Performance of Spatial Modulation in Correlated and Uncorrelated Nakagami Fading Channel

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Abstracr— **Spatial Modulation (SM) has been proposed to avoid interchannel interference and the need of exact time synchronization amongst antennas of the Multiple Input Multiple Output (MIMO) transmission system. In this paper, we study the performance of SM in a Nakagami fading environment. Exact integral expressions for calculating the symbol error rate of multilevel Quadrature Amplitude Modulation (M-QAM) of SM in correlated and uncorrelated Nakagami fading channels are derived. The analytical and simulation results match closely over a wide range of SNR values.**

Index Terms — **MQAM, Nakagami channel, Spatial modulation, MIMO**

I. INTRODUCTION

Multiple Input Multiple Output (MIMO) transmission systems have been proposed to significantly increase the spectral efficiency of future wireless communications. A spectral efficiency of 20-40 bps/Hz can be achieved in the Vertical Bell Laboratory Layered Space Time (VBLAST) architecture when considering an indoor rich scattering propagation condition [1]. However, simultaneous transmission on the same frequency from multiple transmitting antennas causes high interchannel interference (ICI). This significantly increases system complexity as the number of transmitting antennas increases [2].

SM avoids ICI and the need of accurate time synchronization amongst antennas by making only one antenna active at any instant of time and employing the antenna index as additional source of information [3]. The use of transmit antenna number to convey information increases the spectral efficiency by a factor equals to $\log_2$ (the number of transmit antennas) [4].

In SM any group of information bits is mapped into two constellations; signal constellation based on the type of modulation and space constellation to encode the transmit antenna number [3, 4]. At the receiver, maximum ratio combing is used. The detection process consists of two steps. The first one is the transmit antenna estimation while the second one is the transmit symbol estimation.

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A closed form expression to calculate symbol error rate (SER) of SM in i.i.d (identical and independent distribution) Rayliegh fading channels is presented in [3]. In [5] they proposed an optimal detector for the SM. Also they prove that their optimal detector performs better than the one proposed by [4]. However, Nakagami distribution fits empirical results more generally than other distributions, such as Rayleigh or Rice [6]. In this paper we study the performance of SM in Nakagami fading channels. Exact integral expressions for calculating the symbol error rate of square (M-QAM) of SM in correlated and uncorrelated Nakagami fading channels are derived. Analytical and simulation results are in close agreement over a wide range of signal-to-noise ratio (SNR).

II. SYSTEM MODEL

Fig. 1 shows a spatial modulation system model [4]. It has N_t transmit and N_r receive antennas. We use the following notations: bold and capital letters denote matrices, bold and small letters denote vectors, $(.)^H$ and $(.)^T$ denote Hermitian and transpose of a vector or matrix, respectively. The modulator groups the input binary data sequence (a) into symbols of n bits, where $n = \log_2(MN_t)$. Then it maps the resultant symbols into a vector: $\mathbf{x} = \left[\mathbf{x_1}\ \mathbf{x_2}\ ....\mathbf{x}_{N_t}\right]$, where it is assumed that $E_X\left[X^H X\right] = 1$; i.e. unity channel gain. Since one antenna is active, only one of x_j is nonzero in the vector $\mathbf{x}$. For the j^{th} active transmit antenna and the q^{th} symbol from M-ary constellation, the output of the SM mapper can be written as: $\mathbf{x}_{jq} = \left[0\,0.....x_q\ 0\,0....0\right]^T$ [5].

The SM mapping of 3bps/Hz transmission with 4 transmit antennas, considering binary phase shift keying (BPSK), is shown in Table 1 [4]. It can be noticed that the information bits are mapped to ± 1 BPSK and transmitted on one of the four transmit antennas.

The signal is transmitted over a MIMO channel $\mathbf{H} = \left[\mathbf{h_1}\ \mathbf{h_2}\ \cdots\ \mathbf{h}_{N_t}\right]$ and the corresponding channel vector from the j^{th} transmit antenna to all receive antennas is $\mathbf{h}_j = \left[h_{1,j}\ h_{2,j}\ \cdots\ h_{N_r,j}\right]^T$. Each channel in the system is modeled as frequency nonselective slowly Nakagami-m fading channel.

