Using Log Likelihood Relation for BER Analysis of QAM in Space Diversity

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Abstract— **The bit error rate (BER) performance for quadrature amplitude modulation (QAM) with different diversity combining scheme is derived for Rayleigh fading channel using Log-likelihood ratio (LLR). In this paper, four diversity combining techniques such as, Maximal Ratio Combiner (MRC), Selection Combiner (SC), Equal Gain Combiner (EGC), and Switching Combiner (SCC) are considered for the BER analysis of QAM symbol through a Raykeigh fading channel. The average BER performance of QAM symbol through a flat fading channel is derived from the individual bits forming the QAM symbol with MRC, SC, EGC, and SCC space diversity. The analytical and simulated result of the BER performance of the QAM symbol through the same channel with MRC, SC, EGC, and SCC diversity combiner is shown and compared to each other to find which combiner provides better BER performance. . The both analytical and simulation result shows that the probability of bit error decreases with the order of diversity for all the cases of MRC, SC, EGC, and SCC. It is also seen from the results that MRC provides better BER performance than the other combiners (such as SC, EGC, and SCC). It is observed from the results that LLR-space diversity based formulation gives the best BER performance among all.**

Index Terms— **Rayleigh fading channel, QAM, Diversity, Diversity combiner, LLR**

I. INTRODUCTION

In cellular mobile communications, multi-path fading is a common phenomenon in urban areas. Transmitted radio signals from base stations (BSs) undergo reflection, diffraction and scattering from different types of obstacles resulting to their multi-path propagation. The channel impulse response of different paths cause individual signals of the corresponding paths arrive at the receiver with different amplitudes, phases and delays in the air-interface [1]. The fading causes the BER degradation of the transmitted symbol. Generally, equalizer is used to increase the signal strength of the faded signal. The design complexity of the equalizer is increased with the data rate [2]. The next generation wireless communication (3G and beyond) requires high data rate, and equalizer is not suitable to minimize the effect of multi-path fading. Diversity is a technique which searches for the strongest signal from the multi-path radio signals. The property of the technique increases its demand for digital mobile radio services. If one path undergoes very deep fade, other path may provide less fade, and another path may have strong signal. The diversity process selects the path having strong signal.

The difficulty to send high speed data is to achieve the limitation of channel capacity for multi-path fading effect. Two types of diversity are used in digital land mobile system, a) transmit diversity and b) receive diversity. Transmit diversity is more efficient than its receive counterpart to reduce noise and multi-path fading effects [3]. In this technique, modulated complex symbols are transmitted as Space Time Block Coded (STBC) signals. In receive diversity, one transmitter transmits symbol and two or more antennas receive the transmitted symbols that arrive via different multi-paths. In transmit diversity, two or more antennas transmit symbols and one antenna receives the multi-path signal. When the symbols are transmitted through the fading channel, channel bandwidth limitation occurs. The diversity branches carry the same information at the same time period but have uncorrelated multi-path fading; the diversity combiner circuit combines those fading signals or selects the path having the highest signal strength. The multi-path fading appearing in the diversity branches becomes uncorrelated if the spacing between the adjacent antennas at receiver site is properly selected. The decisions to detect symbol are made at the receiver and are unknown to the transmitter.

Generally, the diversity falls into seven categories: i) space, ii) angle, iii) polarization, iv) field, v) frequency, vi) multi-path, and vii) time. Space diversity is achieved by using more than one antennas at transmitter or and receiver sites. In this paper, authors attempt to derive an algorithm to reduce BER of M-ary QAM signals using LLR (Log Likelihood Relation) with space diversity. For space diversity analysis, a number of diversity combiner circuits, such as, Maximal Ratio Combiner (MRC), Selection Combiner (SC), Equal Gain Combiner (EGC), and Switching Combiner (SCC) [4] are used. In MRC, the signal received from different

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branches are weighted, co-phased and then summed to obtain the highest received signal strength. In SC, the signal having the highest instantaneous signal to noise (SNR) is selected from the multi-path. The EGC receives all the multi-path signals, co-phased and summed but not weighted. The SC receiver switched to a diversity branch having SNR greater than a specified threshold value, and switched to another branch when the SNR of the operating diversity branch drop below the threshold value. The MRC provides the lowest BER for multi-path fading channel, but SC is more suitable for mobile radio applications, because of its simple implementation. The quadrature amplitude modulation (QAM) is one of the effective modulation techniques to achieve higher spectral efficiency for the next generation broadband wireless communication system through Rayleigh fading channel under the condition when the receiver is capable of estimating the channel state information (CSI) [2]. Space diversity is used to combat multi-path fading in mobile radio communications, and diversity gain is achieved at the desired level without increasing the corresponding transmission power [5].

The individual bit in QAM symbol is affected independently by the faded channel (frequency selective or frequency non-selective) and noisy channel i.e. Additive White Gaussian Noise (AWGN). The overall BER of QAM symbol is affected by the channel noise because the individual bits are affected independently. The Log-likelihood Ratio (LLR) for individual bit of QAM symbol is derived for flat fading channels [6] without considering diversity combiners. In [10], similar derivations and computations are done using space diversity only, ignoring LLR. In this paper, the authors derive the BER expression for each bit in MQAM symbol, based on both LLR and space diversity simultaneously for a Rayleigh fading channel with MRC, SC and EGC combiner circuits. The MRC weighs each diversity branch in a co-phased manner and thus generates the highest SNR at the receiver site.

