Cooperative Networks: Bit-Interleaved Coded Modulation with Iterative Decoding

Shujaat Ali Khan Tanoli, Imran Khan and Nandana Rajatheva
School of Engineering and Technology, Asian Institute of Technology, Thailand
Email: {shujaat.ali.khan,imran.khan,rajath} @ait.ac.th

***Abstract*— In this paper the performance of bit-interleaved coded modulation-iterative decoding (BICM-ID) based co-operative network is analyzed over Rayleigh, Nakagami-m and Rician fading channels. In this system coding diversity is obtained through BICM and spatial diversity through cooperative relaying network. The analysis is performed for the relays operating in both amplify-and-forward (AF) and decode-and-forward (DF) modes. The bit error rate (BER) bound of multiple-relay cooperative system with the serial concatenated BICM-ID using the set partitioning (SP) and Gray mapped (GM) labeling for M-ary Phase Shift Keying (M-PSK) modulations over Nakagami-m fading channels is obtained. For the numerical results of union bound on BER, moment generating function (MGF) based approach is used. The expression of MGF derived for Nakagami distribution is then extended to Rayleigh and Rician fading channels. The comparisons of BER theoretical and simulation results are also shown. The performance of bit-interleaved low-density parity-check coded modulation with iterative decoding (BILDPCM-ID) based cooperative system, using low-density parity-check (LDPC) code instead of convolutional code, is also analyzed in term of bit-error rate (BER). The performance comparisons of BICM-ID and BILDPCM-ID based systems are shown using the set partitioning (SP) labeling for 8-PSK modulation scheme by performing Monte Carlo simulation.**

***Index Terms*— Bit-interleaved coded modulation, Bit-interleaved low-density Parity-Check coded modulation with iterative decoding (BILDPCM-ID), Cooperative Networks, Nakagami-m distribution, Moment generating function.**

I. Introduction

Wireless communication faces the main challenges of spectral efficiency, link reliability, power on the terminal and complexity. In order to address all such problems, the cooperative communication is most potential candidate in the next generation wireless systems [1] [2].

In cooperative communication, the spatial diversity as in multiple-input multiple-output (MIMO) is achieved by involving a number of relays between source and destination. This involvement helps to achieve the high data rate and make the overall system more reliable in term of bit error rate (BER) and throughput. The relay node operates over two strategies i.e. Amplify and forward (AF) and decode and forward (DF). Both modes of transmission are well-matched for high order modulation (i.e. 8-PSK, 16-QAM etc) and coding rates at source and relay. To obtain the maximum throughput, the transmitter and relay both raise its modulation order depending on the reliability of the channel condition between source-relay and relay-destination [3]. Hence, the communication system efficiently exploits the higher order modulation to further enhance the system throughput, which makes the system more efficient.

Bit-interleaved coded modulation (BICM) is a coding scheme that obtains the code diversity by effectively utilizing the hamming distance structure of binary codes when used in the combination of higher order modulation over fading channels. Hence, the reliability of the coded modulation in fading scenario could be further improved. The performance evaluation and principles of the design for bit-interleaved coded modulation is explained in [4].

In [5], the author identified that pitfall of BICM is reduced free Euclidean distance because of random modulation inbuilt in bit-interleaved scheme. To address this problem, a simpler approach of iterative decoding (ID) is used with a serial concatenation of encoding, bit-wise interleaving and high order modulation. In [6], bit-interleaved space time coded modulation with iterative decoding (BI-STCM-ID) is introduced for MIMO system. Hence, the performance of the BICM can be further improved by iterative decoding (BICM-ID) using hard decision feedback [7] and achieves significantly reduced receiver complexity as compared to turbo codes, as BICM-ID needs only one set of encoder/decoder.

The BICM-ID switches a 2^M-ary signaling channel to M parallel binary channels. With appropriate bit labeling, hence, a large binary Hamming distance between coded bits can be indirectly interpreted in to a large Euclidean distance. Consequently, this comprehends high diversity order, large free Euclidean distance and efficiently combines powerful binary codes with bandwidth efficient modulation. BICM-ID improves BICM by more than 1-dB and provides excellent performance over both Gaussian and fading channels, with iterative decoding [8] [9].

In this regard, BICM-ID will be most promising candidate when cooperative communication employs the higher order modulations over fading channels. The BICM-ID based cooperative system further allows different modulation i.e. 8-PSK, QAM, 16-QAM and also support different types of coding for cooperative transmission instead of conventional convolutional codes like space-time codes (STC) [6], Turbo codes [10] [11] and low density parity

Manuscript received February 11, 2010; revised April 15, 2010.

doi:10.4304/jcm.5.7.562-570

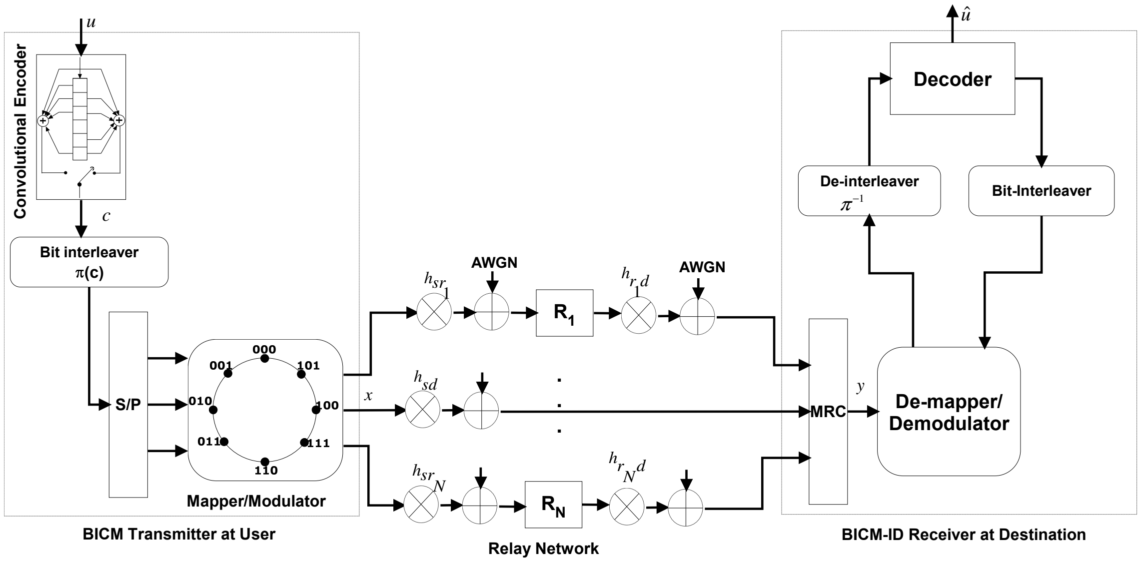

Figure 1. BICM-ID Based Cooperative Network.

check codes (LDPC) [12], [13]. In coded cooperation [14]- [15] the users are considered as the relay devices, that receive and decode the information bits of its neighboring user, re-encode and retransmit to the destination with additional parity bit for its neighbor users. If it is not possible for user to decode the information of its partner then it reverts to the non-cooperative mode.