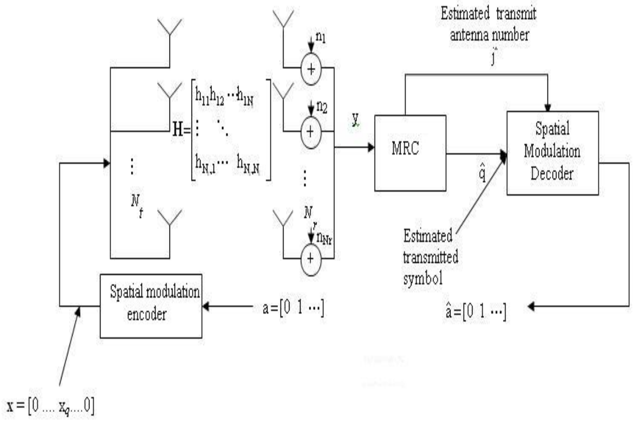

Figure 1. Spatial modulation system model.

Table 1. SM mapping table- 3pbs/Hz with N_t = 4 and BPSK

Input bits	Antenna #	Transmit symbol
000	1	-1
001	1	+1
010	2	-1
011	2	+1
100	3	-1
101	3	+1
110	4	-1
111	4	+1

The received signal $\mathbf{y}=\mathbf{Hx}+\mathbf{n}$, where n is N_r dimension additive white Gaussian (AWGN) noise $\left(\mathbf{n}=\left[n_1\ n_2 .. n_{N_r}\right]^T\right)$. The detection of information bits can be achieved by first estimate the antenna number then estimate the transmitted symbol according to the following rule [4, 5]:

$\hat{j}=\arg_j \max\left|\mathbf{h}_j^H \mathbf{y}\right|$ and $\hat{q}=\arg_q \max \mathrm{Re}\left\{\left(\mathbf{h}_{\hat{j}x_q}\right)^H \mathbf{y}\right\}$ where $\hat{j}$ and $\hat{q}$ are the estimated antenna number and transmitted symbol, respectively. Assuming correct estimate of the antenna number and the transmitted symbol, the original information bits can be retrieved.

III. Performance Analysis

From the previous discussion, it is clear that there are two estimation processes to detect the information symbol. The first is the antenna number estimation and the second is transmitted symbol. Following the same approach in [4] and assuming that the two estimations are independent, the symbol error rate $(\mathrm{P_{sm}}(\mathrm{e}))$ of SM can be given as:

$$P_{sm}(e)=P_a(e)+P_s(e)-P_a(e)P_s(e) \qquad (1)$$

Where $P_a(e)$ is the probability of incorrect estimation of the antenna number and $P_s(e)$ is the probability of incorrect estimation of the transmitted symbol. It is worth noting that if the channel paths are correlated then the two estimation processes will be dependent, thus, the derived equation represents an upper bound for SER. In the following the SER of the antenna number and transmit symbol estimation is considered separately.

A. SER of Transmitted Symbol

In SM since one transmit antenna is active at any instant of time, the estimation of the transmitted symbol for M-QAM modulation is a 1xN_r branch maximum ratio combining detection (MRC). The average SER $(P_s(e))$ of M-QAM with MRC in a slow and nonselective Nakagami fading channel can be found by averaging the error rates for the AWGN channel over the probability density function (pdf) of the SNR in Nakagami fading [8]:

$$P_S(e)=\int_0^{\infty} P_S(e/\gamma)p(\gamma)d\gamma \qquad (2)$$

Where $\gamma=\sum_{k=1}^{N_r}\gamma_k$, and γ_k is the instantaneous SNR in the kth channel with N_r fold MRC diversity. The probability density function of γ at the output of MRC for equal correlation between the branches is given by [9]:

$$p_\gamma(\gamma)=\frac{\left(\frac{\gamma m}{\bar{\gamma}}\right)^{N_r m-1}\exp\left(-\frac{\gamma m}{(1-\rho)\bar{\gamma}}\right){}_1F_1\left(m,N_r m;\frac{N_r m\rho\gamma}{(1-\rho)(1-\rho+N_r\rho)\bar{\gamma}}\right)}{\left(\frac{\bar{\gamma}}{m}\right)(1-\rho)^{m(N_r-1)}(1-\rho+N_r\rho)^m\,\Gamma(N_r m)},\quad \gamma>0 \qquad (3)$$

