polynomials with a finite degree. Furthermore, by taking derivatives on r we can find λ_{max}. In addition, we could analyze the system throughput performance with respect to parameters f, g and, p.

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