Performance Analysis of SFBC and Data Conjugate in MIMO-OFDM System over Nakagami Fading Channel

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Abstract— **An integration of multiple input and multiple output (MIMO) and orthogonal frequency division multiplexing (OFDM) exploit spatial diversity for high data rate transmission of signals over wireless channel. In this paper, analytical models have been developed to suppress signal to noise plus interference ratio incorporating inter carrier interference and average probability of error with the concept of space frequency block code (SFBC) and data conjugate (DC) over Nakagami-m fading channel. Results show that average probability of error performance of MIMO-OFDM system in Nakagami fading channel decreases if fading parameter m is increased. The performance of SFBC has improved about 3 dB at average probability of error of 10^{-4} than MIMO-OFDM with DC system.**

Index Terms— **Average probability of error, Inter carrier interference, Multiple input multiple output, Nakagami-m fading, Orthogonal frequency division multiplexing.**

I. INTRODUCTION

New generation wireless communication systems require high data rate transmission and better quality of service for various types of fading channel. To achieve high data rate, the system has to overcome problems such as multipath fading and interference [1]. Multiple input multiple output (MIMO) system can mitigate multipath fading, increase the system capacity, improve the interference performance and transmission reliability [2].

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation technique and sensitive to normalized frequency offset and phase noise, which destroys the orthogonality among subcarriers and causes inter carrier interference (ICI). Nakagami-m fading covers a wide range of multipath fading channels by varying its fading parameter m. It has greater flexibility in matching some experimental data than that of the Rayleigh, Rician fading [3, 4].

In this paper, three different schemes (i.e., 2x2 MIMO-OFDM, 2x2 MIMO-OFDM with data conjugate, 2x2 MIMO-OFDM with space frequency block code) have been discussed. Detailed mathematical derivation is carried out to find the expression of the signal to noise plus interference ratio (SNIR) incorporating ICI and average probability of error. Performance results are evaluated numerically in terms of average probability error for different numbers of Nakagami-m fading channel and normalized frequency offset.

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II. MIMO-OFDM SYSTEM MODEL

Figure 1 illustrates a block diagram of 2x2 MIMO-OFDM system with the total number of N subcarriers. X_k^t is modulated data using suitable modulation technique like (BPSK, QPSK or M-QAM) for k[th] subcarrier. The modulated data are mapped after modulation in order for the first and second antenna to transmit and modulate the same OFDM symbols. The MIMO-OFDM signal after inverse fast fourier transform (IFFT) at the transmitter can be expressed as [5],

$$x^t(n) = \sum_{k=0}^{N-1} X_k^t e^{j(\frac{2\pi}{N})kn} \quad \text{for } 0 \le n \le N-1, \quad t = 1 \text{ or } 2 \tag{1}$$

Where, $j = \sqrt{-1}$, t means transmitter antenna. The received signal is affected by phase noise and frequency offset which can be expressed as [5],

$$r^\tau(n) = \{\sum_{t=1}^{2}[x^t(n) \otimes h^t(n) + w(n)]\} e^{j[2\pi\Delta f^\tau t + \varphi^\tau(n)]} \quad \tau = 1 \text{ or } 2 \tag{2}$$

In which, Δf^τ and $\varphi^\tau(n)$ are frequency offset and phase noise. τ means received antenna. x(n), h(n), w(n), r(n) are transmitted signal, channel impulse response, AWGN and received signal respectively. The received signal after fast fourier transform (FFT) can be expressed as [5],

$$Y_k^\tau = \frac{1}{N}\sum_{n=0}^{N-1} r^\tau(n) e^{-j[\frac{2\pi}{N}]kn}$$

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$$= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{t=1}^{2}\sum_{l=0}^{N-1} X_l^t H_l^t e^{j[(\frac{2\pi}{N})(l-k+\varepsilon^\tau)n+\varphi^\tau(n)]} + N_k$$

$$= \sum_{t=1}^{2}\sum_{l=0}^{N-1} X_l^t H_l^t Q_{l-k}^\tau + N_k \quad (3)$$

Where, Y_k, X_k and H_k are the frequency domain expression of r(n), x(n), h(n) . N_k is the complex additive white Gaussian noise (AWGN). Here, ε is the normalized frequency offset and is given by $\Delta f^\tau T$. T is the subcarrier symbol period.

