PAPR Distribution Analysis of OFDM Signals with Partial Transmit Sequence

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***Abstract*—Third Generation Partnership Project (3GPP) LTE has adopted OFDMA as the uplink multiple access scheme. One of the major drawbacks is the high PAPR. Not only is the performance of PAPR with PTS technique influenced by the number of subblocks and the phase vector but also by the subblock partitioning. The Partial Transmit Sequence (PTS) technique suffers from the search complexity of finding the optimum set of phase vectors. We propose a suboptimal combination algorithm that reduces the search complexity. The number of commutations in the suboptimal combination algorithm is much lower than the required by the original PTS technique. In this paper, we propose a suboptimal combination algorithm to reduce the searching complexity of finding the optimum set of vectors to minimize PAPR. The performance of PAPR utilizing the PTS technique improves by the use of the proposed suboptimal combination algorithm. We also show that a SC-FDMA system with Interleaved-FDMA or Localized-FDMA performs better than Orthogonal- FDMA in the uplink direction where transmitter power efficiency is of great importance.**

***Index Terms*— Partial Transmit Sequence (PTS); Long-term-evolution (LTE); Orthogonal frequency division multiplexing (OFDM); peak-to-average power ratio (PAPR).**

I. Introduction

Wireless communication has experienced an incredible growth in the last decade. Two decades ago, the number of mobile subscribers was less than 1% of the world's population and in 2001,the number of mobile subscribers was 16% of the world's population [1]. By the end of 2001 the number of countries worldwide having a mobile network has tremendously increased from just 3% to over 90% and the number of mobile subscribers worldwide exceeded the number of fixed-line subscribers in 2002 [2]. As of 2010, the number of mobile subscribers was around 73% of the world's population, which is around to 5 billion mobile subscribers. In addition to mobile phones, Wireless Local Area Network (WLAN) has experienced a rapid growth during the last decade. IEEE 802.11 a/b/g/n is a set of standards that specify the physical and data link layers in ad-hoc mode or access point for current wide use. In 1997 WLAN standard – IEEE 802.11, also known as Wi-Fi, was first developed with speeds of up to 2 Mbps [2]. At present, WLANs are capable of offering speeds up to 600 Mbps by the use of IEEE 802.11n utilizing orthogonal frequency division multiplexing (OFDM) as a modulation technique in the 2.4 GHz and 5 GHz license-free industrial, scientific and medical (ISM) bands. It is important to note that WLANs do not offer the type of mobility, which mobile systems offer.

In our previous work, we analyzed a low complexity clipping and filtering scheme to reduce both the Peak-to-Average-Power-Ratio (PAPR) and the out-of-band-radiation caused by the clipping distortion in downlink systems utilizing OFDM technique [3]. We also modeled a mix of low mobility 1.8mph, and high mobility, 75mph with a delay spread that is constantly slighter than the guard time of the OFDM symbol to predict complex channel gains by the user by means of reserved pilot subcarriers [4]. Single-Carrier-Frequency-Division-Multiple-Access (SC-FDMA) is the modified version of Orthogonal Frequency-Division-Multiple-Access (OFDMA). SC-FDMA is a customized form of OFDMA with comparable throughput performance and complexity. The only dissimilarity between OFDMA and SC-FDMA transmitter is the discrete Fourier transform (DFT) mapper. The transmitter collects the modulation symbols into a block of N symbols after mapping data bits into modulation symbols. DFT transforms these symbols in the time domain into the frequency domain. The frequency domain samples are then mapped to a subset of M subcarriers where M is greater than N. Like OFDM, an M point IFFT is used to generate the time-domain samples of these subcarriers.

OFDMA is a broadband multicarrier modulation scheme where SC-FDMA is a single carrier modulation scheme. Research on multi-carrier transmission started to be an interesting research area [5]-[7]. OFDM modulation scheme leads to better performance than a single carrier scheme over wireless channels. OFDM uses a large number of orthogonal, narrowband sub-carrier that are transmitted simultaneously in parallel ;however, high PAPR becomes an issue that limits the uplink performance more than the downlink due to the low power processing terminals. SC-FDMA adds additional

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advantage of low PAPR compared to OFDM making it appropriate for uplink transmission.

