On The Symbol Error Probability of Distributed-Alamouti Scheme

Trung Q. Duong, Dac-Binh Ha, Hoai-An Tran, Nguyen-Son Vo, and Ahsan Haroon

***Abstract*—Taking into account the relay's location, we analyze the maximum likelihood (ML) decoding performance of dual-hop relay network, in which two amplify-and-forward (AF) relays employ the Alamouti code in a distributed fashion. In particular, using the well-known moment generating function (MGF) approach we derive the closed-form expressions of the average symbol error probability (SEP) for M-ary phase-shift keying (M-PSK) when the relays are located nearby either the source or destination. The analytical result is obtained as a single integral with finite limits and the integrand composed solely of trigonometric functions. Assessing the asymptotic characteristic of SEP formulas in the high signal-to-noise ratio regime, we show that the distributed-Alamouti protocol achieves a full diversity order. We also perform the Monte-Carlo simulations to validate our analysis. In addition, based on the upper bound of SEP we propose an optimal power allocation between the first-hop (the source-to-relay link) and second-hop (the relay-to-destination link) transmission. We further show that as the two relays are located nearby the destination most of the total power should be allocated to the broadcasting phase (the first-hop transmission). When the two relays are placed close to the source, we propose an optimal transmission scheme which is a non-realtime processing, hence, can be applied for practical applications. It is shown that the optimal power allocation scheme outperforms the equal power scheme with a SEP performance improvement by 2-3 dB.**

***Index Terms*—Distributed-Alamouti space–time code, amplify-and-forward (AF) relay, symbol error probability (SEP).**

I. INTRODUCTION

The Alamouti space-time code [1] has been considered as the only orthogonal space-time block code [2] can achieve full rate and full diversity over complex constellation with the symbol-wise maximum likelihood (ML) decoding complexity. Recently, there exists an extension of the Alamouti scheme into cooperative/relay systems where the relays simultaneously construct the Alamouti space-time code in a distributed fashion before relaying the signals to the destination [3]–[7].

Distributed-Alamouti scheme dated back to the work in [3], [4], where two single-antenna relays assisted the source-destination communication by forming the Alamouti space–time block code in the relay networks. The authors conjectured the diversity order of distributed-Alamouti scheme around one from their simulation [3], [4]. In [6], the average bit error rate (BER) of the distributed-Alamouti scheme was shown to be proportional to $\log(\mathsf{SNR})/\mathsf{SNR}^2$.

In [5], the distributed-Alamouti system was created by using only one single-antenna relay in Protocol III, i.e., the relay and source generate a distributed-Alamouti space-time code and each terminal transmits each row of Alamouti code. Recently, exact closed-form expressions for pairwise error probability of this scheme has been analyzed in [7] where it has been shown that a full diversity order is achieved. More recently, the construction of distributed space–time codes using amicable orthogonal design has been generalized in relay networks [8]. It has been showed that the scheme can achieve a full diversity order with the single-symbol ML decoding complexity.

Unlike most of previous works [3], [4], [6], [9], which focused on the average BER of the distributed-Alamouti space–time code for binary phase-shift keying (BPSK) modulation, in this paper, we analyze the average symbol error probability (SEP) for M-ary phase-shift keying (M-PSK) by using the moment generating function (MGF) method [10], [11]. Taking into account the relay's location, we derive the closed-form expressions of average SEP for M-PSK modulation as the relays are close to either the destination or source. Since the asymptotic characteristic of the SEP in the high signal-to-noise (SNR) regime reveals a high-SNR slope of the SEP curve [10]–[12], we show that distributed-Alamouti scheme achieves full diversity, i.e., the second-order of diversity. Our analysis has been validated by comparing with the simulation results. In addition, we further propose the optimal power allocation between the first-hop and the second-hop transmission. It has been shown that the optimal power scheme can increase the SEP performance by 2-3 dB compared to the equal power allocation.

The paper is organized as follows. In Section II, we briefly review the cooperative system in which the two AF relays deploy the Alamouti space–time code in a distributed scheme. We then derive two closed-form expressions of the average SEP and diversity orders when two relays are closely located to either the source or destination in Section III. The optimal power allocation is proposed in Section IV. We show that our analysis agree exactly with the Monte-Carlo simulations and the optimal power allocation outperforms the equal power scheme in Section V. Finally, Section VI concludes our paper.

Notation: Throughout the paper, we shall use the following notation. Vector is written as bold lower case letter and matrix is written as bold upper case letter. The superscripts $*$ and $\dagger$ stand for the complex conjugate and transpose conjugate. $\boldsymbol{I}_n$ represents the $n \times n$ identity matrix. $\|\boldsymbol{A}\|_{\mathrm{F}}$ denotes Frobenius norm of the matrix $\boldsymbol{A}$ and $|x|$ indicates the envelope

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of x. $\mathbb{E}_x\{.\}$ is the expectation operator over the random variable x. A complex Gaussian distribution with mean μ and variance σ^2 is denoted by $\mathcal{CN}(\mu,\sigma^2)$. $\log$ is the natural logarithm. $\Gamma(a,x)$ is the incomplete gamma function defined as $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$ and $\mathcal{K}_0(.)$ is the zeroth-order modified Bessel function of the second kind.

