A Low-Complexity Optimum Multiuser Receiver for DS-CDMA Wireless Systems

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***Abstract*— Multiuser detection is an important technology in wireless CDMA systems for improving both data rate as well as user capacity. However, the computational complexity of multiuser detection prevents the widespread use of this technique. Most of the CDMA systems today and in the near future will continue to use the conventional matched filter with its comparatively low user capacity and a slow data rate. However, if we could lower the computational complexity of multiuser detectors, CDMA systems would offer an increased system capacity with a better data rate. In this paper, a new scheme for reducing the computational complexity of multiuser receivers is proposed. It utilizes the transformation matrix algorithm to improve the performance of multiuser receivers by effectively reducing the bit error rate (BER). In addition to the transformation matrix algorithm, a quantitative analysis of the processing gain for a multiuser DS-CDMA system is presented. The quantitative analysis of the processing gain demonstrates that how the reduced BER could be used to achieve reasonable values of processing gain by which unwanted signals or interference can be suppressed relative to the desired signal at the receiving end. We present that the proposed scheme can reduce the asymptotic computational complexity of multiuser receivers while at the same time effectively eliminates the unwanted signals. The proposed algorithms not only are shown to substantially improve the performance of the multiuser detectors by means of reduced BER but also have a much lower multi-access interference. The performance measure adopted in this paper is the achievable bit rate for a fixed probability of error (10^{-7}) and consistent values of SNR.**

***Index Terms*—Bit error rate, CDMA, DS-CDMA, computational complexity, multiuser receiver.**

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I. Introduction

Multiuser direct-sequence code division multiple access (DS-CDMA) has received wide attention in the field of wireless communications [1]. With the emergence of multiple access techniques, there has been an increase in the interest in performing simultaneous estimation and detection over all users [2, 3]. In CDMA communication systems, several users are active on the same fringe of the spectrum at the same time. Therefore, the received signal results from the sum of all the contributions from the active users [4]. In CDMA systems, where all users spread their transmission over a common transmission bandwidth, a dominant impairment is interference between users, referred to as multiple access interference (MAI) [5]. Much research has been done on the problem of MAI suppression in wireless systems, and a wide rage of algorithms has been proposed for use in receivers such as [6, 7]. MAI can be prevented by selecting mutually orthogonal signature waveforms for all active users. However, it is not possible to ensure a perfect orthogonality among received signature waveforms in a mobile environment, and thus MAI arises.

The optimum multiuser detector was obtained in [8], where it was shown that the near-far problem suffered by the conventional matched filter receiver can be eliminated by a more sophisticated receiver [2]. The complexity of this optimum receiver, which is exponential in the number of users, has motivated the development of various linear and nonlinear sub-optimal near-far resistant multiuser detectors, such as the decorrelating detector [6, 7], the minimum mean square error (MMSE) receiver [9, 10, 11, 12], the multistage detector [13] and the decision-feedback detector [14], to name a few.

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Multiuser detection is a technique to improve the capacity and coverage in a CDMA system. Being a critical component of this technique, the maximum likelihood (ML) multiuser receiver has received extensive study [15, 16, 17]. However, the computational complexity of this receiver prevents the widespread use of this technique. Due to the high computational complexity, most of the CDMA systems today and in the near future will continue to use the conventional matched filter with comparatively low user capacity and a slow data rate.

In this paper, a novel approach for reducing the asymptotic computational complexity of multiuser receivers is proposed that utilizes the transformation matrix (TM) technique to improve the performance of multiuser detectors. By using the proposed algorithm, the computational complexity of multiuser detectors can be reduced by several orders of magnitude. This is done by realizing that much of the processing performed is unnecessary. Since most of the decisions are correct, we can reduce the number of computations by using the transformation matrices only on those coordinates that are most likely lead to an incorrect decision.

The rest of this paper is organized as follows: Section II describes the related work. Section III presents the proposed TM algorithm along with the corresponding computational complexity. The mathematical derivations for generating consistent values of SNR and the standard formulas for BER are presented in Section IV. Section V presents the quantitative analysis of the processing gain (PG) to minimize the MAI. The numerical and simulation results of SNR, PG, and BER performance are provided in Section VI. Finally, Section VII concludes the paper.

II. Related Work

Multiuser receivers can be categorized in the following two forms: optimal maximum likelihood sequence estimation (MLSE) receivers [15, 17] and suboptimal linear and nonlinear receivers [3, 13]. Suboptimal multiuser detection algorithms can be further classified into linear and interference cancellation type algorithms. The linear detectors are designed to eliminate MAI and inter symbol interference (ISI), either in synchronous or asynchronous systems. In case of synchronous CDMA system, two main criteria are employed, namely the zero-forcing (ZF) and the MMSE. Both mechanisms can implement in two possible ways. In the first option, both of them can be implemented to deal simultaneously with ISI and MAI where as in the second option, they deal only with ISI [10]. Moreover, the MAI can also be suppressed by using multiuser detection technique [18], potentially approaching the single user performance. One disadvantage of the linear detector is that the evaluation of the tap coefficients of the filters involves a matrix inversion [19, 20].

Two well known classes of CDMA adaptive multiuser detection are *trained* and *blind* detectors [21, 22]. The trained detector is a robust adaptive detector that does not require the knowledge of spreading code of the desired user [6, 23]. Results have shown [17] that these receivers are robust. The blind detector is a powerful adaptive detector that does not require any preliminary information about the data sequence [21, 22]. Non-linear multiuser receiver involves the estimation and reconstruction of MAI [23] seen by each user with the objective of canceling it from the received signal.

In interference cancellation, MAI is first estimated and then subtracted from the received signal [3]. On the other hand, linear multiuser receivers apply a linear transformation to an observation vector, which serves as soft decision for the transmitted data. In order to mitigate the problem of MAI, Verdu [8] proposed and analyzed the optimum multiuser detector for asynchronous Gaussian multiple access channels. The ML receiver searches all the possible demodulated bits in order to find the decision region that maximizes the correlation metric given by [24]. The practical application of this mechanism is limited by the complexity of the receiver [9, 10]. This optimal detector outperforms the conventional detector, but unfortunately its complexity grows exponentially with a complexity of $O(2)^K$, where K represents number of active users.

Much research has been done to reduce this receiver's computational complexity. Ottosson and Agrell [16] proposed a ML receiver that uses the neighboring decent (ND) algorithm. They implemented an iterative approach using the ND algorithm to locate the region where the actual observations belong. To reduce the computational complexity of optimum receivers, the iterative approach using the ND algorithm performs MAI cancellation linearly. The linearity of the iterative approach increases noise components at the receiving end. Due to the enhancement in the noise components, the SNR and BER of the ND algorithm are more affected by the MAI.