Where $\bar{\gamma}$ is the average signal power and ${}_1F_1()$ is the confluent hypergeometric function. When $\rho=0$, equation (3) can be written as:

$$P_\gamma(\gamma)=\left(\frac{m}{\bar{\gamma}}\right)^{N_r m}\frac{\gamma^{N_r m-1}}{\Gamma(N_r m)}\exp\left(-\frac{m}{\bar{\gamma}}\gamma\right),\quad \gamma>0\,.$$

The exact SER, for M-QAM in AWGN channel is given by [10, (8.12) p. 226]:

$$P_s(e/\gamma)=\frac{4B}{\pi}\int_0^{\pi/2}\exp\left(-\frac{g\gamma}{\sin^2\theta}\right)d\theta-\frac{4B^2}{\pi}\int_0^{\pi/4}\exp\left(-\frac{g\gamma}{\sin^2\theta}\right)d\theta \qquad (4)$$

Where $g=3/[2(M-1)]$, $B=1-1/\sqrt{M}$ and M is the size for QAM constellation. Substitute equation (3) and equation.(4) in equation (2) yields:

$$P_s(e)=E1-E2 \qquad (5)$$

Where E1 and E2 are given in the next page. Let $q=\frac{g}{\sin^2\theta}$ and by using the identities [11, 256(5)] and [9: (A-5)], we have equation (6), as shown at the bottom of the page.

Also by substituting equation .(6) in equation (5) and do more mathematical manipulation and simplification, the symbol error rate of M-QAM modulation in correlated Nakagami fading channel with MRC is given as in equation (7):

$$P_s(e)=\frac{4B}{\pi}\int_0^{\pi/2}\left(\frac{m^{N_r}}{\left(\bar{\gamma}\frac{g}{\sin^2\theta}(1-\rho+N_r\rho)+m\right)\left(\bar{\gamma}\frac{g}{\sin^2\theta}(1-\rho)+m\right)^{N_r-1}}\right)^m - \frac{4B^2}{\pi}\int_0^{\pi/4}\left(\frac{m^{N_r}}{\left(\bar{\gamma}\frac{g}{\sin^2\theta}(1-\rho+N_r\rho)+m\right)\left(\bar{\gamma}\frac{g}{\sin^2\theta}(1-\rho)+m\right)^{N_r-1}}\right)^m d\theta \quad (7)$$

If $\rho = 0$, then the SER for uncorrelated Nakagami fading channel with MRC is given as in equation (8):

$$P_e(s)=\left(\frac{4B}{\pi}\int_0^{\pi/2}\left(\frac{m}{[\bar{\gamma}\frac{g}{\sin^2\theta}+m]}\right)^{N_r m} - \frac{4B^2}{\pi}\int_0^{\pi/4}\left(\frac{m}{[\bar{\gamma}\frac{g}{\sin^2\theta}+m]}\right)^{N_r m} d\theta\right) \quad (8)$$

B. SER of transmit antenna number estimation

The probability of incorrect estimation of the antenna number $(P_e(a))$ is given by [7]:

$$P_a(e) = 2P_r - P_r^2 \quad (9)$$

Where P_r is the average overall probability of incorrect estimation of antenna number considering the real parts of the MQAM constellation, it is assumed that the imaginary part is identical to the real part [7]. Define $P(s_i)$ as the probability of incorrect estimation of antenna number when transmitting s_i, P_r can be written as [7]:

$$P_r = \frac{1}{k}\sum_{i=1}^{k} P(s_i) \quad (10)$$

Where $P(s_i)=\frac{1}{N_t-1}\left(\sum_{i=1}^{N_t-1}\int_0^{x_i} f_{x_{N_t}}(x/s_i,\sigma_n^2)dx\right), k=2^{m/2-1}$