II. System Model

We investigate a dual-hop relay channel shown in Fig. 1, where the channel remains constant for T_{coh} coherence time (an integer multiple of the dual-hop interval) and changes independently to a new value for each coherence time. All terminals are in half-duplex mode and, hence, transmission occurs over two time slots, each with the interval of two symbol periods. We denote the total average transmit power per symbol time of the source and two relays as $\mathcal{P}_{\text{total}}$. Our transmission scheme is divided into two phases. In the first phase (broadcasting phase), the source broadcasts the signals to the two relays with the average transmit power $\mathcal{P}_{\text{s}}$. In the second phase (relaying phase), the two relays generate a distributed-Alamouti code and forward to the destination with the average transmit power $\mathcal{P}_{\text{total}} - \mathcal{P}_{\text{s}}$, while the source remains silent. In the broadcasting phase, the source transmits

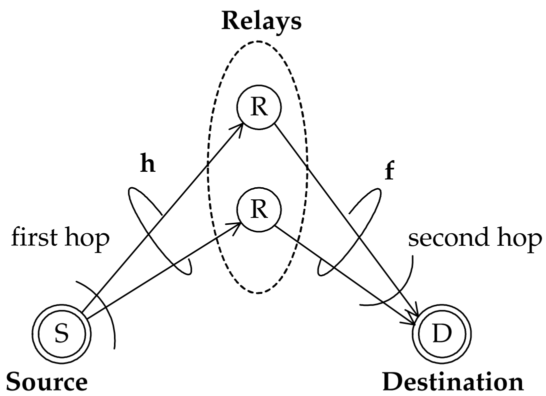

Fig. 1. Dual-hop AF relay channels.

two symbols $\boldsymbol{s} = [\ s_1 \quad s_2\ ]$, selected from M-PSK signal constellation $\mathcal{S}$, with average transmit power per symbol $\mathcal{P}_{\text{s}}$. During the broadcasting phase, the received signals $\boldsymbol{y}_i = [\ y_i(1) \quad y_i(2)\ ]$ at the ith relay is given by

$$\boldsymbol{y}_i = h_i\boldsymbol{s} + \boldsymbol{n}_{\text{R}_i} \tag{1}$$

where $y_i(j)$ is the jth symbol received at the ith relay, $h_i \sim \mathcal{CN}(0,\Omega_{\text{h}})$ is the Rayleigh-fading channel coefficient for the source-ith relay link with the channel mean power Ω_{h}, and $\boldsymbol{n}_{\text{R}_i}$ is complex additive white Gaussian noise (AWGN) of variance N_0, where $i = 1,2$.

During the relaying phase, the two relays construct the Alamouti space–time scheme from the two received signals and then retransmit a scaled version to the destination, whereas the source remains silent. To simplify the relaying operation, the relaying gain is determined only to satisfy the average power constraint with statistical channel state information (CSI) on h_i (not its instantaneous realizations) at the relay. With this *semi-blind* relaying, the output signal of the two relays are defined as

$$\boldsymbol{X} = \begin{bmatrix} x_1(1) & x_2(1) \\ x_1(2) & x_2(2) \end{bmatrix} = G\begin{bmatrix} y_1(1) & -y_2^*(2) \\ y_1(2) & y_2^*(1) \end{bmatrix} \tag{2}$$

where $x_i(j)$ is the jth symbol transmitted from the ith relay and G is the scalar relaying gain defined in the following. The received signal at the destination can be described as

$$r(j) = \sum_{i=1}^{2} f_i x_i(j) + n_D(j) \tag{3}$$

where $f_i \sim \mathcal{CN}(0,\Omega_{\text{f}})$ is the Rayleigh-fading channel coefficient for the ith relay-destination link with the channel mean power Ω_{f} and $n_D(j)$ is the AWGN of variance N_0. Note that all the random variable h_i, f_i, $i = 1,2$, are statistically independent and the variations in Ω_{A}, $\mathsf{A} \in \{h,f\}$, capture the effect of distance-related path-loss in each link. In this section, we assume the source consumes a half of total transmit power, i.e., two relays use the same amount of power as the source. In other words, we have $\mathcal{P}_{\text{s}} = \frac{\mathcal{P}_{\text{total}}}{2}$. The amplifying gain G can be derived from this power constraint, i.e., $\mathbb{E}\left\{\|\boldsymbol{X}\|_{\text{F}}^2\right\} = \mathbb{E}\left\{\|\boldsymbol{s}\|_{\text{F}}^2\right\}$, as follows:

$$4G^2(\Omega_{\text{h}}\mathcal{P}_{\text{s}} + N_0) = 2\mathcal{P}_{\text{s}} \tag{4}$$

yielding

$$G^2 = \frac{1}{4}\left(\frac{\Omega_{\mathsf{h}}}{2} + \frac{1}{\mathsf{SNR}_0}\right)^{-1} \tag{5}$$

where[1] $\mathsf{SNR}_0 = \frac{\mathcal{P}_{\text{total}}}{N_0}$ is the common SNR of each link without fading [13]. The received signals at the destination in (3) now can be equivalently described in the matrix form as

$$\boldsymbol{r} = \boldsymbol{H}\tilde{\boldsymbol{s}} + \boldsymbol{n} \tag{6}$$

where

$$\boldsymbol{r} = \begin{bmatrix} r(1) \\ r^*(2) \end{bmatrix}$$
$$\boldsymbol{H} = G\begin{bmatrix} h_1 f_1 & -h_2^* f_2 \\ h_2 f_2^* & h_1^* f_1^* \end{bmatrix}$$
$$\tilde{\boldsymbol{s}} = \begin{bmatrix} s_1 \\ s_2^* \end{bmatrix}$$