The maximum data rate that can be attained over a given channel is determined by the capacity and bit error rate of that channel. We investigated the channel capacity and bit error rate of Multiple-Input and Multiple-Output-OFDM (MIMO-OFDM) [8]. The use of OFDM scheme is the solution to the increase demand for future bandwidth-hungry wireless applications [9]. Some of the wireless technologies using OFDM are Long-Term Evolution (LTE) which is the standard for fourth generation (4G) cellular technology, Association of Radio Industries Business (ARIB) Multimedia Mobile Access Communication (MMAC) in Japan have adopted the OFDM transmission technology as a physical layer for future broadband WLAN systems, The European Telecommunications Standards Institute (ETSI) Broadband Radio Access Networks (BRAN) in Europe and WLANs. Due to the robustness of OFDM systems against multipath fading, the integration of OFDM technology and Radio-Over-Fiber (RoF) technology made it possible to transform the high-speed RF signal to the optical signal utilizing optical fibers with broad bandwidth [10]. Nevertheless, OFDM suffers from high peak to average power ratio (PAPR) in both the uplink and downlink which results in making the OFDM signal a complex signal [11].

The outcome of high PAPR on the transmitted OFDM symbols results in two disadvantages high bit error rate and inference between adjacent channels. This would imply the need for linear amplification. The consequence of linear amplification is more power consumption. This has limited the optimal use of OFDM as a modulation and demodulation technique on the uplink [12]-[15]. The problem of PARP affects the uplink and downlink channels differently. On the downlink, we can use distinguished PAPR reduction methods; these reduction methods cannot be applied to the uplink due to their difficulty in low processing power devices such as mobile devices. Besides, on the uplink it is important to reduce the cost of power amplifiers as well.

PAPR reduction schemes have been studied for years [16]-[19]. Some of the PAPR reduction techniques are coding techniques, which can reduce PAPR at the expense of bandwidth efficiency and increase in complexity [20]-[21]. The probabilistic technique, which includes Selective Mapping (SLM) Tone Reservation (TR) and Tone Injection, can also reduce PAPR; however, suffers from complexity and spectral efficiency for large number of subcarriers [22]-[23].

II. DISTURBUTION OF OFDM SIGNAL

Fig. 1 illustrates the block diagram of an OFDM system. The discrete time signal after IFFT can be expressed as:

$$x[n] = \frac{1}{N}\sum_{K=0}^{N-1} X[k]e^{\frac{j2\pi}{N}kn} \tag{1}$$

For a sequence of modulated data symbols, $X[k]$, the discrete time domain signal $x[n]$ is the addition of N different time domain signals $e^{j\frac{2\pi}{N}kn}$ where each signal corresponds to different orthogonal subcarriers.

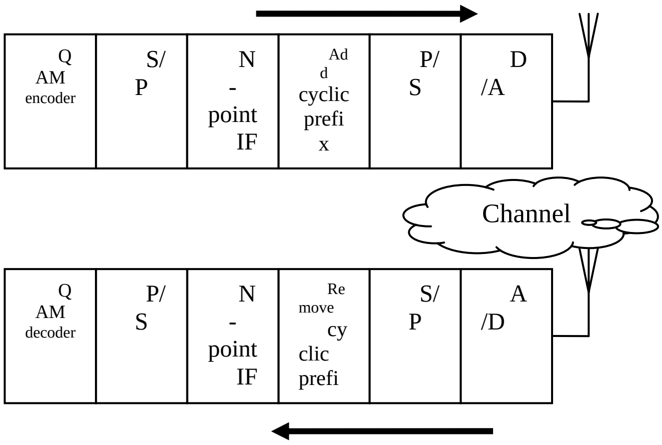

Fig. 1—Block diagram of OFDM system

Fig. 1 shows a block diagram of OFDM transceiever. Each symbol can transmit up to 4000 bits. Serial to parallel (S/P) converts the input data allowing transmission in each OFDM symbol. The modulation scheme and number of subcarriers determine the data allocated to each symbol. The Inverse Fourier Transform IFFT transforms the signal to the time domain for transmission and reduces the amount of calculations dramatically. The cyclic prefix prevents inter-symbol interference (ISI) and inter-carrier interference (ICI) in fading channels.