III. The Proposed Transformation Matrix (TM) Algorithm

We consider a synchronous DS-CDMA system as a linear time invariant (LTI) channel. In a LTI channel, the probability of variations in the interference parameters, such as the timing of all users, amplitude variation, phase shift, and frequency shift, is extremely low. This property makes it possible to reduce the overall computational complexity at the receiving end. Our TM technique utilizes the complex properties of the existing inverse matrix algorithms to construct the transformation matrices and to determine the location of the TPs that may occur in any coordinate of the constellation diagram.

The system may consist of K users. User k can transmit a signal at any given time with the power of W_k.

With the binary phase shift keying (BPSK) modulation technique, the transmitted bits belong to either +1 or -1, (i.e., $b_k \in \{\pm 1\}$). The cross correlation can be reduced by neglecting the variable delay spreads, since these delays are relatively small as compared to the symbol transmission time. In order to detect signals from any user, the demodulated output of the low pass filter is multiplied by a unique signature waveform assigned by a pseudo random number generator.

The optimum multiuser receiver exists and permits to relax the constraints of choosing the spreading sequences with good correlation properties at a cost of increased receiver complexity. Fig. 1 shows the block diagram of an optimum receiver that uses a bank of matched filters and a maximum likelihood Viterbi decision algorithm [25] for signal detection. It should be noted in Fig. 1 that the proposed TM algorithm is implemented in conjunction with the Viterbi decision algorithm with the feedback mechanism. In order to detect signal from any user, the demodulated output of the low pass filter is multiplied by a unique signature waveform assigned by a pseudo random number generator.

A. Transformation Matrix (TM) Algorithm Description

According to original Verdu's algorithm, the outputs of the matched filter $y_1(m)$ *and* $y_2(m)$ can be considered as a single output $y(m)$. In order to minimize the noise components and to maximize the received demodulated bits, we can transform the output of the matched filter, and this transformation can be expressed as follows: $y(m) = Tb + \eta$ where T represents the TM, $b_k \in \{\pm 1\}$ and η represents the noise components. In addition, if the vectors are regarded as points in K-dimensional space, then the vectors constitute a constellation diagram that has *K* total points.

The constellation diagram can be mathematically expressed as: $X = \{\mathrm{T}b\}$ where $b \in \{-1, +1\}$ and X represents the collective computational complexity of a multiuser receiver. The preceding equation is fundamental to the proposed algorithm. According to the detection rule, the constellation diagram can be partitioned into 2^K lines (where the total possible lines in the constellation diagram can be represented as $\int$) that can only intersect each other at the following points: $X = \{\mathrm{Tb}\}_{b \in \{-1,1\}^K} \backslash \int$.

Fig. 2 shows the constellation diagram that consists of three different vectors (lines) with the original vector 'X' that represents the collective complexity of the receiver. *Q*, *R*, and *S* represent vectors or TP within the coverage area of a cellular network (see Fig. 2). In addition, $Q^{\neg}$, $R^{\neg}$, and $S^{\neg}$ represent the computational complexity of each individual TP. In order to compute the collective computational complexity of the optimum receiver, it is essential to determine the complexity of each individual TP.

The computational complexity of each individual TP is represented by $X^{\neg}$ of the TP which is equal to the collective complexity of $Q^{\neg}$, $R^{\neg}$, and $S^{\neg}$. In order to derive the value of the original vector ***X***, we need to perform the following derivations. We consider the original vector with respect to each transmitted symbol or bit.

$$X^{\neg}Q = Xi^{\neg} = \left\langle XQ_i + XR_j + XS_K \right\rangle i^{\neg} =$$
$$\left\langle XQ_i i^{\neg} + XR_j i^{\neg} + XS_K i^{\neg} \right\rangle$$
$$X^{\neg}R = Xj^{\neg} = \left\langle XQ_i + XR_j + XS_K \right\rangle j^{\neg} =$$
$$\left\langle XQ_i j^{\neg} + XR_j j^{\neg} + XS_K j^{\neg} \right\rangle$$
$$X^{\neg}S = Xk^{\neg} = \left\langle XQ_i + XR_j + XS_K \right\rangle k^{\neg} =$$
$$\left\langle XQ_i k^{\neg} + XR_j k^{\neg} + XS_K k^{\neg} \right\rangle$$

The following equation can be derived from the above system:

$$\left\langle \begin{array}{c|c} X^{\neg}Q & i^{\neg} \\ X^{\neg}R & j^{\neg} \\ X^{\neg}S & k^{\neg} \end{array} \right\rangle = \begin{bmatrix} i(i^{\neg}) & j(i^{\neg}) & k(i^{\neg}) \\ i(j^{\neg}) & j(j^{\neg}) & k(j^{\neg}) \\ i(k^{\neg}) & j(k^{\neg}) & k(k^{\neg}) \end{bmatrix} \begin{bmatrix} XQ \\ XR \\ XS \end{bmatrix} \quad (1)$$

Equation (1) represents the following: *QRS* with the unit vectors i, j, and k, and $X^{\neg}Q$, $X^{\neg}R$, and $X^{\neg}S$ with the inverse of the unit vectors $i^{\neg}$, $j^{\neg}$, and $k^{\neg}$. The second matrix on the right hand side of (1) represents **b**, where as the first matrix on the right hand side of (1) represents the actual TM. The TM from the global reference points to a particular local reference point can now be derived from (1):

$$\left\langle \begin{array}{c|c} X^{\neg}Q & i^{\neg} \\ X^{\neg}R & j^{\neg} \\ X^{\neg}S & k^{\neg} \end{array} \right\rangle = T_{L/G} \begin{bmatrix} XQ \\ XR \\ XS \end{bmatrix} \quad (2)$$

Equation (2) can also be written as:

$$T_{L/G} = \begin{bmatrix} ii^{\neg} & ji^{\neg} & ki^{\neg} \\ ij^{\neg} & jj^{\neg} & kj^{\neg} \\ ik^{\neg} & jk^{\neg} & kk^{\neg} \end{bmatrix} \quad (3)$$

In (3), the dot products of the unit vectors of the two reference points are in fact the same as the unit vector of the inverse TM of (2). We need to compute the locations of the actual TP described in (2) and (3). Let the unit vectors for the local reference point be:

