Uplink Power Control in QoS-aware Multi-Service CDMA Wireless Networks

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***Abstract*— In this paper the problem of QoS-driven power control in the uplink of CDMA wireless networks supporting multiple services is considered. Due to the need for supporting simultaneously various services with diverse QoS requirements, each user is associated with a nested utility function which appropriately represents his degree of satisfaction in relation to the expected tradeoff between his QoS-aware actual uplink throughput performance and the corresponding power consumption. Based on this framework, the problem is formulated as a non-convex non-cooperative Multi-Service Uplink Power Control (MSUPC) game where users aim selfishly at maximizing their utility-based performance under the imposed physical limitations. We first prove the existence and uniqueness of a Nash equilibrium point of the MSUPC game, and then a decentralized iterative algorithm to obtain MSUPC game's equilibrium point is introduced. Finally, through modeling and simulation, the operation and features of the proposed framework and the decentralized MSUPC algorithm are evaluated with respect to the fulfillment of assorted services' QoS requirements, while numerical results and relevant discussions that demonstrate the tradeoffs among users' channel conditions, corresponding power consumption and utility-based performance are presented.**

***Index Terms*— CDMA networks, power control, QoS, resource allocation, multimedia services.**

I. INTRODUCTION

With the growing demand for high data rate and support of multiple services with various quality of service (QoS) requirements, the scheduling policy plays a key role in the efficient resource allocation process in future wireless networks. The inner characteristics of wireless communications in code division multiple access (CDMA) cellular systems, in terms of network's scarce radio resources, mobile nodes' physical limitations and users' time-varying channel conditions, have motivated the adoption of power control in both the uplink and downlink communication, for proficient resource allocation and interference management.

Recently, game theoretic approaches for studying power control in the uplink of CDMA networks have attracted significant attention [1], [2], as an alternative to traditional power control approaches that have been based on decisions taken in a centralized [3]-[7] or semi-centralized [8] way at the base station. Power control is modeled as a non-cooperative game where users act as individual players aiming at maximizing selfishly their degree of satisfaction, expressed by appropriately defined utility functions. In principle, utilities assigned to users are functions of the consumed power and the achieved signal to noise ratio, while each user determines his own transmission power level so as to maximize his utility.

Intuitively, the choice of the utility functions has a great impact on the nature of the game and how the users choose their respective actions. In [9] and [10], towards maximizing system's spectral efficiency, users' utilities are expressed as logarithmic, concave functions of their signal-to-interference-plus-noise ratio (SINR). This type of utility functions is proportional to the Shannon capacity for each user, thus treating all interference as white Gaussian noise [11]. In [12], the utility function is assumed to be proportional to the user's throughput and a pricing function based on the normalized received power of the user is proposed. S-modular power control games are studied in [13]. In [14] the authors use a utility function that measures the number of reliable bits that are transmitted (i.e. goodput) per joule of energy consumed. This is further extended in [15] by introducing pricing to improve the efficiency of Nash equilibrium. Joint energy-efficient power control and receiver design has been studied in [16], [17] and a game theoretic approach to energy-efficient power allocation in multicarrier systems is introduced in [18]. Joint network-centric and user-centric power control is discussed in [19], while the stability of distributed power control for CDMA systems is addressed in [20].

In this paper we study the problem of channel-aware power control in the uplink of CDMA networks supporting multiple services (i.e. both non-real-time and real-time) with various and often diverse QoS prerequisites via game theory. Following recent research efforts and approaches, we also adopt a utility-based framework. However, one of the fundamental differences in our work lies in the definition and treatment of users' utility functions, in order to fulfill simultaneously multiple services' QoS performance requirements in an environment where the service performances are interrelated. Specifically, instead of simply expressing the tradeoff between the number of a user's reliably

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transmitted bits and corresponding consumed power, in our study a user's utility that reflects his service QoS-aware performance efficacy as a function of his achieved goodput per joule of consumed energy is considered. To facilitate our goal general nested utilities of users' achieved goodput are adopted with respect to their service type, which may not be concave, therefore leading us to the formulation of a non-convex non-cooperative Multi-Service Uplink Power Control (MSUPC) game. This generalization requires a significantly different treatment than the works presented in [16] and [20].

By exploiting the introduced utility-based framework, we identify specific designing attributes that users' actual throughput utilities must encompass so as to proficiently express their service QoS-aware performance expectations. Furthermore, in our work we place special emphasis on multimedia services QoS prerequisites fulfillment in a time-varying environment. It is noted that in [21] a utility-based framework for reflecting real-rime users' degree of satisfaction in accordance to their achievable goodput performance has also been considered, however the corresponding users' energy consumption has not been taken into account and utility attributes are not studied. In [22], a game-theoretic framework has also been adopted for studying the power control problem in the uplink of CDMA networks supporting delay sensitive real-time services. Nevertheless, in that work and in [23], only linear utilities of users' goodput are considered, while real-time services' QoS requirements are expressed as statistical delay constraints; a consideration which in many cases is not sufficient since as shown in [24], [25] even if the delay constraints of a real-time user are satisfied, the degradation of his service quality may not be avoided due to the possibly bad channel conditions and variations. In this work, strict short-term throughput requirements are adopted for real-time multimedia services, expressed via their actual throughput utilities, in order to simultaneously satisfy both their short-term delay and throughput prerequisites.

It is noted that our approach provides a methodology for setting and analyzing generic MSUPC games in CDMA wireless networks. We identify necessary and sufficient conditions for the existence and uniqueness of a Nash equilibrium point in this non-convex multi-objective MSUPC optimization problem. Moreover, a low-complexity iterative algorithm for reaching MSUPC game's Nash equilibrium is proposed and its convergence is studied. The distributed nature of MSUPC algorithm favors a user-centric QoS-aware uplink architecture in sync with future networking vision that regards as its foundation element a self-optimized wireless node. Furthermore, via modeling and simulation, we extensively study the properties of system's equilibrium, and as a result strong correlations and tradeoffs among real-time services' strict QoS requirements fulfillment, non-real-time services' greedy throughput expectations, system's modulation and coding schemes and users' power and rate limitations are revealed.

The rest of the paper is organized as follows. In section II, the system model and some background information are provided, while in section III users' QoS properties are studied and mapped to appropriate utility functions. The proposed uplink power control non-cooperative game is formulated in section IV and its solution is presented in section V. Section VI, contains a decentralized low complexity iterative algorithm for attaining the proposed game's Nash equilibrium and its convergence is proven. Finally, section VII contains some numerical results and relevant discussions with respect to the performance of the proposed approach, while section VIII concludes the paper.

II. System Model & Background Information

We examine the uplink of a single cell time-slotted CDMA wireless network with $N(t)$ continuously backlogged users at time slot t, where $S(t)$ denotes their corresponding set. A time slot is a fixed time interval and could consist of one or several packets. Users' time-varying channels are affected by fast fading, shadow fading, and long-time scale variations and thus, can be modelled as a stationary time-varying stochastic process. Let us denote by $G_i(t)$ the corresponding path gain of each user $i \in S$ at time slot t. The required information is assumed to be available to all users and the base station at the beginning of each time slot t. In the following, assuming fixed users' channel conditions within the duration of each time slot, we omit the notation of the specific slot t in the notations and definitions we introduce. At the beginning of each time slot t users' Multi-Service Uplink Power Control (MSUPC) algorithm is responsible to make decisions on their transmission power and resulting rate in a decentralized manner. Note that a node's transmission power and rate are also fixed within the duration of a time slot.

Let us denote by P_i and R_i the uplink transmission power and transmission rate of user $i \in S$ in the slot under consideration, which are both upper bounded due to mobile node's physical limitation (i.e. $P_i \le P_i^{Max}$ and $R_i \le R_i^{Max}$). Moreover, the received bit energy to interference density ratio at the base station $\gamma_i = E_b / I_o$ for each user $i \in S$ is given by:

$$\gamma_i\left(R_i, P_i, \overline{P}_{-i}\right) = \frac{W}{R_i} \frac{G_i P_i}{\theta \sum_{j=1}^{N} G_j P_j - \theta G_i P_i + I_0} = \frac{W}{R_i} \frac{G_i P_i}{I_{-i}(\overline{P}_{-i})} \qquad (1)$$

where θ denotes the orthogonality factor (θ=1), W is the system's spreading bandwidth, $\overline{P}_{-i}$ denotes the users' power allocation vector excluding user i and I_0 contains the background noise and intercell interference. Thus, $I_{-i}(\overline{P}_{-i})$ actually denotes the network interference and background noise at the base station when receiving data from user *i* and is given by:

$$I_{-i}(\overline{P}_{-i}) = \theta\sum_{j=1}^{N} G_j P_j - \theta G_i P_i + I_0 \quad (2)$$

In our system we consider two basic types of users, namely non-real-time (*NRT*) users requesting delay-tolerant high-throughput services and real-time (*RT*) users with strict short-term QoS constraints. Throughout the rest of the paper we denote by N_{NRT} (N_{RT}) the number of non-real-time users (real-time users) and by S_{NRT} (S_{RT}) the corresponding set. Due to the need for supporting multiple services with various QoS requirements, each user is associated with a proper utility function U_i which represents his degree of satisfaction in relation to the expected tradeoff between his QoS-aware utility-based actual uplink throughput performance and the corresponding energy consumption per time slot *t*. Therefore, it can be expressed as:

$$U(R_i^*, P_i, \overline{P}_{-i}) = \frac{T_i(R_i^*, P_i, \overline{P}_{-i})}{P_i} = \frac{T_i(R_i^F \cdot f_i(\gamma_i), P_i, \overline{P}_{-i})}{P_i} \quad (3)$$

where $R_i^* \equiv R_i^F \cdot f_i(\gamma_i)$ denotes user's *i* actual uplink transmission rate (i.e. goodput) at the under consideration time slot, R_i^F denotes the user's fixed designed transmission rate (i.e. $R_i^F \equiv R_i$) and f_i is his efficiency function. Finally, user's *i* actual throughput utility T_i is employed to reflect, within his power control algorithm, his degree of satisfaction in terms of service performance expectations and QoS requirements fulfilment for a given achieved goodput, as defined in detail in the following section.