The BICM based relay network using bilayer LDPC codes is introduced in [12]. Where, the fading scenario is ignored. In [16], we briefly analyze the performance of BICM based cooperative networks over AWGN and Rayleigh fading channels. The results are further extended to bit-interleaved space time-coded modulation for user cooperation diversity (CO-BISTCM) and two STC based transmission protocols are proposed for the system [17]. The idea of BICM and LDPC for DF mode is also presented in [18]. In [13], a bit-interleaved low density parity-check (LDPC) coded modulation with iterative demapping and decoding is introduced, using LDPC codes.

In this article, our main contribution is to analyze a single user serial concatenation of BICM-ID with multiple-relay cooperative system. The performance analysis of the proposed system is carried out by obtaining the theoretical bounds and verified with simulation results. The analytical results consist of BER curves of BICM-ID based cooperative system over Rayleigh, Nakagami-m and Rician fading channels for M-PSK modulation schemes. The capacity of the system is analyzed by performing Monte Carlo simulations for various fading channels. The analysis is performed for both AF and DF relaying schemes.

Currently, an explosion of attention has been seen in the field of relay coding to further enhance the capacity performance. So, in this regards we also present a novel approach of bit-interleaved low-density parity-check coded modulation with iterative decoding (BILDPCM-ID) based cooperative system in which we explore the possibility of using LDPC codes (instead of convention convolutional codes) with combination of BICM-ID and AF half duplex relay protocol. LDPC codes are very famous for their capacity-approaching performance for conventional single user communication channels. The main inspiration of LDPC codes is to practically apply the random coding theorem of Shannon by implementing a set of random parity check constraints on information bits [19].

This paper is further organized as follows: the next section presents the system model; section III shows the theoretical results for the MGF of branch metric. The simulation and analytical discussion is made in section V. Finally, section VI includes the conclusion of our work.

II. System Model

The system is composed of three main components: user (U), relay network (R) and destination (D). A single user BICM-ID based multiple-relay cooperative system model is given in Fig. 1 and each block is explained below.

A. BICM Transmitter at User and Relay:

The BICM Transmitter has further three modules discussed as follow:

Encoder: Two encoders are employed for U and R represented by Enc-U and Enc-R respectively. The Enc-U first takes k_c bit information block u from the user and generates n_c-bit codeword $\bar{c}_1 \in C^1$ and similarly $\bar{c}_2 \in C^2$ form Enc-R, where both C^1 and C^2 are binary codes. Note that, each encoder has two types, The convolutional encoder is used for BICM-ID and LDPC encoder for BILDPCM-ID scenarios.

Interleaver: Both code-words $\bar{c}_1$ and $\bar{c}_2$ are then bit interleaved by π-U and π-R respectively, to obtain the interleaved codeword. A one-to-one correspondence is setup by interleaver in BICM technique i.e. $\pi : t \rightarrow (\acute{t}, i)$ here i identifies the bit position in the symbol label, $\acute{t}$ is the time ordering of the modulated symbol and t represents the time ordering of bit sequence before interleaving. Here, bit wise interleaving has the main objective, of maximizing the diversity order of the system by breaking the correlation of sequential fading coefficient [12].

Modulator: After interleaving, the bits are mapped onto symbols by M-PSK binary labeling maper at U and R $\mu_1 : (0,1)^{\ell_1} \rightarrow \chi_1$ and $\mu_2 : (0,1)^{\ell_2} \rightarrow \chi_2$ and modulated over the signal sets $\chi_1 \subset V^{q_1}$ and $\chi_2 \subset V^{q_2}$, where ℓ is the number of bits per symbol, q is the dimension of complex Euclidean spaces V^{q_1} and V^{q_2}. Both modulators at the U and R are memory less and denoted by Mod-U= (μ_1, χ_1) and Mod-R= (μ_2, χ_2) respectively. Where the size of the signal set is given by $|\chi| = M = 2^{\ell}$.

B. The Tanner graph of BILDPCM-ID

The Tanner graph of BILDPCM-ID system shown in Fig.2 represents both BICM-ID and LDPC codes. The corresponding bits and symbols have some restrictions. Here, the message passing occurs back and forth between symbol nodes and bit nodes that can be considered as the procedure of iterative decoding. The interesting thing which can be noticed form BILDPCM-ID Tanner graph is that different bit nodes are attach to each symbol node but there are no cycles between the symbol node and bit nodes except some minimum length of the cycles that termed as girth as shown in Tanner graph. These girths affects the BILDPCM-ID error performance at low coding rates.

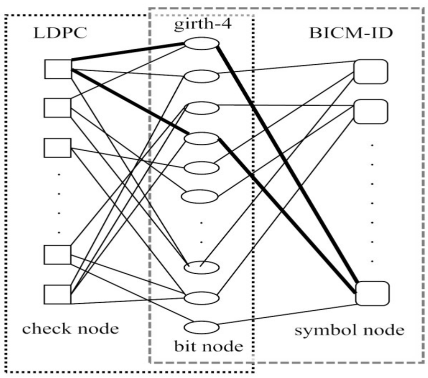

Figure 2. Tanner graph of BILDPCM-ID

C. Relay Network

The modulated signal is transmitted over the relay network. Consider the multiple-relay cooperative system as shown in the Fig. 3. There are Total N number of relays. The relays are operating in amplify-and-forward (AF) mode (transparent relaying) with fixed gain or decode-and-forward (DF) mode (regenerative relaying). In AF mode each relay receives a signal from user and retransmits after amplifying the signal to the destination. In the DF mode, the signal is decoded, re-encoded and finally retransmitted to the destination.

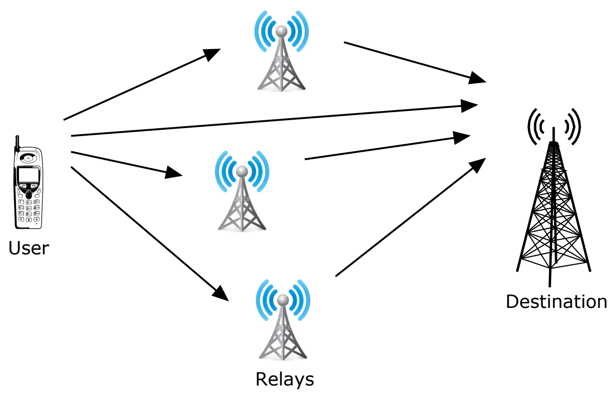

Figure 3. Relay Network.

