Accumulative Interference Modeling for Distributed Cognitive Radio Networks

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***Abstract*—A Cognitive Radio (CR) network should be able to sense its environment to adapt its communication so that it can utilize unused licensed spectrum without interfering with incumbent users. Properly modeling the expected interference from the entire CR network is therefore very important to effectively protect these incumbent users. We model the accumulative interference generated from a large-scale CR network and investigate how the CR network density affects the sensing requirements of the CRs to meet an interference constraint. More specifically, our model considers the impact of discrete network topology, the impact of imperfect sensing and the impact of collisions when the CR uses a distributed channel access scheme. We then apply our model to a CR network based on the IEEE 802.11 standard. We show that the collisions occurring frequently in these networks only have a small on the sensing requirements to protect the incumbent network.**

***Index Terms*—cognitive radio, opportunistic spectrum access, interference modeling, IEEE 802.11**

I. Introduction

Due to the accelerated deployment of broadband communication systems and the current fixed frequency allocation scheme, spectrum is becoming a major bottleneck. However, experiments show that up to 85% of the spectrum remains unused at a given time and location, indicating that a more flexible allocation strategy could solve the spectrum scarcity problem [1]. This observation has recently led to the new paradigm of opportunistic spectrum access, where users can actively search for unused spectrum in licensed bands and communicate using these *white holes*. This vision is supported by regulatory bodies, such as the Federal Communications Commission (FCC) [2] and the European Commission (EC) [3]. The concept is also often referred to as Cognitive Radio (CR)[1] [6].

To enable the concept of opportunistic spectrum sharing, many problems remain to be solved. Most importantly, the CRs have to make sure they do not cause excessive interference to the incumbent users. If no guarantees about the interference can be given, it will be very hard to convince incumbent users to tolerate CRs.

Giving guarantees on the level of interference to the incumbents is however very challenging in the context of wireless communications. This has already been noted in [7], [8], where large margins were introduced in the sensing threshold requirements to account for unpredictable fading and shadowing. Even after considering these margins, a single CR meeting its personal sensing constraints could still cause excessive interference when that CR simultaneously transmits with another CR meeting its sensing constraints. This is referred to as *accumulative interference*.

In this paper we address the problem of accumulative interference from multiple CRs that use distributed channel access. Currently, it is not possible to avoid all CR collisions in such a distributed environment. This is largely motivated by the large amount of literature on collisions for 802.11. Moreover, besides direct collisions caused by the 802.11 contention mechanism, simultaneous transmissions occur when nodes cannot hear each other and hence do not back off for each other.

The fact that simultaneous transmissions from CRs increase the interference to an incumbent user was noted in [7]. The authors model this by assuming CRs are spread out continuously in a sea of CR nodes modeled with a power density. Hence, the authors do not consider the impact of discrete topologies, neglecting the spread around the average interference resulting from a variation in topology instantiation. We will determine how the interference varies as the topology becomes more discrete, i.e., centered around hot spots, as compared to a continuous

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[1]The term *Cognitive Radio* (CR) was first coined in [4] and meant a radio that uses model-based reasoning to autonomously change its transmission parameters based on interaction with the complex environment (radio scene, application and user requirements) in which it operates [5]. In the present paper we focus on a shorter-term and spectrum-centric view of CR, i.e., a radio system that co-exists with incumbent wireless systems by using the same spectrum resources without significantly interfering with these incumbents [6].

sea.

Also, in [7], the impact of imperfect sensing is not considered, and it is assumed that all CRs inside a circular zone around the incumbent base station do not transmit. However, it is very unrealistic to assume that all nodes inside this area will detect the incumbent transmission. This will increase the interference from nodes that should not transmit. Similarly, detection errors will cause CRs outside the silenced area to falsely detect the incumbent transmissions and defer their own transmissions. The latter effect results in a decrease of interference from nodes that were actually allowed to transmit. We will show that by optimally tuning the detection algorithm, both effects can balance out and the resulting interference to the incumbent users is not increased significantly with a realistic sensing performance. It is however very important to verify this accurately.

Finally, as stated in [9], the impact of realistic medium access control strategies has to be considered. In this paper, we will instantiate our model in the particular case where the CR nodes employ the 802.11 distributed channel access mechanism. Referring to the recent Open Spectrum Access (OSA) equipments submitted for standard compliance testing to the FCC [10], it is clear why the choice for 802.11 as the MAC protocol for CR nodes is a very realistic one.

Within the IEEE 802.11 standard, the impact of (accumulative) interference has been recently studied [11], [12]. These studies focus on optimal sensing thresholds so as to optimize the throughput of the 802.11 network. However, since these studies are only concerned with intra-system interference, they significantly differ from the CR case. In the case of opportunistic spectrum sharing the interference to incumbents has to be controlled, while optimizing the throughput of the CR networks is generally considered to be of secondary importance.

In this paper we will hence study the impact of accumulative interference on the incumbent user protection, or alternatively study how such interference increases the sensing requirements of the CR nodes.

In light of [13], where the authors make a distinction between spectrum opportunity and interference constraint[2], this paper focuses on the interference constraint. We assume that when a CR is not able to meet the interference constraint, it will shut down its interface. On the other hand, if it is able to meet the interference constraint, it is assumed that a successful CR communication can be set up.

First, we will detail the considered system in Section II. The model is presented in Section III. In Section III-B the model considers a discrete random topology of CR hot spots to investigate whether the exact topology instance has an impact on the generated interference, independently of the CR sensing performance. In Section III-C we model a CR system with perfect sensing. Next, we will add the detection errors, and see how this impacts the interference to the incumbent in Section III-D. Finally, we will analyze the impact of the channel access scheduling for the CR network, focusing on the realistic assumption of an 802.11 distributed channel access scheme (see Section IV). At the end of the paper, we present our conclusions in Section V.