Q_L^τ is defined as follows,

$$Q_L^\tau = \frac{1}{N}\sum_{n=0}^{N-1} e^{j[(\frac{2\pi}{N})(L+\varepsilon^\tau)n+\varphi^\tau(n)]}$$

$$= \exp[j\{2\pi(L+\varepsilon^\tau)+\varphi^\tau\}(1/2-1/2N)]\frac{\sin[\{2\pi(L+\varepsilon^\tau)+\varphi^\tau\}/2]}{N.\sin[\{2\pi(L+\varepsilon^\tau)+\varphi^\tau\}/2N]} \quad (4)$$

At the receiver side, demodulation results in the recovery of the original signal from the simple relation of $Z_k^{'} = Y_k^1 + Y_k^2$. Here, Y_k^1 and Y_k^2 are the first antenna and the second antenna k^{th} subcarrier data. Finally, the original data could be found through the detection process. For the simplicity of system performance analysis, $Q_L^{\tau,t} = Q_L^\tau$ and the cyclic prefix is not considered.

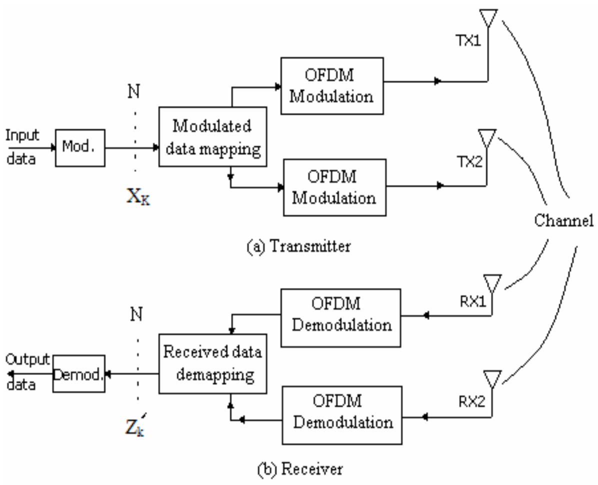

Figure 1: Block diagram of MIMO-OFDM system model
(a) Transmitter (b) Receiver

III. Theoretical Analysis

A. Original in MIMO-OFDM

In original 2x2 MIMO-OFDM, both antennas transmit the same signal as the form of $X_l^1 = X_l^2 = X_k$ the k^{th} subcarrier signal.

The received signal at the receiver 1 (RX1) can be expressed as [6],

$$Y_k^1 = \sum_{l=0}^{N-1} X_l^1 \cdot H_l^1 \cdot Q_{l-k}^1 + \sum_{l=0}^{N-1} X_l^2 \cdot H_l^2 \cdot Q_{l-k}^1 + N_{1k}$$

$$= X_k + X_k\{H_k^1.Q_0^1 + H_k^2.Q_0^1 - 1\}$$

$$+ \sum_{l=0,l\neq k}^{N-1}\{X_l^1.H_l^1.Q_{l-k}^1 + X_l^2.H_l^2.Q_{l-k}^1\} + N_{1k} \quad (5)$$

Similarly, the received signal at the receiver 2 (RX2) can be expressed as [6],

$$Y_k^2 = \sum_{l=0}^{N-1} X_l^1 \cdot H_l^1 \cdot Q_{l-k}^2 + \sum_{l=0}^{N-1} X_l^2 \cdot H_l^2 \cdot Q_{l-k}^2 + N_{2k}$$

$$= X_k + X_k\{H_k^1.Q_0^2 + H_k^2.Q_0^2 - 1\}$$

$$+ \sum_{l=0,l\neq k}^{N-1}\{X_l^1.H_l^1.Q_{l-k}^2 + X_l^2.H_l^2.Q_{l-k}^2\} + N_{2k} \quad (6)$$