The efficiency function f_i represents the probability of a successful packet transmission for user *i*, and is an increasing function of his bit energy to interference ratio γ_i at any time slot. A user's function for the probability of a successful packet transmission at fixed data rate depends on the transmission scheme (modulation and coding) being used, and can be represented by a sigmoidal-like function of his power allocation for various modulation schemes [8], [16]. Therefore, a user's *i*, efficiency function f_i has the following properties:

1) f_i is an increasing function of γ_i.

2) f_i is a continuous, twice differentiable sigmoidal function with respect to γ_i.

3) $f_i(0) = 0$ to ensure that $T_i = 0$ when $P_i = 0$.

4) $f_i(\infty) = 1$.

III. Satisfying Multiple Services QoS Requirements

Towards designing a proficient power control algorithm that efficiently supports multiple services, in this section we formally define users' actual throughput utilities as well as overall utilities per type of service. In general, in our study a user's *i* actual throughput utility

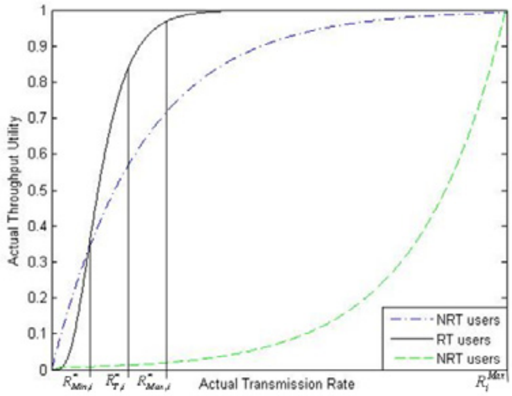

Figure 1. Actual throughput utilities as a function of user's achieved goodput.

$T_i(R_i^*, P_i, \overline{P}_{-i}) \equiv T_i(R_i^*)$ is a nested (due to the existence of f_i), non-convex function of his achieved goodput R_i^*, properly selected to reflect within his overall utility U_i and respective power control policy, his service performance requirements and QoS prerequisites under the imposed physical limitations. We further assume that T_i has the following properties.

Assumptions:

a) T_i is an increasing function of R_i^*.

b) T_i is twice continuously differentiable.

c) $T_i(0)=0$ to ensure that $U_i = 0$ when $R_i^* = 0$.

d) T_i is bounded above (i.e. $T_i(R_i^*) \leq 1$).

e) T_i is a sigmoidal-like[1], a strictly concave, or a strictly convex function of R_i^* within its corresponding definition set $[0, R_{Max,i}^*]$, where $R_{Max,i}^*$ denotes user's *i* maximum achievable goodput.

Note that typically, most utility functions that have been used in wireline [26] or wireless [8], [21] networks can be represented by the latter three types of functions. Specific examples of the actual throughput utility functions for both real-time and non-real-time users are included in section VII. In the rest of this section, we formally define the designing attributes that a user's actual throughput utility should possess, in order to proficiently express his service QoS-aware performance expectations.

A. Non-Real-Time Data Services

Concerning NRT data services, T_i can be either a strictly convex or a strictly concave function of user's *i* achieved goodput R_i^* in order to convey his greedy, high throughout performance expectations. On the other hand, due to physical limitations the maximum goodput a NRT user *i* can achieve ($R_{Max,i}^*$) is inherently constrained by his transmitter's limitations (i.e. $R_i^* \leq R_i^{Max}\ \forall i \in S_{NRT}$, therefore

[1]A function *f(x)* is a sigmoidal function if it has a unique inflection point, x_{Infl} and $d^2 f(x)/\partial x^2\big|_{x<x_{Infl}} > 0$ and $d^2 f(x)/\partial x^2\big|_{x>x_{Infl}} < 0$.

$R^*_{Max,i} = R_i^{Max}$ when f_i (∞) = 1) which accordingly determines his maximum feasible degree of satisfaction. Therefore, to incorporate a NRT user's rate limitations in his actual throughput utility, $R_i^* \equiv R_i^{Max} \cdot f_i(\gamma_i)$ since $f_i(\gamma_i) \le 1$ and thus T_i $(R^*_{Max,i}) = 1$ (Fig.1). Without loss of generality we consider:

Definition 1. NRT Users' Actual Throughput Utility $T_i(R_i^*) \equiv T_i(R_i^{Max} f(\gamma_i))$ $\forall i \in S_{NRT}$, where $R_i^* \in [0, R_i^{Max}]$ and $\gamma_i \ge 0$, is a strictly convex or strictly concave function of R_i^* with $T_i(R^*_{Max,i}) = T_i$ $(R_i^{Max})=1$. ■

B. Real-Time Multimedia Services

With respect to RT services, either regarding delay-adaptive (e.g. audio and video) or rate-adaptive real-time applications [26], T_i is considered as a sigmoidal function of user's i achieved goodput R_i^* to reflect his QoS performance requirements that primarily consist of short-term throughput [24] and delay constraints. These can be expressed via a properly selected sigmoidal actual throughput utility, shaped in accordance to the RT user's constant target actual rate per time slot and an appropriate margin factor [27]. The target actual uplink rate $R^*_{T,i}$, indicates the ideal value of a user's per time slot (to fulfill his delay constraints) actual transmission rate at which his service throughput QoS requirements are fulfilled, in order his short-term throughput requirement to be achieved as well, while the Margin Factor (MF_i) determines acceptable bounds of his actual achieved throughput deviations with respect to the target actual rate.

Specifically, a RT user's margin factor determines the minimum acceptable levels of his expected actual throughput (i.e. $R^*_{Min,i} = R^*_{T,i} - MF_i$). Moreover, we argue that when the user's achieved actual transmission rate remains within the range of $[R^*_{Min,i}, R^*_{T,i})$, his actual throughput utility must be a slowly increasing function of his actual data rate. On the other hand, when $R_i^* < R^*_{Min,i}$ then his actual throughput utility should be a rapidly increasing function of R_i^*, indicating his need to occupy additional system resources. Therefore, $R^*_{Min,i}$ shall determine the unique inflection point of user's $i \in S_{RT}$ actual throughput utility. The previous design option gives to a user's power control mechanism operating over fast fading channels environment, the enhanced flexibility of decreasing his actual uplink throughput up to a certain level ($R^*_{Min,i}$), if required due to potentially bad instantaneous channel conditions, without however excluding the user from transmitting data at that corresponding time slot. The later would occur, if a step function of a RT user's actual transmission rate was used to reflect his corresponding degree of satisfaction.

Furthermore, we argue that an additional increment of a RT user's actual data rate from his target value, must not correspond to an analogous increase of his actual throughput degree of satisfaction and thus, the latter should tend asymptotically to its maximum value, as $R_i^* \to R^*_{Max,i}$ (i.e. T_i $(R^*_{Max,i}) = 1$). The previous argument is based on the observation that since a RT service's QoS prerequisites are fulfilled when the required target bit rate per time slot is achieved, an additional improvement of user's actual throughput performance will not further improve his degree of satisfaction. Moreover, by restricting up to a specific level ($R^*_{Max,i}$) the achieved actual uplink data rates of RT users, that due to their temporarily good transmission environment could receive more recourses than required, we get the advantage of reallocating the excess system resources to the un-favored RT users (e.g. with temporally bad transmission environment) or NRT users, in order to increase their performance satisfaction.

For practical purposes, as is the case of NRT services, due to users' hardware limitations, we consider that a RT user's actual throughput utility has reached a value close to the maximum when $R^*_{Max,i} = R^*_{T,i} + MF_i$ $\forall i \in S_{RT}$. Therefore, when $i \in S_{RT}$ we set $R_i^* \equiv (R^*_{T,i} + MF_i) \cdot f_i(\gamma_i)$ in order $T_i(R^*_{T,i} + MF_i) = 1$, since $f_i(\gamma_i) \le 1$ (Fig.1). Without loss of generality we declare:

Definition 2. RT Users' Actual Throughput Utility $T_i(R_i^*) \equiv T_i((R^*_{T,i} + MF_i) f(\gamma_i))$ $\forall i \in S_{RT}$, where $R_i^* \in [0, R^*_{T,i} + MF_i]$ and $\gamma_i \ge 0$, is a sigmoidal function of R_i^* with unique inflection point $R^*_{Infl,i}$ as follows:

$$R^*_{Infl,i} = \left\{ R_i^* : \left. \frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} \right|_{R_i^* = R^*_{Min,i}} = 0 \,\middle|\, R^*_{Min,i} = R^*_{T,i} - MF_i \right\} \tag{4}$$

with T_i $(R^*_{Max,i}) = T_i$ $(R^*_{T,i} + MF_i) = 1$. ■

Finally, we should underline that by properly selecting a RT user's modulation and coding scheme (i.e. efficiency function f), his short-term throughput and thus, his delay prerequisites can be assured, as shown in [27].

In accordance to (3), the overall utility function of each user $i \in S$ can be expressed as follows:

$$U_i(P_i, \bar{P}_{-i}) = \frac{T_i(R_i^*)}{P_i} = \begin{cases} \dfrac{T_i(R_i^{Max} \cdot f_i(\gamma_i))}{P_i} & i \in S_{NRT} \\ \dfrac{T_i((R^*_{T,i} + MF_i) \cdot f_i(\gamma_i))}{P_i} & i \in S_{RT} \end{cases} \tag{5}$$

where $R_i^* \in [0, R_i^{Max}]$ $\forall i \in S_{NRT}$ and $R_i^* \in [0, R^*_{T,i} + MF_i]$ $\forall i \in S_{RT}$ since $f_i(\gamma_i) \in [0,1]$ $\forall \gamma_i \ge 0$. Intuitively, we consider that $U_i(P_i)$ is a non-negative, bounded above function for $P_i \ge 0$, for normalization purposes only, and thus $\lim_{P_i \to 0^+} U_i(P_i) = 0^+$ since if the user does not transmit data, then his satisfaction must be 0.