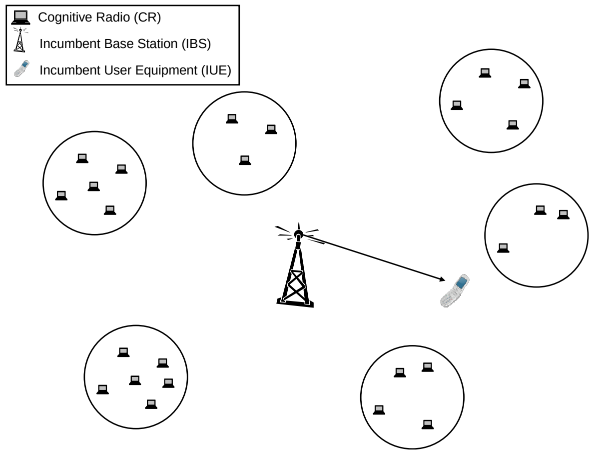

Fig. 1. The considered topology. An incumbent system has to withstand interference from several CR networks. These CRs are assumed to be randomly distributed on a plane following a Poisson point process with density δ.

II. System Description

When considering the possible interference to the incumbent users, the assumptions on the system model have to be established. We first introduce the topology assumed both for the incumbent users and the CRs in Section II-A. In Section II-B, the propagation and interference models are introduced.

The system model typically assumed for CR is based on the assumptions of the IEEE 802.22 standard, since this is the first CR system that is being standardized. In these networks, the incumbent users are TV broadcast stations, covering a very large area. The CR power is often assumed to be orders of magnitude smaller than the incumbent transmission powers.

In this paper we want to relax this system view and consider a broader range of scenarios. The goal is to illustrate how much interference is to be expected in *any* scenario where a CR network using distributed channel access coexists with *any* incumbent network.

A. Topology

We consider an incumbent system that is surrounded by several CRs in Fig. 1. CR hot spots, i.e., local groups of CRs, are randomly distributed on a plane following a Poisson point process with density δ. The hot spot size varies from a single CR per hot spot, which could mimic a single CR in a home, to 10 CRs per hot spot, which is more typical in a coffee shop. This allows us to model topologies where the CRs are more or less spread out and topologies where CRs are clustered in hot spots.

[2] *A channel is an opportunity to a pair of secondary users if they can communicate successfully without violating the interference constraint imposed by the primary network [13]*

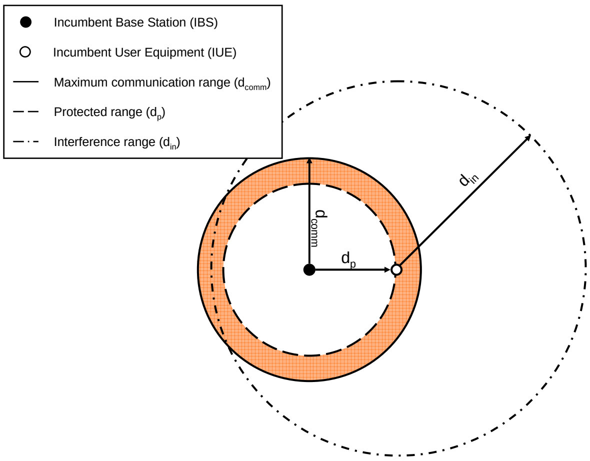

Fig. 2. The system topology for the incumbent users. Incumbent User Equipments (IUEs) are spread around an Incumbent Base Station (IBS). CRs within the interference range d_{in} from the border of the protection region can potentially harm the communication of the incumbents inside the protected area.

As stated in [8], the introduction of CRs will inevitably reduce the communication range of the incumbent system. The authors use the concept of a protected area, the area in which the incumbent system desires to operate unharmed (see Fig. 2). The protected area is defined through its radius, the protected range d_p. If not otherwise stated, the protected range is chosen to be 95% of the maximal communication distance d_{comm} of the incumbent system before the introduction of CRs. We assume that all incumbent receivers are within this protection range.

B. Propagation and Interference Models

The received signal power R is a decreasing function of the distance d between the Incumbent Base Station (IBS) and the Incumbent User Equipment (IUE). Let S denote the transmit power of the sender and α_1 the path loss exponent (typically ranging between 2 and 4). The received signal power can then be expressed as:

$$R = S - 10\alpha_1 \log_{10}(d) - \beta_1 \text{ [dB]}, \tag{1}$$

where β_1 represents system losses and effectively hides all non-distance-related components, such as the frequency dependency introduced by the antenna. Similarly, the interference power, I, at the IUE from one CR at a distance d_{cr} from the IUE is given by:

$$I = S_{cr} - 10\alpha_2 \log_{10}(d_{cr}) - \beta_2 \text{ [dB]}. \tag{2}$$

Without loss of generalization, we will assume $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$.

Packets can be decoded if the received Signal-to-Interference-and-Noise-Ratio ($SINR$) exceeds a certain threshold $SINR_t$. For a given scenario, the lowest possible $SINR$ at an IUE in the protected area is at the edge of this area, since it will have the lowest possible received power R from the IBS. Hence, if we can protect a link in this worst-case situation, we are protecting any link within the protected area:

$$SINR|_{d_p} \geq SINR_t = \frac{R|_{d_p}}{\sigma_0^2 + I_t}, \tag{3}$$

where σ_0^2 is the noise power and I_t the interference threshold. Throughout this paper we will assume thermal noise:

$$\sigma_0^2 = kTW, \tag{4}$$

where k is the Boltzmann constant, T the environment temperature and W the used bandwidth.

Let us now introduce the interference range d_{in} (see Fig. 2), which is defined as the maximum distance for which a single CR transmission will harm the incumbent system ($I|_{d_{in}} = I_t$):

$$d_{in} = \left(\frac{S_{cr}}{10^{\frac{\beta}{10}} I_t} \right)^{\frac{1}{\alpha_2}}. \tag{5}$$

A rigorous study of CR networks also needs to specify an outage probability as well as an interference threshold [14]. In the remainder of this paper, we assume an outage probability of 0.01%.

III. Proposed Interference Model in a Cognitive Radio Setting

In this section we model the effect of direct and accumulative interference for the system presented in Section II, to effectively protect the incumbent system. After defining the problem in Section III-A, we compute the impact of interference in case the CRs do not sense the incumbent and all transmit at the same time (see Section III-B). Next, we introduce perfect sensing in Section III-C. Finally, we consider the impact of imperfect sensing in Section III-D.