In the receiver, the decision variable $Z_k^{'}$ of the final signal are achieved as follows [6],

$$Z_k^{'} = Y_k^1 + Y_k^2$$

$$= 2X_k + 2X_k\{(Q_0^1 + Q_0^2) - 1\}$$

$$+ \sum_{l=0,l\neq k}^{N-1}\{(X_l^1 + X_l^2).Q_{l-k}^1 + (X_l^1 + X_l^2).Q_{l-k}^2\} + N_k \quad (7)$$

The desired received signal (DRS) is generated by the signal of k^{th} subcarrier. All subcarriers are rotated by the same angle simultaneously. Usually it affects all the subchannels equally. Consider *l*=*k*, the desired received signal power is expressed as [6],

$$\sigma_{DRS}^2 = |X|^2[\{\sin((2\pi\varepsilon^1 + \varphi^1)/2)\}^2(34.7738)$$

$$+ \{\sin((2\pi\varepsilon^2 + \varphi^2)/2)\}^2(34.7738)] \quad (8)$$

ICI is corrupted by adjacent subcarrier signal. Consider $l \neq k$, the ICI power is expressed as [6],

$$\sigma_{ICI}^2 = |X|^2[\{\sin((2\pi\varepsilon^1 + \varphi^1)/2)\}^2(0.6704)$$

$$+ \{\sin((2\pi\varepsilon^2 + \varphi^2)/2)\}^2(0.6704)] \quad (9)$$

The average probability of error for binary phase shift keying (BPSK) modulated in MIMO-OFDM system over Nakagami fading channel is given [6],

$$P_b = \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{SNR.[\{\sin((2\pi\varepsilon^1+\varphi^1)/2)\}^2(34.7738)}{m(\sin^2\varphi).[1+SNR.[\{\sin((2\pi\varepsilon^1+\varphi^1)/2)\}^2(0.6704)} \cdot \frac{+\{\sin((2\pi\varepsilon^2+\varphi^2)/2)\}^2(34.7738)]}{+\{\sin((2\pi\varepsilon^2+\varphi^2)/2)\}^2(0.6704)]}\right)^{-m} d\varphi \quad (10)$$

B. Data Conjugate (DC) in MIMO-OFDM

From the Figure 1, block diagram has been some modified. The mapping scheme for DC method in MIMO-OFDM is applied such that on the first antenna the original data has been transmitted and the second antenna the conjugate data has been transmitted. At the receiver side, the original signal can be recovered from the simple relation of $Z_k^{'} = Y_k^1 + Y_k^{2^*}$. So, the DC signals are remapped as the form of $X_l^1 = X_k, X_l^2 = X_k^*$

The received signal at RX1 can be expressed as,

$$Y_k^1 = \sum_{l=0}^{N-1} X_l^1 \cdot H_l^1 \cdot Q_{l-k}^1 + \sum_{l=0}^{N-1} X_l^2 \cdot H_l^2 \cdot Q_{l-k}^1 + N_{1k}$$

$$= X_k.H_k^1.Q_0^1 + \sum_{l=0,l\neq k}^{N-1} X_l^1.H_l^1.Q_{l-k}^1 + X_k^*.H_k^2.Q_0^1 + \sum_{l=0,l\neq k}^{N-1} X_l^2.H_l^2.Q_{l-k}^1 + N_{1k} \quad (11)$$

Similarly, the received signal at RX2 can be expressed as,

$$Y_k^2 = \sum_{l=0}^{N-1} X_l^1 \cdot H_l^1 \cdot Q_{l-k}^2 + \sum_{l=0}^{N-1} X_l^2 \cdot H_l^2 \cdot Q_{l-k}^2 + N_{2k}$$

$$= X_k.H_k^1.Q_0^2 + \sum_{l=0,l\neq k}^{N-1} X_l^1.H_l^1.Q_{l-k}^2 + X_k^*.H_k^2.Q_0^2 + \sum_{l=0,l\neq k}^{N-1} X_l^2.H_l^2.Q_{l-k}^2 + N_{2k} \quad (12)$$