IV. The Non-Cooperative Multi-Service Uplink Power Control Game

As the system evolves, at the beginning of each time slot a user's uplink power control algorithm is responsible for determining an appropriate uplink transmission power level towards maximizing his overall degree of satisfaction, which is reflected by the corresponding values of his utility. In this section, the main goals of the proposed MSUPC algorithm are analyzed and formally defined as a generic optimization problem in a game theoretic framework.

Since, each user in the network aims at the maximization of the expectation of his utility U_i, the corresponding goal of user's *i* MSUPC algorithm can be defined as the maximization of the objective function:

$$\max_{P_i} E[U_i(P_i,\overline{P}_{-i})] \quad s.t.\ 0 \le P_i \le P_i^{Max} \tag{6}$$

Assuming independent and identically distributed (i.i.d.) communications channels, the maximization of the average utility in expression (6) is obtained by maximizing the utility at every time slot *t*. Moreover, due to the users' selfish operation, in terms of pursuing optimal power values towards their individual utility maximization, the overall network uplink power control problem at each time slot can be formulated as a non-cooperative multi-service uplink power control game (MSUPC game).

Let $G=[S,\{A_i\},\{U_i\}]$ denote the proposed non-convex non-cooperative game, where *S* is the set of all users and $A_i=[0,P_i^{Max}]\times\Re^N$ is the strategy set of the i^{th} user. Furthermore, for fixed transmission powers of the rest of the users, the resulting per time slot non-cooperative game can be expressed as the following maximization problem:

$$\max_{P_i} U_i = \max_{P_i} U_i(P_i,\overline{P}_{-i}) \quad s.t.\ 0 \le P_i \le P_i^{Max} \qquad for\ i=1,\ldots,N \tag{7}$$

In other words, each player-user in game *G* picks a transmission power from his strategy set A_i and receives a payoff U_i in accordance to his best response policy $B_i(\overline{P}_{-i})=\max_{P_i\in A_i} U_i(P_i,\overline{P}_{-i})$.

We adopt Nash equilibrium approach towards seeking the solution of the non-convex non-cooperative MSUPC game, which is most widely used for game theoretic problems. A Nash equilibrium point is a set of power vectors, such that no user has the incentive to change his power level, since its utility cannot be further improved by making any individual changes on its value, given the powers of other users. Therefore:

Definition 3. The power vector $\overline{P}^*=\left(P_1^*,\ldots,P_N^*\right)$ is a Nash equilibrium of the MSUPC game, if for every $i\in S$ $U_i(P_i^*,\overline{P}_{-i}^*)\ge U_i(P_i,\overline{P}_{-i}^*)$ for all $P_i^*\in A_i$. ∎

V. Towards a Unique Nash Equilibrium for the MSUPC Game

In this section we study the existence and uniqueness of a Nash equilibrium point for the proposed MSUPC game. The following results reveal that users' actual throughput utilities must comply with specific attributes regarding their form at the boundaries of their corresponding definition sets, in order to assert overall utilities' soundness, and thus for a unique Nash equilibrium of game MSUPC to exist. The absence of the Nash equilibrium means that the system is inherently unstable.

A. Properties of Users' Actual Throughput Utilities

Following the previous discussions and analysis, we initially study the properties of both RT and NRT users' actual throughput and overall utility functions over their corresponding definition domains. The following two lemmas determine the attributes of user's nested T_i actual throughput utility as a function of the achieved SINR. Specifically, for RT services, in accordance to definition 2, the following lemma holds.

Lemma 1. For RT users (i.e. $i\in S_{RT}$) given: a) an efficiency function $f_i(\gamma_i)$, which is a sigmoidal function of γ_i and has a unique inflection point $\gamma_{Infl,i}^f$, and b) an actual throughput utility function $T_i(R_i^*)$, which is a sigmoidal function of R_i^* and has a unique inflection point $R_{Infl,i}^*\equiv R_{Min,i}^*=R_{T,i}^*-MF_i$, where $R_i^*\equiv R_{Max,i}^*\cdot f_i(\gamma_i)\equiv(R_{T,i}^*+MF_i)\cdot f_i(\gamma_i)$, then function $T_i(\gamma_i)=T_i((R_{T,i}^*+MF_i)\cdot f_i(\gamma_i))$ is also a sigmoidal function of γ_i ($\gamma_i\ge 0$) with a unique inflection point $\gamma_{Infl,i}^{\mathrm{T}}$ such that:

$$\gamma_{Infl,i}^{\mathrm{T}}:\begin{cases} f_i^{-1}\left(\dfrac{R_{Infl,i}^*}{R_{Max,i}^*}\right)\le\gamma_{Infl,i}^{\mathrm{T}}\le\gamma_{Infl,i}^{f} & when\ f_i^{-1}\left(\dfrac{R_{Infl,i}^*}{R_{Max,i}^*}\right)\le\gamma_{Infl,i}^{f}\\ \gamma_{Infl,i}^{f}\le\gamma_{Infl,i}^{\mathrm{T}}\le f^{-1}\left(\dfrac{R_{Infl,i}^*}{R_{Max,i}^*}\right) & otherwise\end{cases} \tag{8}$$

where $f_i^{-1}\left(R_{Infl,i}^*/R_{Max,i}^*\right)$ is the mapping of function's $T_i(R_i^*)$ inflection point in γ_i axis.

Proof: See Appendix A. ∎

For NRT services, with respect to definition 1, the following lemma holds.

Lemma 2. For NRT users (i.e. $i\in S_{NRT}$) given: a) an efficiency function $f_i(\gamma_i)$, which is a sigmoidal function of γ_i and has a unique inflection point $\gamma_{Infl,i}^f$, and b) an actual throughput utility function $T_i(R_i^*)$, which is a strictly concave (strictly convex) function of R_i^*, where $R_i^*\equiv R_{Max,i}^*\cdot f_i(\gamma_i)\equiv R_i^{Max}\cdot f_i(\gamma_i)$, then the corresponding actual throughput utility function $T_i(\gamma_i)=T_i(R_i^{Max}\cdot f_i(\gamma_i))$ is as follows:

$$A)\quad if \lim_{\gamma_i \to 0^+} \frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^+ \quad (\lim_{\gamma_i \to \infty} \frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^-) \tag{9}$$

a sigmoidal function of γ_i ($\gamma_i \geq 0$) with unique inflection point $\gamma^{T}_{Infl,i}$ for which:

$$\gamma^{T}_{Infl,i} \leq \gamma^{f}_{Infl,i} \quad (\gamma^{f}_{Infl,i} \leq \gamma^{T}_{Infl,i}) \tag{10}$$

$$B)\quad if \lim_{\gamma_i \to 0^+} \frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^- \quad (\lim_{\gamma_i \to \infty} \frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^+). \tag{11}$$

a strictly concave (strictly convex) function of γ_i ($\gamma_i \geq 0$).

Proof: See Appendix B. ■

B. Properties of Overall Users' Utilities

In accordance to lemma 2, when $T_i(R_i^*)$ is a strictly concave (strictly convex) function of R_i^* and equality (11) holds, then a NRT user's actual throughput utility is also a strictly concave (strictly convex) function of γ_i ($\gamma_i \geq 0$). The following lemma demonstrates that in such cases the corresponding overall utility contradicts with the fundamental assumptions with respect to the definition of U_i in section III (i.e. $\lim_{P_i \to 0^+} U_i(P_i) = 0^+$ and $U_i(P_i)$ bounded above for $P_i \geq 0$), and therefore these cases are not feasible.

Lemma 3. When a NRT user's actual throughput utility $T_i(R_i^*)$ is a strictly concave (strictly convex) function of R_i^* and $\lim_{\gamma_i \to 0^+} \partial^2 T_i(\gamma_i) / \partial \gamma_i^2 = 0^-$ ($\lim_{\gamma_i \to \infty} \partial^2 T_i(\gamma_i) / \partial \gamma_i^2 = 0^+$) holds, then for the corresponding $U_i(P_i, \overline{P}_{-i})$ $i \in S_{NRT}$ as a function of P_i, either $\lim_{P_i \to 0^+} U_i(P_i) \neq 0^+$ or $U_i(P_i) \leq 0$ for $P_i \geq 0$ ($U_i(P_i)$ is not bounded above for $P_i \geq 0$), which in any case contradicts with the basic assumptions in the definition of U_i.

Proof: See Appendix C. ■

With respect to lemma 3, we argue that a NRT user's actual throughput utility must comply only with the properties in relation (9). The intuition behind condition (9) is that the properties of a user's utility in the domain of power (i.e. the basic resource that a user spends when transmitting data) should also hold when his utility is transformed in the field of γ_i, which reflects the corresponding resource expended when these data are received by the base station. The latter occurs only when conditions (9) hold. Such an argument is founded on the interference sensitive nature of the uplink in a CDMA system, where the individual amount of resources (i.e. transmission power) that a user spends is not the only factor that determines his degree of satisfaction, due to others simultaneous transmissions that affect his achieved SINR and thus his actual service performance. Moreover, when a user's actual throughput T_i utility attributes follow (11) the MSUPC game does not have a unique Nash equilibrium, as it is extensively analysed later.

Based on lemma 1 and lemma 2 and taking into consideration the logical limitations in the definition of a user's overall utility function U_i (i.e. bearing in mind in the rest of the paper that user's actual throughput utilities are in line with equalities (9)), the following lemma characterizes the overall utility maximization of a single user when others' transmission powers are fixed.