The SNR expression for MRC is applied to find the BER expression based on LLR analysis of each bit. The average BER for QAM in Rayleigh fading channels can also be determined [4]. The SC and EGC use the average SNR for determining the average BER of QAM. A simulation work is done to find the BER performance of M-ary QAM for a Rayleigh fading channel using MRC, SC, EGC, and SCC. Here, the BER expression of individual bits of QAM is used to find the average BER of the QAM symbol through a fading channel. Finally, we compare the analytical and simulated average BER performance of QAM symbol for Rayleigh fading channel for MRC, SC, EGC, and SCC diversity combiners. The analytical and simulated values of BER values are determined based on our joint LLR and space diversity derivations. The results are compared with those of other derivations reported in [5] and [10].

II. PROBABILITY OF BER USING LLR SCHEME

Let us consider 16-QAM case, where the 4 bits (r_1, r_2, r_3, r_4) are mapped on to a complex symbol $a = a_1 + ja_0$. If the modulated symbol undergoes multi-path fading, then the received signal y can be written as:

$$y = h\,a + n. \tag{1}$$

where, h is the complex channel impulse response, which is assumed to be Rayleigh distributed, and $n = n_1 + jn_0$ is the complex noise with zero mean and variance $\sigma^2/2$.
The LLR of bit $r_i = 1, 2, 3, 4$ of the received symbol can be given [6] by:

$$LLR(r_i) = \log\left(\frac{\sum_{\alpha\in S^{(1)}_i} P_r\{a=\alpha|y,h\}}{\sum_{\beta\in S_i^{(0)}} P_r\{a=\beta|y,h\}}\right). \tag{2}$$

For independent symbol using Bayes' rule, we have,

$$LLR(r_i) = \log\left(\frac{\sum_{\alpha\in S^{(1)}_i} f_{y|h,a}\{y|h,a=\alpha\}}{\sum_{\beta\in S_i^{(0)}} f_{y|h,a}\{y|h,a=\beta\}}\right). \tag{3}$$

$$f_{y|h,a}\{y|h,a=\alpha\} = \frac{1}{\sigma\sqrt{\pi}}\exp\left(\frac{-1}{\sigma^2}\|y-h\alpha\|^2\right),$$

Now, (3) can be written as:

$$LLR(r_i) = \log\left(\frac{\sum_{\alpha\in S^{(1)}_i}\exp\left(-\frac{1}{\sigma^2}\|y-h\alpha\|^2\right)}{\sum_{\beta\in S_i^{(0)}}\exp\left(-\frac{1}{\sigma^2}\|y-h\beta\|^2\right)}\right). \tag{4}$$

Using the identity

$\log\left(\sum_j \exp(-X_j)\right) \approx -\min_j(X_j)$, we write (4) as:

$$LLR(r_i) = \frac{1}{\sigma^2}\left\{\min_{\beta\in S^{(0)}_i}\|y-h\beta\|^2 \min_{\alpha\in S^{(1)}_i}\|y-h\alpha\|^2\right\}. \tag{5}$$

We define a variable, $z \overset{\Delta}{=} \frac{y}{h} = a + \frac{n}{h} = a + \hat{n}$, where $\hat{n}$ is Gaussian complex random variable. Normalizing $LLR(r_i)$ by $4/\sigma^2$, we get:

$$LLR(r_i) = \frac{\|h\|^2}{4}\left\{\min_{\beta\in S^0_i}\|z-\beta\|^2 - \min_{\alpha\in S^{(1)}_i}\|z-\alpha\|^2\right\}$$

$$LLR(r_i) = \frac{\|h\|^2}{4}\left\{\begin{array}{l}\min_{\beta\in S^0_i}\left\{\|\beta\|^2 - 2z_I\beta_I - 2z_Q\beta_Q\right\} - \\ \min_{\alpha\in S^{(1)}_i}\left\{\|\alpha\|^2 - 2z_I\alpha_I - 2z_Q\alpha_Q\right\}\end{array}\right\}. \tag{6}$$

Where, $z = z_I + jz_Q$, $\alpha = \alpha_I + j\alpha_Q$ and $\beta(k) = \beta_I + j\beta_Q$. The LLR for bit r_1, r_2, r_3, and r_4 becomes:

$$LLR(r_1)=\begin{cases}-\|h\|^2 z_I d, \forall |z_I| \le 2d \\ 2\|h\|^2 d(d-z_I), \forall z_I > 2d \\ -2\|h\|^2 d(d+z_I), \forall z_I < -2d\end{cases}. \quad (7)$$

$$LLR(r_2)=\begin{cases}-\|h\|^2 z_Q d, \forall |z_Q| \le 2d \\ 2\|h\|^2 d(d-z_Q), \forall z_Q > 2d \\ -2\|h\|^2 d(d+z_Q), \forall z_Q < -2d\end{cases}. \quad (8)$$

$$LLR(r_3)=\|h\|^2 d\{|z_I|-2d\}. \quad (9)$$

$$LLR(r_4)=\|h\|^2 d\{|z_Q|-2d\}. \quad (10)$$

Here, $2d$ is the minimum Euclidean distance, which can be computed according to [7] by:

$$d=\sqrt{\frac{3m.E_b}{2(M-1)}}. \quad (11)$$

where, m=$\log_2$M, and M represents the number of bits per symbol and E_b represents the average energy per bit.