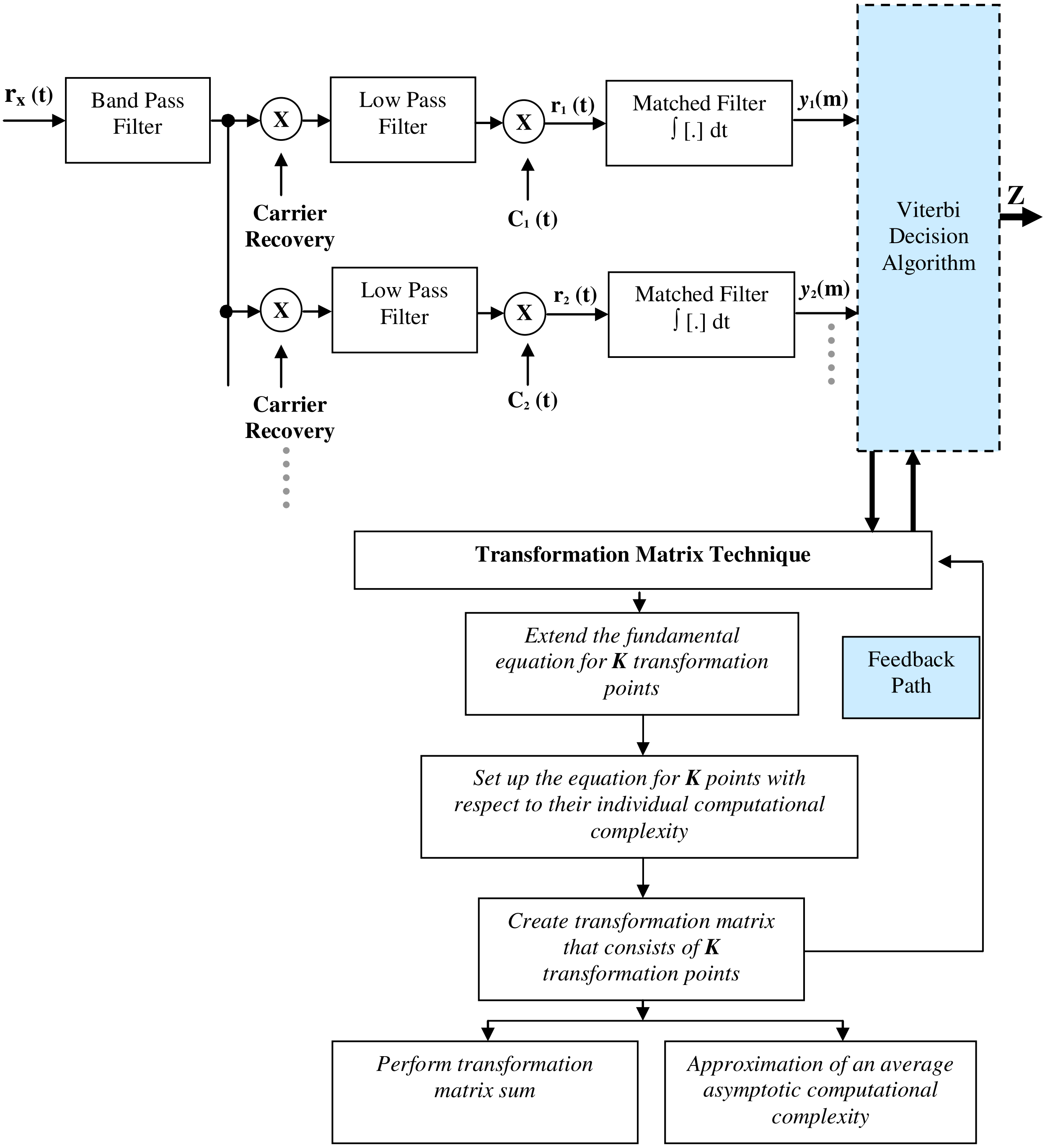

Figure 1. Implementation of proposed transformation matrix (TM) algorithm with the optimum multiuser receiver

$$
\begin{aligned}
i^{\neg} &= \left[T_{11}i, T_{12}j, T_{13}k\right] \\
j^{\neg} &= \left[T_{21}i, T_{22}j, T_{23}k\right] \\
k^{\neg} &= \left[T_{31}i, T_{32}j, T_{33}k\right]
\end{aligned} \tag{4}
$$

Since, $i^{\neg}\left(i+j+k\right)=i^{\neg}$, where $\left(i+j+k\right)=1$. The same argument is true for the rest of the unit vectors. Therefore, (4) can be rewritten as:

$$
\begin{aligned}
i^{\neg} &= \left[T_{11}, T_{12}, T_{13}\right] \\
j^{\neg} &= \left[T_{21}, T_{22}, T_{23}\right] \\
k^{\neg} &= \left[T_{31}, T_{32}, T_{33}\right]
\end{aligned} \tag{5}
$$

By substituting the values of $i^{\neg}$, $j^{\neg}$, and $k^{\neg}$ from (5) into (3), we obtain

$$
T_{L/G} = \begin{bmatrix}
i\left\langle T_{11}i, T_{12}j, T_{13}k \right\rangle & j\left\langle T_{11}i, T_{12}j, T_{13}k \right\rangle & k\left\langle T_{11}i, T_{12}j, T_{13}k \right\rangle \\
i\left\langle T_{21}i, T_{22}j, T_{23}k \right\rangle & j\left\langle T_{21}i, T_{22}j, T_{23}k \right\rangle & k\left\langle T_{21}i, T_{22}j, T_{23}k \right\rangle \\
i\left\langle T_{31}i, T_{32}j, T_{33}k \right\rangle & j\left\langle T_{31}i, T_{32}j, T_{33}k \right\rangle & k\left\langle T_{31}i, T_{32}j, T_{33}k \right\rangle
\end{bmatrix}
$$