Lemma 4. User's $i \in S$ utility function $U_i(P_i, \overline{P}_{-i})$ is a quasi-concave function of his own power P_i. Moreover, considering that other users' transmission powers are fixed, $U_i(P_i, \overline{P}_{-i})$ for $P_i \in [0, P_i^{Max}]$ has a unique global maximization point:

$$P_i^* = \begin{cases} \min\{\dfrac{\gamma_i^*(R_{T,i}^* + MF_i) I_{-i}(\overline{P}_{-i})}{WG_i}, P_i^{Max}\}, & i \in S_{RT} \\ \min\{\dfrac{\gamma_i^* R_i^{Max} I_{-i}(\overline{P}_{-i})}{WG_i}, P_i^{Max}\}, & i \in S_{NRT} \end{cases} \tag{12}$$

where γ_i^* is the unique positive solution of the equation $\dfrac{\partial T_i(\gamma_i)}{\partial \gamma_i} \cdot \gamma_i - T_i(\gamma_i) = 0$.

Proof: See Appendix D. ■

Lemma 4 reveals that users' goal to maximize their utility-based performance and thus their degree of satisfaction in terms of QoS-aware service performance and corresponding energy consumption, can be translated to a constant attempt of meeting specific SINRs γ_i^* (at the base station), which eventually leads to an SINR-balanced network. Moreover, according to (12), if a user's maximum transmission power is not sufficient for reaching the targeted γ_i^*, due to his potentially bad channel conditions, then the best policy $B_i(\overline{P}_{-i})$ is to transmit with maximum power.

C. Existence and Uniqueness of a Nash Equilibrium

The following proposition asserts the existence and the uniqueness of a Nash equilibrium point of the proposed uplink power control game, and hence determines nodes' transmission power vector at equilibrium.

Proposition 1. The Nash equilibrium of the non-cooperative game (7) is given by $\overline{P}^* = (P_1^*, ..., P_N^*)$, where P_i^* is the unique global maximization point of the overall user's i utility function, given by:

$$P_i^* = \min\{\frac{\gamma_i^*(R_{T,i}^* + MF_i) I_{-i}(\overline{P}_{-i})}{WG_i}, P_i^{Max}\} \qquad if\ i \in S_{RT},$$

and by:

$$P_i^* = \min\{\frac{\gamma_i^* R_i^{Max} I_{-i}(\overline{P}_{-i})}{WG_i}, P_i^{Max}\} \qquad if\ i \in S_{NRT}.$$

Furthermore, γ_i^* results from the unique positive solution of equation $(\partial T_i(\gamma_i)/\partial\gamma_i)\cdot\gamma_i - T_i(\gamma_i) = 0$.

Proof: In accordance to lemma 4, the power level that corresponds to the maximization of user's $i \in S$, utility-based performance, given other users' power levels, equals to the power level that maximizes the utility in (5) and thus, is given by (12), where γ_i^* is the unique positive solution of $\partial T_i(\gamma_i)/\partial\gamma_i \cdot \gamma_i - T_i(\gamma_i) = 0$. Up to here, it has been argued that at Nash equilibrium point, if such a point exists, each user's transmission power is pursuing a targeted SINR value which depends not only on the modulation, coding, and packet size being used (expressed through his appropriate efficiency function f_i) but also, is affected by RT or NRT users' QoS requirements (reflected by their actual throughput utility).

Moreover, following [14] and [15], the existence of a Nash equilibrium of the game in (7) can be shown via the quasi-concavity of each node's utility function in his own power. In lemma 4 we have already shown that $U_i(P_i, \overline{P}_{-i})$ is a quasi-concave function in $P_i \in [0, P_i^{Max}]$, and hence Nash equilibrium always exists. Finally, for an S-shaped actual throughput utility function T_i (given that T_i obeys (9)), $\partial T_i(\gamma_i)/\partial\gamma_i \cdot \gamma_i - T_i(\gamma_i) = 0$ has a unique solution γ_i^*, which is the unique global maximizer of user's i utility function. Due to the uniqueness of γ_i^* and the one-to-one relationship between uplink transmission power and corresponding SINR, the above Nash equilibrium is unique. ■

Proposition 1 indicates that if all users adopt a best response policy (7) governed by (12), game's MSUPC unique Nash equilibrium [30] will be always reached. Such a strategy is formally described by the proposed algorithm in the following section. Concluding this section's analysis we provide a necessary and sufficient condition for the existence of a unique Nash equilibrium in MSUPC game. In accordance to proposition 1, the existence and uniqueness of game's MSUPC Nash equilibrium is founded on the quasi-concaveness property of users' utilities U_i as a function of P_i and therefore, the S-shape property of their corresponding actual throughput utilities T_i as functions of the achieved SINR γ_i (as lemma 4 reveals). Moreover, a generic actual throughput utility of a user, as defined in section III, is always an S-shaped sigmoidal function of γ_i if and only if relation (9) holds, following lemmas 1 and 2. Therefore, the satisfaction of conditions in (9) not only asserts the proper definition of a user's utility U_i, as argued in lemma 3, but also reassures system's stability via guaranteeing the existence of a unique Nash equilibrium for the MSUPC game (7).

Proposition 2. MSUPC non-cooperative game always has a unique Nash equilibrium if every user's i actual throughout utility T_i, as defined in section III, possesses the following additional attributes:

a) $\lim_{\gamma_i \to 0^+} \dfrac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^+$ if T_i is strictly concave in R_i^*,

b) $\lim_{\gamma_i \to \infty} \dfrac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^-$ if T_i is strictly convex in R_i^*.

Proof: Following proposition 1 and lemmas 3 and 4 the proposition holds. Note that when T_i is a sigmoidal function in R_i^* no additional attributes are required for the existence and uniqueness of the MSUPC game's Nash equilibrium. ■

VI. MSUPC ALGORITHM & CONVERGENCE

In this section, based on our previous analysis, an iterative and decentralized uplink power control algorithm for reaching the Nash equilibrium for the MSUPC game G at every time slot t is presented and its convergence is discussed.

MSUPC Algorithm

(1) At the beginning of time slot t, user i, $i \in S$ transmits with a randomly selected feasible power (i.e. $0 \le P_i^{*(0)} \le P_i^{Max}$) power. Set $k=0$ and hence $P_i^{*(0)}$, $i \in S$.

(2) Given the uplink transmission powers of other users, which is implicitly reported by the base station when broadcasts its overall interference $I^{(k)}(\overline{P}^{(k)})$, the user computes $I_{-i}^{(k)}(\overline{P}_{-i}^{(k)})$ and refines his power, i.e. computes $P_i^{*(k+1)}$ in accordance to (12).

(3) If the powers converge (i.e. $\left|P_i^{*(k+1)} - P_i^{*(k)}\right| \le 10^{-5}$) then stop.

(4) Otherwise, set $k=k+1$, go to step 2.

MSUPC algorithm can be characterized as a single-valued low complexity best response algorithm for every user starting from a randomly selected feasible power of his non-empty orthogonal strategic space A_i (i.e $P_i^{*(0)}\ \forall i \in S$) [30]. Under the assumption that P_i^{Max} is sufficiently large in order γ_i^* can be achieved by all users, the convergence of MSUPC algorithm is always reached in accordance to (proposition 1, in [16]). In the general case however, the convergence of MSUPC algorithm can be drawn based on the supermodularity attribute of the under consideration game (following propositions 1 and 3 and theorem 1, in [28]). A formal definition of a supermodular game for the case of single dimensional user strategy sets is stated in the following [15].

Definition 4. A game $G = [S, \{A_i\}, \{U_i\}]$, where S is the set of users/players and $A_i = [0, P_i^{Max}] \times \Re^N$ is the strategy set of the i^{th} user is supermodular if for each user i, $U_i(P_i, \overline{P}_{-i})$ has nondecreasing differences (NDD) in $(P_i, \overline{P}_{-i})$ [15]. ■

As explained in [28], the sufficient conditions for a supermodular game are:

D1. The strategy A_i is a nonempty and compact sublattic.

D2. The utility function U_i is continuous in all players' strategies, is supermodular in player i's own strategy, and has increasing differences between any component of player i's strategy and any component of any other player's strategy.

Regarding MSUPC game, since each user's i strategic space A_i is a closed single-dimension non-empty space, property D1 holds. Moreover, by definition U_i is continuous in all users' strategies for all $i \in S$. Each user's U_i utility function is (trivially) supermodular in his own one-dimensional strategy space. The remaining increasing difference condition of U_i for any component of player i's strategy and any component of any other player's strategy (i.e. $\frac{\partial^2 U_i(P_i, \bar{P}_{-i})}{\partial P_i \partial P_j} \geq 0 \ \ \forall i, j \in S$) holds according to Theorem 1 in [28], since U_i is a quasi-concave function of his own power P_i and continuous inversely proportional function of others' power $\bar{P}_{-i}$ (thus satisfying the same properties as the utility used in [28], (i.e. proposition 2(ii) equation (9)).

In accordance to proposition 3 in [28], in a supermodular game G: *i)* If the users' best responses are single-valued, and each user uses the myopic best response updates starting from the smallest (largest) elements of his strategy space, then the strategies monotonically converge to the smallest (largest) Nash equilibrium, *ii)* If the Nash equilibrium is unique, the myopic best response updates globally converge to that Nash equilibrium from any initial strategies.

VII. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we provide some numerical results illustrating the operation and features of the proposed framework and the MSUPC algorithm. Initially, for demonstration mainly purposes, we assume fixed, pre-defined users' channel conditions, in order to identify and comprehend the proposed game's G equilibrium properties for various users' channel states and services' QoS characteristics. Then, assuming time-varying Rayleigh fading channels, system's long-term properties under the proposed MSUPC algorithm are illustrated and the tradeoffs among user's average channel conditions, corresponding power consumption and utility-based performance are revealed. Finally, the correlations between real-time multimedia and data users' service performance are studied, in terms of achieved uplink actual throughput, power consumption and utility-based performance, when both types of services share the same available resources under time-varying channels.