Fig. 2 shows the individual time domain Quadrature Phase Shift Keying (QPSK) modulated subcarrier signals for N=8. The PAPR worsens as the number of subcarriers increases.

Figure. 3 shows the PAPR characteristics of the OFDM signal which includes the distributions of $x[n]$ as well as the imaginary and real parts for N=16.

Fig. 3 also shows that the real and imaginary parts of $x[n]$ follow a Gaussian distribution while $x[t]$ follow a Rayleigh distribution.

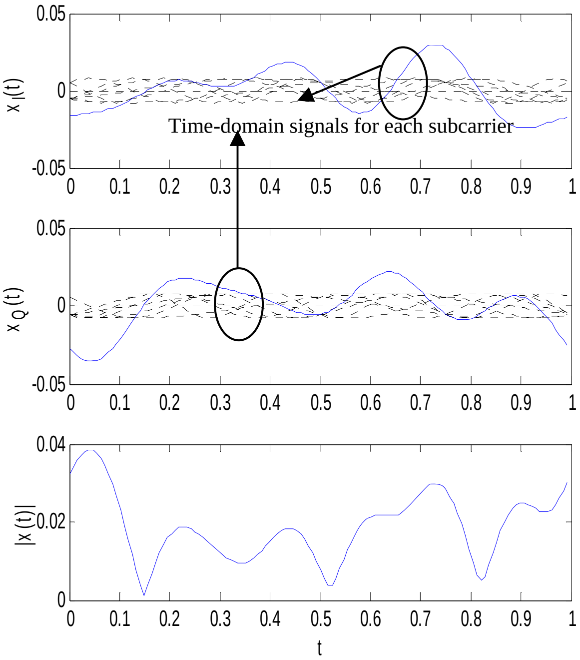

Fig. 2—Time domain OFDM signals for N=16

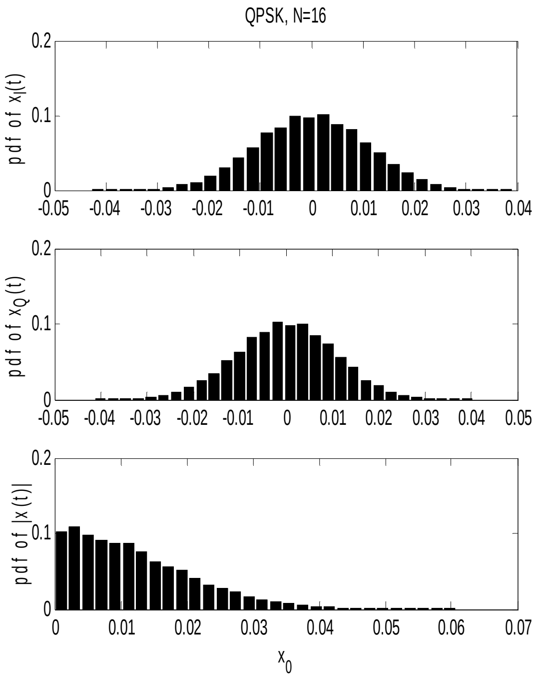

Fig. 3—Magnitude distribution of OFDM signal for N=16

Before examining the reduction of PAPR, let us consider a single-carrier system where N=1. Fig. 4 shows the passband signal with a single carrier frequency of 1 Hz and an oversampling factor of 8. The baseband signal's average and peak power values are the same that is PAPR is 0 dB , however, the passband signal's PAPR is 3.01 dB as shown in Fig. 4.