1) Relay Links: We consider that all components operate on TDMA based two time frame protocol for transmission and reception as discussed in Section 2.2.2. The proposed system consist of three types of links $U \rightarrow R_n$, $U \rightarrow D$, and $R_n \rightarrow D$ where n is n-th relay among the multiple relays. For simplicity we assume that the links are itemize with j= $(0, 1, 2)$ respectively.

Each terminal U, R and D are equipped with single antenna for transmission and reception. The mechanism of various signal transmission is designed on the bases of positive integers q_1, q_2, and q_D dimensional complex Euclidean spaces V^{q_1},V^{q_2} and V^{q_D} respectively and transmission-reception of the single antenna can be modeled by allocating all these constraints to one. Similar to the MIMO channel model, the relay link with high order complex modulated signal (i.e. MPSK, and MQAM) can also be designed by setting all dimension $q_1 = 1$,$q_2 = 1$ and $q_D = 1$ for a single antenna on U, R and D.

Now the transmission/reception is carried out in two time frames. In first time frame, U transmits signal sequence $\bar{x}_1 = (x_{1,1}, \cdots, x_{1,f_1})$ to R and D, while in the second time frame R forwards the signal sequence $\bar{x}_1 = (x_{2,1}, \cdots, x_{2,f_2})$ to the destination. Where f_1 and f_2 are the transmissions during first and second time frames.

y_{sr_n} and y_{sd} are the signals sequences received at R and D respectively in the first time frame while y_{r_nd} is the received signals sequences at D in the second time frames (discussed detail in later section) from corresponding links $U \rightarrow R_n$, $U \rightarrow D$, and $R_n \rightarrow D$. Here the number of relays $n = 1 \cdots N$ where n is n-th relay and N denotes the total number of relays. These links considered to be follow the frequency-flat fading channel distribution. The perfect channel state information (CSI) is assumed at the receiver of relay and destination and transmitter have no channel knowledge.

2) Channel Model: As shown by [5], [4], It is assuming in BICM system model that, ideal interleaving permits the transmission of code sequence through parallel binary input channel, where each bit position in the labeling map corresponds to that of parallel binary input channel.

Keeping in view, we can also consider the BICM with ideal interleaving assumption in our BICM-ID based cooperative communication system. So, in this regards our system is composed of $N+1$ sets of orthogonal parallel channels corresponding to $U \rightarrow D$ and $R_n \rightarrow D$ links with characteristics of independent and memory less binary input, where each bit location in the χ_j corresponds to each channel in a set of parallel channels. The conditional probability density function (PDF) of binary input channel when selected i bit position in label mapping is given by [4]:

$$p_{\theta^j}(y^j|b,i) = \frac{1}{2^{\ell_j - 1}} \sum_{z \in \chi_j(i;b)} p_{\theta^j}(y^j|z) \tag{1}$$

where θ^j denotes channel state, $i = (1, 2, \cdots, \ell_j)$, $b \in (0,1)$, $j = 1,2$ and y^j is received signal sequence at D from j-th link.

It is assumed that the fading channels remain the same over the duration of two time slots i.e. slow-flat fading coefficients. Channel coefficients follow Rayleigh, Nakagami-m and Rician distributions. For Nakagami fading factor, $m = 1$ and $m = (K+1)^2/(2K+1)$ [20] (where K is the Rice factor), the channel coefficients follows Rayleigh and Ricean distribution, respectively. Nakagami-m distribution is a versatile statistical representation, that can model a variety of fading channels, like Rayleigh, Rician and one-sided Gaussian distribution. It is also assumed that perfect synchronization and channel state information (CSI) are available at the receiver. h_{xy} is the path gain through $X \rightarrow Y$ link and for $|h_{xy}|$ follows Nakagami distribution with probability density function (pdf) given by [21]:

$$\begin{aligned} f(|h_{xy}|) &= \left(\frac{m}{E(|h_{xy}|^2)}\right)^m \frac{2|h_{xy}|^{2m-1}}{\Gamma(m)} \times \\ & \exp\left(\frac{-m|h_{xy}|^2}{E\{|h_{xy}|^2\}}\right) \end{aligned} \tag{2}$$

where $E\{|h_{xy}|^2\} = 1$, Let $\alpha_{sd} = |h_{sd}|^2$, $\alpha_{rd} = |h_{rd}|^2$ and $\alpha_{sr} = |h_{sr}|^2$ are gamma distributed random variables and its pdf is given by:

$$f(\alpha_{xy}) = \frac{2m^m \alpha_{xy}^{m-1}}{\Gamma(m)} \exp(-m\alpha_{xy}) \tag{3}$$

where $\Gamma(.)$ is a gamma function. It is assumed that all the terminals transmit through orthogonal channels [22] using time division, frequency division or code division. The received signals at the destination are $(N+1)$ independent copies of transmitted signal.

3) Transmission Protocol: The transmission protocol used here is proposed by Nabar *et al.* [23]. In first time frame U broadcasts its information to R and D, and in second time frame only R operating in AF or DF mode, forwards the signal to D and U kept silence during second time frame. By doing so, we save the resources in second time frame.

The received signal at the destination in time slot 1 is given as:

$$y_{sd} = h_{sd}\sqrt{E_{sd}}x + w_{sd} \tag{4}$$

where , E_{sd} is the transmitted bit energy and w_{sd} is additive white Gaussian noise at terminal D, i.e. $w_{sd} \sim \mathcal{CN}(0, N_0)$. The received signal at the n-th relay in time slot 1 is given by:

$$y_{sr_n} = h_{sr_n}\sqrt{E_{sr_n}}x + w_{r_n} \tag{5}$$

where, $w_{r_n} \sim \mathcal{CN}(0, N_0)$.

Amplify-and-Forward (AF) scheme: By using the AF relay scheme in second time slot, the relay normalizes the received signal from source and forwards to the destination. The received signal through n-th relay (R_n) in time slot 2 is given as [24]:

$$y_{r_n d} = \frac{1}{\omega_n}\sqrt{\frac{E_{rd}E_{sr}}{E_{sr}+N_0}} h_{r_n d} h_{sr_n} x + w_{r_n d} \tag{6}$$

where $\omega_n = \sqrt{\frac{E_{rd}|h_{r_n d}|^2}{E_{sr}+N_0} + 1}$, is a noise normalization factor for n-th relay path, $w_{r_n d} \sim \mathcal{CN}(0, N_0)$ for $n = 1, 2, \cdots, N$ and $E\{|h_{sr_n}|^2\} = 1$. Source (S) transmits same signal to all the relays (R) with same power (E_{sr}). It is assumed that all the relays are transmitting the signal to the destination with the same power i.e. E_{rd}. Hence the amplification factor is the same for all the relays (N). N is also the number of independent signals received at D in the second time slot.