Throughout our study we consider the uplink of a single cell time-slotted CDMA system, supporting *N=10*

TABLE I
SIMULATION PARAMETER VALUES

Parameter ($\forall i \in S$)	Value
P_i^{Max}	*2 (Watt)*
$P_i^{*(0)}$	*2 (Watt)*
W	*10^6 (Hz)*
I_0	*$5*10^{-16}$*
n	*4*

TABLE II
QOS PREREQUISITES OF CLASS 1(NRT USERS)

Users' Class / Characteristic	Class 1 (data users)
c_i	*0.001*
R_i^{Max}	*2.4(Mbps)*
$\gamma^*_{M=500}$	*5.8 (7.6 dB)*

TABLE III
QOS PREREQUISITES OF CLASS 2 (RT USERS)

Users' Class / Characteristic	Class 2 (multimedia users)
R_T^*	*128 (Kbps)*
MF	*10 (Kbps)*
M_T	*800*
A_T	*112*
$\gamma^*_{M=500}$	*8.9 (9.5 dB)*

continuously backlogged users. Each simulation lasts *10.000* time slots. Unless otherwise explicitly indicated, we model users' path gains as: $G_i = K_i / d_i^n$ where d_i is the distance of user i from the base station, n is the distance loss exponent, and K_i is a log-normal distributed random variable with mean 0 and variance σ^2 = *8(dB)*, which represents the shadowing effect [8]. Henceforth, we consider the following values of system parameters, as shown in Table I.

Two classes of users are considered, namely class 1 and class 2. Class 1 represents data users and class 2 multimedia users, who require video conference services. The two classes are differentiated in accordance to their services' characteristics as they are reflected in the form of their corresponding actual throughput utility function and their parameters. In the following, multimedia users' QoS prerequisites are expressed through a sigmoidal actual throughput utility function (i.e. $T_i(R_i^*) = (1 - e^{-(R_i^* - A_i)})^{M_{T,i}}$) and data users' through a strictly concave function (i.e. $T_i(R_i^*) = \log(c_i R_i^* + 1)$). Moreover, one common modulation and coding scheme is considered for both classes of users, which is expressed through the sigmoidal efficiency function $f_i(\gamma_i) = (1 - e^{-\gamma_i})^M \quad \forall i = 1, \ldots N$. The previous characteristics are summarized in Tables II, III.

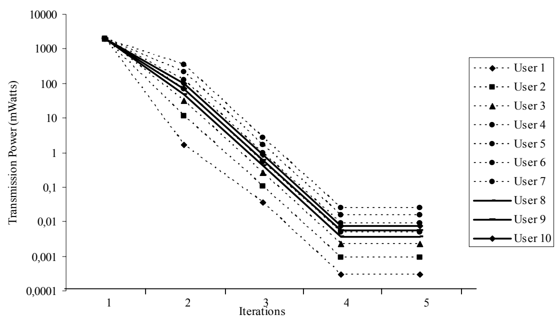

Figure 2. Users' transmission powers convergence at a given time slot

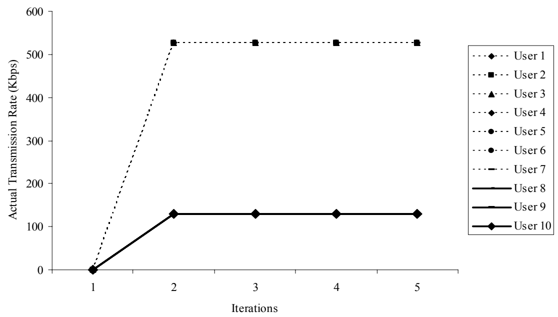

Figure 3. Users' actual throughput at a given time slot

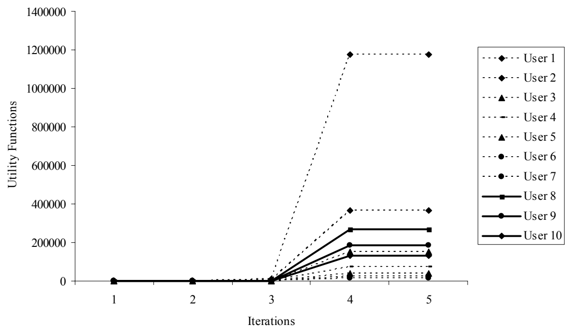

Figure 4. Users' utilities at a given time slot

A. Game's Nash Equilibrium Properties

In the following, we study MSUPC algorithm's convergence at game's Nash equilibrium for a specific time slot. Fixed channels conditions are assumed during a time slot and we set $G_i = 1/d_i$ for each user *i*, where $d_i=d_{i-1}+100$ *(m)* for $i=2,..,10$ and $d_1=300$ *(m)*. In this way we emulate a scenario where users' channel conditions are worse as their ID value (i.e. $i=1,\ldots, 10$) increases. For demonstration purpose only, users *1* to *7* are assumed to belong to class 1, while users *8* to *10* belong to class 2.

Fig.2, Fig.3 and Fig.4 illustrate users' transmission powers, actual uplink transmission rates and utility values respectively, as a function of the iterations required for users' MSUPC algorithm to converge at game's G equilibrium point $\overline{P}^*$. The corresponding results reveal that for users in the same class their powers at equilibrium are inversely proportional to their instantaneous channel conditions (Fig.2). Moreover, only five iterations are required for reaching equilibrium, which is an important attribute of our approach due to slots' small length (i.e. 1.67 msec approximately in HDR systems [5]) in channel aware wireless systems. Furthermore, even if users' utilities converge at values proportional to their channels' states (Fig.4), their actual achieved uplink throughput is such that their QoS expectations are met even for the most demanding users in terms of low instantaneous channel gains G_i and *QoS* prerequisites (i.e. RT users $i = 8, 9, 10$ achieved goodput is *128 Kbps*). The latter observation reveals MSUPC algorithms' flexibility in proficiently managing the drawbacks emerging from users' "near – far" effect, even when various services are simultaneously supported.

Moreover, the fact that for both classes of users their actual uplink data rates at equilibrium are in line with their services' expectations, not only reveals the proper functionality of their actual throughput utilities (T_i) in introducing the services' QoS demands in the MSUPC algorithm, but also demonstrates the utilities ability to distinguish services with diverse QoS prerequisites.

B. Users' Power Consumption & Overall Utility-based / Actual Throughput Performance Trade-off

We first study the trade-off among a user's utility-based performance, QoS requirements fulfillment and corresponding power consumption as a function of his average channel state and service type, as the system evolves under the MSUPC algorithm. Therefore assuming Rayleigh users' time-varying channel conditions we let the system evolve for *10.000* timeslots.

Fig.5 (Fig.8), Fig.6 (Fig.9) and Fig.7 (Fig.10) illustrate users' average power consumption, actual throughput rate and utility based performance respectively as a function of users' ID, when distributing them around the base station with step distance $d_i=d_{i-1}+50$ ($d_i=d_{i-1}+100$) *(m)* for $i=2,..,10$ and $d_1=300$ *(m)*. As in the previous scenario, users *1* to *7* are assumed to belong to class 1, while users *8* to *10* belong to class 2. In the case of class 2 users, the results reveal that as their mean channel conditions become worse (i.e. increasing number of RT-class 2 users' ID) and in order to satisfy their QoS requirements (Fig.6) (i.e. actual achievable uplink transmission rate of *128 Kbps* per time slot with corresponding margin factor *10 Kbps*), their energy consumption increases (Fig.5) which eventually leads to their overall utility-based performance decrement (Fig.7). On the other hand, as non-real-time users' mean channel conditions become worse (i.e. increasing number of NRT-class 1 users' ID) both their overall utility and actual throughput performance decreases, while their power consumption increases.

More specifically, the results show that for both NRT and RT users, their QoS expectations are fulfilled in terms of high throughput performance and fixed transmission rates, respectively. This reveals the proper functionality of their actual throughput utilities (T_i) in introducing the services' QoS demands in the MSUPC mechanism, and thus demonstrates their utilities ability to distinguish and support various services' QoS performance metrics. Specifically, regarding data users

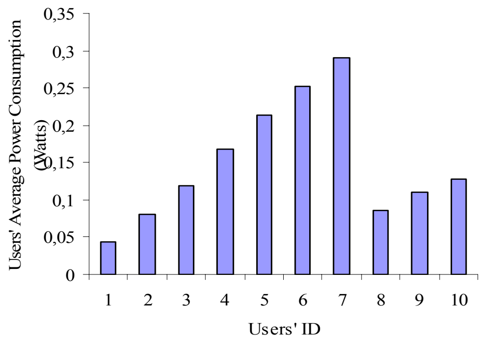

Figure 5. Users' average power consumption (step distance 50m)

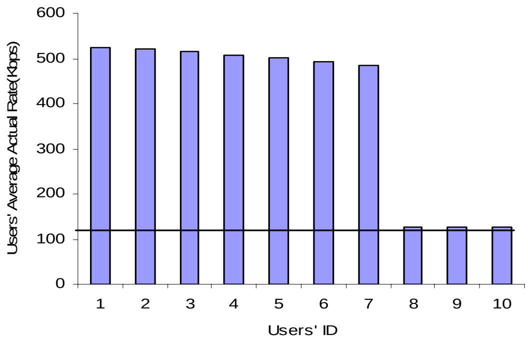

Figure 6. Users' average actual throughput rate (step distance 50m)

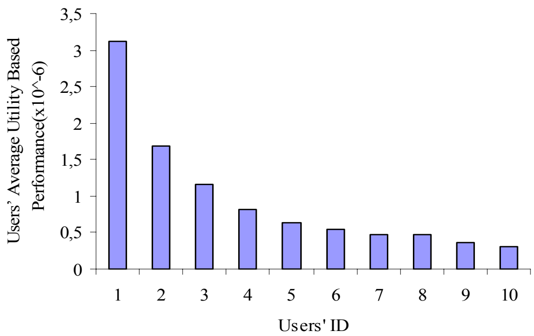

Figure 7. Users' average utility based performance (step distance 50m)