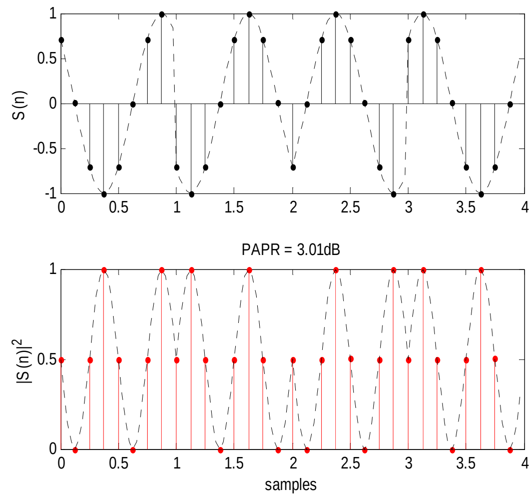

Fig. 4—Passband signal

Note that the PAPR varies in the passband signal depending on the carrier frequency. As a result, when measuring the PAPR of a single-carrier system, then we must be taken into consideration the carrier frequency of the passband signal [24-25].

III. System Model

The PTS technique partitions the data block of N symbols into Z disjoint sublocks as follows:

$$X = [X^{0}, X^{1}, X^{2}, ..., X^{z-1}]^{T} \qquad (2)$$

Where X^{i} are the subcarriers, which are of equal size and consecutively located. In the PTS technique, scrambling is applied to each subblock where in the selective mapping technique scrambling is applied to all subcarriers. Each subblock is multiplied by a phase factor $b^{z} = e^{j\phi z}$, z =1,2,3,…, z, the IFFT becomes

$$x = IFFT\{\sum_{z=1}^{Z} b^{z} X^{z}\} = \sum_{z=1}^{Z} b^{z} IFFT\{X^{z}\} = \sum_{z=1}^{Z} b^{z} x^{z} \qquad (3)$$

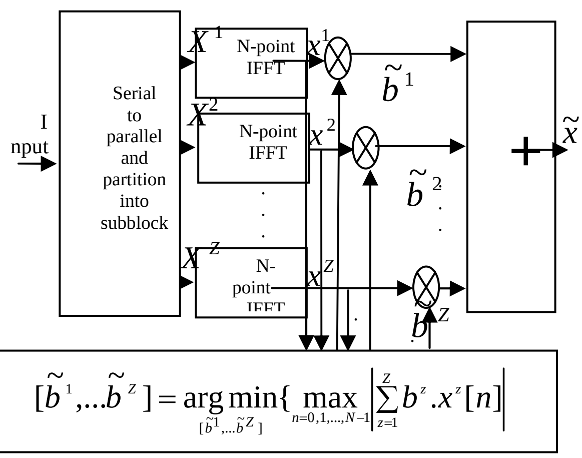

Fig. 5—Block diagram of partial transmit sequence (PTS) technique for PAPR reduction

Where x^Z is referred to as a partial transmit sequence. The phase vector is selected so that PAPR can be minimized as follows:

$$[\tilde{b}^{1},\dots\tilde{b}^{z}] = \underset{[\tilde{b}^{1},\dots\tilde{b}^{Z}]}{\arg\min} \{ \max_{n=0,1,\dots,N-1} \left| \sum_{z=1}^{Z} b^{z}.x^{z}[n] \right| \tag{4}$$

Then the time domain signal with the lowest PAPR vector can be expressed as follows:

$$\tilde{x} = \sum_{z=1}^{Z} \tilde{b}^{z} x^{z} \tag{5}$$

Original PTS technique. Fig. 6 shows the CCDF of PAPR for a quadrature amplitude modulation (QAM)/OFDM system with PTS technique when the number of subblocks varies. It can be seen that the PAPR improves as the number of subblocks increases.

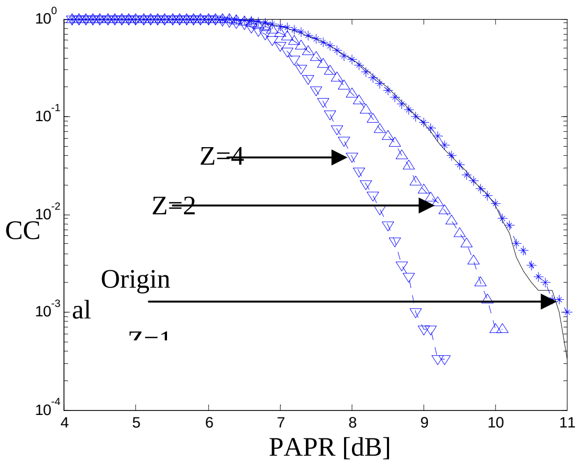

Fig. 6—PAPR performance of a 16 QAM/OFDM system with PTS technique when the number of subblocks vary