In the receiver, the final decision variable $Z_k^{'}$ of the k^th symbol is found by,

$$\begin{aligned} Z_k^{'} &= Y_k^1 + Y_k^{2^*} \\ &= X_k.H_k^1.Q_0^1 + \sum_{l=0,l\neq k}^{N-1} X_l^1.H_l^1.Q_{l-k}^1 + X_k^*.H_k^2.Q_0^1 \\ &+ \sum_{l=0,l\neq k}^{N-1} X_l^2.H_l^2.Q_{l-k}^1 + X_k^*.(H_k^1)^*.Q_0^{1*} \\ &+ \sum_{l=0,l\neq k}^{N-1} (X_l^1)^*.(H_l^1)^*.Q_{l-k}^{2*} + X_k.(H_k^2)^*.Q_0^{2*} \\ &+ \sum_{l=0,l\neq k}^{N-1} (X_l^2)^*.(H_l^2)^*.Q_{l-k}^{2*} + N_k \\ &= X_k.(H_k^1.Q_0^1 + (H_k^2)^*.Q_0^{2*}) + X_k^*.(H_k^2.Q_0^1 + (H_k^1)^*.Q_0^{2*}) \\ &+ \sum_{l=0,l\neq k}^{N-1} \{X_l^1.H_l^1.Q_{l-k}^1 + (X_l^1)^*.(H_l^1)^*.Q_{l-k}^{2*}\} \\ &+ \sum_{l=0,l\neq k}^{N-1} \{X_l^2.H_l^2.Q_{l-k}^1 + (X_l^2)^*.(H_l^2)^*.Q_{l-k}^{2*}\} + N_k \end{aligned} \quad (13)$$

Now, considering Normalized frequency offset is not zero. When $\varepsilon_\tau = \Delta f^* T \neq 0$, equation (13) can be written as,

$$\begin{aligned} Z_k^{'} &= X_k + X_k.(H_k^1.Q_0^1 + (H_k^2)^*.Q_0^{2*} - 1) + X_k^*.(H_k^2.Q_0^1 + (H_k^1)^*.Q_0^{2*}) \\ &+ \sum_{l=0,l\neq k}^{N-1} X_l^1.H_l^1.Q_{l-k}^1 + (X_l^1)^*.(H_l^1)^*.Q_{l-k}^{2*} \\ &+ \sum_{l=0,l\neq k}^{N-1} X_l^2.H_l^2.Q_{l-k}^1 + (X_l^2)^*.(H_l^2)^*.Q_{l-k}^{2*} + N_k \\ &= X_k + X_k.(Q_0^1 + Q_0^{2*} - 1) + X_k^*.(Q_0^1 + Q_0^{2*}) \\ &+ \sum_{l=0,l\neq k}^{N-1} X_l^1.Q_{l-k}^1 + (X_l^1)^*.Q_{l-k}^{2*} \\ &+ \sum_{l=0,l\neq k}^{N-1} X_l^2.Q_{l-k}^1 + (X_l^2)^*.Q_{l-k}^{2*} + N_k \end{aligned} \quad (14)$$

In order to evaluate the statistical properties [7], assuming average channel gain

$$E\left[\left|H_l^1\right|^2\right] = E\left[\left|H_l^2\right|^2\right] = 1 \text{ and}$$

$$E\left[\left|X_l^1\right|^2\right] = E\left[\left|X_l^2\right|^2\right] = |X|^2 \quad (15)$$

The desired received signal power can be represented by,

$$\begin{aligned} \sigma^2_{DRS(DC)} &= E[\left|X_k\right|^2].E[\left|H_k^1\right|^2].\left|Q_0^1\right|^2 \\ &+ E[\left|X_k\right|^2].E[\left|H_k^{2*}\right|^2].\left|Q_0^{2*}\right|^2 \\ &= |X|^2.\left|Q_0^1\right|^2 + |X|^2.\left|Q_0^{2*}\right|^2 \end{aligned} \quad (16)$$