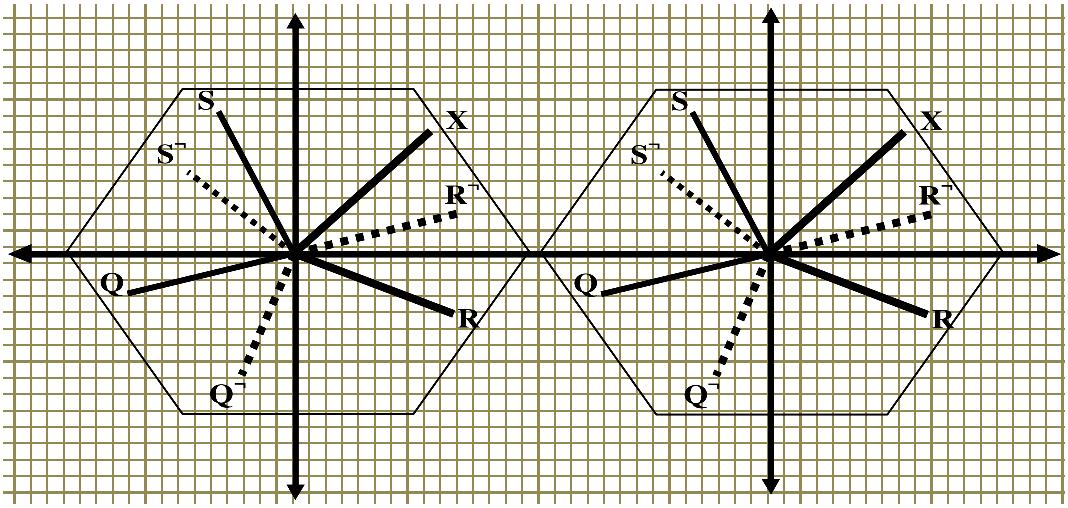

Figure 2. A constellation diagram consisting of three different vectors

$$T_{L/G} = \begin{bmatrix} T_{11}, T_{12}, T_{13} \\ T_{21}, T_{22}, T_{23} \\ T_{31}, T_{32}, T_{33} \end{bmatrix} \tag{6}$$

Substituting $\boldsymbol{T_{L/G}}$ from (6) into (2), yields

$$\left\langle \begin{matrix} X^{\neg}Q \\ X^{\neg}R \\ X^{\neg}S \end{matrix} \right\rangle = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} XQ \\ XR \\ XS \end{bmatrix} \tag{7}$$

Equation (7) corresponds to the following standard equation that used for computing the computational complexity at the receiving end: *X*= *Tb* where $b \in \{-1, +1\}^k$.

If the target of one transformation $(U: Q \to R)$ is the same as the source of other transformation $(T: R \to S)$, then we can combine two or more transformations and form the following composition: *TU: Q→S, TU(Q)=T[U(Q)]*.This composition can be used to derive the collective computational complexity at the receiving end using (7). Since we assumed that the transmitted signals are modulated using BPSK which can at most use 1 bit out of two bits (i.e., $b_k \in \{\pm 1\}$), consider the following set of TP to approximate the number of demodulated received bits that need to search out by decision algorithm:

$$\begin{bmatrix} y(m) \\ y(m+1) \\ \vdots \\ \\ \\ y(K) \end{bmatrix} = \begin{bmatrix} Tb(0) & Tb(-1) & 0 & \cdots & 0 \\ Tb(1) & Tb(0) & Tb(-1) & \cdots & 0 \\ 0 & Tb(1) & Tb(0) & \cdots & 0 \\ & \ddots & \ddots & & \\ & & & \ddots & \\ \vdots & \vdots & \ddots & & \vdots \\ 0 & \cdots & \cdots & Tb(1) & Tb(0) \end{bmatrix} + \begin{bmatrix} \eta(m) \\ \eta(m+1) \\ \\ \vdots \\ \\ \eta(m+k) \end{bmatrix} \tag{8}$$

Equation (8) is derived using our fundamental equation of TM (i.e., $y = Tb + \eta$). Our approach is to assume terms $\eta(m)$ and $\eta(m+k)$ in (7) not equal to zero. This condition is fulfilled by periodically inserting a nonzero-energy bit in the information bit sequence. Therefore, the interference due to the cross-correlation of the actual symbols with the past and future symbols in the asynchronous channels can be accounted.

Using (7), a simple matrix addition of the received demodulated bits can be used to approximate the number of most correlated TP. The entire procedure for computing the number of demodulated bits that need to be searched out by the decision algorithm can be used to approximate the number of most correlated signals for any given set of TP. This is because we need to check whether or not the TP are closest to either (+1, +1) or (-1, -1). The decision regions or the coordinates where the TP lie for (+1, +1) and (-1, -1) are simply the corresponding transformation matrices that store the patterns of their occurrences. If the TP do not exist in the region of either (+1, +1) or (-1, -1), then it is just a matter of checking whether the TP are closest to (+1, -1) or to (-1, +1).

The minimum search performed by the decision algorithm is conducted if the TP exist within the incorrect region. Since the minimum search saves computation by one degree, the decision algorithm has to search at least 4^k demodulated bits. This implies that the total number of demodulated bits that need to be searched out by the decision algorithm can not exceed by 5^K - 4^K. Thus, the total number of most correlated pairs has an upper bound of 5^K - 4^K.

The computational complexity of any multiuser receiver can be quantified by its time complexity per bit [6]. The collective computational complexity of the proposed algorithm is achieved after performing the TM sum. This implies that both quantities *T* and *b* from our fundamental equation can be computed together and the generation of all the values of the demodulated received bits b can be done through the sum of the actual TM T that approximately takes O $(5/4)^k$ operations with an asymptotic constant. Using the Newton approximation method given in MATLAB, we can directly come to an approximation of $O\,(5/4)^k$.

IV. Quantification Of Signal-to-Noise Ratio (SNR) And Bit Error Rate (BER)

In this section, we derive a closed form expression for both SNR and BER. In all subsequent derivations, we use different properties and mathematical expressions of the proposed TM algorithm.

A. *Quantifying SNR for the TM Algorithm*

Consider the following points:

a) $\aleph$ is a computational complexity that belongs to a certain coverage area.

b) If SNR (we represent SNR by γ) is uniformly distributed among all the active user's signals with respect to computational complexity.

c) A certain cellular coverage area has $\boldsymbol{K}$ users.

Based on the above points, we can give the following mathematical hypothesis:

$$\aleph_i \in \{\aleph_1, \aleph_2, \aleph_3, \ldots\ldots\ldots\ldots\ldots, \aleph_K\}$$

where $\aleph_{1,}\aleph_{2,}\aleph_{3,}\ldots\ldots\ldots\ldots\ldots\aleph_K$ indicates the computational complexity-domain and

$$h_i \in \{h_1, h_2, h_3, \ldots\ldots\ldots\ldots\ldots, h_K\}$$

where $h_1, h_2, h_3, \ldots\ldots\ldots\ldots\ldots, h_K$ indicates the user-domain.