The probability of error for bit r_1, P_{b1}, is according to [3]:

$$P_{b1}=P_{b1|a_I=-d.}P_r\{a_I=-d\}+ P_{b1|a_I=-3d.}P_r\{a_I=-3d\}+P_{b1|a_I=d.}P_r\{a_I=d\} +P_{b1|a_I=-d.}P_r\{a_I=3d\}. \quad (12)$$

where, $P_{b1|a_I=-d,h}=Q\left(\sqrt{\frac{4E_b\|h\|^2}{5N_0}}\right)$.

It can be written as

$$P_{b1|a_I=-d}=Q\left(\sqrt{\frac{4E_b\|h\|^2}{5N_0}}\right)=\frac{1}{\sqrt{2\pi p_1}}\exp(-\frac{1}{2}p_1),$$

$$p_1=\sqrt{\frac{4E_b}{5N_0}}. \quad (13)$$

Similarly, $P_{b1|a_I=-3d}$ can be written as:

$$P_{b1|a_I=-3d}=Q\left(\sqrt{\frac{36E_b\|h\|^2}{5N_0}}\right)=\frac{1}{\sqrt{2\pi p_2}}\exp(-\frac{1}{2}p_2),$$

$$p_2=\sqrt{\frac{36E_b}{5N_0}}. \quad (14)$$

Therefore P_{b1} is

$$P_{b1}=\frac{1}{2\sqrt{2\pi p_1}}\exp(-\frac{1}{2}p_1)+\frac{1}{2\sqrt{2\pi p_2}}\exp(-\frac{1}{2}p_2). \quad (15)$$

For 16-QAM, P_{b1}=P_{b2}, and the error probability Pb3 and Pb_4 are:

$$P_{b3}=\frac{1}{2}\left(2P_{b1|a_I=-d}+P_{b1|a_I=-3d}-\frac{1}{\sqrt{2\pi p_3}}\exp(-\frac{1}{2}p_3)\right)$$

where, $p_3=\sqrt{\frac{100E_b}{5N_0}}$. (16)

The average bit error rate of 16-QAM symbol

$$P_b=\frac{1}{2}(P_{b1}+P_{b3}). \quad (17)$$

III. SYSTEM MODEL

If a signal S(t) is transmitted at symbol period T, the received signal for a postdetection diversity receiver can be written as [9]

$$r_k(t)=h_kS(t)+n_k(t),\ k=1, 2, \ldots, L \quad (18)$$

where h_k (t) is the channel impulse response for the kth diversity branch and $n_k(t)$ is the AWGN noise for the kth fading channel.

A. Maximal Ratio Combining (MRC)

With MRC, each diversity branch is weighted by their respective fading gain, co-phased, and added. The received signal vectors

$$\bar{r}=(r_1, r_2, \ldots\ldots, r_L). \quad (19)$$

from each of the L diversity branch are added to provide maximum SNR. If each branch has fading gain h_k, then the diversity combiner generates the sum [8]

$$\bar{r}=\sum_{k=1}^{L}h_k^* r_k. \quad (20)$$

The envelope of the composite signal is

$$\alpha_M=\sum_{k=1}^{L}\alpha_k^{\ 2}. \quad (21)$$

where, $h_k=\alpha_k\exp(-j\theta_k)$. The weighted sum of the branch noise power of each branch has the same average noise power N_0 expresses by:

$$N=N_0\sum_{k=1}^{L}\alpha_k^{\ 2}. \quad (22)$$

The SNR at the detector device is, $\gamma_M=\frac{r^2{}_M}{2N_T}$

The maximum value of γ_M, according to [9] is:

$$\gamma_M = \frac{1}{2}\sum_{k=1}^{L}\frac{r^2{}_k}{N_0} = \sum_{k=1}^{L}\gamma_k \ .$$

The individual average SNR for each diversity branch is given by

$$SNR = \Gamma = \frac{E_b}{N_0}\alpha^2 . \qquad (23)$$

where, α^2=1.

The average SNR ($\bar{\gamma}_M$) equals to the sum of the individual average SNR [8]:

$$\bar{\gamma}_M = \sum_{k=1}^{L}\gamma_k = \sum_{k=1}^{L}\Gamma = L\Gamma . \qquad (24)$$

The probability of error for distance -d ($P_{b1|a_I=-d}$) for MRC space diversity is given by applying the average SNR from (24) into (12), and then we get:

$$P_{b1|a_I=-d} = Q\left(\sqrt{L\frac{4E_b}{5N_0}}\right)$$
$$= \frac{1}{\sqrt{2\pi p_1}}\exp(-\frac{1}{2}p_1), p_1 = \sqrt{L\frac{4E_b}{5N_0}} . \qquad (25)$$

Similarly, the error for distance -3d ($P_{b1|a_I=-3d}$) from (13), we have:

$$P_{b1|a_I=-3d} = Q\left(\sqrt{L\frac{36E_b}{5N_0}}\right)$$
$$= \frac{1}{\sqrt{2\pi p_2}}\exp(-\frac{1}{2}p_2), p_2 = \sqrt{L\frac{36E_b}{5N_0}} . \qquad (26)$$