Hence, the ICI power is,

$$\begin{aligned} \sigma^2_{ICI(DC)} &= E\left[\left|I_{ICI}\right|^2\right] \\ &= \sum_{l=0,l\neq k}^{N-1} E[\left|X_l^1\right|^2].E[\left|H_l^1\right|^2].\left|Q_{l-k}^1\right|^2 \\ &+ \sum_{l=0,l\neq k}^{N-1} E[\left|X_l^{1^*}\right|^2].E[\left|H_l^{1^*}\right|^2].\left|Q_{l-k}^{2*}\right|^2 \\ &+ \sum_{l=0,l\neq k}^{N-1} E[\left|X_l^2\right|^2].E[\left|H_l^2\right|^2].\left|Q_{l-k}^1\right|^2 \\ &+ \sum_{l=0,l\neq k}^{N-1} E[\left|X_l^{2^*}\right|^2].E[\left|H_l^{2^*}\right|^2].\left|Q_{l-k}^{2*}\right|^2 \\ &= 2\sum_{l=1}^{N-1} |X|^2.\left|Q_l^1\right|^2 + |X|^2.\left|Q_l^{2*}\right|^2 \end{aligned} \quad (17)$$

The signal to noise ratio (SNR) can be calculated as [8],

$$SNR = \frac{|X|^2}{\sigma_n^2} \quad (18)$$

The signal to noise plus interference ratio (SNIR) of DC can be calculated as [8],

$$SNIR = \frac{\sigma^2_{DRS\ (DC)}}{\sigma_n^2 + \sigma^2_{ICI\ (DC)}} = \frac{\dfrac{\sigma^2_{DRS\ (DC)}}{\sigma_n^2}}{1 + \dfrac{\sigma^2_{ICI\ (DC)}}{\sigma_n^2}} = \frac{SNR.[|Q_0^1|^2 + |Q_0^{2*}|^2]}{1 + SNR.[2\sum_{l=1}^{N-1}|Q_l^1|^2 + 2\sum_{l=1}^{N-1}|Q_l^{2*}|^2]} \quad (19)$$

The average probability of error for BPSK modulated in Nakagami fading channel is given by [9],

$$P_b = \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{SNIR}{m(\sin^2\varphi)}\right)^{-m} d\varphi \quad (20)$$

The average probability of error for BPSK modulated SFBC in MIMO-OFDM system over Nakagami fading channel is given,

$$P_b = \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{SNR[|Q_0^1|^2 + |Q_0^{2*}|^2]}{m(\sin^2\varphi).[1 + SNR\{2\sum_{l=1}^{N-1}|Q_l^1|^2 + 2\sum_{l=1}^{N-1}|Q_l^{2*}|^2\}]}\right)^{-m} d\varphi \quad (21)$$

C. SFBC in MIMO-OFDM

From figure 1, the data symbols are mapped according to the Alamouti space frequency block code (SFBC) [10, 11] and signals are mapped as the form of $X_l^1 = X_k or - X_{k+1}^*, X_l^2 = X_{k+1} or X_k^*$

The average probability of error for BPSK modulated SFBC in MIMO-OFDM system over Nakagami fading channel is given [12],

$$P_b = \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{SNR[|Q_0^1|^2 + |Q_0^{2*}|^2]}{m(\sin^2\varphi).[1 + SNR[\{|Q_0^1|^2 - |Q_0^{2*}|^2\} + 2\sum_{l=1}^{N-1}|Q_l^1|^2 + 2\sum_{l=1}^{N-1}|Q_{l-1}^{2*}|^2]]}\right)^{-m} d\varphi \quad (22)$$

IV. Result and Discussion

To analyze the average probability of error performance of MIMO-OFDM system, simulation is performed according to the theoretical analysis for 2 transceivers configuration, 64 subcarriers and BPSK modulation. The period of IFFT (N=64), normalized frequency offset (ε = 0.05), phase noise (φ = 0.025), k = 0 in equation have been taken into consideration.