The collective computational complexity can be expressed as:

$$\aleph = \sum_{i=1}^{K} \aleph_i \text{ where } i = 1, 2, 3, \ldots\ldots\ldots\ldots, K$$

Since each user has h_{th} part of the computational complexity such as: $h_1 \in \aleph_1, h_2 \in \aleph_2, \ldots.., h_{K-1} \in \aleph_{K-1}, h_K \in \aleph_K$. This implies that each active user in a certain area of a cellular network has an average of $\aleph/K$ computational complexity. Since SNR is uniformly distributed among all the user's signals at the receiving end, each user experiences an average of γ/K SNR. In order to achieve maximum positive values of SNR for most of the values of K, we propose that the inverse of the computational complexity should equal the difference between the inverse-normalization factor and the product of inverse-normalization factor and SNR with respect to the collective computational complexity of the system. This hypothesis leads us to the following equation:

$$\frac{K}{\aleph} = C^{-1} - C^{-1}\frac{\gamma}{\aleph} = \frac{1}{C}\left[1 - \frac{\gamma}{\aleph}\right] \tag{9}$$

where C in (9) represents the normalization factor, $K/\aleph$ is the inverse of the computational complexity, and $\gamma/\aleph$ represents the SNR with respect to the collective computational complexity.

The main objective of (9) is to ensure that we achieve maximum positive values of SNR for most of the values of K. Using the complexity and the user-domain, we can make an argument that the inverse of an average SNR should at least greater than zero. This argument guarantees that the system does not work with a non-positive value of SNR.

Proof for $\gamma/\aleph$:

In order to prove $\gamma/\aleph$, consider the following points:

a) The coverage area of a cellular network has $\aleph$ computational complexity.

b) Each user has an average of k_{th} part of the computational complexity from the complexity domain (that is, $\aleph/K$).

c) The average γ is uniformly distributed among K active users (that is, γ/K)

If the above points are valid, the following equation must be true not only for the complexity-domain but also for the user-domain: $\dfrac{\aleph}{K} = C + \dfrac{\gamma}{K}$.

We present our hypothesis that the difference between the average computational complexity and the average SNR should equal to the normalization factor. The main objective of the above equation is to get maximum positive values of SNR for most of the values of K. We can write the following equation:

$$\frac{\gamma}{\aleph} = 1 - C\left(\frac{K}{\aleph}\right) \tag{10}$$

Since the right hand side of (10) represents the inverse of the average computational complexity with the normalization factor, the number of required operations can not be less than zero. It should be noted that the right hand side of (10) always gives us a positive value of SNR for any value of K which is greater than 10.

Using the complexity and the user domain, we can make an argument that the inverse of an average SNR should be at least greater than zero. This argument guarantees that the system does not work with a non positive value of SNR. This is essential, since a negative SNR significantly degrades the BER performance. Recall (9): $K/\aleph = 1/C\left[1-\gamma/\aleph\right] = 1/C\left[\left(\aleph-\gamma\right)/\aleph\right]$

$$\gamma = \aleph - CK \tag{11}$$

Equation (11) represents SNR by determining the difference between the power of the transmitted signal from the computational complexity-domain and the number of users from the user-domain. Equation (11) can also be used to compute the values of SNR in an ideal situation only if MAI does not affect the received signals by K-1 users. However, in a practical DS-CDMA system, this assumption does not exist. Therefore, we should consider that the variations in the network load for an additive white Gaussian noise (AWGN) channel introduces the presence of variance (we represent variance by σ^2) that represents MAI. The selection of variance is entirely dependent on the network load.

In order to compute the values of SNR in decibels (dB), we need to change linear quantity into decibels (dB) by multiplying it to the base-10 logarithmic function as well as with the variance.

$$\gamma = 10\sigma^2 \log_{10}\left(\aleph - CK\right) \tag{12}$$

We consider the precomputed values of variance given in [23]. Furthermore, the normalization factor represents a varying quantity that can be used to approximate the different values of SNR with respect to the difference between the average complexity and the average SNR.

B. A Closed Form Expression for BER

The use of DS-CDMA technology in communication system causes interference to other signals present on the channel. The occurrence of MAI substantially increases as more signals simultaneously access the channel. Since different signals are slightly correlated to each other, the increase in the occurrence of MAI reduces the BER performance. This fact makes the BER performance as one of the important measures that determines the maximum number of simultaneously transmitting signals.

We modeled the cellular network as a LTI synchronous DS-CDMA system in which users utilize an AWGN multipath channel. Due to an AWGN channel and the linearity property, the different signal components do not experience deep fades. If the signal changes during the transition, the receiver receives the following signal:

$$\Re(t) = \mathrm{A}e^{-j\theta} + s(t) + \eta(t)$$

where A is an attenuation factor, θ is a phase shift, $s(t)$ is the desired signal, and $\eta(t)$ is the additive Gaussian noise.

Due to LTI characteristics, the proposed algorithm is independent of the phase shift, which permits us to ignore it by simply setting the value of θ to zero. Therefore, the receiver receives the following signal:

$$\Re(t) = s(t) + \eta(t) + A$$

Since the attenuation factor A is uncorrelated with $\eta(t)$, we can use the value of SNR directly in the BER formula. Consider (13) that can be used to determine the BER in an AWGN channel for a system where the transmitted bits are modulated using the BPSK modulation technique.

$$\text{BER} = Q\left[1/\sqrt{1/SNR}\right] \tag{13}$$

Since the attenuation factor and the white noise are uncorrelated, the SNR can be directly placed in (13) as follows:

$$\text{BER} = Q\left[1/10\left(SNR\right) + \sigma^2\right]^{-1/2} \tag{14}$$

where $Q(x)$ is the Gaussian Q function [4]. For simplicity, (14) can also be written as:

$$\text{BER} = Q\left[\sqrt{10SNR}/\sqrt{1+10\sigma^2 SNR}\right] \tag{15}$$

The second term in (15) represents the SNR degradation due to MAI. This term depends on the cross-correlation between the spreading code as well as the number of users.

V. Processing Gain (PG) For TM Algorithm

It is observed that the PG has no effect on wideband thermal noise. In addition, a spread system requires the same transmitter power as an un-spread system on the AWGN when MAI is absent. However, we consider an AWGN channel where the MAI can severally affect the BER performance.

A. Quantitative Analysis of Processing Gain (PG)

For a DS-CDMA system, the PG can be viewed as ratio between the signal power and the interference power at the receiver. In a DS-CDMA based cell communication system, the interference is caused due to the cross-correlation between the spreading code as well as the number of users. Based on the above discussion, we can give the following mathematical hypothesis:

$$PG = S_P / I_P \tag{16}$$

where S_P and I_P represents signal and interference power respectively.