It is easy to see that the channel matrix $\boldsymbol{H}$ is complex orthogonal, i.e.,

$$\boldsymbol{H}^\dagger\boldsymbol{H} = G^2\sum_{i=1}^{2}|h_i|^2|f_i|^2\boldsymbol{I}_2,$$

and

$$\boldsymbol{n} = G\begin{bmatrix} f_1 n_{\text{R}_1}(1) & -f_2 n_{\text{R}_2}^*(2) \\ f_1^* n_{\text{R}_1}^*(2) & f_2^* n_{\text{R}_2}(1) \end{bmatrix} + \begin{bmatrix} n_D(1) \\ n_D^*(2) \end{bmatrix}$$

which is the zero-mean AWGN of the variance $N_0\left(G^2\sum_{i=1}^{2}|f_i|^2 + 1\right)$. It is easy to see that the ML

[1] It is worth mentioning that we denoted the common SNR without fading as $\frac{\mathcal{P}_{\text{s}}}{N_0}$ in [12] whereas we define $\frac{\mathcal{P}_{\text{total}}}{N_0}$. This definition facilities the power allocation scheme between two hops in the next section.

decoding of the symbol vector $\boldsymbol{s}$ turns out very simple as two symbols in $\boldsymbol{s}$ (s_1 and s_2) are independently decomposed from one another. Therefore, the instantaneous receive SNR per symbol is readily written as

$$\gamma = \frac{\mathcal{P}_{\mathsf{s}}}{N_0} \frac{G^2 \sum_{i=1}^{2} |h_i|^2 |f_i|^2}{G^2 \sum_{i=1}^{2} |f_i|^2 + 1} \tag{7}$$

To examine the statistical characteristic of γ given in (7) we consider two special cases:

- If the relays are much closer to the destination than the source, then we may have $\Omega_{\mathsf{h}} \ll \Omega_{\mathsf{f}}$ and $G^2 \approx \frac{1}{2\Omega_{\mathsf{h}}}$ with the high SNR_0. Therefore, we can assume that $G^2 \sum_{i=1}^{2} |f_i|^2 \gg 1$. In this special case, substituting $\mathcal{P}_{\mathsf{s}}/N_0 = \mathsf{SNR}_0/2$ in (7) we can express γ as follows:

$$\gamma = \frac{\mathsf{SNR}_0}{2} \frac{\sum_{i=1}^{2} |h_i|^2 |f_i|^2}{\sum_{i=1}^{2} |f_i|^2} \tag{8}$$

- On the other hand, if the relays are near the source, then the following expression may hold $G^2 \sum_{i=1}^{2} |f_i|^2 \ll 1$. Therefore, the instantaneous receive SNR γ is as follows

$$\gamma = \frac{\mathsf{SNR}_0}{2} G^2 \sum_{i=1}^{2} |h_i|^2 |f_i|^2 \tag{9}$$

III. Closed-Form Expression of the Average SEP and Diversity Order

In this section, on account of the statistical behavior of the instantaneous receive SNR for two special cases shown in previous section and applying the well-known MGF approach, we can derive the closed form expressions of the average SEP for M-PSK modulation and then deduce the diversity order of distributed-Alamouti scheme in such cases. Our analysis can be easily extended to the binary and other M-ary modulation schemes (see [14] and references therein).

A. When the relays are close to the destination

From the expression of the instantaneous receive SNR in (8), the MGF of γ can be given by

$$\phi_\gamma(\nu) \triangleq \mathbb{E}_\gamma \{\exp(-\nu\gamma)\} = \mathbb{E}_{h_i, f_i} \{\exp(-\nu\gamma)\} \tag{10}$$

Since $\mathsf{A}_i \sim \mathcal{CN}(0, \Omega_{\mathsf{A}})$, $\mathsf{A} \in \{h, f\}$ and $i = 1, 2$, it is obvious that $|\mathsf{A}_i|^2$ obeys an exponential distribution with the hazard rate $1/\Omega_{\mathsf{A}}$. The probability density function (p.d.f.) of $|\mathsf{A}_i|^2$ can be expressed as

$$p_{|\mathsf{A}_i|^2}(x) = \frac{1}{\Omega_{\mathsf{A}}} \exp(-x/\Omega_{\mathsf{A}}) \tag{11}$$

Since h_i and f_i are statistically independent, the MGF of γ in (10) can be written as

$$\begin{aligned}
\phi_\gamma(\nu) &= \mathbb{E}_{f_i} \{\mathbb{E}_{h_i} \{\exp(-\nu\gamma)\}\} \\
&= \mathbb{E}_{f_i} \left\{ \prod_{i=1}^{2} \left(1 + \tfrac{\nu \mathsf{SNR}_0}{2} \frac{|f_i|^2}{|f_1|^2 + |f_2|^2}\right)^{-1} \right\} \\
&= \mathbb{E}_Z \left\{ \left(1 + \xi \frac{Z}{Z+1}\right)^{-1} \left(1 + \xi \frac{1}{Z+1}\right)^{-1} \right\} \\
&\overset{(a)}{=} \int_0^\infty \left[(1 + (1+\xi) z)(1 + \xi + z)\right]^{-1} dz \\
&= \frac{2 \log(1+\xi)}{\xi(2+\xi)}
\end{aligned} \tag{12}$$

where $Z = \frac{|f_1|^2}{|f_2|^2}$, $\xi = \frac{\Omega_{\mathsf{h}} \mathsf{SNR}_0 \nu}{2}$, and (a) follows immediately from Theorem 2 in Appendix. Using the well-known MGF approach [10], [11] along with (12), we obtain the average SEP of the distributed-Alamouti scheme with M-PSK in relay channels as[2]

$$\begin{aligned}
P_{\mathsf{e}} &= \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \phi_\gamma\left(\frac{g_{\text{MPSK}}}{\sin^2\theta}\right) d\theta \\
&= \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \frac{2 \sin^4\theta \log\left(1 + \frac{\psi}{\sin^2\theta}\right)}{\psi\left(2\sin^2\theta + \psi\right)} d\theta
\end{aligned} \tag{13}$$

where $\psi = \frac{1}{2} \Omega_{\mathsf{h}} \mathsf{SNR}_0 g_{\text{MPSK}}$ and $g_{\text{MPSK}} = \sin^2(\pi/M)$.