Decode-and-Forward (DF) scheme: In DF scheme, the relay forwards the decoded signal, when it successfully decodes the signal in time slot 2. Here we assume ideal case the relay knows whether the transmitted symbol is decoded correctly or not. The received signal at the destination from n-th relay is given by [24]:

$$y_{r_n d} = \sqrt{\hat{E}_{r_n d}} h_{r_n d} x + w_{r_n d} \tag{7}$$

where $\hat{E}_{r_n d} = E_{rd}$ on correctly decoding the signal, otherwise $\hat{E}_{r_n d} = 0$.

General: In general the input/output equations can also be written as:

$$\mathbf{Y} = \mathbf{H}\mathbf{x} + \mathbf{W} \tag{8}$$

where

$$\mathbf{Y}^T = \begin{pmatrix} y_{sd} & y_{r_1 d} & \cdots & y_{r_N d} \end{pmatrix}_{1\times(N+1)}$$

The channel gain is represented as:

$$\mathbf{H}^T = \begin{pmatrix} A & B_1 & \cdots & B_N \end{pmatrix}_{1\times(N+1)}$$

$$\mathbf{W}^T = \begin{pmatrix} w_{sd} & w_{r_1 d} & \cdots & w_{r_N d} \end{pmatrix}_{1\times(N+1)}$$

where $A = \sqrt{E_{sd}}h_{sd}$ and

$$B_n = \begin{cases} \frac{1}{\omega_n}\sqrt{\frac{E_{rd}E_{sr}}{E_{sr}+N_0}} h_{r_n d} h_{sr_n}; & \text{for AF} \\ \sqrt{\hat{E}_{r_n d}} h_{r_n d}; & \text{for DF} \end{cases}$$

where $n = 1, 2, \cdots, N$ and ^{T} denotes the transpose of the matrix.

D. BICM-ID Receiver at Relay and Destination

Similar to BICM Transmitter, The reversed components are installed at the R and D such as:
Demodulator: Bit metric calculation is carried out by employing two demodulators Dem-R and Dem-D at R and D respectively.
Interleaver: The third interleaver π-D is installed at the D to reduce the error propagation in iterative decoding as shown in Fig.1 by removing the correlation among sequentially coded bits and the bit associated with the same channel symbol [7].
Deinterleaver: Two deinterleavers π^{-1}-R and π^{-1}-D are at R and D respectively.
Decoder: The decoder Dec-R is at R and Dec-D is at D. Note that the iterative decoding implements on D, hence only Dec-D involves in iterative decoding but both decoders Dec-R and Dec-D separately decodes convolutional and LDPC codes for BICM-ID and BILDPCM-ID scenarios repectively.

At the destination maximal ratio combiner (MRC) is used to combine the received signal coherently and then noise normalization is performed. The BICM decoder with iterative decoding is installed at the destination. The block diagram of the BICM-ID decoder is shown in the Fig. 1. The hard decision of the information bits is made from the soft information. The de-mapper generates the extrinsic logarithmic likelihood ratios (LLRs). The LLRs are then de-interleaved and fed to the SISO a-posteriori decoder. The iterative decoding is carried by feeding back the posterior probability (MAP) through the interleaver to de-mapper for the next iteration. The output of demapper will be given as:

$$L_k = \log \frac{\sum_{x_k \in S_k^{(1)}} f(Y|x_k)p(x_k)}{\sum_{X_k \in S_k^{(0)}} f(Y|x_k)p(x_k)}$$

where $p(x_k)$ is the probability of signal $x_k \in S$ and $f(Y|x_k) = Dp(Y|x_k)$, D is a constant and $p(Y|x_k)$ is the probability that Y is received given that x_k is transmitted. $S_k^{(1)}$ and $S_k^{(0)}$ represent the set of symbols having k-th bit equal to 1 and 0, respectively. Fig. 8 shows the gray and set partitioning (SP) labeling maps for 8-PSK modulation. At the second pass decoding, given the feedback of bits 2 and 3, the constellation of bit 2 is confined to a pair of points shown in Fig. 8. Therefore as far as bit 1 is concerned, 8-PSK channel is translated into a binary channel with a BPSK constellation selected by the two feedback bits from the four possible signal pairs. Similarly we can proceed for bits 2 and 3.

III. Moment Generating Function (MGF) of Branch Metric

We review the bit metric for the viterbi decoding in BICM-ID based cooperative system over Nakagami-m fading channels and evaluate the conditional MGF of the metric difference. The BICM-ID based cooperative system uses viterbi decoding with branch metric, given

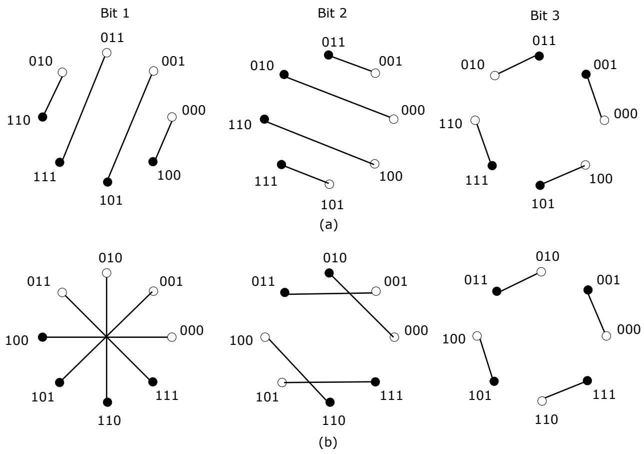

Figure 4. Labeling methods for 8-PSK a) Gray mapped (GM) b) Set partitioning (SP) [8].

as [4]:

$$\lambda_b^i = \min_{x \in \chi_b^i} \|\mathbf{Y} - \mathbf{H}\mathbf{x}\|^2 \tag{9}$$

where $\|.\|$ denotes the Euclidean norm of matrix. The metric difference relative to component x and z is given as:

$$\Delta(x,z) = \|\mathbf{Y} - \mathbf{H}\mathbf{x}\|^2 - \|\mathbf{Y} - \mathbf{H}\mathbf{z}\|^2 \tag{10}$$

For BER calculation, the MGF of metric difference can be given as [25]:

$$\Phi_{\Delta(x,z)}(s) = E\{\exp(-s\Delta(x,z))\} \tag{11}$$

The total MGF can be written as:

$$\Phi_{\Delta(x,z)} = \Phi_{\Delta(x,z)_{direct}} \times \Phi_{\Delta(x,z)_{2-hop}} \tag{12}$$

where $\Phi_{\Delta(x,z)_{direct}}$ is the MGF for direct path and $\Phi_{\Delta(x,z)_{2-hop}}$ is the MGF for 2-hop path. The MGF for direct path is given as:

$$\Phi_{\Delta(x,z)_{direct}} = E_{\alpha_{sd}} \left\{ e^{-sE_{sd} d_E^2 \alpha_{sd}(1-2sN_0)} \right\} \tag{13}$$

where, $d_E^2 = \|x - z\|^2$ is the squared Euclidean distance.

