A Simple Amplify-and-Forward Relaying Scheme Based on Clipping and Forwardsing for Dual-Hop Transmissions

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Abstract—In this paper, we propose a simple relaying protocol, namely clip-amplify-forward (CAF), for dual-hop transmissions. In the CAF relaying protocol, the received signal at the relay node is clipped firstly and then amplified with a fixed amplification gain before retransmission. Only a hard clipper and a linear amplifier are needed at the relay node, where no channel estimation is needed. Simulation results show that with a relatively small amplification gain, the CAF relaying protocol has very close performance to the AF relaying protocol while enjoying the simplicity of implementation. A simple design criterion for the amplification gain is also presented.

Index Terms—Clip-amplify-forward, relaying, clipping, linear amplification.

I. INTRODUCTION

The earliest work on relay channels could trace back to [1], where the author modeled a classical three-node communication system. Lower and upper bounds on the channel capacity for specific non-faded relay channels were then reported in [2]. Very recently, relayed transmission has gained tremendous research interests due to its advantage of inherent spatial diversity for wireless communications and many cooperative relaying strategies have been proposed. Among the most popular relaying strategies are decode-and-forward (DF) and amplify-and-forward (AF) [3]-[6]. In DF schemes, demodulation of the received symbol at the relay is followed by modulation with its own power constraint $P_R$. With BPSK modulation, the relay function for DF can be expressed as

$$f_{DF}(r) = \sqrt{P_R} \cdot \text{sign}(r) \tag{1}$$

where $r$ is the relay-received signal after channel phase compensation. The limitations with this protocol are: 1) complexity of decoding is involved at the relay; 2) the relay transmitted signal carries no information about the degree of uncertainty in choosing the optimal demodulated symbol. With the AF relaying protocol, the relay simply forwards the received signal after scaling it to satisfy its power constraint. The relay function for AF can be written as

$$f_{AF}(y_r) = \frac{P_R}{P_S|h_{sr}|^2 + 1} \cdot y_r \tag{2}$$

where $P_S$ is the source transmit power, $h_{sr}$ is the instantaneous channel gain of source-relay (S-R) link, and $y_r$ is the received signal at the relay. The advantage of this protocol is that the relay provides soft information to the destination. However, in order for the relay to work, 1) the relay node needs to perform channel estimation to know the S-R link channel gain (i.e., $h_{sr}$) to find the scaling factor $\sqrt{P_R/(P_S|h_{sr}|^2 + 1)}$; 2) complex amplification operation is involved at the relay node since the amplification gain changes with the channel conditions.

From a practical point of view, the benefits of cooperation are more or less offset by the cost of cooperation in terms of the required processing complexity at the relay nodes. To make relay nodes as simple as possible while retaining acceptable performance, we propose in this paper a simple relaying protocol for dual-hop transmissions, namely clip-amplify-forward (CAF), where only a hard clipper and a linear amplifier with fixed amplification gain are needed at the relay node. With this protocol, the relay node can be completely blind to the source signals as well as the channel state information (CSI). Further more, since the amplification gain is fixed, it is much easier to implement in practice. The proposed relaying protocol can be employed when the system uses constant-amplitude modulation such as phase-shift keying (PSK) and continuous phase modulation (CPM). Simulation results show that with proper clipping threshold and amplification gain, the CAF relaying protocol has very close performance to the AF relaying protocol, in addition to its simplicity of implementation.

II. DESCRIPTION OF CLIP-AMPLIFY-FORWARD

A. System Model

We consider a scheme consisting of one source node, one relay node and one destination node. For simplicity, we assume that there is no direct link between the source and destination nodes and the to-be-presented results apply to the case with direct link as well as more complex relaying schemes involving multiple hops or relays. Constant-amplitude modulation is assumed throughout this paper. We consider time-division multiplexed (TDM) transmissions. In the first time slot, source node sends data to the relay node. The received signal at the relay node is clipped firstly and then amplified with a fixed amplification gain before retransmission. Only a hard clipper and a linear amplifier are needed at the relay node, where no channel estimation is needed. Simulation results show that with a relatively small amplification gain, the CAF relaying protocol has very close performance to the AF relaying protocol while enjoying the simplicity of implementation.
node is modeled as

\[ y_r = \sqrt{P_S} h_{sr} x + N_1 \]  

(3)

where \( x \) is the source transmitted symbol with \(|x| = 1\) and \( N_1 \) is the additive noise received at the relay node. In the second time slot, the relay node performs clipping on the received signal and then amplifies it before retransmitting it as shown in Figure 1. The CAF relaying function can be written as

\[
J_{CAF}(y_r) = \begin{cases} \sqrt{P_C} \cdot \exp(j\phi) \cdot G, & \text{if } |y_r| > \sqrt{P_C} \\ y_r \cdot G, & \text{if } |y_r| \leq \sqrt{P_C} \end{cases}
\]