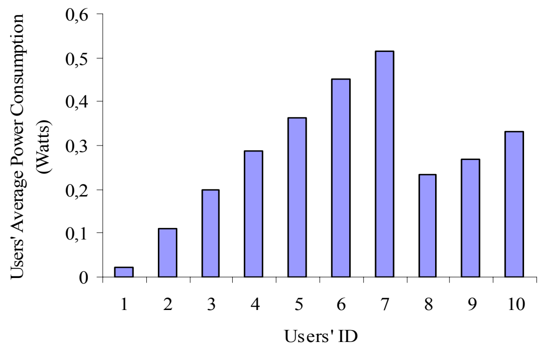

Figure 8. Users' average power consumption (step distance 100m)

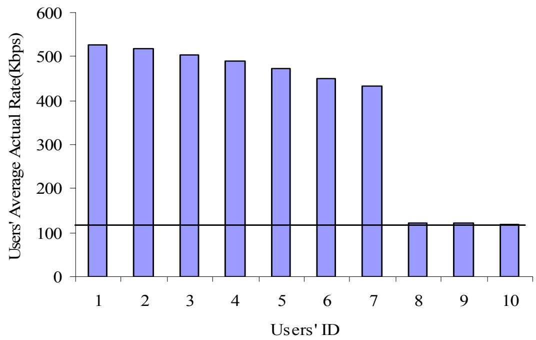

Figure 9. Users' average actual throughput rate (step distance 100m)

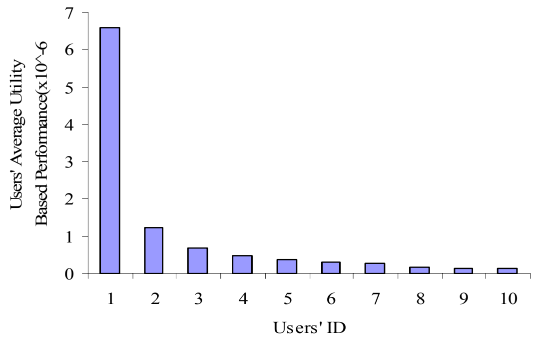

Figure 10. Users' average utility based performance (step distance 100m)

(i.e. $i=1,\dots,7$) their optimal service-performance – energy-consumption tradeoff (i.e. utility maximization) is achieved for high data rates even for the most distant from the base station users. Such a behavior is driven by their actual throughput utility types that led them to get high transmission powers in system's non-cooperative game, due to the corresponding high goodput performance gains they attain. On the other hand, data services utility maximization is steering them to lower energy consumption while fulfilling their fixed expected data rates, since an additional increment on their power and thus, their achieved goodput, would not correspond to an analogous increment of their actual throughput as well as overall utility, as explained before. Such a behavior eventually reduces overall uplink interference towards improving both types of users' performance.

Finally, a parallel and close examination of the two sets of figures, Fig.5-Fig.7 and Fig.8-Fig.10, allows us to better identify the correlations in the behavior and performance of real-time multimedia and non-real-time data users, when both types of services coexist under a system with time-varying available radio resources. We conclude that as channel's conditions become worse (Fig.8-Fig.10), the achieved actual transmission rate of NRT users decreases (even for those close to the base station), while the corresponding rate of RT users remains greater than their minimum actual data rate, even if those users' channel conditions are worse. Such a behavior is driven by users' actual throughput utility types that favor RT users to achieve the targeted uplink actual transmission rate when the appropriate modulation and coding scheme is selected (as shown in proposition 2, in [27]). Moreover, in such a case data users' still maintain high achieved goodput, but at the cost of increased unfairness among them in terms of power consumption and achieved utility.

VIII. Concluding Remarks

In this paper we studied the issue of efficient power control in the uplink of a CDMA wireless network, while satisfying multiple services' QoS prerequisites. To proficiently reflect multiple services' various and often diverse QoS prerequisites a generic utility-based framework was introduced and analyzed.

The corresponding non-convex utility-based multi-service optimization problem was formulated and solved through a game theoretic framework. Towards this direction, first the existence and uniqueness of a Nash equilibrium of the proposed game was studied, and then a decentralized low-complexity algorithm for reaching the unique Nash equilibrium point of the proposed game has been presented. Furthermore, the efficacy of the proposed framework in supporting both real-time multimedia services' and non-real-time data services' QoS demands, while achieving their utility-based performance maximization has been demonstrated and evaluated via analysis and simulation.

The introduced utility-based framework and proposed approach provides us with an enhanced flexible framework which can be used to uniformly treat and theoretically analyze different access technologies (e.g. CDMA and WLAN) types of resources, as well as multiple user services' diverse expectations. Therefore, the study and analysis of heterogeneous wireless networks' joint resource allocation and QoS provisioning mechanisms under a common utility framework towards realizing their seamless integration becomes of high research and practical importance and is part of our current work.

Appendix A - Proof of Lemma 1

In accordance to the definition of a sigmoidal function the following properties hold:

$$\frac{\partial f_i(\gamma_i)}{\partial \gamma_i} > 0 \tag{13}$$

$$\frac{\partial T(R_i^*)}{\partial R_i^*} > 0 \tag{14}$$

$$\left.\frac{\partial^2 f_i(\gamma_i)}{\partial \gamma_i^2}\right|_{\gamma_i<\gamma_{Infl,i}^f} > 0 \quad , \quad \left.\frac{\partial^2 f_i(\gamma_i)}{\partial \gamma_i^2}\right|_{\gamma_i>\gamma_{Infl,i}^f} < 0 \tag{15}$$

$$\left.\frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2}\right|_{R_i^*<R_{Infl,i}^*} > 0 \ , \ \left.\frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2}\right|_{R_i^*>R_{Infl,i}^*} < 0 \tag{16}$$

$$\lim_{R_i^*\to 0^+} \frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} = 0^+ \quad , \quad \lim_{R_i^*\to \infty} \frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} = 0^- \tag{17}$$

$$\lim_{\gamma_i\to 0^+} \frac{\partial^2 f_i(\gamma_i)}{\partial \gamma_i^2} = 0^+ \quad , \quad \lim_{\gamma_i\to \infty} \frac{\partial^2 f_i(\gamma_i)}{\partial \gamma_i^2} = 0^- \tag{18}$$

Regarding the first derivative of $T_i(\gamma_i)$ we have:

$$\frac{\partial T_i(\gamma_i)}{\partial \gamma_i} = \frac{\partial T_i(R_{Max,i}^* f_i(\gamma_i))}{\partial \gamma_i} = \frac{\partial T_i(R_{Max,i}^* f_i(\gamma_i))}{\partial (R_{Max,i}^* f_i(\gamma_i))} \cdot \frac{\partial (R_{Max,i}^* f_i(\gamma_i))}{\partial \gamma_i}$$
$$= R_{Max,i}^* \frac{\partial T_i(R_i^*)}{\partial R_i^*} \cdot \frac{\partial f_i(\gamma_i)}{\partial \gamma_i} > 0$$

according to (13) and (14). Considering the second derivative of $T_i(\gamma_i)$ we have:

$$\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = \frac{\partial^2 T_i(R_{Max,i}^* f_i(\gamma_i))}{\partial \gamma_i^2} = R_{Max,i}^* \frac{\partial T_i\left(\frac{\partial T_i(R_{Max,i}^* f_i(\gamma_i))}{\partial (R_{Max,i}^* f_i(\gamma_i))} \cdot \frac{\partial f_i(\gamma_i)}{\partial \gamma_i}\right)}{\partial \gamma_i} =$$

$$R_{Max,i}^* \left[R_{Max,i}^* \frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} \cdot \left(\frac{\partial f_i(\gamma_i)}{\partial \gamma_i}\right)^2 + \frac{\partial^2 f_i(\gamma_i)}{\partial \gamma_i^2} \frac{\partial T_i(R_i^*)}{\partial R_i^*} \right] \tag{19}$$

Initially, considering the bounds in the definition domain of $T_i(\gamma_i)$, $\gamma_i \in [0,\infty)$ we can easily see that:

$$\lim_{\gamma_i\to 0^+} \frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = R_{Max,i}^* \cdot$$
$$\left[R_{Max,i}^* \lim_{R_i^*\to 0^+} \frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} \cdot \left(\lim_{\gamma_i\to 0^+} \frac{\partial f_i(\gamma_i)}{\partial \gamma_i} \right)^2 + \lim_{\gamma_i\to 0^+} \frac{\partial^2 f_i(\gamma_i)}{\partial \gamma_i^2} \cdot \lim_{R_i^*\to 0^+} \frac{\partial T_i(R_i^*)}{\partial R_i^*} \right] = 0^+$$

and thus,

$$\lim_{\gamma_i\to \infty} \frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = R_{Max,i}^* \cdot$$
$$\left[R_{Max,i}^* \lim_{R_i^*\to R_{Max,i}^*} \frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} \cdot \left(\lim_{\gamma_i\to \infty} \frac{\partial f_i(\gamma_i)}{\partial \gamma_i} \right)^2 + \lim_{\gamma_i\to \infty} \frac{\partial^2 f_i(\gamma_i)}{\partial \gamma_i^2} \cdot \lim_{R_i^*\to R_{Max,i}^*} \frac{\partial T_i(R_i^*)}{\partial R_i^*} \right] = 0^-$$

since $\lim_{R_i^*\to R_{Max,i}^*} \frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} \neq \infty$ and $\lim_{R_i^*\to R_{Max,i}^*} \frac{\partial T_i(R_i^*)}{\partial R_i^*} \neq \infty$.

Furthermore, with respect to the corresponding relevance (position) of the inflection points of the initial utilities (i.e. $\gamma_{Infl,i}^f$ and $f_i^{-1}\left(R_{Infl,i}^* / R_{Max,i}^*\right)$) on the axis of γ_i ($\gamma_i \geq 0$), we must study the properties of the second derivative of $T_i(\gamma_i)$ as a function of γ_i in two different cases.

Case I.1. When $f_i^{-1}\left(R_{Infl,i}^* / R_{Max,i}^*\right) \leq \gamma_{Infl,i}^f$.