AF Scheme: For AF scheme, the MGF for 2-hop path is given as:

$$\Phi_{\Delta(x,z)_{2-hop}} = E_{\alpha_{sr_n}} \left\{ e^{-\sum_{n=1}^{N} \frac{se_1 d_E^2}{e_2+1} \alpha_{sr_n} \alpha_{r_n d}(1-2sN_0)} \right\}$$

where $e_1 = \frac{E_{rd}E_{sr}}{E_{sr}+N_0}$ and $e_2 = \frac{E_{rd}}{E_{sr}+N_0}$. Equation (12) should be averaged over all channel realizations to get the unconditional MGF, by assuming that α_{sd}, α_{sr} and α_{rd} to be independent random variables. When α_{rd} is given, (11) can be written as:

$$\Phi_{\Delta(x,z)|\alpha_{r_n d}}(s) = E_{\alpha_{sd}} \left\{ e^{-sE_{sd} d_E^2 \alpha_{sd}(1-2sN_0)} \right\} \times E_{\alpha_{sr_n}} \left\{ e^{-\sum_{n=1}^{N} \frac{se_1 d_E^2}{e_2 \alpha_{r_n d}+1} \alpha_{sr_n} \alpha_{r_n d}(1-2sN_0)} \right\} \tag{14}$$

$$\Phi_{\Delta(x,z)|\alpha_{r_nd}}(s) = \frac{1}{\left(1-\frac{sE_{sd}d_E^2}{m}(2sN_0-1)\right)^m} \times \frac{1}{\left(1-\frac{se_1d_E^2}{e_2m}(2sN_0-1)\right)^{mN}} \prod_{n=1}^{N}\left(1+\sum_{v=1}^{m}\frac{C_v}{(\alpha_{r_nd}+\lambda(s))^v}\right) \tag{17}$$

$$\Phi_{\Delta(x,z)}(s) = \frac{1}{\left(1-\frac{sE_{sd}d_E^2}{m}(2sN_0-1)\right)^m\left(1-\frac{se_1d_E^2}{e_2m}(2sN_0-1)\right)^{mN}} \times\left(1+\sum_{v=1}^{m}m^mC_v\lambda^{m-v}\psi(m,m-v+1,m\lambda)\right)^N \tag{18}$$

$$\Phi_{\Delta(x,z)} = \left(\frac{1}{1-sE_{sd}d_E^2(2sN_0-1)}\right)\times\left(-\frac{(1+e_1)}{k}.e^{\frac{1}{k}}.E_i(-\frac{1}{k})\right)^N \tag{19}$$

After averaging (11) with respect to α_{sd} and α_{sr}, (14) can be written as:

$$\Phi_{\Delta(x,z)|\alpha_{r_nd}}(s) = \frac{1}{\left(1-\frac{sE_{sd}d_E^2}{m}(2sN_0-1)\right)^m}\times \prod_{n=1}^{N}\frac{1}{\left(1\frac{se_1d_E^2}{(e_2\alpha_{r_nd}+1)m}\alpha_{r_nd}(2sN_0-1)\right)^m} \tag{15}$$

After some simple mathematical steps, (15) can be written as:

$$\Phi_{\Delta(x,z)|\alpha_{r_nd}}(s) = \frac{1}{\left(1-\frac{sE_{sd}d_E^2}{m}(2sN_0-1)\right)^m}\times \frac{1}{\left(1-\frac{se_1d_E^2}{e_2m}(2sN_0-1)\right)^{mN}}\prod_{n=1}^{N}\left(1+G(\alpha_{r_nd})\right) \tag{16}$$

where $G(\alpha_{r_nd}) = \frac{g_{m-1}\alpha_{r_nd}^{m-1}+\cdots+g_1\alpha_{r_nd}+g_0}{(\alpha_{r_nd}+\lambda(s))^m}$ with $\lambda(s) = \frac{1}{e_2-\frac{se_1d_E^2}{m}(2sN_0-1)}$ and $g_{m-1},\cdots,g_1,g_0$ are real constants. After decomposing into partial fraction, (16) can be written as (17). where $C_v = \frac{1}{(m-v)!}\frac{d^{m-v}}{d\alpha_{r_nd}^{m-v}}\left((\alpha_{r_nd}+\lambda(s))^m G(\alpha_{r_nd})\right)|_{\alpha_{r_nd}=-\lambda(s)}$.
By averaging (17) over α_{r_nd}, we get the unconditional MGF as given in (18) for Nakagami-m fading channels. where $k = e_1 + se_2d_E^2(2sN_0-1)$, $\lambda(s) = \frac{1}{e_2-\frac{se_1d_E^2}{m}(2sN_0-1)}$, $E_i(.)$ is an exponential integral function and $\psi(.,.,.)$ is the confluent hypergeometric function of the second kind [26].
For $m=1$, i.e. Rayleigh distribution, (18) takes the form as (19). $E_i(.)$ is an exponential integral function [26]. For Rician distribution, $m=(K+1)^2/(2K+1)$ [20] in (18).

DF Scheme: For DF scheme, the MGF for 2-hop path is given as:

$$\Phi_{\Delta(x,z)_{2-hop}} = E_{\alpha_{r_nd}}\left\{e^{-\sum_{n=1}^{N}\hat{E}_{r_nd}\alpha_{r_nd}(1-2sN_0)}\right\}$$

After averaging (12) with respect to α_{sd} and α_{r_nd}, the unconditional MGF can be written as:

$$\Phi_{\Delta(x,z)}(s) = \frac{1}{\left(1-\frac{sE_{sd}d_E^2}{m}(2sN_0-1)\right)^m}\times \prod_{n=1}^{N}\frac{1}{\left(1-\frac{s\hat{E}_{r_nd}d_E^2}{m}(2sN_0-1)\right)^m} \tag{20}$$

In most of the cases the relay decodes correctly at high SNR values, i.e. $\hat{E}_{r_nd}=E_{rd}$. Hence at higher SNR (20) can be written as:

$$\Phi_{\Delta(x,z)}(s) = \frac{1}{\left(1-\frac{sE_{sd}d_E^2}{m}(2sN_0-1)\right)^m}\times \frac{1}{\left(1-\frac{sE_{rd}d_E^2}{m}(2sN_0-1)\right)^{mN}} \tag{21}$$

In (21), $m=1$ and $m=(K+1)^2/(2K+1)$ [20] for Rayleigh and Rician distributions, respectively.