B. When the relays are close to the source

In this section, we consider the case when the two relays are close to the source. Following the same steps as in Section III-A and from (9), the MGF of γ can be described as

$$\phi_\gamma(\nu) = \mathbb{E}_{h_i, f_i} \left\{ \prod_{i=1}^{2} \exp\left(-\tfrac{G^2 \mathsf{SNR}_0 \nu}{2} |h_i|^2 |f_i|^2\right) \right\} \tag{14}$$

$$= \left[\int_0^\infty \exp\left(-\tfrac{G^2 \mathsf{SNR}_0 \nu}{2} t\right) p_T(t)\, dt\right]^2 \tag{15}$$

$$= \left[\int_0^\infty \frac{2}{\Omega} \exp\left(-\tfrac{G^2 \mathsf{SNR}_0 \nu}{2} t\right) \mathcal{K}_0\left(2\sqrt{\frac{t}{\Omega}}\right) dt\right]^2 \tag{16}$$

$$= \left[\lambda \exp(\lambda)\, \Gamma(0, \lambda)\right]^2 \tag{17}$$

where $T = |h_i|^2 |f_i|^2$, $\Omega = \Omega_{\mathsf{h}} \Omega_{\mathsf{f}}$, $\lambda = \left(\frac{1}{2} G^2 \mathsf{SNR}_0 \nu \Omega\right)^{-1}$, (16) follows immediately from Theorem 3 in Appendix, and (17) can be obtained from the change of variable $v = \frac{1}{2} G^2 \mathsf{SNR}_0 \nu t$ along with [15, eq. (8.353.4)]. Therefore, following the same approach as in the previous subsection, the SEP is given by

$$P_{\mathsf{e}} = \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \left[\tau \sin^2\theta \exp\left(\tau \sin^2\theta\right) \Gamma\left(0, \tau \sin^2\theta\right)\right]^2 d\theta \tag{18}$$

where $\tau = \left(\frac{1}{2} G^2 \mathsf{SNR}_0 \Omega g_{\text{MPSK}}\right)^{-1}$.

[2]The result can be applied to other binary and M-ary signals in a straightforward way (see, e.g., [10]).

C. High-SNR Characteristic: Diversity Order

We now assess the effect of cooperative diversity on the SEP behavior in a high-SNR regime. The diversity impact of AF cooperation on a high-SNR slope of the SEP curve can be quantified by the following theorem.

Theorem 1 (Achievable Diversity Order): The AF cooperation of distributed-Alamouti scheme provides maximum diversity order as relays move close to both ends, i.e.,

$$D \triangleq \lim_{\mathsf{SNR}\to\infty} \frac{-\log P_{\mathrm{e}}}{\log(\mathsf{SNR})} = 2. \quad (19)$$

Proof: See Appendix C ■

IV. Optimal Power Allocation

In this section, taking into account the relay's location, we propose an optimal power allocation between the broadcasting and relaying phase based on the minimization of the upper bound on SEP. We do not assume the equally distributed power allocation between the broadcasting and relaying phase as in the previous section. In other words, the total average transmit power of the two relays per symbol time is $\mathcal{P}_{\text{relay}} = \mathcal{P}_{\text{total}} - \mathcal{P}_{\text{s}} \neq \mathcal{P}_{\text{s}}$. Our goal is to find the optimal solution to allocate the power to the source-to-destination and relay-to-destination link under the total transmit power constraint $\mathcal{P}_{\text{total}}$. It is also noted that we should replace SNR_0 in ξ of eq. (12) and θ of eq. (17) by $\mathcal{P}_{\text{s}}/N_0$. Without the loss of generality, we assume a unit total transmit power, i.e., $\mathcal{P}_{\text{total}} = 1$, and hence, $N_0 = 1/\mathsf{SNR}_0$. As a result, the SEP expressions given in (13) and (18) as the two relays are closely located to the destination and source are upper bounded, respectively, as follows:

$$P_{\mathrm{e}} \leq \frac{2\log\left(1+\tilde{\xi}\right)}{\tilde{\xi}\left(2+\tilde{\xi}\right)} \quad (20)$$

where $\tilde{\xi} = \frac{1}{\mathsf{SNR}_0}\Omega_{\mathrm{h}}\mathcal{P}_{\mathrm{s}} g_{\mathrm{MPSK}}$, and

$$P_{\mathrm{e}} \leq \left[\tilde{\lambda}\exp\left(\tilde{\lambda}\right)\Gamma\left(0,\tilde{\lambda}\right)\right]^2 \quad (21)$$

where $\tilde{\lambda} = \left(\frac{1}{2}\frac{(1-\mathcal{P}_{\mathrm{s}})\mathcal{P}_{\mathrm{s}}\mathsf{SNR}_0 g_{\mathrm{MPSK}}\Omega_{\mathrm{h}}\Omega_{\mathrm{f}}}{\left(\Omega_{\mathrm{h}}\mathcal{P}_{\mathrm{s}}+\mathsf{SNR}_0^{-1}\right)}\right)^{-1}$.