As we know that the signal to noise ration (SNR) can be defined as a ration between signal power and noise power. By taking this into account, one can infer that an increase in the noise power causes an increase in the bit error rate. This relationship can be expressed as:

$$N_P \propto BER \qquad (17)$$

where N_P represent the noise power

Based on (17), we can say that the effect caused on the SNR due to the values of N_P is the same as the effect caused by the BER values. Therefore, (17) can also be written as:

$$N_P \triangleq BER \qquad (18)$$

where "$\triangleq$" represents the estimated value of a quantity for some large number '*n*'. By using (17) and (18), we can rewrite (16) as:

$$PG = (SNR)(BER)/I_P \qquad (19)$$

The BER in (19) satisfies the characteristics of (17) and (18) for high values of BER. On the other hand, the higher values of SNR represent the reduction in the noise power as well as yield better values for PG. Recall (13) that can be used to determine the BER in an AWGN channel: BER $= Q\left[1/\sqrt{1/SNR}\right]$

$$\text{BER} = Q\left[1/10(SNR)+\sigma^2\right]^{-1/2} \qquad (20)$$

Recall (20) for an AWGN channel with the BPSK modulation technique, (19) can also be modified as:

$$PG = \frac{(SNR)Q\left[\frac{1}{10(SNR)}+\sigma^2\right]^{-1/2}}{I_P} \qquad (21)$$

The second term of (21), σ^2, represents MAI that caused due to the cross corelation between the spreading code and the number of users and can vary due to the variations in the network load for an AWGN channel. Thus the selection of variance is entirely dependent on the network load. For simplicity, this can also be written as:

$$PG = \frac{(SNR)Q\left[\frac{\sqrt{10SNR}}{\sqrt{1+10\sigma^2 SNR}}\right]}{I_P} \qquad (22)$$

B. Counter Proof for the Proposed Model

In this section, we provide a proof for analyzing the correctness of (22) by considering the same set of derivations for computing the PG on an AWGN channel in the absence of MAI. We expect that the absence of MAI leads us to an equivalent mathematical equation like (22) that should not contain the parameters for a variance.

In a spread-spectrum system, PG can be defined as a ratio of a SNR of a processed signal to the SNR of the unprocessed signal. This relationship can be expressed as:

$$PG = \frac{SNR_{PR}}{SNR_{UP}} \qquad (23)$$

where the subscripts PR and UP stand for *processed* and *unprocessed* signals, respectively.

According to our initial assumption, the cellular network is modeled as a LTI system that permits us to clearly distinguish the unprocessed input signal to the processed output signal This allows us to ignore the possibility of noise at the input signal. The leads us to the following mathematical expression:

$$PG = 1/N_P = \frac{SNR}{S_P} \qquad (24)$$

We use the same hypothesis that we presented to derive (17) and (18) in order to derive (25).

$$S_P \triangleq \left(1/BER\right) \Rightarrow PG = SNR(BER) \qquad (25)$$

Recall our previous derivations of BER for an AWGN channel, (25) can be written as:

$$PG = (SNR)Q\left[\frac{\sqrt{10SNR}}{\sqrt{1+10\sigma^2 SNR}}\right] \qquad (26)$$

Equation (26) gives the value of PG where the MAI is not caused due to the variation in the network load. By comparing (26) with (22), we can observe that the values of PG in (26) is much higher than the values we can get from (22) because of the absence of MAI. According to our initial assumption, if k_{th} signal changes during the transition, the output of the corelater is given as:

$$\Re_k = \mathrm{A}e^{-j\theta} + s_k N + \eta_k \qquad (27)$$

In (27), the first, second, and third term represent the MAI component, the desired signal component, and the noise component, respectively. Our model is not affected by a phase shift and frequency shift. Therefore, this simplifies (27) as:

$$\Re_k = s_k N + \eta_k + A \qquad (28)$$

where N is usefully interpreted as the PG. The second term in (28) is a zero mean Gaussian random variable with variance. The third term of (28) is a MAI component that can be defined as:

$$A = \sum_{k=2}^{K} A_k U_k \cos(\phi_k) \qquad (29)$$

where A_k represents the envelop of a complex Gaussian process with unit variance in each quadrature component and U_k represents a non-faded amplitude of the k_{th} signal. In (29), ϕ_k is a uniform random variable that represents the phase difference of the k_{th} user. Even though, the right hand side of (29) is independent and represents a Gaussian distribution process over a range of 0 to 2π, the left hand side is not a pure Gaussian function. Thus, to approximate the presence of processing gain and MAI in the received signal, (28) and (29) can be used as follows:

$$\Re_k = s_k\left[(SNR)Q\left(\frac{\sqrt{10SNR}}{\sqrt{1+10\sigma^2 SNR}}\right)\right]+ \sum_{k=2}^{K} A_k U_k + \eta_k \quad (30)$$

The first two terms of (30) can be used to approximate the PG and the MAI for a user *k*. It should be noted that the second term gives an average variance of the MAI over all possible operating conditions that can be used to compute the required SNR for a desirable BER performance. We use (30) in conducting the simulation result and performing the experimental verification by giving the non-folded amplitude of the k_{th} signal and computing the corresponding MAI as a Gaussian random variable with zero mean and conditional variance that represents by σ^2. Similarly, first terms of (30) can be used to approximate the values of achievable PG with respect to the required SNR for a desirable BER performance.

VI. Performance Analysis Of The Proposed TM Algorithm

In this section, we use the results in [16, 24] to compare the computational complexity, SNR, and the BER performance of the proposed TM algorithm with the ND and the ML multiuser detection algorithms. The system is modeled in MATLAB and the results are presented in the subsequent sections.

A. Complexity Analysis for the TM Algorithm

The numerical results show the asymptotic computational complexities with respect to the number of users as shown in Fig. 3 and Fig. 4 for 100 and 500 users, respectively. As the number of users increases in the system, the computational complexity differences among the three approaches will be obvious.

The computational complexity for a network that consists of 100 users is shown in Fig. 3. The complexity curve for the proposed algorithm is growing in a linear order rather than in an exponential order. The computational linearity of the proposed algorithm comes by employing the TM technique that avoids considering all the decision variables and thus provides much better performance over the ND and the ML algorithms. Fig. 4 shows the computational complexities of three different algorithms for a network consisting of 500 users. As we increase the number of users in the system, more transformation matrixes will be used to determine that which coordinate(s) or decision region(s) within the constellation diagram is most likely to produce errors. Only the selected coordinate(s) or decision region(s) will be considered and thus reduce the amount of computations performed by the receiver.