A. Direct and accumulative interference

As mentioned in Section II-B, the IBS will not be able to communicate with a receiver inside the protected region if:

- at least one CR inside the interference area is active (direct interference) or
- the accumulated power from all the CRs outside the interference area exceeds the interference threshold (accumulative interference).

The total probability of interference p_{in} can be derived from the probability of direct interference $p_{in,d}$ and the probability of indirect (or accumulative) interference $p_{in,a}$ as follows:

$$p_{in} = p_{in,d} + (1 - p_{in,d}) p_{in,a}. \tag{6}$$

B. Maximum power without sensing

We look here at the case where all CRs transmit at the same time without sensing the IBS. This is the simplest, though unrealistic, case and we introduce it mainly to explain our interference computation method. Since we are dealing with a Poisson point process with parameter δ, the probability that k transmitters are present in a certain region A, can be written as:

$$p_A(k) = \frac{1}{k!} (\delta|A|)^k e^{-\delta|A|}, \quad (7)$$

where $|A|$ denotes the area of A. The probability $p_{in,d}$ that a link gets interfered by a single CR transmission (direct interference) is the probability that at least one node inside the interference range is transmitting. Hence, we can express this as:

$$p_{in,d} = 1 - e^{-\delta\pi d_{in}^2}. \quad (8)$$

The probability $p_{in,a}$ that a link gets interfered by accumulative interference from outside the interference range is the probability that the accumulated power from all the CRs exceeds the interference threshold. To simplify analysis, we will consider the normalized interference power, I_n:

$$I_n = \frac{I}{I_t} = \frac{d_{in}^\alpha}{r^\alpha}. \quad (9)$$

The system will hence be interfered with accumulatively if $I_n \geq 1$. Each transmission inside the interference range is assumed to have a normalized interference power equal to 1 (i.e., with one single transmission inside the interference range the system is interfered).

To find this total accumulated power, we need to integrate over the area outside the interference range, similar to the technique proposed in [11], to compute the interference in 802.11 systems. We consider a thin ring R_i with inner radius, $r_i = d_{in} + (i-1)\Delta r$ and outer radius $r_i + \Delta r$. The number of nodes in this ring is Poisson distributed. However, since each transmission only accounts for $\frac{d_{in}^\alpha}{r_i^\alpha}$ (the normalized received power at the IUE from a transmission inside R_i), we are now dealing with a scaled Poisson process [15].

Based on the above, the mean and the variance of the accumulative interference can be expressed as an integration over the means and variances of these scaled Poisson processes:

$$\mu_a = \int_{d_{in}}^{\infty} \delta 2\pi r \frac{d_{in}^\alpha}{r^\alpha} dr = \frac{2\pi\delta d_{in}^2}{\alpha - 2}, \quad (10)$$

$$\sigma_a^2 = \int_{d_{in}}^{\infty} \delta 2\pi r \left(\frac{d_{in}^\alpha}{r^\alpha}\right)^2 dr = \frac{\delta\pi d_{in}^2}{\alpha - 1}. \quad (11)$$

Now, we only need to choose the appropriate distribution for the accumulative interference. As the Gamma distribution is widely used to model continuous variables that are always positive and have skewed probability density functions, the summation of an infinite number of scaled Poisson processes (which is such a positive skewed distribution) is well approximated by such a Gamma distribution [16]. Other possibilities include a Gaussian distribution with Edgeworth expansions [17] or a log-normal distribution [18].

The Gamma distribution is formed with the sum of exponential variables and has two parameters: a scale parameter θ and a shape parameter k. Using (10) and (11), we can find the parameters that completely define

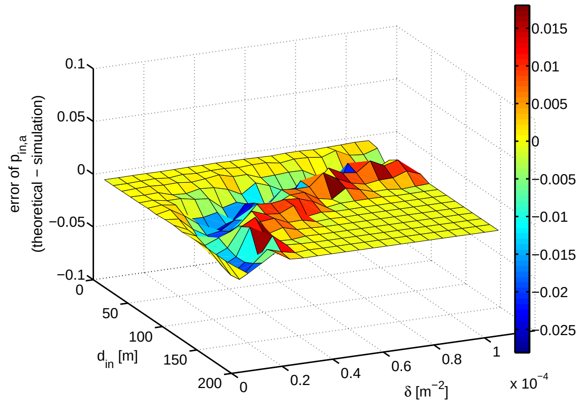

Fig. 3. A comparison between analytical and simulation results. This comparison shows that the Gamma distribution is a sufficient analytical approximation for the summation of scaled Poisson point processes. The approximation error is small for a broad range of tested ranges of the parameters δ and d_{in}.

the desired Gamma distribution [16]:

$$k_a = \frac{\mu_a^2}{\sigma_a^2}, \quad (12)$$

$$\theta_a = \frac{\sigma_a^2}{\mu_a} \quad (13)$$

The probability that a link is interfered through accumulative interference, $p_{in,a}$ is then equal to

$$p_{in,a} = 1 - F_\Gamma(1; k_a, \theta_a), \quad (14)$$

where $F_\Gamma(x; k, \theta)$ denotes the cumulative distribution function of the Gamma distribution with shape parameter k and scale parameter θ.

In Fig. 3, the absolute error between simulation outcome and the analytical prediction is shown. During simulation we distribute terminals according to a spatial Poisson process and calculate the power that simultaneous transmissions from all terminals generate. We see that the maximum of this absolute error is smaller than 2.5%.

By using (8) and (14), we can find the total probability of interference through (6). In Fig. 4, we plot the cumulative distribution of the normalized interference. It is shown that with a probability $p_{in} = 14.7\%$, the incumbent system is harmed for the parameters considered. This high interference probability is a consequence of the fact that the CR network does not sense for the incumbent and because all CR terminals are transmitting simultaneously. In the next subsection, we show how to relax those assumptions.