and $f_x(x\,|\,s_i,\sigma_n^2)=\frac{1}{\sigma_n\sqrt{2\pi}}\left[e^{-\frac{(x-s_i)^2}{2\sigma_n^2}} - e^{-\frac{(x+s_i)^2}{2\sigma_n^2}}\right].$

$$E1=\frac{4B}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\infty}\frac{\exp\left(-\frac{g\gamma_s}{\sin^2\theta}\right)(\gamma)^{N_r m-1}\exp\left(-\frac{\gamma m}{(1-\rho)\bar{\gamma}}\right){}_1F_1\left(m,N_r m;\frac{N_r m\rho\gamma}{(1-\rho)(1-\rho+N_r\rho)\bar{\gamma}}\right)}{\left(\frac{\bar{\gamma}}{m}\right)^{N_r m}(1-\rho)^{m(N_r-1)}(1-\rho+N_r\rho)^m\,\Gamma(N_r m)}d\gamma d\theta$$

$$E2=\frac{4B^2}{\pi}\int_0^{\frac{\pi}{4}}\int_0^{\infty}\frac{\exp\left(-\frac{g\gamma_s}{\sin^2\theta}\right)(\gamma)^{N_r m-1}\exp\left(-\frac{\gamma m}{(1-\rho)\bar{\gamma}}\right){}_1F_1\left(m,N_r m;\frac{N_r m\rho\gamma}{(1-\rho)(1-\rho+N_r\rho)\bar{\gamma}}\right)}{\left(\frac{\bar{\gamma}}{m}\right)^{N_r m}(1-\rho)^{m(N_r-1)}(1-\rho+N_r\rho)^m\,\Gamma(N_r m)}d\gamma\, d\theta$$

$$\int_0^{\infty}\frac{\exp\left(-\frac{g\gamma_s}{\sin^2\theta}\right)\left(\frac{\gamma m}{\bar{\gamma}}\right)^{N_r m-1}\exp\left(-\frac{\gamma m}{(1-\rho)\bar{\gamma}}\right){}_1F_1\left(m,N_r m;\frac{N_r m\rho\gamma}{(1-\rho)(1-\rho+N_r\rho)\bar{\gamma}}\right)}{\left(\frac{\bar{\gamma}}{m}\right)(1-\rho)^{m(N_r-1)}(1-\rho+N_r\rho)^m\,\Gamma(N_r m)}d\theta \quad (6)$$

$$=\frac{\left(1-\frac{N_r m\rho}{\bar{\gamma}(1-\rho)(1-\rho+N_r\rho)\left(q+\frac{m}{\gamma(1-\rho)}\right)}\right)^{-m}}{\left[\frac{\bar{\gamma}q}{m}+\frac{1}{1-\rho}\right]^{N_r m}(1-\rho)^{m(N_r-1)}(1-\rho+N_r\rho)^m}$$

It is clear that knowing $P(s_i)$, P_r can be computed using equation (10), then it is used to calculate $P_a(e)$ as in equation (9). Both $P_s(e)$ as in equation (8) and $P_a(e)$ as in equation (9) are used to calculate the SER of SM in uncorrelated Nakagami fading channel. Finally, the upper bound of the SER of SM in correlated Nakagami fading channel is computed using equation (7) and equation (9).

IV. RESULTS AND DISCUSSION

This section presents simulation and analytical SER performance of SM over correlated and uncorrelated Nakagami fading channels. Fig. 2 shows simulated and analytical SER performance for SM 16QAM and 64QAM 4×4 on uncorrelated Nakagami fading channels $(\rho = 0)$ with fading parameter $m = 1$. This fading parameter corresponds to a Rayleigh fading channel. It is clear that simulation and analytical results match closely. These results are identical with that shown in Fig. 2 in [3].