We have the following sub-cases:

I.1.A. For $0 \leq \gamma_i \leq f^{-1}\left(R_{Infl,i}^* / R_{Max,i}^*\right) \leq \gamma_{Infl,i}^f$ in accordance to (13), (14), (15), (16) it can be seen that $\partial^2 T_i(\gamma_i)/\partial \gamma_i^2 > 0$.

I.1.B. For $f_i^{-1}\left(R_{Infl,i}^* / R_{Max,i}^*\right) \leq \gamma_{Infl,i}^f \leq \gamma_i$ in accordance to (13), (14), (15), (16) it can be seen that $\partial^2 T_i(\gamma_i)/\partial \gamma_i^2 < 0$.

I.1.C. For $f_i^{-1}\left(R_{Infl,i}^* / R_{Max,i}^*\right) \leq \gamma_i \leq \gamma_{Infl,i}^f$ in accordance to previous sub-cases (*I.1.A*) and (*I.1.B*), we know that $\partial^2 T_i(\gamma_i)/\partial \gamma_i^2$ has at least one interruption point with the axis of zero, when $\gamma_i \in [f_i^{-1}\left(R_{Infl,i}^* / R_{Max,i}^*\right), \gamma_{Infl,i}^f]$. In order to show that $T_i(\gamma_i)$ is a sigmoidal function, we need to prove that there is only one interruption point with the zero axis, denoted as $\gamma^{\mathrm{T}}_{Infl,i}$. Through the examination of (19), when $\gamma_i \in [f_i^{-1}\left(R^*_{Infl,i}/R^*_{Max,i}\right), \gamma^f_{Infl,i}]$, since $\partial^2 f_i(\gamma_i)/\partial\gamma_i^2\big|_{\gamma_i<\gamma^f_{Infl,i}} > 0$, the second part of the sum in (19) is always positive. Therefore, only the sign of $\partial^2 T_i(R_i^*)/\partial(R_i^*)^2$ determines, and hence can alter, the sign of $\partial^2 T_i(\gamma_i)/\partial\gamma_i^2$. However, since $\partial^2 T_i(R_i^*)/\partial(R_i^*)^2$ alters its sign once, we can conclude that $\partial^2 T_i(\gamma_i)/\partial\gamma_i^2$ has a unique cutting point with the axis of zero, and therefore following the previous analysis we conclude that when $f_i^{-1}\left(R^*_{Infl,i}/R^*_{Max,i}\right) \le \gamma^f_{Infl,i}$, $T_i(\gamma_i)$ is a sigmoidal function of γ_i, with a unique inflection point $\gamma^{\mathrm{T}}_{Infl,i}$, and hence $\gamma^{\mathrm{T}}_{Infl,i} \in [f_i^{-1}\left(R^*_{Infl,i}/R^*_{Max,i}\right), \gamma^f_{Infl,i}]$.

Case I.2. When $\gamma^f_{Infl,i} \le f_i^{-1}\left(R^*_{Infl,i}/R^*_{Max,i}\right)$.

Following the same thread of analysis we can easily conclude that when $\gamma^f_{Infl,i} \le f_i^{-1}\left(R^*_{Infl,i}/R^*_{Max,i}\right)$, $T_i(\gamma_i)$ is a sigmoidal function of γ_i, with a unique inflection point $\gamma^{\mathrm{T}}_{Infl,i}$, and hence $\gamma^{\mathrm{T}}_{Infl,i} \in [\gamma^f_{Infl,i}, f_i^{-1}\left(R^*_{Infl,i}/R^*_{Max,i}\right)]$. ■

APPENDIX B - PROOF OF LEMMA 2

In the following a common proof is provided for both the cases where $T_i(R_i^*)$ is a strictly concave or strictly convex function of R_i^*. Whenever needed, we use parenthesis to provide the appropriate expressions for the case of strictly convex function, while the corresponding expressions for the case of strictly concave function appear without parenthesis. In accordance to the definition of a strictly concave (strictly convex) function the following properties hold:

$$\frac{\partial T_i(R_i^*)}{\partial R_i^*} > 0 \tag{20}$$

$$\frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} < 0 \quad \left(\frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2} > 0\right) \tag{21}$$

Moreover, for the sigmoidal efficiency function $f_i(\gamma_i)$ properties (13), (14) and (18) still hold. Regarding the first derivative of $T_i(\gamma_i)$, as shown in lemma 1, we have:

$$\frac{\partial T_i(\gamma_i)}{\partial \gamma_i} = R^*_{Max,i}\frac{\partial T_i(R_i^*)}{\partial R_i^*}\cdot\frac{\partial f_i(\gamma_i)}{\partial \gamma_i} > 0$$

according to (20) and (13). Considering the second derivative of $T_i(\gamma_i)$ and following lemma 1, we have:

$$\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = R^*_{Max,i}\left[R^*_{Max,i}\frac{\partial^2 T_i(R_i^*)}{\partial (R_i^*)^2}\cdot\left(\frac{\partial f_i(\gamma_i)}{\partial \gamma_i}\right)^2 + \frac{\partial^2 f_i(\gamma_i)}{\partial \gamma_i^2}\frac{\partial T_i(R_i^*)}{\partial R_i^*}\right] \tag{22}$$

At this point, we should study the properties of the second derivative of $T_i(\gamma_i)$ as a function of γ_i, $\gamma_i \in [0,\infty)$. Therefore, in the following due to the existence of $\gamma^f_{Infl,i}$ we examine the values of the sign of the second derivative of $T_i(\gamma_i)$ and its corresponding properties at the limits of its definition set (i.e. lower (upper) bound of γ_i) in the following complementary subsets in the axis of γ_i i.e. $[0,\gamma^f_{Infl,i}]$ and $[\gamma^f_{Infl,i},\infty)$.

Case II.1. When $\gamma_i > \gamma^f_{Infl,i}$ ($\gamma_i < \gamma^f_{Infl,i}$).

We can easily see that $\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} < 0$ $\left(\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} > 0\right)$ according to (15), (20), (21).

Case II.2. When $\gamma_i < \gamma^f_{Infl,i}$ $\left(\gamma_i > \gamma^f_{Infl,i}\right)$.

We consider two sub-cases (II.2.A and II.2.B) with respect to values of $T_i(\gamma_i)$ where $\gamma_i \in [0,\infty)$ in the lower (upper) bound of its definition domain:

II.2.A: When $\lim_{\gamma_i\to 0^+}\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^+$ $\left(\lim_{\gamma_i\to\infty}\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^-\right)$.

In such case, there always exists an interval within $[0,\gamma^f_{Infl,i}]$ ($[\gamma^f_{Infl,i},\infty)$), where $\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} > 0$ $\left(\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} < 0\right)$. Consequently, we can argue, with respect to the outcome of *case II.1*, that $\partial^2 T_i(\gamma_i)/\partial\gamma_i^2$ has at least one interruption point with the axis of zero in the interval $[0,\gamma^f_{Infl,i}]$ ($[\gamma^f_{Infl,i},\infty)$). Moreover, the uniqueness of such a point can be proven as follows.

Studying (22), when $0 < \gamma_i < \gamma^f_{Infl,i}$ $\left(\gamma_i > \gamma^f_{Infl,i}\right)$, the second part of the sum in (22) is always positive (negative), following (15). Therefore, only the sign of $\partial^2 T_i(R_i^*)/\partial(R_i^*)^2$ contributes to the change of the sign of $\partial^2 T_i(\gamma_i)/\partial\gamma_i^2$. However, since $\partial^2 T_i(R_i^*)/\partial(R_i^*)^2$ alters its sign only once, we conclude that $\partial^2 T_i(\gamma_i)/\partial\gamma_i^2$ has a unique intersection point with the axis of γ_i.
Therefore with respect to *Case II.1* and subcase *II.2.A* we can conclude the following:

Conclusion 1: When $\lim_{\gamma_i\to 0^+}\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^+$ $\left(\lim_{\gamma_i\to\infty}\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^-\right)$, $T_i(\gamma_i)$ is a sigmoidal function of $\gamma_i \in [0,\infty)$, with a unique inflection point $\gamma^{\mathrm{T}}_{Infl,i}$, and hence $\gamma^{\mathrm{T}}_{Infl,i} \in [0,\gamma^f_{Infl,i}]$ $\left(\gamma^{\mathrm{T}}_{Infl,i} \in [\gamma^f_{Infl,i},\infty)\right)$.

II.2.B: When $\lim_{\gamma_i\to 0^+}\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^-$ $\left(\lim_{\gamma_i\to\infty}\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^+\right)$.

In such case, $\partial^2 T_i(\gamma_i)/\partial\gamma_i^2$ has no interruption point with the the axis of γ_i in the interval $[0,\gamma^f_{Infl,i}]$ ($[\gamma^f_{Infl,i},\infty)$). To prove the above argument we must observe that from Case II.1 and the fact that $\lim_{\gamma_i\to 0^+}\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^-$ $\left(\lim_{\gamma_i\to\infty}\frac{\partial^2 T_i(\gamma_i)}{\partial \gamma_i^2} = 0^+\right)$, $\partial^2 T_i(\gamma_i)/\partial\gamma_i^2$ either never equals to zero in the interval $[0,\gamma^f_{Infl,i}]$ ($[\gamma^f_{Infl,i},\infty)$) or equals to zero (i.e. interrupts with the axis of γ_i) more than one times. The latter is not valid due to the fact that $\partial^2 T_i(\gamma_i)/\partial\gamma_i^2$ can alter its sign only once when $\gamma_i \in [0,\infty)$ because its sign is determined by $\partial^2 T_i(R_i^*)/\partial(R_i^*)^2$ which alters it sign only once.