Hence, the probability of error for 1st bit in the QAM symbol is $P_{b1} = \frac{1}{2}(P_{b1|a_I=-d} + P_{b1|a_I=-3d})$. Using (25) and (26) the bit error is expressed as:

$$P_{b1} = \frac{1}{2\sqrt{2\pi p_1}}\exp(-\frac{1}{2}p_1) + \frac{1}{2\sqrt{2\pi p_2}}\exp(-\frac{1}{2}p_2). \qquad (27)$$

Now, we know P_{b3}=P_{b4}. By the similar argument as above, the probability of error for 3rd and 4th bit can be given as:

$$P_{b3} = \frac{1}{2}\left(2P_{b1|a_I=-d} + P_{b1|a_I=-3d} - \frac{1}{\sqrt{2\pi p_3}}\exp(-\frac{1}{2}p_3)\right),$$
$$\text{where, } p_3 = \sqrt{L\frac{100E_b}{5N_0}} . \qquad (28)$$

From (27) and (28), the average bit error for MRC space diversity is given as:

$$P_b = \frac{1}{2}(P_{b1} + P_{b3})$$
$$= \frac{3}{4\sqrt{2\pi p_1}}\exp(-\frac{1}{2}p_1) + \frac{1}{2\sqrt{2\pi p_2}}\exp(-\frac{1}{2}p_2)$$
$$-\frac{1}{4\sqrt{2\pi p_3}}\exp(-\frac{1}{2}p_3) \qquad (29)$$

B. Selection Combining (SC)

In SC the detector select the highest SNR path, in this case the diversity combiner performs the operation

$$\bar{r} = \underbrace{\max}_{|h_k|}|r_k| . \qquad (30)$$

The instantaneous SNR (γ_k) of kth branch has the probability density function [8] is expressed as:

$$p(\gamma_k) = \frac{1}{\Gamma}\exp(-\frac{\gamma_k}{\Gamma}) . \qquad (31)$$

where Γ is the mean SNR of each branch [9] is:

$$\text{SNR}=\Gamma= (E0/N0)\ \bar{\alpha}^2 . \qquad (32)$$

We assume $\bar{\alpha}^2$=1. The average SNR with SC [8] is:

$$\bar{\gamma} = \Gamma\sum_{k=1}^{L}\frac{1}{k} . \qquad (33)$$

Let us define the average SNR by $\beta = \bar{\gamma} = \frac{E_b}{N_0}\sum_{k=1}^{L}\frac{1}{k}$, then, from (27), we have the bit error for 1st bit with SC expressed as:

$$P_{b1} = \frac{1}{2\sqrt{2\pi q_1}}\exp(-\frac{1}{2}q_1) + \frac{1}{2\sqrt{2\pi q_2}}\exp(-\frac{1}{2}q_2)$$
$$\text{where, } q_1 = \sqrt{\frac{4E_b}{5LN_0}} \text{ and } q_2 = \sqrt{\frac{36E_b}{5LN_0}}$$

Now from (28), the error for 3rd and 4th bit of QAM symbol with the assumption $P_{b3} = P_{b4}$ can be given as:

$$P_{b3} = \frac{1}{2}\left(2P_{b1|a_I=-d} + P_{b1|a_I=-3d} - \frac{1}{\sqrt{2\pi q_3}}\exp(-\frac{1}{2}q_3)\right), \text{ where}$$
$$q_3 = \sqrt{\frac{100E_b}{5LN_0}} .$$

The average probability of bit error for SC space diversity through Rayleigh fading channel can be given as

$$P_b = \frac{1}{2}\left(P_{b1} + P_{b3}\right)$$
$$= \frac{3}{4\sqrt{2\pi q_1}}\exp(-\frac{1}{2}q_1) + \frac{1}{2\sqrt{2\pi q_2}}\exp(-\frac{1}{2}q_2)$$
$$-\frac{1}{4\sqrt{2\pi q_3}}\exp(-\frac{1}{2}q_3). \quad (34)$$

C. Equal Gain Combiner (EGC)

The EGC is same as MRC but different from it that the diversity branches are not weighted. The combiner generates the sum [8].

$$\bar{r} = \sum_{k=1}^{L}\exp(-\theta_k)\bar{r}_k . \quad (35)$$

The envelope of the composite signal is

$$\alpha_E = \sum_{k=1}^{L}\alpha_k . \quad (36)$$

The resulting SNR is

$$\bar{\gamma}_M = \frac{\alpha^2{}_E E_b}{LN_0} = \frac{1}{L}\sum_{k=1}^{L}\gamma_k = \frac{1}{L}\times L\Gamma = \Gamma . \quad (37)$$

The probability of error due to distance -d ($P_{b1|a_I=-d}$) for EGC space diversity is given by applying the average SNR from (37) into (12), we have

$$P_{b1|a_I=-d} = Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$$
$$= \frac{1}{\sqrt{2\pi p_1}}\exp(-\frac{1}{2}p_1), p_1 = \sqrt{\frac{4E_b}{5N_0}} . \quad (38)$$

Similarly, the error probability for distance -3d ($P_{b1|a_I=-3d}$) from (13), we have:

$$P_{b1|a_I=-3d} = Q\left(\sqrt{\frac{36E_b}{5N_0}}\right)$$
$$= \frac{1}{\sqrt{2\pi p_2}}\exp(-\frac{1}{2}p_2), p_2 = \sqrt{\frac{36E_b}{5N_0}} . \quad (39)$$