Here we obtain the expressions for multiple relay network and for both AF and DF operating modes. It is clear from the MGF expressions that a diversity order of $m(N+1)$ is obtained for N-relay network in Nakagami-m fading.

IV. Bit Error Rate (BER) Bound

The performance analysis in our work is based on the union bound analysis assuming Error free feedback [30] and moment generating function (MGF) approach used by [9], [27], [28] and [29]. The probability to decode a received sequence as a codeword x with an error weight d (hamming distance) given that a transmitted codeword is z is known as pairwise error probability (PEP). The PEP union bound for BICM can also be expressed in the form of moment generating function (MGF) approach, given as [4]:

$$f(d,\mu,\chi) \le \frac{1}{2\pi j}\times\int_{\alpha-j\infty}^{\alpha+j\infty}[\psi_{ub}(s)]^d\frac{ds}{s} \tag{22}$$

where d is hamming distance of code and

$$\psi_{ub}(s) = \frac{1}{\ell 2^{\ell}} \sum_{i=1}^{\ell} \sum_{b=0}^{1} \sum_{x \in \chi_b^i} \sum_{z \in \chi_{\bar{b}}^i} \Phi_{\Delta_{(x,z)}}(s) \tag{23}$$

$\Phi_{\Delta_{(x,z)}}(s)$ is the Laplace transform (MGF) of the metric difference $\Delta(x,z)$ between x and z.

As the iterative decoding give us a significant gain, so it is very much interesting for us to evaluate the analytical bound for the error free feedback performance which we term as error floor (*ef*)to which the BICM-ID performance converges at low BER.

Known ideal feedback for each $x \in \chi_b^i$, as there is only one term in $x \in \chi_{\bar{b}}^i$ whose label has the same binary bit values as those of x except at the i-th bit location that term is $\tilde{z} = \tilde{z}(x)$, where $\bar{b}$ is the compliment of b and $\tilde{z} = \tilde{z}(x)$denotes the nearest neighbor of x. Therefore, the PEP of the error floor of BICM-ID can be obtained by removing the innermost summation in (11), and can be written as [9]:

$$f(d,\mu,\chi) \leq \frac{1}{2\pi j} \times \int_{\alpha - j\infty}^{\alpha + j\infty} [\psi_{ef}(s)]^d \frac{ds}{s} \tag{24}$$

where

$$\psi_{ef}(s) = \frac{1}{\ell 2^{\ell}} \sum_{i=1}^{\ell} \sum_{b=0}^{1} \sum_{x \in \chi_b^i} \Phi_{\Delta_{(x,\tilde{z})}}(s) \tag{25}$$

The union bound of probability of bit error as shown in the Fig. 5 and 6 code of rate $R = k_c/n_c$ is given as [4]:

$$P_b \leq \frac{1}{k_c} \sum_{d=d_H}^{\infty} W_1(d) f(d,\mu,\chi) \tag{26}$$

where the minimum Hamming distance d_H and $W_1(d)$ is the total input weight of error events at d . As the harmonic mean of the minimum squared Euclidean distance can also be increased by increasing in the euclidean distance between signals through iterative decoding, therefore, the error floor of BICM-ID is the horizontally shifted version of the performance curve of BICM without feedback.

V. RESULTS AND DISCUSSION

Now in this section, we present the analytical and simulation results. The analysis is performed in terms of BER and achievable rates for the system with $1/2$ convolutional encoder of generator sequences $g = [133 \quad 171]$, QPSK and 8-PSK modulation schemes with set partitioning (SP) and Gray labeled mapping. The simulation results are obtained for uncorrelated Rayleigh, Nakagami and Rician fading channels by simulating 10^7 information bits using MATLAB and the numerical results of error floors are calculated from (18) and (21). The simulation results are taken for SISO decoder using the log-MAP algorithm with iteration for decoding. The analysis are based on Monte Carlo simulations.

Fig. 5 shows simulation results with analytical error free feedback (EF) bounds for Nakagami-2 fading channels. It presents the performance of the system with 2 relays operating in AF mode. These curves are simulated upto 5 iterations using set-partitioning (SP) mapped 8-PSK modulation. The results show that iterative decoding helps converging the BER curves to the bound obtained, at 3-rd iteration the BER converges to the theoretical EF bound given in (18). The error floor effect occurs at BER less than 10^{-4}. It is clear from the results that the BER curves are very tight to the asymptotic performances at medium and high SNR. In these regions the theoretical expression can be used.

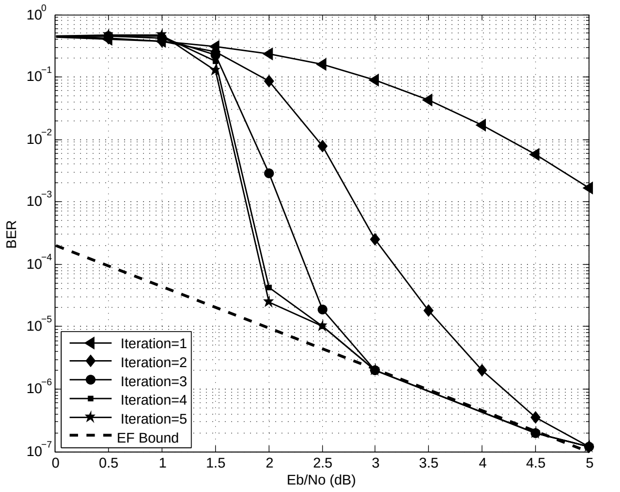

Figure 5. BER curves of BICM based 2-relay cooperative network over Nakagami-2 fading channels, 8-PSK and SP labeling.

Fig. 6 shows the EF bound and the simulation results for the same system over Rician fading channel for different number of iterations. The BER curves converges at SNR 2dB in 3-rd iteration. In this case the error floor occurs at SNR less than 10^{-4}, similar to the previous case.

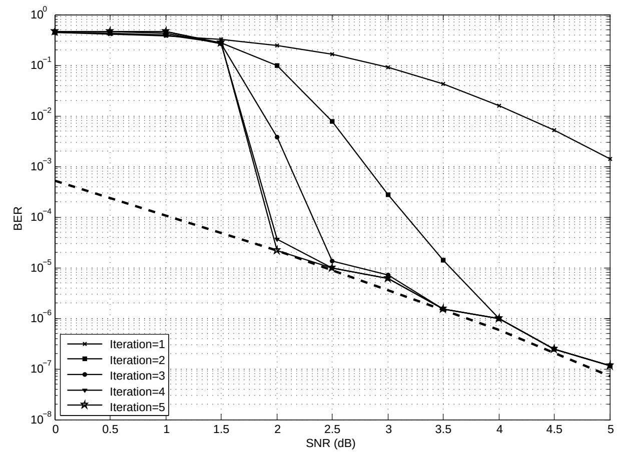

Figure 6. BER curves of BICM based 2-relay cooperative network for over Rician fading channels, $K = 10$, 8-PSK and SP labeling.