Fig. 3 shows the effect of the fading parameter m on the SER performance of SM. It depicts the SER performance of SM 16QAM 4×4 over uncorrelated Nakagami fading channel with different values of the fading parameter: $m = 0.75, 1, 2$ and 5. It can be noticed that as m decreases, i.e., the fading severity increases, the SER deteriorates. Also, the performance is significantly improves when there is a strong line of sight path, i.e., $m = 5$, corresponds to Rician fading with Rice factor $K \approx 10\,dB$.

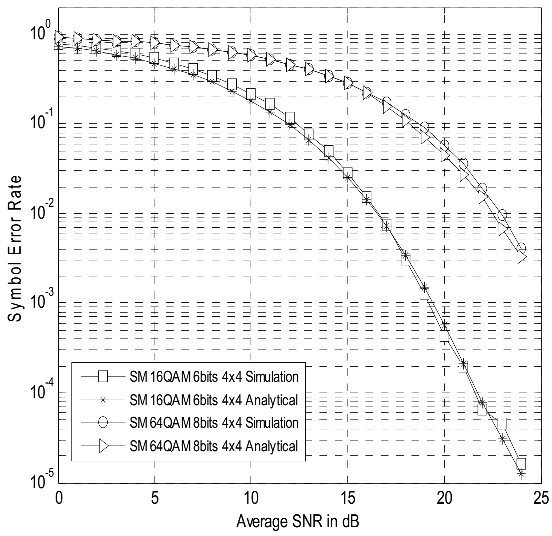

Figure 2. Analytical and simulated SER for SM 16QAM and 64QAM with N_t=4 and N_r=4 in uncorrelated $(\rho = 0)$ Nakagami fading channels with fading parameter m = 1

In Fig. 4 and Fig. 5, the diversity gain of SM considering uncorrelated Nakagami fading channels when m= 1and 2 is shown, respectively. Comparing between the two the Figures indicates that the diversity gain is more noticeable when m=1, i.e., when fading severity increases. This is because the difference between instantaneous received SNR on various diversity branches will be less when the fading parameter (m) increases.

The analytical SER performance for SM 16QAM 4×4 with equal correlation between the branches for $(\rho = 0, 0.2, 0.4$ and $1)$ in Nakagami fading channels and considering fading parameter $m = 2$ is shown in Fig. 6. It can be observed that the performance degrades as the correlation coefficient increases, becomes significant when there is full correlation between the diversity branches, i.e. $\rho = 1$. Table 2 gives specific results for the diversity improvement over correlated and uncorrelated Nagakami fading for SM 16QAM when $P_{sm}(e)=10^{-3}$.

Table 2: Diverity gain in dB for 16QAM SM with N_r=4

ρ	m=1	m=2	m=4
0	14.9	7.9	5
0.5	14.2	7.45	4.7
0.9	11.4	5.4	3.6

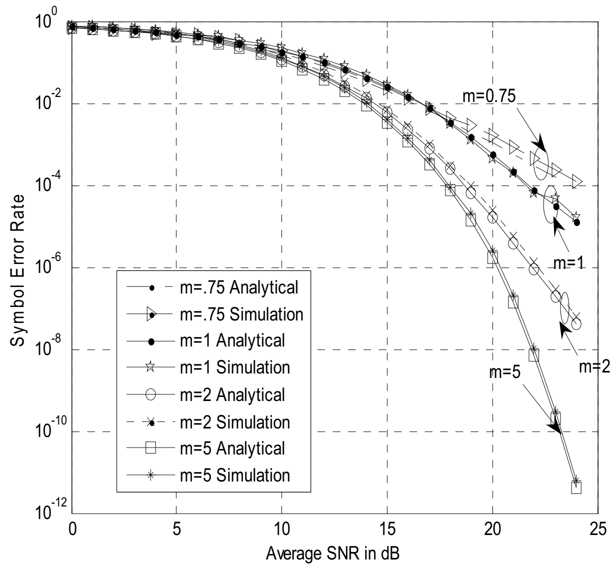

Figure 3. Analytical and simulated SER for SM 16QAM with N_t=4 and N_r=4 (6 bps/Hz)on uncorrelated $(\rho = 0)$ Nakagami fading channels with fading parameter m = 0.75, 1, 2 and 5.