IV. PAPR Reduction Schemes for Uplink Transmission

There are two channel allocation schemes for SC-FDMA systems; i.e., the localized and interleaved schemes where the subcarriers are transmitted subsequently, rather than in parallel. In the following simulation results, we compared different allocation schemes of SC-FDMA systems and their PAPR. These types of allocation schemes are subject to intersymbol interference when the signal suffers from sever multipath propagation. In SC-FDMA this type of interference can be substantial and usually an adaptive frequency domain equalizer is placed at the base station. This type of arrangement makes sense in the uplink of cellular systems due to the additional benefit that SC-FDMA adds in terms of PAPR. In this type of arrangement, i.e, single carrier system the burden of linear amplification in portable terminals is shifted to the base station at the cost of complex signal processing that is frequency domain equalization.

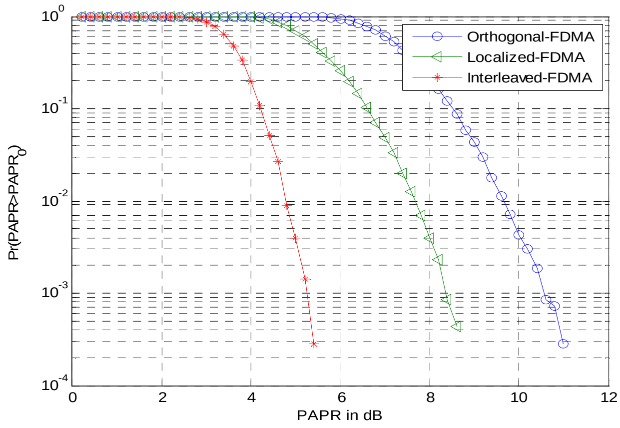

Fig. 7. (a) Performance of PAPR using QPSK

Figure 7 show the performance of PAPR while the number of subcarriers is 256 and the number of subcarriers assigned to each unit or mobile device is 64. This simulation helps in evaluating the performance of PAPR with different mapping schemes and modulation techniques. In LFDMA each user transmission is localized in the frequency domain where in the DFDMA each user transmission is spread over the entire frequency band making it less sensitive to frequency errors and diversifies frequency.