Therefore with respect to *Case II.1* and subcase *II.2.B* we can conclude the following:

Conclusion 2: When $\lim_{\gamma_i\to 0^+}\frac{\partial^2 T_i(\gamma_i)}{\partial\gamma_i^2}=0^-$ $\left(\lim_{\gamma_i\to\infty}\frac{\partial^2 T_i(\gamma_i)}{\partial\gamma_i^2}=0^+\right)$, $T_i(\gamma_i)$ is a strictly concave (strictly convex) function of γ_i, $\gamma_i \in [0,\infty)$. Conclusions 1 and 2 complete the proof. ■

APPENDIX C - PROOF OF LEMMA 3

In this proof, for simplicity in the presentation let us denote the overall utility function of NRT users as:

$$U_i(P_i)=\frac{T_i(P_i)}{P_i},\ \text{s.t. } 0\le P_i\le P_i^{Max}$$

Moreover, let us underline that due to the one to one relationship between γ_i and P_i, the properties of $T_i(\gamma_i)$ as a function of γ_i revealed in lemmas 1 and 2 also hold for $T_i(P_i)$ as a function of P_i.

The first derivative of $U_i(P_i)$ can be expressed as:

$$\frac{\partial U_i(P_i)}{\partial P_i}=\frac{\partial\left(\frac{T_i(P_i)}{P_i}\right)}{\partial P_i}=\frac{\frac{\partial T_i(P_i)}{\partial P_i}P_i-T_i(P_i)}{P_i^2}=-\frac{b_i(P_i)}{P_i^2}$$

where $b_i(P_i)=T_i(P_i)-\frac{\partial T_i(P_i)}{\partial P_i}P_i$. Functions $T_i(P_i)$ and $\frac{\partial T_i(P_i)}{\partial P_i}$ have been assumed as continuous functions of P_i, therefore $b_i(P_i)$ is also a continuous function of P_i. For any two values $v\in\{1,2\}$ of user's i power P_i, denoted as $P_{v,i}\in[0,\infty)$, the following properties hold for a strictly concave (strictly convex) $T_i(P_i)$ function:

$$T_i(P_{2,i})<T_i(P_{1,i})+\left.\frac{\partial T_i(P_i)}{\partial P_i}\right|_{P_i=P_{1,i}}(P_{2,i}-P_{1,i}),\ \forall P_{1,i},P_{2,i}\in[0,\infty)$$
,
$$\left(T_i(P_{2,i})>T_i(P_{1,i})+\left.\frac{\partial T_i(P_i)}{\partial P_i}\right|_{P_i=P_{1,i}}(P_{2,i}-P_{1,i}),\ \forall P_{1,i},P_{2,i}\in[0,\infty)\right)$$

due to the strict concavity (strict convexity) of $T_i(P_i)$. Letting $P_{2,i}=0$ and $P_{1,i}=P_{v,i}$ for $P_i\ge 0$ we conclude that:

$$T_i(0)<T_i(P_{v,i})+\left.\frac{\partial T_i(P_i)}{\partial P_i}\right|_{P_i=P_{v,i}}(0-P_{v,i})$$

$$\left(T_i(0)>T_i(P_{v,i})+\left.\frac{\partial T_i(P_i)}{\partial P_i}\right|_{P_i=P_{v,i}}(0-P_{v,i})\right)$$

and thus,

$$0<T_i(P_{v,i})-\left.\frac{\partial T_i(P_i)}{\partial P_i}\right|_{P_i=P_{v,i}}P_{v,i}\quad\left(0>T_i(P_{v,i})-\left.\frac{\partial T_i(P_i)}{\partial P_i}\right|_{P_i=P_{v,i}}P_{v,i}\right)$$

$$b_i(P_{v,i})>0\qquad\left(b_i(P_{v,i})<0\right)$$

therefore,

$$\frac{\partial U_i(P_i)}{\partial P_i}=-\frac{b_i(P_i)}{P_i^2}<0\qquad\left(\frac{\partial U_i(P_i)}{\partial P_i}=-\frac{b_i(P_i)}{P_i^2}>0\right)\tag{23}$$

Following the previous analysis,

a) when $T_i(P_i)$ is a strictly concave function of P_i, we can observe that $U_i(P_i)$ is a decreasing function of P_i, $P_i\in[0,P_i^{Max})$ in accordance to (23). In such case if $\lim_{P_i\to 0^+}U_i(P_i)=0^+$, that would imply that i) $U_i(P_i)<0$ when $P_i\in[0,P_i^{Max})$ or ii) $\lim_{P_i\to 0^+}U_i(P_i)\ne 0^+$ in order $U_i(P_i)\ge 0$ when $P_i\in[0,P_i^{Max})$, which contradicts with the basic assumptions in the definition of U_i (section III).

b) when $T_i(P_i)$ is a strictly convex function of P_i, we can observe that $U_i(P_i)$ is an increasing function of $P_i\ge 0$, in accordance to (23). Therefore, not bounded above for $P_i\ge 0$, which also contradicts with the basic assumptions in the definition of U_i (section III).

APPENDIX D - PROOF OF LEMMA 4

Via lemmas 1, 2 and following lemma 3, it is easily shown that $T_i(\gamma_i),\ \forall i\in S$ is a sigmoidal function of $\gamma_i(\gamma_i\ge 0)$, regarding both RT multimedia and NRT data users. Moreover, due to the strict linear relationship between γ_i and P_i in accordance to (1), it follows immediately that a user's actual throughput utility is also a sigmoidal function of its own power. Therefore, the following properties of $T_i(\gamma_i)=T_i(P_i,\overline{P_{-i}})$ as a function of P_i, when $P_i\ge 0$ hold:

1. Its domain is the non-negative part of the real-line, that is, the interval $[0,\infty)$.

2. Its range is the interval $[0,1)$, according to the properties of $T_i(\gamma_i)$, $i\in S$ (assumptions c and d in section III).

3. It is an increasing function of P_i, $i\in S$, following lemmas 1, 2 and respecting lemma 3.

4. It is strictly convex over the interval $[0,P^{T}_{Infl,i})$, and strictly concave over the interval $[P^{T}_{Infl,i},P^{i}_{Max}]$, where from (1) as well as lemmas 1 and 2 it is straightforward that $P^{T}_{Infl,i}=\frac{\gamma^{T}_{Infl,i}(R^*_{T,i}+MF_i)I_{-i}(\overline{P_{-i}})}{G_iW}$ $\forall i\in S_{RT}$ and $P^{T}_{Infl,i}=\frac{\gamma^{T}_{Infl,i}R_i^{Max}I_{-i}(\overline{P_{-i}})}{G_iW}$ $\forall i\in S_{NRT}$. In this paper, it is assumed that $P_i^{Max}>>P^{T}_{Infl,i}$ $\forall i\in S$. The validity of the

previous property follows directly by taking into account the outcome of lemma 1 and 2, while respecting lemma 3.

5. It has a continuous derivative, since functions $T_i(R_i^*)$ and $f_i(\gamma_i)$ are assumed to have continuous derivatives too.

Since the previous properties hold for $T_i(P_i,\overline{P_{-i}})\ \forall i \in S$, the ratio $T_i(P_i,\overline{P_{-i}})/P_i,\ \forall i \in S$ is a quasi-concave function of $P_i \in [0, P_i^{Max}]$ [29]. Therefore, a user's actual throughput utility $U_i(P_i,\overline{P_{-i}})\ \forall i \in S$ is a quasi-concave function of its own power.

Moreover, the first order optimization condition of $U_i(P_i,\overline{P_{-i}})$ can be expressed as follows:

$$\frac{\partial U_i(P_i,\overline{P_{-i}})}{\partial P_i} = \frac{\partial\left(\frac{T_i(\gamma_i)}{P_i}\right)}{\partial P_i} = \frac{\frac{\partial T_i(\gamma_i)}{\partial P_i}P_i - T_i(\gamma_i)}{(P_i)^2} = 0$$

Therefore, $U_i(P_i,\overline{P_{-i}})\ \ \forall i \in S$ is maximized when:

$$\frac{\partial T_i(\gamma_i)}{\partial \gamma_i}\gamma_i - T_i(\gamma_i) = 0 \quad (24)$$

due to the fact that $\frac{\partial \gamma_i}{\partial P_i} = \frac{\gamma_i}{P_i}$ according to (1) and the strict linear relationship between γ_i and P_i. Moreover, due to the sigmoidal type of function $T_i(\gamma_i),\ \forall i \in S$, (24) has a unique solution γ_i^*, as proven in [29]. Therefore, the unique value $\hat{P}_i$ that maximizes $U_i(P_i,\overline{P_{-i}})$ when $P_i \geq 0$, can be represented, according to (1), as follows:

$$\hat{P}_i = \begin{cases} \dfrac{\gamma_i^*(R_{T,i}^* + MF_i)I_{-i}(\overline{P_{-i}})}{G_i W} & i \in S_{RT} \\ \dfrac{\gamma_i^* R_i^{Max} I_{-i}(\overline{P_{-i}})}{G_i W} & i \in S_{NRT} \end{cases}$$

Finally, if the maximum allowable value of P_i is less than $\hat{P}_i$, $U_i(P_i^{Max},\overline{P_{-i}}) = T_i(P_i^{Max},\overline{P_{-i}})/P_i^{Max}\ \ \forall i \in S$ is the highest value of $U_i(P_i,\overline{P_{-i}})$, since the utility function U_i is increasing over the interval $[0,\hat{P}_i]$. Therefore, the smallest of the values of $\hat{P}_i, P_i^{Max}$ is the global maximizer of $U_i(P_i,\overline{P_{-i}})\ \ \forall i \in S$ when $P_i \in [0, P_i^{Max}]$ i.e.

$$P_i^* = \begin{cases} \min\left(\dfrac{\gamma_i^*(R_{T,i}^* + MF_i)I_{-i}(\overline{P_{-i}})}{G_i W}, P_i^{Max}\right), & i \in S_{RT} \\ \min\left(\dfrac{\gamma_i^* R_i^{Max} I_{-i}(\overline{P_{-i}})}{G_i W}, P_i^{Max}\right), & i \in S_{NRT} \end{cases}$$

which concludes the proof. ■

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