We now consider an optimal power allocation between the first hop and second hop transmission depending on relays' positions by utilizing a similar approach as in [16]. Under the specific common SNR without fading ($\mathsf{SNR}_0 = \mathcal{P}_{\text{total}}/N_0$) and mean channel fading parameters $(\Omega_{\mathrm{h}}, \Omega_{\mathrm{f}})$, the cost functions, i.e., two upper bounded expressions of SEP, depends only on the average transmit power $\mathcal{P}_{\mathrm{s}}$. It turns out that we need to minimize the two upper bounds on SEP given in (20) and (21) subject to $\mathcal{P}_{\mathrm{s}}$ $(0 \leq \mathcal{P}_{\mathrm{s}} \leq 1)$. It can be easily see that the second derivative of the right-hand side in (20) and (21) is positive with respective to $\mathcal{P}_{\mathrm{s}}$, hence, the two upper bounds are strictly convex. The convexity implies that the minimum value is a global minimization. Fortunately, the cost functions do not depend on the instantaneous fading parameters, e.g., instantaneous SNR, we can *a posteriori* solve the optimization problem with respect to $\mathcal{P}_{\mathrm{s}}$ and store the resolvable values (optimal values $\mathcal{P}_{\mathrm{s}}$) into the look-up table for the future usages. This non-realtime operation leads our optimal power allocation algorithm realistic for practical applications. This algorithm can be easily implemented with any symbolic mathematical software packages such as MAPLE or MATHEMATICA. In this paper, we apply the MATHEMATICA built-in function "FindMinimum" to obtain the optimal values of $\mathcal{P}_{\mathrm{s}}$. The results are given in terms of the percentage of total average transmit power $\mathcal{P}_{\text{total}}$.

For the case as the two relays are close to the destination, it is easy that the right-hand side of (20) is monotonically decreased subject to $\mathcal{P}_{\mathrm{s}}$. It means that we should allocate all power for the first-hop transmission $(\mathcal{P}_{\mathrm{s}} = 100\%)$[3]. Intuitively, we can observe similar results through examining the formulas in (20). The right-hand side of (20) contains only the fading parameter of the source-relay link (Ω_{h}), hence, it turns out that the signals are transmitted through a fading channel of source-relay link and a "non-fading" channel of relay-destination link. Thereby, fading compensation should be mostly assigned for the first-hop transmission to overcome the deleterious fading effect. However, since our transmission scheme occurs in two hops a certain amount power should be remained for the second-hop transmission. As shown in the next section, we can set a high-fixed value to $\mathcal{P}_{\mathrm{s}}$, e.g. $\mathcal{P}_{\mathrm{s}} = 98\%$, over the whole range of SNR.

When the relays are located nearby the source, applying the "FindMinimum" function on (21), we can find the optimal value of $\mathcal{P}_{\mathrm{s}}$ shown in Table I for two cases $\epsilon = 0.2$ and $\epsilon = 0.3$, where ϵ is the ratio between the distance of the source to the two relays and the distance of the two relays to the destination.

TABLE I
OPTIMAL POWER ALLOCATION FOR THE DISTRIBUTED-ALAMOUTI SYSTEM AS THE TWO RELAYS ARE CLOSE TO THE SOURCE

SNR_0 [dB]	$\mathcal{P}_{\mathrm{s}}$	
	$\epsilon = 0.2$	$\epsilon = 0.3$
0	0.136435	0.253017
5	0.082243	0.165566
10	0.048101	0.101617
15	0.027654	0.060051
20	0.015746	0.034727
25	0.008917	0.019839
30	0.005034	0.011255

V. Numerical Results

In this section, we validate our analysis by comparing with the Monte-Carlo simulations. In the following numerical examples, we consider the AF relay protocol employing the Alamouti code as in Section II. We also assume collinear geometry for locations of three communicating terminals. The path-loss of each link follows an exponential decay model: if the distance between the source and destination is equal to d, then $\Omega_0 \propto d^{-\alpha}$ where the exponent $\alpha = 4$ corresponding to a typical non line-of-sight propagation. Then, $\Omega_{\mathrm{h}} = \epsilon^{-\alpha}\Omega_0$ and $\Omega_{\mathrm{f}} = (1-\epsilon)^{-\alpha}\Omega_0$. We also assume the unit channel mean

[3]The result has been double-checked by applying the MATHEMATICA "FindMinimum" function on (20). We also obtain the optimal value of $\mathcal{P}_{\mathrm{s}}$ as 1.

power of the source-to-destination link, i.e., $\Omega_0 = 1$ for all numerical examples.

A. Equal Power Allocation

Fig. 2 and Fig. 3 show the SEP of QPSK versus SNR_0 when the two relays approach the destination ($\epsilon = 0.7$ and $\epsilon = 0.8$) and the source ($\epsilon = 0.2$ and $\epsilon = 0.3$), respectively. As can clearly be seen from both figures, analytical and simulated SEP curves match exactly. Observe that the SEP slops for $\epsilon = 0.7$ and $\epsilon = 0.8$ are identical in the high SNR regime, as speculated in Theorem 1. The same observation can be obtained for $\epsilon = 0.2$ and $\epsilon = 0.3$. For comparison, two numerical examples demonstrate that the SEP performance is decreased when the two relays are located nearby both ends, e.g., the SEP for $\epsilon = 0.3$ is slightly less than that for $\epsilon = 0.2$ and a 3dB-gain can be obtained with $\epsilon = 0.7$ compared to the case with $\epsilon = 0.8$.