Fig. 7 shows the comparison of BER curves for partially and perfectly estimated decode-and-forward (DF) cooperative network with $m = 1$ and $m = 3$. In perfect DF, we assume the ideal case the relay knows whether the transmitted symbol is decoded correctly or not. The variations of BER is shown for iterations 1 and 2.

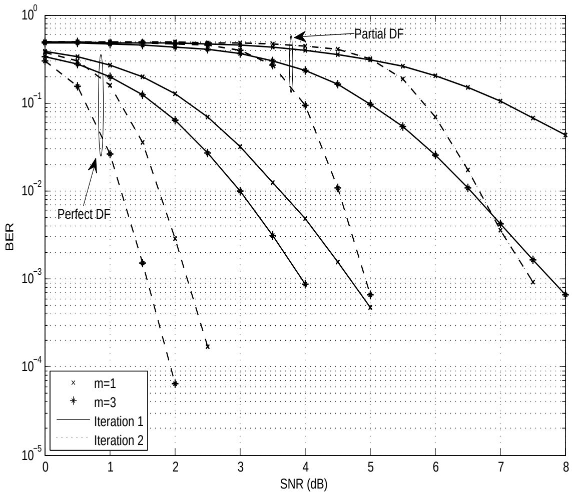

Figure 7. Comparison of Perfect and Partial DF over Nakagami-m fading channels ($m = 1, 3$), with SP labeling 8PSK and $N = 1$

For the performance comparison, the Monte Carlo simulation results are provided for BICM-ID and BILDPCM-ID based multiple-relay cooperative system over Nakagami-m fading channels. LDPC codes of rate $1/2$ length 2304 bits (LDPC short codes (SC)) from WiMax standard and 64800 bits (LDPC long codes (LC)) from DVB-S2 are used for BILDPCM-ID system. The LDPC internal iterations are set to be 20 for BILDPCM-ID system. The performance comparison of BILDPCM-ID and BICM-ID based single-relay cooperative network is shown in Fig. **??**. The simulation results are obtained for Nakagamim $(m = 1, 2)$ fading channels. From the curves, it is clear that in the BILDPCM-ID scenario the increase the m factor will increase the gap of about 1dB which is much larger than that of BICM-ID. The BILDPCM-ID gives significant improvement in the performance as compare to BICM-ID at low SNR. The results are shown for a single iteration of decoding at the end-receiver.

VI. Conclusion and Future Direction

We analyze the performance of BICM-ID based cooperative network over versatile Nakagami-m fading channels in terms of BER and achievable rates. The same theoretical results are extended to Rayleigh and Rician fading channels. The derivation of expression for the theoretical bounds is based on the MGF approach. MGFs of the metric difference of BICM-ID based multiple relay network with orthogonal channels are derived. Maximal ratio combining (MRC) is used at the destination to get the advantage of spatial diversity. The analysis is obtained for M-PSK modulation schemes. It is clear from simulation results that a significant gain is achieved in the performance of the system due to the cooperative and code diversities by introducing the number of cooperating relays and bit-interleaved coded modulation with iterative decoding on user and destination side, respectively.

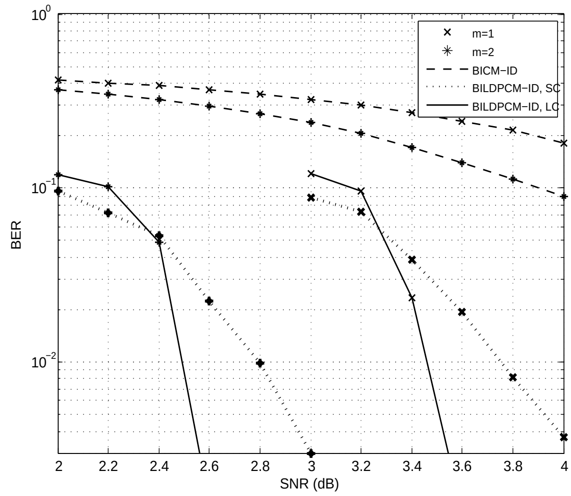

Figure 8. Performance comparison of BILDPCM-ID (short and long LDPC codes) and BICM-ID based single-relay cooperative network, for $m = 1, 2$ and 8-PSK.

Similarly, BICM-ID and cooperation brings a significant improvement to the performance of the system. The impact on the BER curves due to various number of iterations are also shown. The results show that iterative decoding converges the performance to the theoretical bound obtained.

The performance comparisons of BICM-ID and BILDPCM-ID based cooperative systems are also apresented, using the set partitioning (SP) labeling for 8-PSK modulation scheme by performing Monte Carlo simulation. we explored a novel approach of capacity-approaching performance for cooperative single user communication channels by exploiting iteratively decoded BI-LDPC codes and got significant improvement in the performance. Our work can be extended for higher order modulation like 16-QAM, 64-QAM. We can extend this analysis under the effects of other constraints like signal labeling and block length.

Acknowledgment

The first and second authors would like to thank the Higher Education Commission, Pakistan and UET Peshawar, Pakistan respectively, for providing the funds to carry out their higher studies.

References

[1] A. Nosratinia, T.E. Hunter and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, pp. 74–80, 2004.
[2] A. Sendonaris, E. Erkip and B. Aazhang, "User cooperation diversity, part I: System description," *IEEE Trans. on Commun.*, vol. 51, no.11, pp. 1927–1938, Nov. 2003.
[3] R. Hoshyar and R. Tafazolli, "BER performance analysis of a cooperative BICM system based on post-BSC model," Proc. IEEE 19th International Symposium PIMRC 2008. pp. 1–5, Sept. 2008.
[4] G. Caire, G. Taricco and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Inf. Theo.*, vol. 44, no. 3, May 1998.

[5] E. Zehavi, "8-PSK trellis codes for a Rayleigh channel," *IEEE Trans. Commun.*, vol. 40, pp. 873–884, May 1992.

[6] I. Shahid and N. Rajatheva, "Asymptotic BER bounds of iteratively decoded bit-interleaved space-time coded modulation under correlated Rayleigh and Rician fading channels," *IEEE Commun. Lett.*, vol. 12, no. 10, pp. 770–772, Oct. 2008.

[7] X. Li and J.A. Ritcey, "Bit-interleaved coded modulation with iterative decoding," *IEEE Commun. Lett.*, vol. 1, pp. 169–171, Nov. 1997 .

[8] X. Li and J.A. Ritcey, "Trellis-coded modulation with bit-interleaving and iterative decoding," *IEEE J. on Select. Areas in Commun.*, vol. 17, no. 4, April 1999.