C. Maximum power with perfect sensing

In the case of perfect sensing, we assume that a CR perfectly detects the incumbent signal if the Signal-to-Noise Ratio (SNR) of the received incumbent signal is larger than a chosen threshold, SNR_d. The probability of detection, p_d is hence:

$$p_d(SNR) = \begin{cases} 1 & \text{if } SNR \geq SNR_d \\ 0 & \text{if } SNR < SNR_d. \end{cases}$$

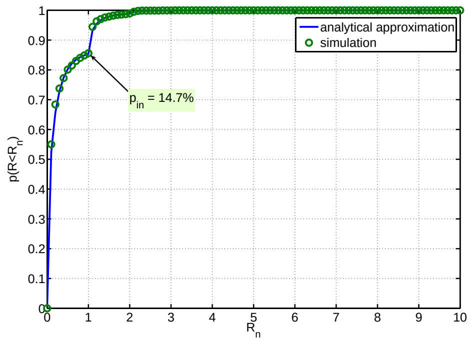

Fig. 4. The cumulative distribution function of the normalized interference power (including direct interference). The probability of interference is high because the nodes don't sense in this scenario.

Let us define d_s as the maximum distance at which a CR detects the incumbent. If an incumbent is detected, the CR will switch off its interface or move to another channel where it will not interfere with the sensed incumbent.

In [7] the authors give an approximation of the total interference power when CRs are sensing given a silence distance d_s from the IBS. This approximation however only holds if $d_p \gg d_{in}$. We present here the exact formula, with mean and variance. Again, we will consider the direct interference first, which is a Poisson process with density δ. The probability of direct interference then writes:

$$p_{in,d} = 1 - e^{-\delta(|A_i \setminus A_s|)}, \tag{15}$$

where A_s is the silence area around the IBS (defined through d_s) and A_i the interference area around the IUE (defined through d_{in}).

The accumulative term can now be found in a similar way as described in Section III-B. We will however need to account for the area that has been silenced. This turns down to integrating between $r_{\min}$ and $r_{\max}$, where the integration rings intersect with the silenced area. For values of r larger than $r_{\max}$ no nodes are silenced. When r is smaller than $r_{\min}$, all nodes are silenced. One can then write:

$$r_{\min} = \max(|d_s - d_p|, d_{in}), \tag{16}$$
$$r_{\max} = \max(d_s + d_p, d_{in}). \tag{17}$$

We can then compute μ_a and σ_a^2 as

$$\begin{aligned} \mu_a &= \int_{r_{\min}}^{r_{\max}} 2\delta\theta(r) r \frac{d_{in}^\alpha}{r^\alpha} dr + \int_{r_{\max}}^{\infty} 2\delta\pi r \frac{d_{in}^\alpha}{r^\alpha} dr \\ &= 2\delta d_{in}^\alpha \int_{r_{\min}}^{r_{\max}} \theta(r) r^{1-\alpha} dr + \frac{2\pi\delta d_{in}^\alpha}{(\alpha-2) r_{\max}^{\alpha-2}} \end{aligned} \tag{18}$$

$$\begin{aligned} \sigma_a^2 &= \int_{r_{\min}}^{r_{\max}} 2\delta\theta(r) r \frac{d_{in}^{2\alpha}}{r^{2\alpha}} dr + \int_{r_{\max}}^{\infty} 2\delta\pi r \frac{d_{in}^{2\alpha}}{r^{2\alpha}} dr \\ &= 2\delta d_{in}^{2\alpha} \int_{r_{\min}}^{r_{\max}} \theta(r) r^{1-2\alpha} dr + \frac{\pi\delta d_{in}^{2\alpha}}{(\alpha-1) r_{\max}^{\alpha-2}} \end{aligned} \tag{19}$$

where

$$\theta(r) = \arccos\left(\frac{d_p^2 + d_s^2 - r^2}{2 d_s d_p}\right). \tag{20}$$

Analytical expressions can be obtained by approximating θ as:

$$\theta(r) \approx \frac{\pi(r - |d_s - d_p|)}{d_s + d_p - |d_s - d_p|}. \tag{21}$$

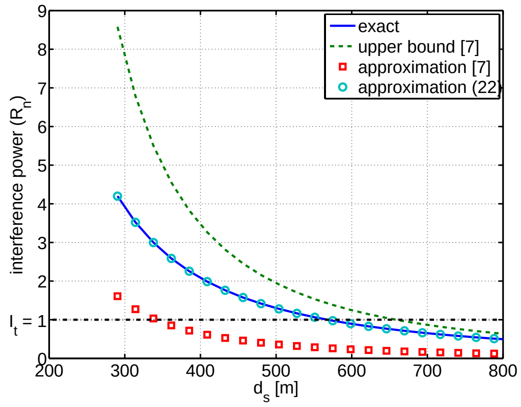

Fig. 5. The accumulative interference power as a function of the silenced distance, d_s. The approximation presented in [7] is not valid for the general case. We present the exact formula, as well as an analytically tractable approximation. We can see that the approximation from [7] gives a too optimistic sensing bound.

In Fig. 5, we plot the approximation and upper bound derived in [7]. We see that the approximation of [7] is not sufficient, because the assumption $d_p \gg d_{in}$ no longer holds. Our proposed approximation however is quite accurate. We can also see that the approximation of [7] is too optimistic regarding the sensing bound.

Using (12) and (13) we can find the parameters for the new Gamma distribution and compute p_{in} as we did in Section III-B. In Fig. 6, we can see the obvious result that increasing d_s (i.e., decreasing the sensing threshold) reduces interference to the incumbent system. Depending on the node density the level of accumulative interference varies a lot (more nodes generate more interference), and hence also the optimal sensing range. It is hence important to establish sensing bounds as function of the expected CR density. We also note that for small densities the curve drops less steeply than for large densities. This is because the homogeneous power density presented in [7] becomes more appropriate for larger densities, since variations in topology are now averaged out by the large number of users (small σ^2).

D. Maximum power with imperfect sensing

We will now introduce the effect of a realistic sensing implementation, which has a non-perfect outcome. In Table I, the probability of detection p_d is expressed as a function of the number of samples N, the received power R, the noise power σ_0^2 and the probability of false alarm p_{fa}. Many MAC protocols for CR networks will try to avoid self-interference from the CRs, so that the $SINR$ is merely determined by the noise σ_0^2 from the environment (see Fig. 7) [19].