B. Optimal Power Allocation

We show the SEP of QPSK as the function of transmit power at the source in Fig. 4 when the relays are close to the destination ($\epsilon = 0.7$ and 0.8) with $\mathsf{SNR}_0 = 20$ and 30 dB. As can be seen from the figure, $\mathcal{P}_\mathrm{s} = 1$ yields the best performance for all cases (strongly agree with above analysis).

For a practical application, we assign 98% total transmit power for the first-hop transmission. As displayed in Fig. 5, optimal power allocation can improve by 2.5 dB compared with equal power allocation

When the relays are nearby the source,the transmit power of the first-hop transmission $\mathcal{P}_\mathrm{s}$ is selected from the look-up Table I to improve the SEP performance. At the low SNR regime, approximately 14% and 25% of the total power should be allocated in the broadcasting phase for $\epsilon = 0.2$ and $\epsilon = 0.3$, respectively. However, we observe from the Table I that in the high SNR regime the broadcast phase requires a small percentage of power. This is actually reasonable since the relay-to-destination link is much worse than source-to-relay link, hence, most of power should be allocated in the second-hop transmission to compensate for the loss of fading. In addition, under the same condition (SNR_0) the relaying protocol with $\epsilon = 0.3$ needs more power in the broadcasting phase than that with $\epsilon = 0.2$. To clearly illustrate the effect of power allocation on system performance, we show in Fig. 6 the SEP of QPSK modulation versus SNR when the relays are close to the source. We observe from Fig. 5 and Fig. 6 that the optimal scheme achieves the improvement of SEP performance by 2-3 dB over the conventional one.

VI. Conclusions

In this paper, using the well-known MGF approach, we have derived the closed-form expressions for SEP of distributed-Alamouti scheme taking into consideration the relay's location. We further show that the distributed-Alamouti scheme achieves a full diversity by assessing the high SNR behavior of SEP performance. Our analysis has been validated by the simulation results. We also proposed an optimal power allocation between the first-hop and second-hop transmission. It has been shown that when the relays are closely located to the destination, most power should be assigned for the first-hop transmission. On the other hands, as the relays are nearby the source, more transmit power should be allocated for the second-hop transmission. The proposed scheme increases the SEP performance by 2-3 dB over the equal power allocation.

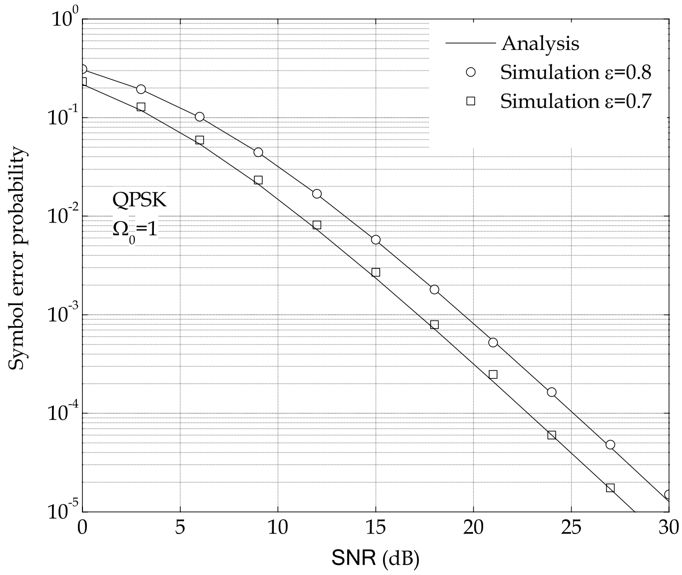

Fig. 2. Symbol error probability of QPSK versus SNR in AF relay channels employing Alamouti scheme when $\epsilon = 0.7$ and $\epsilon = 0.8$ (the two relay are close to the destination).

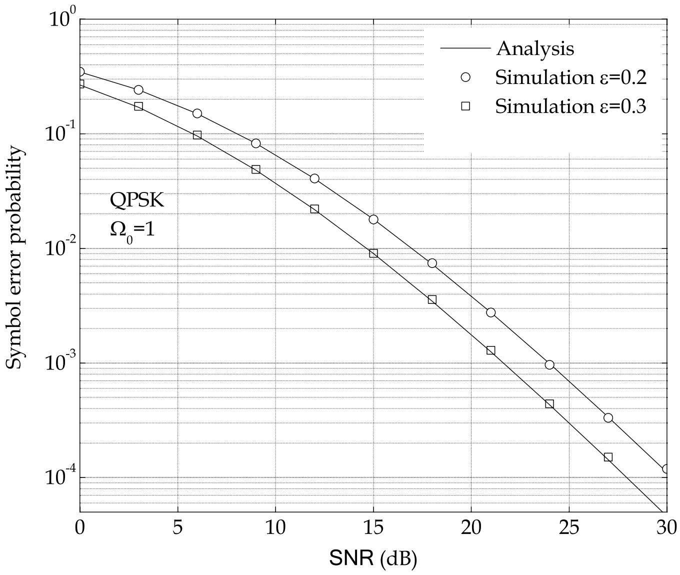

Fig. 3. Symbol error probability of QPSK versus SNR in AF relay channels employing Alamouti scheme when $\epsilon = 0.2$ and $\epsilon = 0.3$ (the two relays are close to the source).