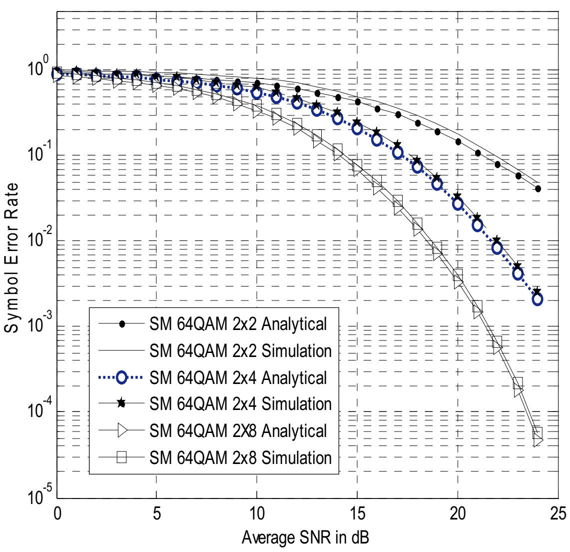

Figure 4. Analytical and simulated SER for SM 64QAM over uncorrelated ($\rho = 0$) Nagakami fading with m=1, Nt=2 and N_r=2, 4 and 8.

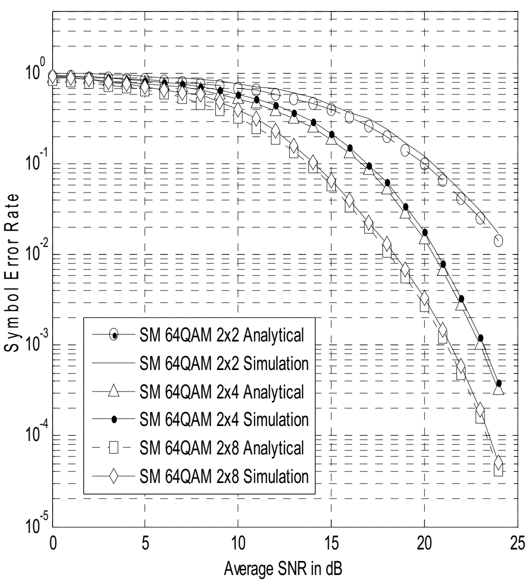

Figure 5. Analytical and simulated SER for SM 64QAM over uncorrelated ($\rho = 0$) Nagakami fading with m=2, Nt=2 and N_r=2, 4 and 8.

V. CONCLUSION

Exact analytical expressions to calculate the SER performance of SM over uncorrelated Nakagami fading environment are derived in this paper. The analytical and simulation results match closely. Also an upper bound for SER performance of SM in equal correlated Nagakami fading channel is derived. The analytical results presented in this paper will provide a convenient tool for design and analysis of SM systems in Nakagamim fading environment under MRC diversity.

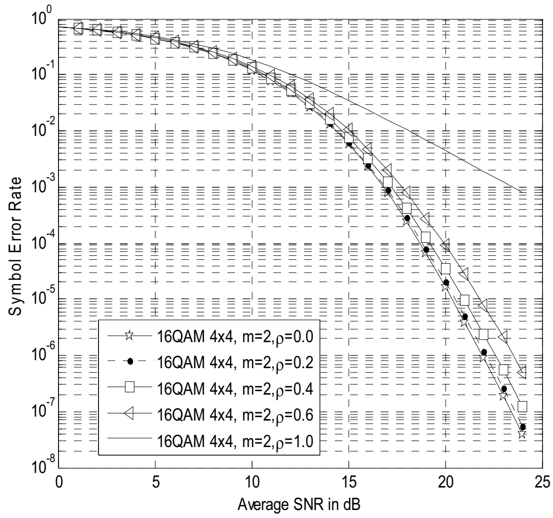

Figure 6. Analytical SER for SM 16QAM 4×4 over correlated Nakagami fading channels with fading parameter $m = 2$ and $(\rho = 0, 0.2, 0.4 \text{ and } 1)$.

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