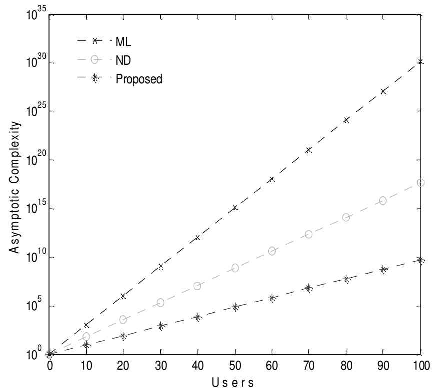

Figure 3. The asymptotic computational complexities versus intermediate number of users

B. Performance Analysis of SNR

MAI causes SNR degradation for a particular value of E_b/N_o. SNR degradation depends on the number of user. An increase in *K* would degrade the performance because it would increase the cross correlation between the received signals from all the users. Mathematically, we can express this as: $K \propto$ MAI $\propto$ high BER $\propto 1/SNR$. However, a large increase in value of *K* causes MAI to reach its peak value which in turns limits the divergence of SNR for the proposed algorithm.

Recall that the first two terms of (30) which can be used to approximate SNR for a desirable BER performance for *k* users. Specifically, we use the first

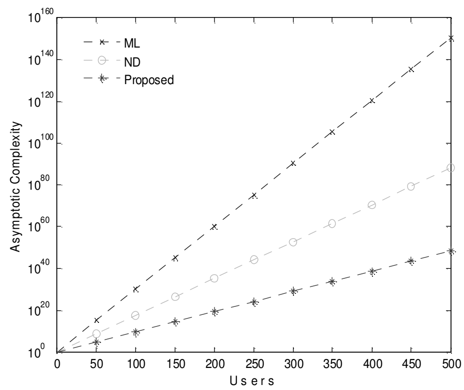

Figure 4. The asymptotic computational complexities versus large number of users

term of (30) to approximate the SNR values with respect to k users for a desirable BER performance. Moreover, we use the second term of (30) to achieve an average variance of the MAI over all possible operating conditions. In our simulation for both scenarios, we use one (i.e., C = 1) as a normalization factor that remains same for all the investigated algorithms. The choice of a small value of σ^2 is entirely based on the load of the coverage area and it is selected through a random process for a certain range of users.

Lightly-Loaded Network: When system has K = 52 users, the divergence rate for the proposed algorithm increases (see Fig. 5). It can be clearly observed from Fig. 5 that the linear increase in SNR for the proposed TM algorithm is more uniform and smoother over the ND and the ML algorithms. Furthermore, the importance of variance can not be ignored since Fig. 5 clearly depicts that a random amount of variance is more affected on the ND and the ML algorithms than on the proposed algorithm. This is because both ML and the ND algorithms have comparatively larger complexity-domains which take more time to perform required iterations to detect the received signals and thus give more time to variance to effect comprehensively on the received SNR. Moreover, for a lightly-loaded network, it can be expected that the selection of variance within the specified range does not meet the threshold value. The random amount of variance is more likely unstable for a lightly-loaded network than in a heavily-loaded network and thus may cause a serious degradation in the values of SNR.

Heavily-Loaded Network: In Fig. 6, it can be seen that the ND algorithm comparatively gets high values of SNR than the ML algorithm for a heavily-loaded network (typically when K > 55) when compare to a lightly-loaded network. This is because the computational complexity for a heavily-loaded case is much greater than for a lightly-loaded case. This forces both ML and the ND algorithms to minimize the factor of divergence and hence maximize the convergence. Since we assume that the selection of variance is random within the specified range, it remains stable after a certain value of

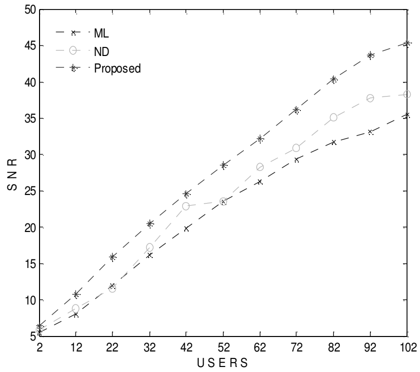

Figure 6. Approximate value of SNR (dB) versus number of users (K =102) with a random amount of variance

K that limits the divergence of SNR. Another important point that can be observed from Fig. 6 is that the graph for the proposed algorithm converges to approximately 45 dB after 100 users and only a slight increase in the value of SNR can be expected for very large values of K.

C. Analysis of PG for the Proposed Algorithm

In (30), we present an approximation of PG and the MAI for the characteristic function involving both the Q function and the SNR. MAI has a Gaussian-like shape that decays exponentially with respect to the non-faded amplitude of the k_{th} signal as shown in Fig. 7. The results of Fig.7 shows that the second term of (30) are extremely well behaved in the sense that they are smooth, strictly non negative, and decay exponentially due to the higher values of PG. In harmony with our expectations, as the number of users, K, increased, the PG of the system degraded as shown in Fig.8. This degradation in the PG is caused due to the decrease in SNR which consequently degrades the rate at which MAI diverges with respect to the PG. However, the performance degradation of the MAI was small compared to the increase in K.

D. Performance Analysis of BER

The standard performance criterion in digital communications is the probability of BER. Some *voice-band modem* applications, such as the transfer of

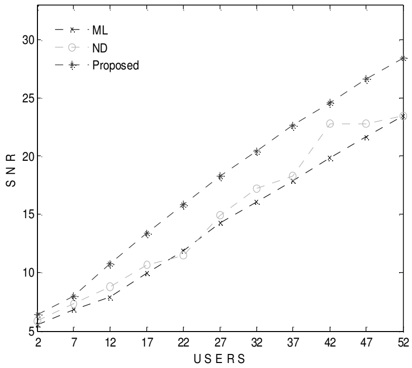

Figure 5. Approximate value of SNR (dB) versus number of users (K =52) with a random amount of variance.