Appendix

A. Auxiliary Results

The following lemmas will be useful in the paper

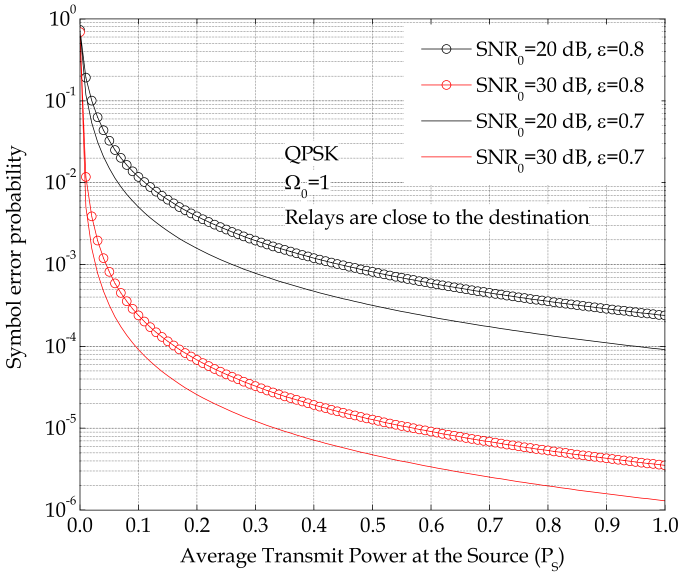

Fig. 4. Symbol error probability of QPSK versus $\mathcal{P}_s$ in AF relay channels employing Alamouti scheme at $\mathsf{SNR}_0 = 20$dB and $\mathsf{SNR}_0 = 30$dB. The two relays are located nearby the destination ($\epsilon = 0.7$ and $\epsilon = 0.8$).

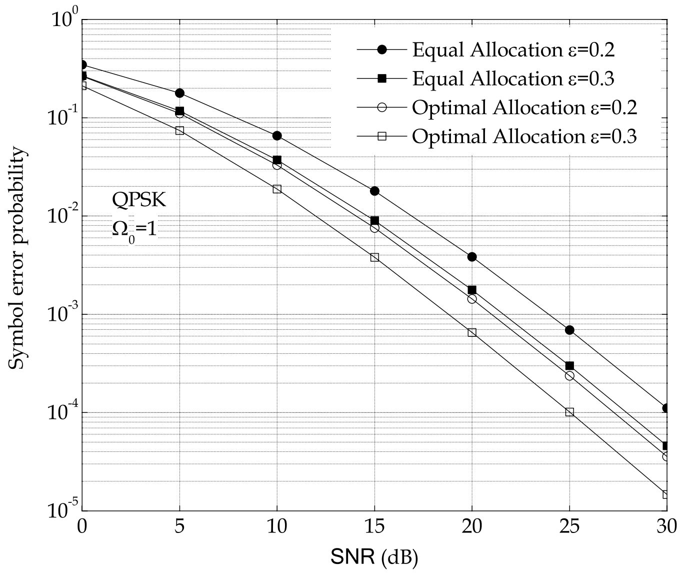

Fig. 6. Symbol error probability of QPSK versus SNR in AF relay channels employing Alamouti scheme with equal and optimal power allocation. The two relays are located nearby the source ($\epsilon = 0.2$ and $\epsilon = 0.3$).

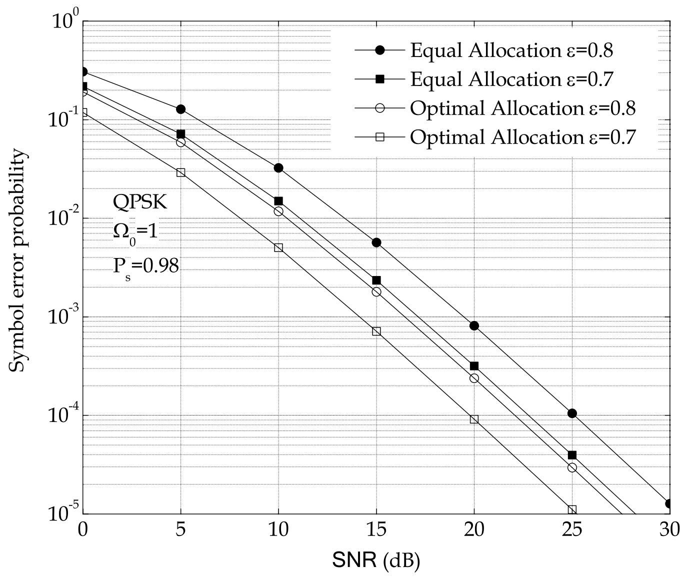

Fig. 5. Symbol error probability of QPSK versus SNR in AF relay channels employing Alamouti scheme with equal and optimal power allocation ($\mathcal{P}_s = 0.98$). The two relays are located nearby the destination ($\epsilon = 0.7$ and $\epsilon = 0.8$).

Lemma 1: Let $a > 0$ be a finite constant and

$$f(x) = \frac{2\log(1+ax)}{ax(2+ax)}, \quad x > 0 \tag{22}$$

We have

$$f_{x\uparrow} \triangleq \lim_{x\to\infty} \frac{-\log f(x)}{\log x} = 2 \tag{23}$$

Proof: It follows immediately from (22) and (23) that

$$f_{x\uparrow} = \underbrace{\lim_{x\to\infty} \frac{-\log\log(1+ax)}{\log x}}_{\overset{(b)}{=}0} + \underbrace{\lim_{x\to\infty} \frac{\log(ax)}{\log x}}_{\overset{(c)}{=}1} + \underbrace{\lim_{x\to\infty} \frac{\log\left(1+\frac{ax}{2}\right)}{\log x}}_{\overset{(d)}{=}1} = 2 \tag{24}$$

where (b), (c), and (d) follow immediately by applying l'Hôspital rule. ■

Lemma 2: Let

$$g(\zeta) = \left[\zeta \exp(\zeta)\,\Gamma(0,\zeta)\right]^2, \quad \zeta > 0 \tag{25}$$