To find p_{in}, we again compute a Poisson component (for direct interference) and a Gamma component (for accumulative interference). Because nodes are silenced

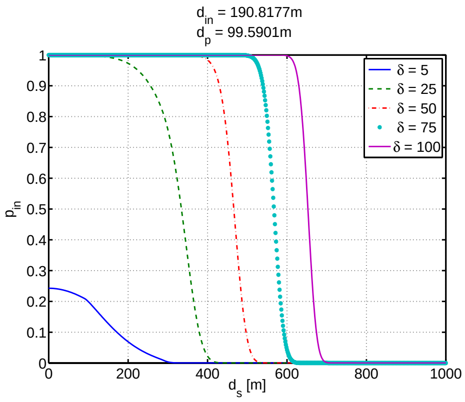

Fig. 6. The probability of interference as function of sensing range d_s for a number of CR densities. A higher CR density generates more accumulative interference and hence more interference for a given sensing range. As a result, larger CR densities need a larger sensing range to protect the incumbent.

TABLE I
THE PROBABILITY OF DETECTION FOR DIFFERENT TECHNIQUES [20]

	p_d
Energy Detection	$Q\left(\frac{Q^{-1}(p_{fa})-\frac{R}{\sigma_0^2\sqrt{N}}}{1+\frac{R}{\sigma_0^2}}\right)$
Matched Filter	$Q\left(Q^{-1}(p_{fa})-\sqrt{2N\frac{R}{\sigma_0^2}}\right)$

differently at different distances from the IBS (depending on the received SNR), we are now dealing with a heterogeneous Poisson process [21]. For the direct interference, we can model this heterogeneous Poisson process as a homogeneous Poisson process where the value for $\lambda|A|$ is equal to:

$$\lambda|A| = \int_0^{d_{in}}\int_0^{2\pi}\delta(1-p_d(r,\theta))2\pi r dr. \quad (22)$$

Using (7) we can then find $p_{in,d}$.

The accumulative component can be found through integrating mean and variances of the different Poisson processes

$$\mu_a = \int_{d_{in}}^{\infty}\delta\frac{d_{in}^{\alpha}}{r^{\alpha}}\int_0^{2\pi}(1-p_d(r,\theta))rd\theta dr, \quad (23)$$

$$\sigma_a^2 = \int_{d_{in}}^{\infty}\delta\frac{d_{in}^{2\alpha}}{r^{2\alpha}}\int_0^{2\pi}(1-p_d(r,\theta))rd\theta dr. \quad (24)$$

We only work out the equations for the matched filter detection, since doing the exercise for energy detection is

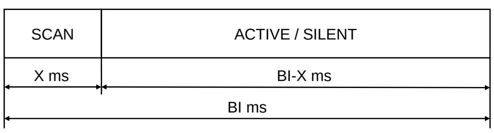

Fig. 7. A typical MAC protocol for CR networks. The CRs scan in a synchronized way so that intra-interference is avoided [19].

very similar. The noise power is given in (4). To compute the SNR, we recall the propagation model to compute the received power

$$R(r,\theta) = \frac{S_{ibs}10^{\frac{\beta}{10}}}{d(r,\theta)^{\alpha}}. \quad (25)$$

Further, $d(r,\theta)^2$ is determined as follows:

$$d(r,\theta)^2 = r^2 + d_p^2 + 2rd_p\cos\theta. \quad (26)$$

We define $d_{\min}$ and $d_{\max}$ for which $p_d(d)$ becomes respectively $(1-\epsilon)$ or $(p_{fa}+\epsilon)$, and hence determine the area where the detection process has impact on the behavior of the CR

$$d_{\min} = \left(\frac{2NS_{ibs}}{\sigma_0^2(Q^{-1}(p_{fa})-Q^{-1}(1-\epsilon))^2}\right)^{\frac{1}{\alpha}}, \quad (27)$$

$$d_{\max} = \left(\frac{2NS_{ibs}}{\sigma_0^2(Q^{-1}(p_{fa})-Q^{-1}(p_{fa}+\epsilon))^2}\right)^{\frac{1}{\alpha}} \quad (28)$$

The range for the integrals to compute the variance and the mean of the Gamma distribution can be found as follows. We determine $r_{\min}$ and $r_{\max}$ for which the lowest and highest detection probability p_d is either $(1-\epsilon)$ or $(p_{fa}+\epsilon)$.

$$r_{\min} = \max(d_{\min}-d_p, d_{in}), \quad (29)$$

$$r_{\max} = \max(d_{\max}+d_p, d_{in}). \quad (30)$$

We assume that for $r < r_{\min}$ all the cognitive radios detect the signal perfectly ($p_d = 1$) and that for $r > r_{\max}$ none of the cognitive radios can detect the signal ($p_d = p_{fa}$). The integrals then simplify to:

$$\mu_a = \int_{r_{\min}}^{r_{\max}}\delta\frac{d_{in}^{\alpha}}{r^{\alpha-1}}\int_0^{2\pi}(1-p_d(r,\theta))d\theta dr + \int_{r_{\max}}^{\infty}2\pi r(1-p_{fa})\delta\frac{d_{in}^{\alpha}}{r^{\alpha}}dr, \quad (31)$$

$$\sigma_a^2 = \int_{r_{\min}}^{r_{\max}}\delta\frac{d_{in}^{2\alpha}}{r^{2\alpha-1}}\int_0^{2\pi}(1-p_d(r,\theta))d\theta dr + \int_{r_{\max}}^{\infty}2\pi r(1-p_{fa})\delta\frac{d_{in}^{2\alpha}}{r^{2\alpha}}dr. \quad (32)$$

or

$$\mu_a = \int_{r_{\min}}^{r_{\max}}\delta\frac{d_{in}^{\alpha}}{r^{\alpha-1}}\int_0^{2\pi}(1-p_d(r,\theta))d\theta dr + \frac{2\pi(1-p_{fa})\delta d_{in}^{\alpha}}{(\alpha-2)\,r_{\max}^{\alpha-2}}, \quad (33)$$

$$\sigma_a^2 = \int_{r_{\min}}^{r_{\max}}\delta\frac{d_{in}^{2\alpha}}{r^{2\alpha-1}}\int_0^{2\pi}(1-p_d(r,\theta))d\theta dr + \frac{\pi(1-p_{fa})\delta d_{in}^{2\alpha}}{(\alpha-1)r_{\max}^{2\alpha-2}}. \quad (34)$$

The integrals presented here are now numerically solvable. Using these results, we can determine the appropriate Gamma distribution. Using this Gamma distribution, we can find $p_{in,a}$ and p_{in}.