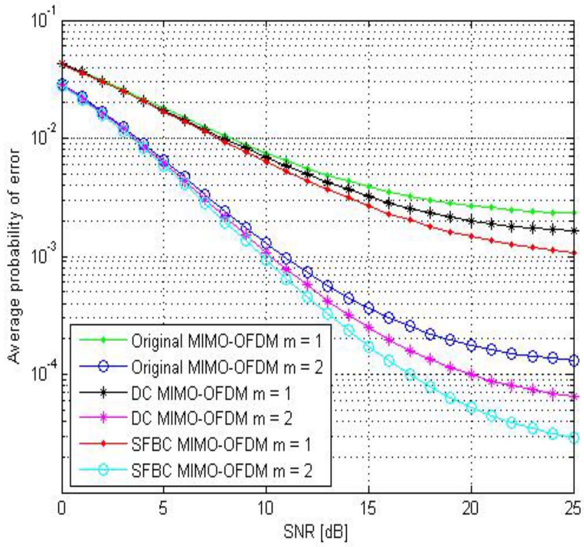

Figure 2: Effect of Nakagami fading parameter on average probability of error with different SNR

Figure 2 illustrates the comparison among three different techniques for various values of Nakagami fading parameter. From the Figure 2, we observe that as the value of Nakagami fading parameter increases, the average probability of error increases too. It is also noticed that SFBC has less average probability of error compared to other methods. For Nakagami fading parameter (m=2), the values of average probability of error for original, DC and SFBC in MIMO-OFDM are approximately 3.5×10^{-4}, 2.4×10^{-4} and 1.7×10^{-4} respectively at SNR = 15dB.

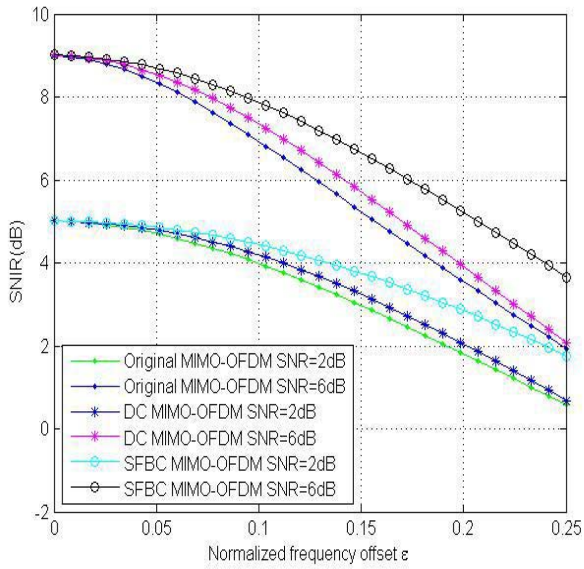

Figure 3: Effect of SNR on SNIR with different normalized frequency offset

The effects of SNR on SNIR with different normalized frequency offset are plotted in Figure 3. It is observed that SFBC has better performance than original and DC in MIMO-OFDM systems. SNIR decreases with increasing normalized frequency offset and also for a particular normalized frequency offset value of SNIR increases as the SNR increases. For instance, at $\varepsilon = 0.15$, SFBC, DC and original in MIMO-OFDM value of SNIR approximately are 6.52 dB, 5.54dB and 5.06 dB at fading parameter m=2, whereas original MIMO-OFDM has the least SNIR.

V. CONCLUSION

A detailed theoretical analysis is carried out to determine the detrimental effects of phase noise and frequency offset and its reduction scheme in MIMO-OFDM systems using DC and SFBC. It is observed that average probability of error has been reduced in Nakagami fading channel for larger values of m parameter. When SFBC is used in the proposed system, SNIR increase by about 1dB and 1.5dB in DC and original MIMO-OFDM system respectively for a fixed 6 dB SNR.

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