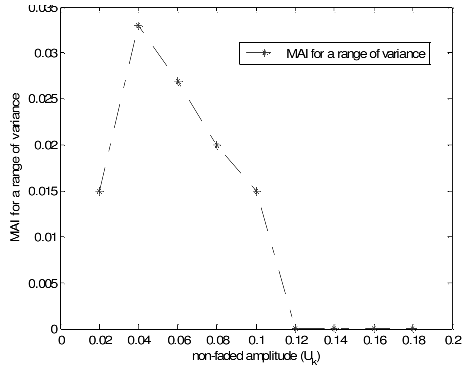

Figure 7. MAI for a range of σ^2 with K=5, and SNR=14 dB, N is computed using the first term of (30)

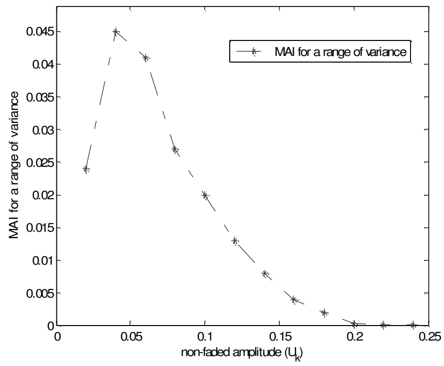

Figure 8. MAI for a range of σ^2 with K=10, and SNR=12 dB, *N* is computed using the first term of (30)

financial data, permit error rates no greater than 10^{-5}, whereas other applications such as digitized voice in cellular networks tolerates error rates as high as 10^{-2} to 10^{-3}.

Lightly-Loaded Network: Fig. 9 and Fig. 10 show a plot of three BER versus SNR curves. These curves were plotted in an AWGN channel for a small range of users using (15). For the first few values of SNR, the ND algorithm almost approaches the ML algorithm whereas the proposed algorithm still maintains a reasonable performance difference. This can be seen in Fig. 9 that the proposed algorithm achieves less than 10^{-2} BER for SNR = 8 dB which is quite closed to the required reasonable BER performance for a voice communication system. The BER curve in Fig. 10 is calculated using the precomputed-values of SNR. Fig. 10 shows a slight improvement over the BER performance of Fig. 9 for all SNR values above 9 dB. Even for small values of SNR, the proposed algorithm gives better performance than the ML and the ND algorithms. As the value of SNR increases, the BER performance of the proposed algorithm over the ND and the ML algorithms becomes more and more substantial because the probability of having more divergent values of SNR increases.

Heavily-Loaded Network: For a heavily-loaded network, the BER performance of the proposed algorithm in an AWGN channel is shown in Fig. 11 and Fig. 12. In harmony with our expectations, as the number of users, *K*, increased, the BER performance of the proposed algorithm degraded. The BER performance of the proposed algorithm below 10 dB is almost similar to that of the ND and the ML algorithms as shown in Fig. 11. Fig. 12 shows the plot of BER versus SNR curves for synchronous DS-CDMA system for large values of *K*. We can see that as the number of users increases in the system, the BER advantage of the proposed algorithm over the ND and the ML algorithms decreases as shown in Fig. 12. The BER performance analysis for a heavily-loaded case corresponds to the fact that as the BER increases, the range of the precomputed desire values of an average SNR decreases and hence the BER performance of the proposed algorithm slightly deteriorates.

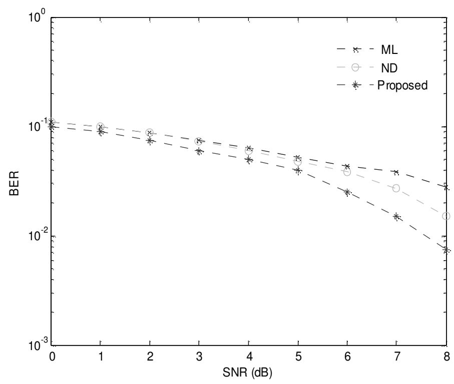

Figure 9. Comparison of BER versus SNR (dB) curves for a synchronous BPSK/DS CDMA system in a Gaussian channel for a small value of *K*.

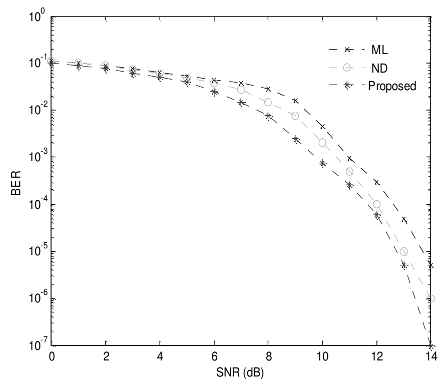

Figure 10. Comparison of BER versus SNR (dB) curves for a synchronous BPSK/DS CDMA system in a Gaussian channel for a small value of *K*.

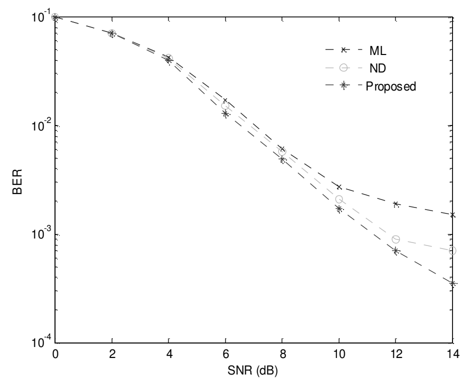

Figure 11. Comparison of BER versus SNR (dB) curves for a synchronous BPSK/DS CDMA system in a Gaussian channel for a large value of *K*.

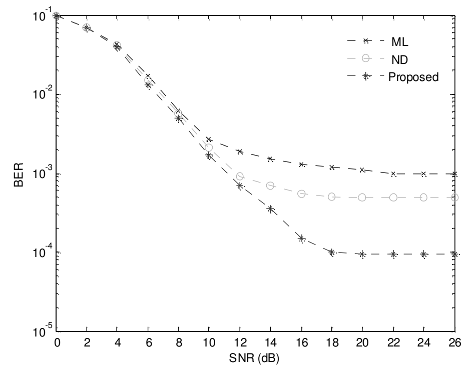

Figure 12. Comparison of BER versus SNR (dB) curves for a synchronous BPSK/DS CDMA system in a Gaussian channel for a large value of *K*.

VII. CONCLUSION

In this paper, a novel approach for reducing the computational complexity of multiuser receivers is proposed, which utilizes the Transformation Matrix technique to improve the performance of multiuser detectors. Furthermore, we present a new mathematical model for computing the values of SNR. The main advantage of the proposed mathematical model for SNR is that it guarantees that the receiver does not process the signals that have non-positive values of SNR. In order to show the consistency and the correctness of the proposed approach, we presented simulation results for computing SNR with different ranges of users. The simulation results for SNR demonstrate the consistency of the desired values required to achieve an optimal BER performance. Furthermore, we presented a quantitative analysis of PG for DS-CDMA systems. The simulation results of PG demonstrate that the unwanted signals or interference can be effectively reduced relative to the desired signal at the receiving end. In addition, we present BER results for both lightly and heavily loaded networks. The simulation results for the BER suggest that the proposed algorithm achieves better BER performance for all values of SNR than the other well-known multiuser detection algorithms.

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