Hence, the probability of error for 1st bit in the QAM symbol is $P_{b1} = \frac{1}{2}(P_{b1|a_I=-d} + P_{b1|a_I=-3d})$. Using (38) and (39) the bit error is expressed as:

$$P_{b1} = \frac{1}{2\sqrt{2\pi p_1}}\exp(-\frac{1}{2}p_1) + \frac{1}{2\sqrt{2\pi p_2}}\exp(-\frac{1}{2}p_2). \quad (40)$$

The probability of error for 3rd and 4th bit can be given with the assumption $P_{b3}=P_{b4}$ as:

$$P_{b3} = \frac{1}{2}\left(2P_{b1|a_I=-d} + P_{b1|a_I=-3d} - \frac{1}{\sqrt{2\pi p_3}}\exp(-\frac{1}{2}p_3)\right),$$
$$\text{where } p_3 = \sqrt{\frac{100E_b}{5N_0}} . \quad (41)$$

Using (40) and (41), the average bit error for EGC comes as:

$$P_b = \frac{1}{2}\left(P_{b1} + P_{b3}\right)$$
$$= \frac{3}{4\sqrt{2\pi p_1}}\exp(-\frac{1}{2}p_1) + \frac{1}{2\sqrt{2\pi p_2}}\exp(-\frac{1}{2}p_2) .$$
$$-\frac{1}{4\sqrt{2\pi p_3}}\exp(-\frac{1}{2}p_3) \quad (42)$$

D. Switching Combiner (SCC)

The Switching Combiner searches a diversity branch that has a signal-to-noise ratio exceeding a specified threshold. This diversity branch is selected and used until the SNR drops to below the threshold, and when the SNR drops then the combiner switches to another diversity branch which has a SNR exceeding the threshold. Here, we analyzed two-branch switch combining. Let the bit energy-to-noise ratios associated with the two branches are denoted by γ_1 and γ_2, and the switching threshold is γ , the probability that γ_i is less than the threshold is

$$\Pr(\gamma_i \le \bar{\gamma}) = 1 - \exp(-\bar{\gamma}/\Gamma) . \quad (43)$$

Now, the probability that the two independent diversity branches receive signals which are simultaneously less than the specific threshold is

$$\Pr(\gamma_1, \gamma_2 \le \bar{\gamma}) = [1 - \exp(-\bar{\gamma}/\Gamma)]^2 . \quad (44)$$

To determine the average SNR of the received signal, it is first necessary to compute the derivative of the CDF of the above expression

$$P(\bar{\gamma}) = \frac{2}{\Gamma}[1 - \exp(-\bar{\gamma}/\Gamma)]\times\exp(-\bar{\gamma}/\Gamma) . \quad (45)$$

The mean SNR for SCC may be expressed as

$$\bar{\gamma}^{sw} = \int_0^\infty \bar{\gamma}P(\bar{\gamma})d\bar{\gamma}$$
$$= \int_0^\infty \bar{\gamma}\frac{2}{\Gamma}[1 - \exp(-\bar{\gamma}/\Gamma)]\times\exp(-\bar{\gamma}/\Gamma)d\bar{\gamma}$$
$$= \Gamma\sum_{k=1}^{2}\frac{1}{k} = \frac{E_b}{N_0}\sum_{k=1}^{2}\frac{1}{k} . \quad (46)$$

The bit error for 1st of QAM symbol with SCC is obtained from (27), we have

$$P_{b1} = \frac{1}{2\sqrt{2\pi q_1}}\exp(-\frac{1}{2}q_1) + \frac{1}{2\sqrt{2\pi q_2}}\exp(-\frac{1}{2}q_2)$$

here, $q_1 = \sqrt{\frac{2E_b}{5N_0}}$, and $q_2 = \sqrt{\frac{18E_b}{5N_0}}$.

The error for 3rd and 4th bit of QAM symbol with the assumption $P_{b3} = P_{b4}$ for Switching Combiner can be given as:

$$P_{b3} = \frac{1}{2}\left(2P_{b1|a_I=-d} + P_{b1|a_I=-3d} - \frac{1}{\sqrt{2\pi q_3}}\exp(-\frac{1}{2}q_3)\right)$$

where, $q_3 = \sqrt{\frac{50E_b}{5N_0}}$.

The average BER for SCC space diversity through Rayleigh fading channel is given as

$$P_b = \frac{1}{2}\left(P_{b1} + P_{b3}\right)$$

$$= \frac{3}{4\sqrt{2\pi q_1}}\exp(-\frac{1}{2}q_1) + \frac{1}{2\sqrt{2\pi q_2}}\exp(-\frac{1}{2}q_2)$$

$$-\frac{1}{4\sqrt{2\pi q_3}}\exp(-\frac{1}{2}q_3). \quad (47)$$

The switching combiner switches to the alternate branch when the SNR drops below the specific threshold that is the selection of diversity branches does not depend on the whether the alternate branch is above or below the threshold. In a particular switching state the SNR is same as for selection combiner as shown from (33) and (46). The performance of switching combiner is same in the switching state as selection combiner for two diversity branches. The probability of bit error in term of the threshold value for SSC can be expressed as in [8]:

$$P_b = \frac{1}{2(1+\Gamma)}(q + (1-q)e^{-\bar{\gamma}}). \quad (48)$$

where q is the probability that a diversity branch having instantaneous SNR less than the threshold, and is expressed as:

$$q = \Pr[\gamma_i \langle \bar{\gamma}] = 1 - e^{-\bar{\gamma}/\Gamma}.$$

IV. RESULTS

The simulation results for the BER performance of MRC, SC and EGC space diversity with 16-QAM for 2×2 diversity system for Rayleigh fading channel is shown in Fig.1. The BER performance of QAM through Rayleigh fading channel is same as SC until the average SNR of the selected diversity branch drop to the threshold level. This result due to the fact that the combiner provides the same average SNR as for SC until the combiner switches to the diversity branch having SNR of that branch higher than the threshold value. The simulation results show that MRC provides better BER performance of QAM symbol through the same fading channel than the other diversity combiner such as SC and EGC. The MRC weighted, co-phased, and added the diversity branches and the strong signal at the output of the combiner, and the highest average SNR as a result. Hence, the diversity significantly improves the wireless channel performance.

The analytical BER performance of 16-QAM symbol for Rayleigh fading channel with MRC space diversity is obtained from (29), and is shown in Fig. 2 for $L = 1\ to\ 8$. It is seen from Figs. 1 and 2 that at E_b/N_0 =10 dB with 2 transmit and 2 receive antenna system, the simulated and analytical BER is 0.0015 and 0.0028 respectively. The analytical BER performance of 16-QAM symbol for Rayleigh fading channel with SC space diversity is obtained from (34), and is shown in Fig. 3 for $L = 1\ to\ 8$. It is seen from Figs. 1 and 3 that at E_b/N_0=10 dB with 2 transmit and 2 receive antenna system for Rayleigh fading channel, the simulated and analytical values of BER are 0.13 and 0.2 respectively.

Similarly, the analytical BER performance of 16-QAM symbol for L = 1 to 8 for Rayleigh fading channel with EGC space diversity is obtained from (42), and is shown in Fig. 4. The simulated and analytic BER values of EGC are 0.093 and 0.12 respectively at E_b/N_0 =10 dB with 2×2 space diversity. The theoretical BER performance results of MRC, SC, and EGC from Figs.2, 3, and 4 show that the BER for MRC space diversity are higher than those of SC and EGC values, because MRC weights and adds all branch signals while SC selects one branch having the highest SNR and EGC does not weight each branch but the only co-phased signal. The BER expression of QAM for SCC as in (48) is plotted in Fig. 5 for several values of the threshold. The performance for threshold $\bar{\gamma}$=0 is the same as using no diversity. The BER performance increases with the value of the threshold level up to $\bar{\gamma}$=5, and the performance changes for $\bar{\gamma}$>5.

The simulated BER performance of BPSK signal with MRC, SC, and EGC through the same fading channel as considered in the previous simulation study for 16-QAM symbol is shown in Fig. 6. . The theoretical BER performance result of BPSK is shown in Fig. 7 for

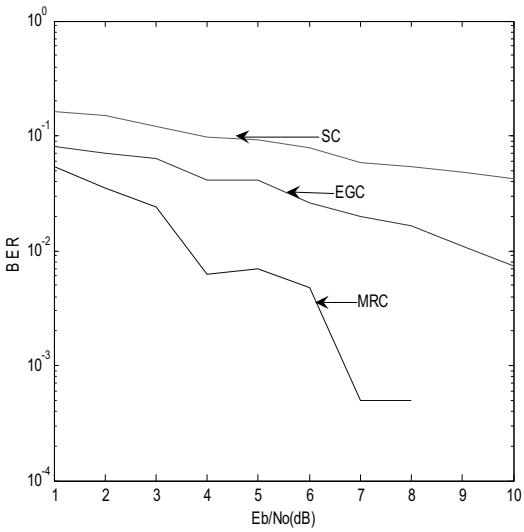

Figure1. Simulated BER performance of MRC, SC, and EGC with 2 transmits and 2 receive antennas by transmitting the 16-QAM symbol through a Rayleigh fading channel.

the three combiners according to the mathematical derivation in [8]. In Fig. 6 the MRC, SC, and EGC provides BER=0.019, 0.17, and 0.016 for E_b/N_0=5 dB. The mathematical BER expression in [7] shows that MRC, SC, and EGC provides BER=0.015, 0.13, and 0.016 for E_b/N_0=5 dB, this result is shown in Fig. 7. The BER expression is derived in section II based on LLR scheme, this expression shows that the BER is very close to the simulation result for 16-QAM with MRC, SC, and EGC. We may find the BER expression for BPSK using LLR scheme, in Fig. 8, which shows the theoretical BER using LLR scheme for BPSK through a Rayleigh fading channel. This simulation result shows that MRC, SC, and EGC provides BER for 16 QAM to 0.014, 0.09, and 0.075 for E_b/N_0=5.5 dB. The analytical BER performance of 16-QAM for MRC is also shown in Fig. 1 based on (22) [10]. The log of BER is -2.0 for SNR 11.5 dB is equivalent to 0.1353 for E_b/N_0=5.5 dB for 2 transmit and 2 receive antennas. The LLR based BER for MRC in (29) is 0.017 which is very close to the simulation result having value0.014 for E_b/N_0=5.5 dB.