Let $\zeta = \alpha \frac{\beta x+1}{x^2}$ and $\alpha, \beta > 0$ be finite constants. We have

$$g_{x\uparrow} \triangleq \lim_{x\to\infty} \frac{-\log g(x)}{\log x} = 2 \tag{26}$$

Proof: Substituting $\zeta = \alpha\frac{\beta x+1}{x^2}$ into (25), it follows immediately from (26) that

$$\begin{aligned} g_{x\uparrow} = &-2\left[\lim_{x\to\infty} \frac{\log\left(\frac{\beta x+1}{x^2}\right)}{\log x} + \lim_{x\to\infty}\frac{\beta x+1}{x^2\log x} + \lim_{x\to\infty}\frac{\log\Gamma\left(0,\alpha\frac{\beta x+1}{x^2}\right)}{\log x}\right] = -2\Bigg[-2 + \underbrace{\lim_{x\to\infty}\frac{\log(1+\beta x)}{\log x}}_{\overset{(e)}{=}1} \\ &+ \underbrace{\lim_{x\to\infty}\frac{\beta x+1}{x^2\log x}}_{\overset{(f)}{=}0} + \underbrace{\lim_{x\to\infty}\frac{\log\Gamma\left(0,\alpha\frac{\beta x+1}{x^2}\right)}{\log x}}_{\overset{(g)}{=}0}\Bigg] = 2 \end{aligned} \tag{27}$$

where (e), (f) follow immediately by l'Hôspital rule and (g) follows from the Laguerre[4] polynomial series representation of incomplete gamma function [15] together with l'Hôspital rule . ■

B. A Ratio and Product Distribution

Theorem 2 (Ratio Distribution): Let

$$X \sim \Upsilon(1/\Omega)$$
$$Y \sim \Upsilon(1/\Omega)$$

be statistically independent and identically distributed (i.i.d.) exponential r.v.'s. Suppose the ratio Z of the form

$$Z = \frac{X}{Y} \tag{28}$$

Then, we obtain the p.d.f. of random variable Z as

$$p_Z(z) = (z+1)^{-2} \tag{29}$$

Proof: Note that

$$\begin{aligned} p_Z(z) &= \int_0^\infty y p_{XY}(yz, y)\, dy \\ &= \int_0^\infty \frac{y}{\Omega^2} \exp\left(-y\frac{z+1}{\Omega}\right) dy \\ &= (z+1)^{-2} \end{aligned} \tag{30}$$

■

Theorem 3 (Product Distribution): Let

$$X \sim \Upsilon(1/\Omega_x)$$
$$Y \sim \Upsilon(1/\Omega_y)$$

be statistically independent and not necessarily identically distributed (i.n.i.d.) exponential r.v.'s. Suppose the product T of the form

$$T = XY \tag{31}$$

Then, we have

$$p_T(t) = \frac{2}{\Omega_x\Omega_y}\mathcal{K}_0\left(2\sqrt{\frac{t}{\Omega_x\Omega_y}}\right) \tag{32}$$

where $\mathcal{K}_0(.)$ is the zeroth-order modified Bessel function of the second kind.

Proof: Note that

$$\begin{aligned} F_T(t) &= \Pr\{XY \le t\} \\ &= \mathbb{E}_X\left\{F_{T|X}(t)\right\} \\ &= \mathbb{E}_X\left\{1 - \exp\left[-\frac{t}{x\Omega_y}\right]\right\} \\ &= 1 - \frac{1}{\Omega_x}\int_0^\infty \exp\left[-\frac{t}{x\Omega_y} - \frac{x}{\Omega_x}\right] dx \end{aligned} \tag{33}$$

[4]The incomplete gamma function can be described as $\Gamma(0,x) = e^{-x}\sum_{n=0}^{\infty}\frac{L_n(x)}{n+1}$ where $L_n(x) = \sum_{m=0}^{n}(-1)^m(n!x^m)/(m!(n-m)!m!)$ is the Laguerre polynomial of order n.

The p.d.f. of T follows immediately from differentiating (33) with respect to t.

$$\begin{aligned} p_T(t) &= \frac{1}{\Omega_x\Omega_y}\int_0^\infty \frac{1}{x}\exp\left[-\frac{t}{x\Omega_y} - \frac{x}{\Omega_x}\right] dx \\ &= \frac{2}{\Omega_x\Omega_y}\mathcal{K}_0\left(2\sqrt{\frac{t}{\Omega_x\Omega_y}}\right) \end{aligned} \tag{34}$$

■

where the last equality follows from the change of variable $u = \frac{x}{\Omega_x}$ along with [15, eq. (8.432.6)] as desired.

C. Proof of Theorem 1

Since the asymptotic behavior of the MGF $\phi_\gamma(\nu)$ at large SNR reveals a high-SNR slope of the SEP curve, we have [11], [12]

$$D = \lim_{\mathsf{SNR}\to\infty}\frac{-\log\phi_\gamma(g_{\mathrm{MPSK}})}{\log(\mathsf{SNR})}. \tag{35}$$

Hence, it follow immediately from (12) and Lemma 1 with $a = \Omega_{\mathrm{h}} g_{\mathrm{MPSK}}$ that $D = 2$ if the two relays are close to the destination. Also, from (17) and Lemma 2 with $\alpha = \frac{2}{\Omega_{\mathrm{h}}\Omega_{\mathrm{f}} g_{\mathrm{MPSK}}}$ and $\beta = \Omega_{\mathrm{h}}$, we can obtain $D = 2$ as the two relays are located nearby the source.

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