[9] X. Li, A. Chindapol and J.A. Ritcey, "Bit-interleaved coded modulation with iterative decoding and 8PSK signaling," *IEEE Trans. Commun.*, vol. 50, no. 8, pp. 1250–1257, Aug. 2002.

[10] S. Lin and D.J. Costello, *Error control coding*, Prentice Hall, 2004.

[11] H.T. Nguyen, H.H. Nguyen and T. Le-Ngoc, "A bandwidth-efficient coded cooperative communications system," Proc. of the 64th IEEE VTC '06, pp. 1–5, Montreal, Canada, Sep. 2006.

[12] P. Razaghi, M. Aleksic and W. Yu, "Bit-interleaved coded modulation for the Relay channel using Bilayer LDPC codes," 10th Canadian Workshop on Info. Theory, 2007.

[13] Q. Xie, K. Peng, J. Song and Z. Yang, "Bit-Interleaved LDPC-coded modulation with iterative demapping and decoding," IEEE 69th Vehicular Technology Conference, 2009 (VTC Spring 2009), Barcelona, April 2009.

[14] M. Janani, A. Hedayat, T.E. Hunter and A. Nosratinia, "Coded cooperation in wireless communications: Space-time transmission and iterative decoding," *IEEE Trans on Sig. Proc.*, vol. 52, pp. 362–372, 2004.

[15] A. Stenfanov and E. Erkip, "Cooperative coding for wireless networks," *IEEE Trans. on Commun.*, vol. 52, pp. 1470–1476, 2004.

[16] S. A. K. Tanoli, I. Khan and N. Rajatheva, "Asymptotic BER bounds for BICM-based multiple relay cooperative network under AWGN and Rayleigh fading Channels," Proc. 4-th Int. Conf. on Commun. and Net. in China (ChinaCom'09), August 2009.

[17] I. Khan, S. A. K. Tanoli and N. Rajatheva, "Performance bounds for bit-interleaved space-time coded modulation with iterative decoding based cooperative network," *Journal of Communications*, Vol 5, No 4 (2010), 297-306, Apr 2010.

[18] G. Kramer, "Distributed and layered codes for relaying," The 39th Asilomar Conf. on Sig., Sys. and Comp., pp. 1752–1756, Oct. 2005.

[19] P. Razaghi and W. Yu, " Bilayer LDPC codes for the Relay Channel," IEEE International Conference on Communications, ICC '06. vol. 4, pp. 1574–1579, June 2006.

[20] R. Hasanizadeh and S. Zokaei, "Optimal power allocation to diversity branches of cooperative MISO sensor networks," *Intl. J. of Sig.*, IJSP, vol. 3, no. 4, pp. 281–288, 2006.

[21] M. Nakagami, "The m-distribution -A general formula for intensity distribution of rapid fading," Statistical Methods in Radio Wave Propagation, W. G. Hoffman, Ed. Oxford, U. K.: Pergamon, 1960.

[22] S. Ikki and M.H. Ahmed, "Performance analysis of cooperative diversity wireless networks over Nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 11, no. 4, pp. 334–336, April 2007.

[23] R.U. Nabar, H. Bolckei and F.W. Kneubuhler, "Fading relay channels performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[24] K.J.R. Liu and A.K. Sadek, *Cooperative communications and networking,* Cambridge University Press, 2009.

[25] A. Nasri, R. Schober and L. Lampe, "Performance evaluation of BICM-OFDM systems impaired by UWB interference," Proc. IEEE Intl. Conf. on Commun. (ICC 2008), Bejing, May 2008.

[26] I.S. Gradshteyn and I.M. Ryzhik, *Table of integrals, series and products*, 6th Ed., New York: Academic Press, 2000.

[27] C. Polprasert and J.A. Ritcey, "Effect of imperfect CSI on iteratively decoded BICM over Nakagami fading channels," Proc. IEEE Commun. Comp. and signal Processing, (PACRIM 2005), pp. 89–92, Aug. 2005.

[28] N.H. Tran and H.H. Nguyen, "Signal mapping of 8-ary constellations for BICM-ID systems over a Rayleigh fading channel," *IEICE Trans. Commun.*, vol. E88-B, no. 10, October 2005.

[29] N.H. Tran and H.H. Nguyen, "A novel multi-dimensional mapping of 8-PSK for BICM-ID," *IEEE Trans. on Commun.*, vol. 6, no. 3, March 2007.

[30] X. Li and J.A. Ritcey, "Bit-interleaved coded modulation with iterative decoding–Approaching Turbo–TCM performance without code concatenation," Proc. CISS'98, March 1998.

Shujaat Ali Khan Tanoli received his B.Sc. degree in electrical engineering from COMSATS Institute of Information Technology, Pakistan in 2006 and M.Sc. in Telecommunications Engineering from the Asian Institute of Technology, Thailand, in 2009. He is currently working towards the PhD degree at the School of Engineering and Technology, Asian Institute of Technology, Thailand. He is student member of IEEE.

Mr. Tanoli's research interests include performance analysis of wireless communications systems, OFDM, OFDMA, MIMO, BICM-ID based systems and cooperative networks.

Imran Khan received his B.Sc. degree (Honors) in electrical engineering from NWFP University of Engineering and Technology, Peshawar, Pakistan in 2003 and M.Sc. in telecommunications engineering from the Asian Institute of Technology, Thailand, in 2007. He is currently working towards the PhD. degree at School of Engineering and Technology, Asian Institute of Technology, Thailand. Earlier he has been working as Lecturer at NWFP University of Engineering and Technology, Peshawar, Pakistan since 2004. He is student member of IEICE and IEEE.

Mr. Khan's research interests include performance analysis of wireless communications systems, OFDM, OFDMA, MIMO, BICM-ID based systems and cooperative networks.

Nandana Rajatheva received the B.Sc. degree in electronic and telecommunication engineering (with first class honors) from the University of Moratuwa, Moratuwa, Sri Lanka, and the M.Sc. and Ph.D. degrees from the University of Manitoba,Winnipeg, MB, Canada, in 1987, 1991, and 1995, respectively. Currently, he is an Associate Professor of telecommunications in the School of Engineering and Technology, Asian Institute of Technology, Pathumthani, Thailand. Earlier, he was with the University of Moratuwa, Sri Lanka, where he became a Professor of Electronic and Telecommunication Engineering in June 2003. From May 1996 to December 2001, he was with TC-SAT as an associate professor. He is an editor of International Journal of Vehicular Technology (Hindawi) and senior member of IEEE (since 2001), Comsoc and VTS.

Dr. Rajatheva's research interests include digital and mobile communications, cooperative diversity, relay systems, OFDMA resource allocation, cognitive radio: detection/estimation techniques, space time processing MIMO systems and distributed video coding (DVC).