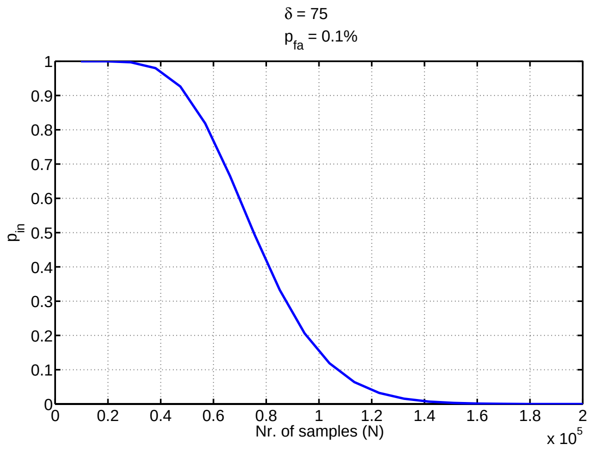

Fig. 8. The probability of interference as a function of the number of samples taken in the scan period. Taking more samples results in a better detection. Taking more samples results in missed spatial opportunity (similar to increasing d_s) and a lower throughput (see Fig. 7).

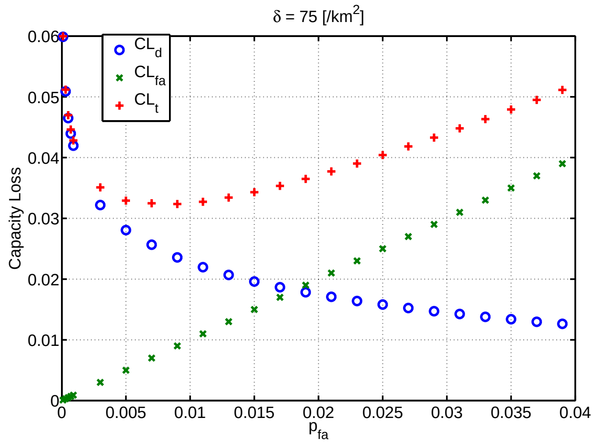

Fig. 9. A fundamental trade-off exists between capacity loss due to false alarms and the capacity loss due to the sensing is present in the typical MAC protocols for CRs. When using adaptive selection of p_{fa}, further gains can be made.

We plot the resulting probability of interference as a function of the detection overhead in Fig. 8. Clearly, when the sampling length N increases, p_{in} decreases. We also note that even for high densities the sensing overhead to achieve a near-zero probability of interference remains acceptable (150kSamples at 40MSamples/s is only 4ms sampling overhead).

As shown in Fig. 8, this curve has been calculated for a p_{fa} of 0.1%. The question now remains if, from a capacity point of view, this preset of p_{fa} was optimal, since the detection process has an inherent trade-off between the number of samples needed versus the probability of false alarm [22]. If we consider the MAC protocol shown in Fig. 7, we can compute the total capacity loss, CL_t as:

$$CL_t = 1 - (1 - CL_d)(1 - CL_{fa}), \tag{35}$$

where CL_d is the overhead of the detection process and CL_{fa} is the capacity loss due to false alarms. The detection process causes a throughput decrease, as all nodes need to silence the channel simultaneously. Hence, CL_d is defined as the ratio between the scan time and the beacon interval (see Fig. 7). False alarms cause a throughput decrease, because the network closes down the channel unnecessarily. Thus, these losses can be calculated as:

$$CL_d = \frac{N}{f_s BI}, \tag{36}$$

$$CL_{fa} = p_{fa}, \tag{37}$$

where N is the number of samples needed, f_s is the sampling frequency and BI is the length of the beacon interval. In Fig. 9, we see that the optimal choice for p_{fa} is 0.9%, which results in a total capacity loss of around 3.2%, which is very small. Of course, as mentioned in the introduction, we don't consider additional margins due to fading or shadowing as done in [8]. The actual resulting overhead will hence be larger. The idea here is to show that with a smart selection of p_{fa} the total capacity loss can be reduced with 50% (as compared to the worst point in Fig. 9).

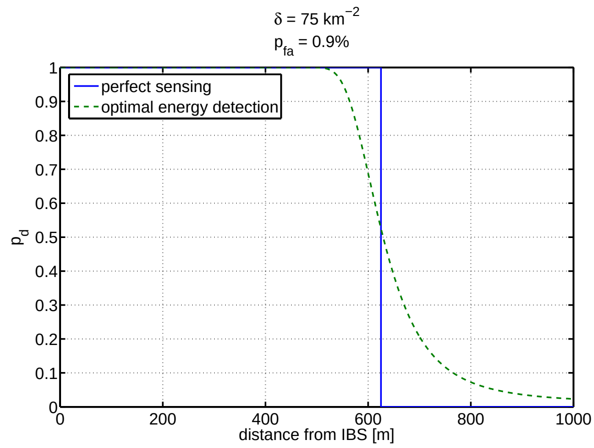

Fig. 10. When using a practical sensing implementation, CRs inside the theoretical silenced area can have a p_d that is less than 1. This is not necessarily causing a severe increase of the interference, because this is compensated by nodes outside the silenced area having a p_d larger than 0. The definition of p_d for imperfect scenarios includes false alarms through the equations presented in Table I

In Fig. 10, we compare the probability of detection for a perfect sensing implementation and a practical sensing implementation, both for bounds leading to a $p_{in} < 0.1\%$. We note that the optimal selection for a practical sensing technique allows the probability of detection to decrease even before the silencing area. However, the effect of these missed detections inside the silenced area is compensated with false alarms outside the silenced area.