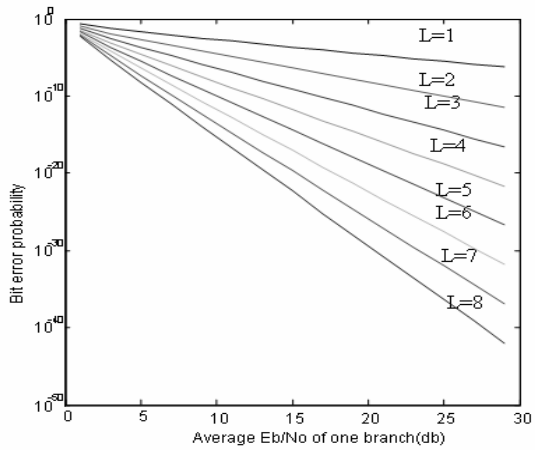

Figure2. The analytical BER Performance of 16-QAM with MRC space diversity in Rayleigh fading channels evaluated using (29).

Hence, our derived LLR based BER performance for space diversity is more accurate than the work [10].

Simulation is done in MATLAB platform and environment. Here, the simulation is performed by sending a 16-QAM symbol through a multipath fading channel. The channel noise is considered as AWGN with mean zero and variance σ^2. In the simulation, the phase offsets taken are 25^0 and 30^0 respectively for two-branch transmit diversity. E_b/N_0 is varied from 1 to 10.0. The 2×2 transmit diversity is considered in the simulation. The transmitted symbols are detected by MRC, SC, EGC, and SCC using MATLAB software. The analytical BER performance for MRC, SC, EGC, and Switched combiner with 16-QAM for different values of diversity order is computed by solving (29), (34), (42), and (48) using MATLAB. The simulated BER results for MRC, SC, and EGC with BPSK are obtained by sending a BPSK symbol through the above fading channel. In all the cases, bit error is calculated by counting number of error bits at the receiver with respect to transmitter.

V. CONCLUSION

In this paper, we compare the BER performance of different diversity combiners specially MRC, SC and EGC. The theoretical and simulated BER performance values are shown in this work. The theoretical values are calculated using LLR space diversity approach and the simulation values are found by sending a 16- QAM symbol through Rayleigh fading channel. Both theoretical and analytical BER performance values show that the MRC provides the highest BER performance than those of the other two combiners. The BER performances are also calculated for MRC and SC space diversity for Rayleigh fading channel in work [5]. The simulation results in Fig.1 show that the BER values at E_b/N_0 =10 dB for MRC and SC are 0.0015 and 0.13 respectively for 2×2 diversity.

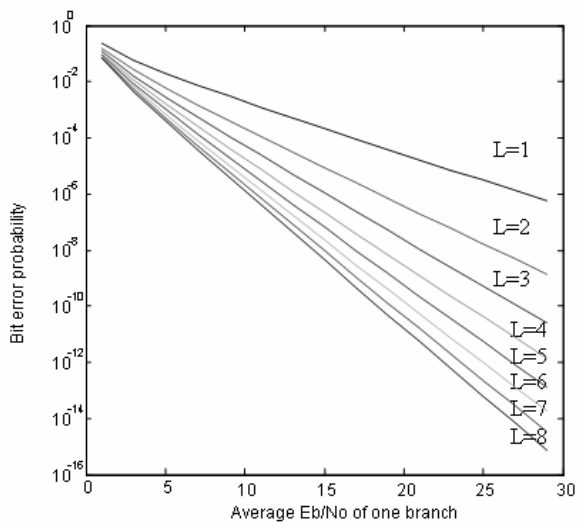

Figure 3. The BER Performance of 16-QAM with SC space diversity in Rayleigh fading channels according to (34).

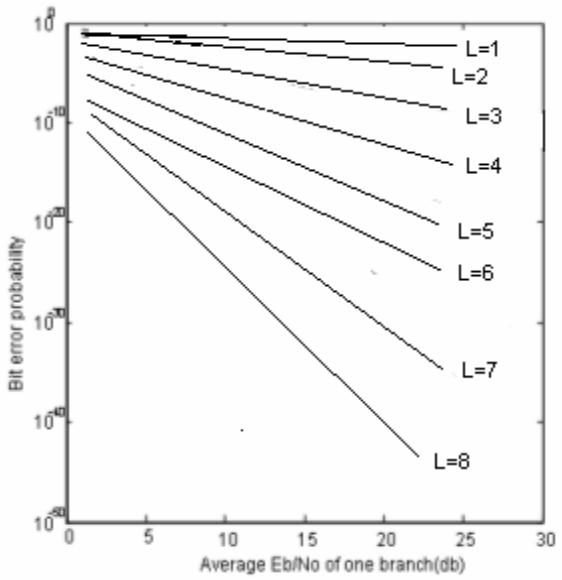

Figure 4. The BER Performance of 16-QAM with EGC space diversity in Rayleigh fading channels based on (42).