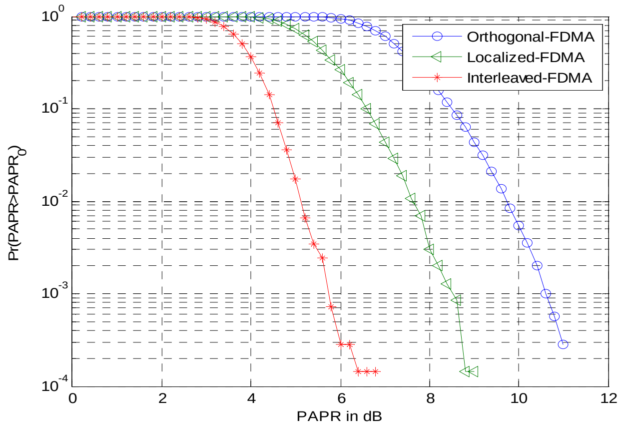

Fig. 7. (b) Performance of PAPR using 8 QPSK

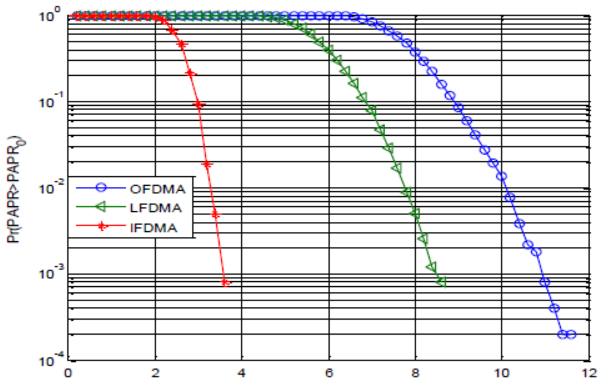

Fig. 7. (c) Performance of PAPR using 16 QAM

The four figures of 7 show that when the single carrier is mapped either by LFDMA or DFDMA, it outperforms OFDMA due to the fact that in an uplink transmission, mobile terminals work differently then a base station in terms of power amplification. In the uplink transmission PAPR is more of a significant problem then on the downlink due to the type and capability of the amplifiers used in base station and mobile devices. For instance, when a mobile circuit's amplifier operates in the non-linear region due to PAPR, the mobile devise would consume more power and become less power efficient whereas base stations do not suffer from this consequence. Therefore, OFDM works better in the downlink transmission in terms of PAPR.

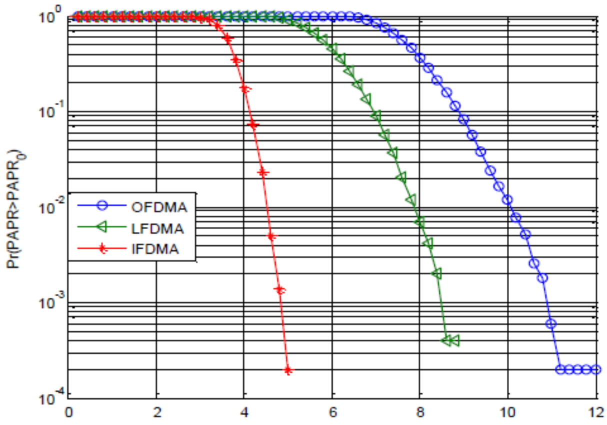

Fig. 7. (d) Performance of PAPR using 64 QAM

Our results show the effect of using Discrete Fourier Transform spreading technique to reduce PAPR for OFDMA,LFDMA and OFDMA with N=256 and N =64. A comparison is shown in Figure 7 a,b,c and d utilizing different modulation schemes. The reduction in PAPR is significant when DFT is used. For example, Figure 7(b) where Orthogonal-FDMA, Localized-FDMA and Interleaved-FDMA have the values of 3.9 dB, 8.5 dB and 11 dB, respectively. The reduction of PAPR in IFDMA utilizing the DFT-spreading technique compared to OFDMA without the use of DFT is 6.1 dB. Such reduction is significant in the performance of PAPR. A single carrier frequency division multiple access systems with Interleaved- FDMA and Localized-FDMA perform better than OFDMA in the uplink transmission. Although Interleaved-FDMA performs better than OFDMA and LFDMA, LFDMA is preferred due to the fact that assigning subcarriers over the whole band of IFDMA is complicated while LFDMA doesn't require the insertion of pilots of guard bands [26-30].

IV. Conclision

We have shown the distribution of OFDM signal and the fact that PAPR worsens as the number of subcarriers increases. The PAPR characteristics of the OFDM signal includes the distributions of the discrete time domain signal $x[n]$ as well as the imaginary and real parts. Our simulation results show that the real and imaginary parts of discrete time signal $x[n]$ follow a Gaussian distribution while the continuous time signal $x[t]$ follows a Rayleigh distribution. Besides, we show when measuring the PAPR of a single-carrier system, we must take into consideration the carrier frequency of the passband signal.

The PTS technique requires Z IFFT operation for each block. The performance of PAPR with PTS is affected by the number of subblocks, the phase vector and by the subblock partitioning. There are three different subblock partitioning schemes: interleaved, adjacent and pseudo-random. We proposed a suboptimal combination algorithm. The number of commutations for equation (2) in the suboptimal combination algorithm is Z, which is much lower than the required by the original PTS technique. Finally yet importantly, our results show that PAPR improves as the number of subblocks increases.

It was also shown that a SC- FDMA system with Interleaved-FDMA or Localized FDMA performs better than Orthogonal-FDMA in the uplink transmission where transmitter power efficiency is of great importance in the uplink. LFDMA and IFDMA result in lower average power values due to the fact that OFDM and OFDMA map their input bits straight to frequency symbols where LFDMA and IFDMA map their input bits to time symbols.

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