E. Not all the CRs transmit at the same time

In reality not all CRs transmit at the same time. Hence, taking the maximum power they could generate as the design input is too conservative. Non-simultaneous transmissions can be approximated by another thinning of the Poisson process. If we assume that nodes are unsynchronized, the probability at a given time that a node transmits is merely its duty cycle:

$$dc = \frac{T_{active}}{T_{active} + T_{inactive}}. \quad (38)$$

Such channel access is achieved when the CRs implement a (slotted) Aloha distributed channel access. Introducing this thinning, under the assumption that all the cognitive radios have the same duty cycle, can easily be done by multiplying δ with dc in all the previous equations. As a result, the probability of direct and accumulative interference decreases.

IV. INSTANTIATION: 802.11

The 802.11 standard is the predominant wireless technology that is being used for data transfer, because of its ease of deployment and relatively high data rates. Many new techniques are being proposed to improve the base 802.11 standard, and standardization on 802.11 and its subgroups is hence very active even today. More importantly, in the context of CR, test devices presented to the FCC by key players were based on this 802.11 channel access [10]. It is hence important to consider the impact of 802.11 channel access on the possibility of interference.

When implementing 802.11 Distributed Coordination Function (DCF) it cannot be assumed that all the CRs are transmitting in an unsynchronized fashion. For 802.11, synchronization of stations is caused by the carrier sense where all the nodes pause their transmission attempts if they sense the medium busy. This carrier sensing is mostly implemented through energy detection with a certain threshold. To capture this effect, we can say that following a transmission all the nodes in the neighborhood are silenced.

We will use the model from Bianchi and assume that the cells or hot spots are not interfering with each other [23]. For simplicity reasons, we fix the number of CRs per hot spots to n and only look at a perfect sensing scenario.

Bianchi transforms the 802.11 with Binary Exponential Backoff to a p-persistent MAC, with an attempt probability p_a that depends on the conditional collision probability, p_c. As a result, the attempt probability p_a is given after solving this system of non-linear equations:

$$p_a = \frac{2(1-2p_c)(1-p_c)}{(1-2p_c)(W+1)+p_cW(1-(2p_c)^m)}, \quad (39)$$

$$p_c = (1-p_a)^n. \quad (40)$$

In reality, the conditional collision probability will be higher, due to hidden collisions and interference between hot spots. However, since those effects were not considered in the original model proposed by Bianchi [23], we also neglect them here.

As we need to differentiate collisions and successful transmissions in a cell, we split the duty cycle for an 802.11 cell with bidirectional transmissions in two parts. The duty cycle for successful transmissions, dc_s and the duty cycle for collisions, dc_c are respectively:

$$dc_s = \frac{p_{tr}(1-p_c)T^\star_{\mathrm{succ}}}{T_v}, \quad (41)$$

$$dc_c = \frac{p_{tr}p_cT^\star_{\mathrm{coll}}}{T_v}. \quad (42)$$

When a transmission in the network takes place (p_{tr}) the network will transmit during $T^\star_{\mathrm{succ}}$ if the transmission was successful $(1-p_c)$ or during $T^\star_{\mathrm{coll}}$ if the transmission was not successful (p_c). In (41) and (42), the times $T^\star_{\mathrm{succ}}$ and $T^\star_{\mathrm{coll}}$ are normalized with the virtual slot time, T_v, that is calculated as the average slot length:

$$T_v = (1-p_{tr})T_{\mathrm{slot}} + p_{tr}(1-p_c)T_{\mathrm{succ}} + p_{tr}p_cT_{\mathrm{coll}}, \quad (43)$$

where T_{slot} is the duration of one backoff slot, T_{succ} the duration of one successful slot and T_{coll} the duration of one collision slot[3]. The probability that a transmission occurs in a network with n terminals is given by:

$$p_{tr} = 1-(1-p_a)^n. \quad (44)$$

This transmission will only be successful if no other terminal in the network is transmitting simultaneously:

$$1-p_c = \frac{np_a(1-p_a)^{n-1}}{p_{tr}}. \quad (45)$$

Finally, we present T_{succ}, $T^\star_{\mathrm{succ}}$, T_{coll}, $T^\star_{\mathrm{coll}}$ for the IEEE 802.11 MAC protocol using DCF without RTS/CTS handshake:

$$T_{\mathrm{succ}} = T_H + T_{DATA} + SIFS + ACK + DIFS, \quad (46)$$

$$T^\star_{\mathrm{succ}} = T_H + T_{DATA} + ACK, \quad (47)$$

$$T_{\mathrm{coll}} = T_H + T_{DATA} + DIFS, \quad (48)$$

$$T^\star_{\mathrm{coll}} = T_H + T_{DATA}, \quad (49)$$

where T_H is the transmission time of the header, T_{DATA} the transmission time of the data. $SIFS$ is the Short Interframe Space and $DIFS$ is the DCF Interframe Space.

As in [12], we will only consider collisions between two transmitting CRs, since the amount of higher-order collisions (more than 2 stations transmit at the same time) is negligible and will only complicate our analysis.

We first look at the probabilities of direct interference for successful transmissions. These probabilities do not change as compared to the perfect sensing model

$$p^{(1)}_{in,d} = 1 - e^{-\delta dc_s(|A_i \setminus A_s|)}. \quad (50)$$

For collisions the interference area grows, as the total power being sent from the cell now doubles. Let's define $A_i^{(2)}$ as the interference area around the receiver, defined by the interference range of $d_{in}^{(2)}$ for collisions. Using (5), this distance can be found as:

$$d_{in}^{(2)} = 2^{\frac{1}{\alpha}} d_{in} \quad (51)$$

[3]Note that $T_{\mathrm{succ}} \neq T^\star_{\mathrm{succ}}$ and $T_{\mathrm{coll}} \neq T^\star_{\mathrm{coll}}$ as T_{succ} and T_{succ} contain idle periods. See (46-49)

The probability of direct interference from a collision is then

$$p_{in,d}^{(2)} = 1 - e^{-\delta d c_c (|A_i^{(2)} \setminus A_s|)}. \quad (52)$$