In work [5], the BERs for MRC and SC at E_b/N_0 =10 dB are 0.03 and 0.24 respectively. The LLR based mathematical BER performance in this paper provides more accurate result than the work [5] [10] because LLR finds the more accurate error probability than the other method. We get the BER values at E_b/N_0 =10 dB to be 0.0028 and 0.2 for MRC and SC from the LLR space diversity based BER derivation as in (29) and (34). The key contribution of our paper is that our LLR based BER performance is more accurate with the simulated result than the BER derivation in work [5]. The SCC provides the same BER performance as the SC when the receiver switches to the diversity branch whose SNR exceeds the threshold.

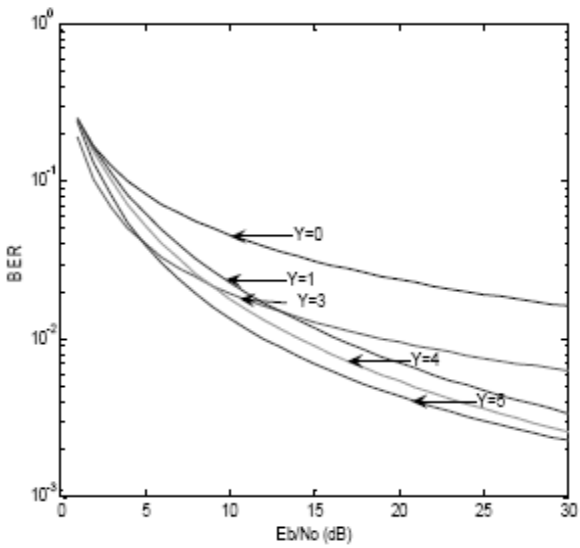

Figure5. BER performance of QAM through a Rayleigh fading channel for 2-branch switched combining.

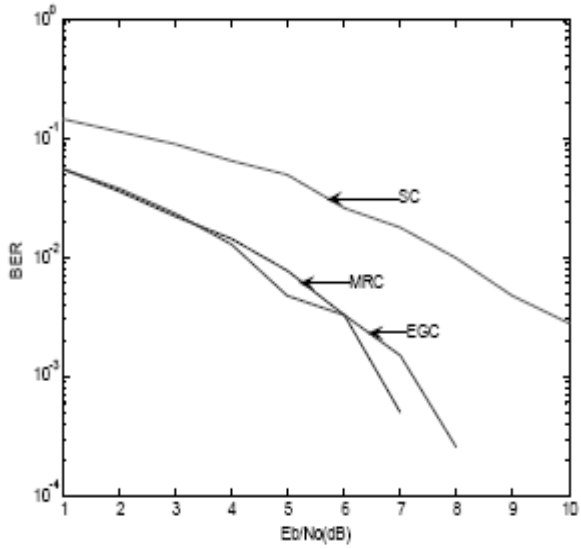

Figure 6. Simulation result for BER performance of BPSK through Rayleigh fading channel with MRC, SC, and EGC.

The main advantage of SCC over all the other diversity combiner is that it needs only one detector. The BER expression for BPSK using LLR scheme in this paper is very close to the simulation result for MRC, SC, and EGC in comparison to those in [2]. Now, the summary is that our derived BER performance result using LLR space diversity provides better BER improvement result over performance reported in [5] and [10].

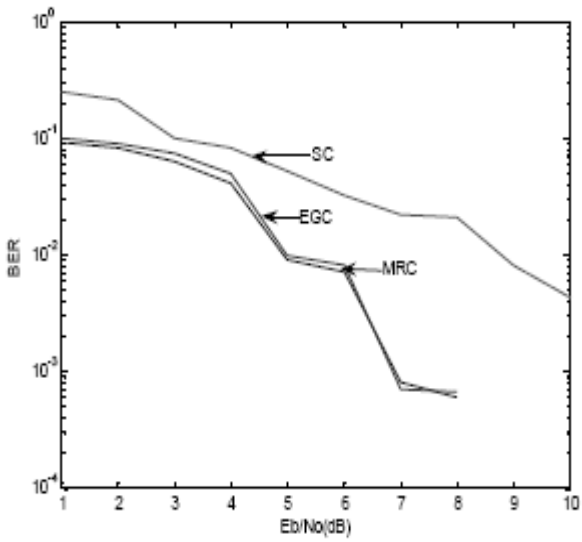

Figure7. BER performance of BPSK with MRC, SC, and EGC through Rayleigh fading channel according to [8].

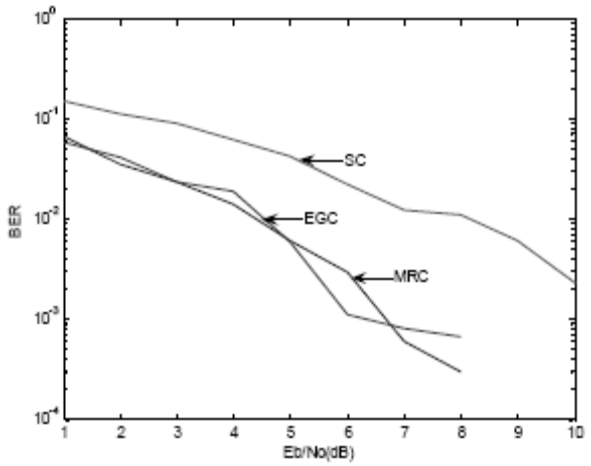

Figure8. LLR scheme based BER performance of BPSK with MRC, SC, and EGC through Rayleigh fading channel.

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