Now we need to find the probability for accumulated interference in between hot spots. Again, we approximate this as a Gamma distribution. We find the mean, μ, and the variance, σ^2, through the combination of two parts: one for successful transmissions ($\mu^{(1)}$, $(\sigma^{(1)})^2$) and for collisions ($\mu^{(2)}$, $(\sigma^{(2)})^2$). The same principle as for the direct interference applies, resulting in a double distance for collisions. We first determine the ranges for the integrals

$$\begin{aligned} r_{\min}^{(1)} &= \max(|d_s - d_p|, d_{in}), & (53)\\ r_{\max}^{(1)} &= \max(d_s + d_p, d_{in}), & (54)\\ r_{\min}^{(2)} &= \max(|d_s - d_p|, d_{in}^{(2)}), & (55)\\ r_{\max}^{(2)} &= \max(d_s + d_p, d_{in}^{(2)}). & (56) \end{aligned}$$

These are the resulting means

$$\begin{aligned} \mu^{(1)} &= 2d_{in}^{\alpha} \int_{r_{\min}^{(1)}}^{r_{\max}^{(1)}} \theta(r) r^{1-\alpha} dr \\ &\quad + \frac{2\pi\delta d_{in}^{\alpha}}{(\alpha-2)\left(r_{\max}^{(1)}\right)^{\alpha-2}}, & (57)\\ \mu^{(2)} &= 4d_{in}^{\alpha} \int_{r_{\min}^{(2)}}^{r_{\max}^{(2)}} \theta(r) r^{1-\alpha} dr \\ &\quad + \frac{4\pi\delta d_{in}^{\alpha}}{(\alpha-2)\left(r_{\max}^{(2)}\right)^{\alpha-2}} & (58)\\ \mu &= \mu^{(1)} + \mu^{(2)}. & (59) \end{aligned}$$

And the variances

$$\begin{aligned} \left(\sigma^{(1)}\right)^2 &= 2d_{in}^{2\alpha} \int_{r_{\min}^{(1)}}^{r_{\max}^{(1)}} \theta(r) r^{1-2\alpha} dr \\ &\quad + \frac{\pi\delta d_{in}^{2\alpha}}{(\alpha-1)\left(r_{\max}^{(1)}\right)^{\alpha-2}}, & (60)\\ \left(\sigma^{(2)}\right)^2 &= 8d_{in}^{2\alpha} \int_{r_{\min}^{(2)}}^{r_{\max}^{(2)}} \theta(r) r^{1-2\alpha} dr \\ &\quad + \frac{4\pi\delta d_{in}^{2\alpha}}{(\alpha-1)\left(r_{\max}^{(2)}\right)^{\alpha-2}}, & (61)\\ \sigma^2 &= \left(\sigma^{(1)}\right)^2 + \left(\sigma^{(2)}\right)^2. & (62) \end{aligned}$$

With μ and σ^2, we can again find the appropriate values k and θ for the Gamma distribution. With these parameters, we can find the probability of accumulative interference as in (14). We can then determine the total probability of interference for 802.11 enabled CR

$$\begin{aligned} p_{in} &= p_{in,d}^{(1)} + (1 - p_{in,d}^{(1)}) p_{in,d}^{(2)} \\ &\quad + (1 - p_{in,d}^{(1)})(1 - p_{in,d}^{(2)}) p_{in,a}. & (63) \end{aligned}$$

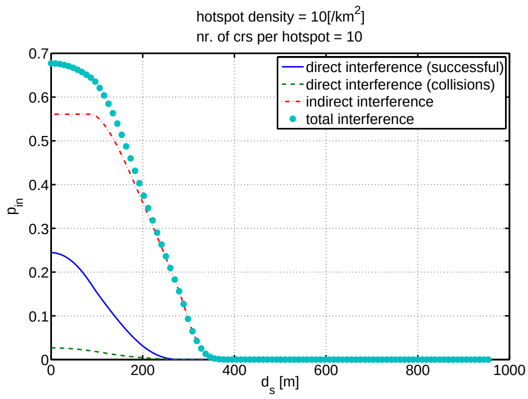

Fig. 11. The interference probability for CR networks using the IEEE 802.11 MAC protocol. The interference probability broken up in all its subparts. In this scenario, the accumulative component completely determines the required silenced distance.

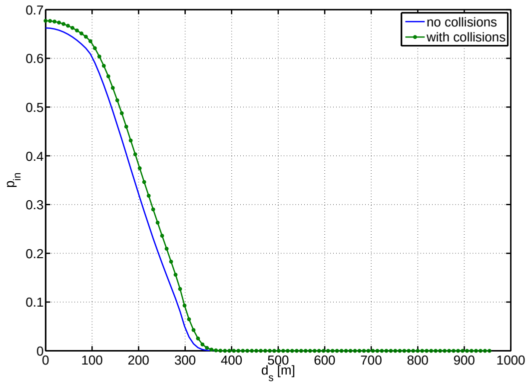

Fig. 12. The interference probability with and without collisions. We see that the collisions have only a slight impact on the interference probability. Hence, we can conclude that the current 802.11 technology is a viable candidate as a driver for new CR networks.

In Fig. 12 we can see that the direct interference from collisions is not so high as compared to the successful direct interference. If we look at Fig. 12, we see that an increase of cognitive radios per hot spot indicates that the area that needs to be silenced in order to avoid interference to an incumbent system grows, since collisions can not be avoided in the 802.11 protocol. However, only a slightly larger silence distance (around 6%) is needed to cope with the increased interference.

V. Conclusions

In this paper, a model for accumulative interference from a CR network was presented. With this model we show that an incumbent system can be protected by silencing the CRs in an appropriate area surrounding the incumbent base station. We presented a more accurate model as the one presented in [7] and showed that the approximation made there results in a too optimistic prediction for the interference.

We also evaluated the interference for practical sensing techniques, giving a lower bound on the samples needed for the detection process, in order to effectively protect the incumbent system. We also noted that a trade-off exists between the capacity loss due to false alarms and that of the detection process. With the model we could find an optimal value for the probability of false alarm, so as to minimize capacity loss under the constraint that the incumbent system remains protected. This optimal value will be different for each scenario.

As a last step, we evaluated the impact of the 802.11 MAC protocol on the interference coming from the CR network. It was shown that through carrier sense, interference power from the CR network is reduced. Although collisions cannot be avoided, carrier sense was shown to be an efficient technique to protect incumbent systems from accumulative interference, increasing the sensing bound only with 6% as compared to a perfect TDMA system.

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