(4)

where \( \sqrt{P_C} \) is the clipping threshold, \( \phi \) is the phase of \( y_r \) (i.e., \( \exp(j\phi) = \frac{y_r}{|y_r|} \)) and \( G \) is the fixed amplification gain. Note that the amplifier always works in its linear range since the power of the output signal after the hard clipper never exceeds \( P_C \). Consequently, the received signal at the destination node can be modeled as

\[
y_d = \begin{cases} \sqrt{P_C} G h_{rd} \exp(j\phi) + N_2, & \text{if } |y_r| > \sqrt{P_C} \\ y_r G h_{rd} + N_2, & \text{if } |y_r| \leq \sqrt{P_C} \end{cases}
\]

(5)

where \( h_{rd} \) is the channel coefficient of relay-destination (R-D) link and \( N_2 \) is the additive noise at the destination. We assume that all the noise terms in this paper including those involved in (1) and (2) are additive zero-mean white circular complex Gaussian random variables with unity variance (i.e., \( \sigma^2 = 1 \)). Without loss of generality, we assume that all channels are subject to independent Rayleigh fading and uniform path loss, with a path loss exponent \( \alpha \). We define \( E\{|h_{sr}|^2\} = \sigma^2_{sr} \) and \( E\{|h_{rd}|^2\} = \sigma^2_{rd} \), where \( E\{\cdot\} \) denotes the expectation operator.

To decode source signals, the destination node needs to know both S-R and R-D link channel gains. The following procedure can be implemented for the destination to discover the entire network CSI. We let the source and relay nodes transmit training symbols \( T_{sr} \) and \( T_{rd} \), respectively. The destination can discover R-D link channel gain \( h_{rd} \) using training symbol \( T_{rd} \). Upon receiving \( T_{sr} \), we let the relay append the S-R received training symbol \( T_{sr} \) into the R-D transmit packet. Then the S-R gain is found by noting that when \( T_{sr} \) is received at the destination it will contain the product \( h_{sr} \cdot h_{rd} \). Since \( h_{rd} \) is known at the destination, it can be compensated for, and then \( h_{sr} \) can be found [7].

**B. Signal-to-Noise Ratio at Destination**

Now we analyze the received signal-to-noise ratio (SNR) at the destination in a CAF system. We consider the following two cases.

**Case 1 (|y_r| > \sqrt{P_C})**

In this case, we have \( y_d = \sqrt{P_C} G h_{rd} \exp(j\phi) + N_2 \). Therefore, the received signal power is given by

\[
E \left\{ \frac{\sqrt{P_S} P_C G h_{sr} h_{rd}^*}{|y_r|} \cdot \frac{\sqrt{P_S} P_C G h_{sr}^* h_{rd}}{|y_r|^*} \right\} = E \left\{ \frac{P_S P_C |h_{sr}|^2 |h_{rd}|^2 G^2}{|y_r|^2} \right\} \geq \frac{P_S P_C |h_{sr}|^2 |h_{rd}|^2 G^2}{P_S |h_{sr}|^2 + 1}.
\]

(6)

The noise power is given by

\[
E \left\{ \frac{N_1 h_{rd} \sqrt{P_C} G}{|y_r|} + N_2 \right\} \cdot \left( \frac{N_1 h_{rd}^* \sqrt{P_C} G}{|y_r|^*} + N_2^* \right) \leq \frac{P_S G^2 |h_{rd}|^2}{P_S |h_{sr}|^2 + 1} + 1.
\]

(7)

The above inequalities follow Jensen’s inequality. The instantaneous SNR at the destination is then given by

\[
\gamma_{CAF,1} \geq \frac{P_S P_C |h_{sr}|^2 |h_{rd}|^2 G^2}{P_S G^2 |h_{rd}|^2 + P_S |h_{sr}|^2 + 1}.
\]

(8)

**Case 2 (|y_r| \leq \sqrt{P_C})**

Since in this case \( y_d = G h_{rd} y_r + N_2 \), the received signal power is given by

\[
E\{\sqrt{P_S} h_{sr} h_{rd} G x\} = P_S G^2 |h_{sr}|^2 |h_{rd}|^2.
\]

(9)
And the noise power is given by
\[ E \{ (N_1 h_{rd} G + N_2) \cdot (N_1^* h_{rd}^* G + N_2^*) \} = G^2 |h_{rd}|^2 + 1. \] (10)

The instantaneous SNR at the destination in this case is thus given by
\[ \gamma_{CAF,2} = \frac{P_S G^2 |h_{sr}|^2 |h_{rd}|^2}{G^2 |h_{rd}|^2 + 1}. \] (11)

Recall that the destination SNR in an AF relaying system is given by [8]
\[ \gamma_{AF} = \frac{P_S P_R |h_{sr}|^2 |h_{rd}|^2}{P_R |h_{rd}|^2 + P_S |h_{sr}|^2 + 1}. \] (12)

III. CHOOSING PROPER G AND PC

From (12), the outage probability of an AF relaying system is given by
\[ P_{o,AF} = P \left( \gamma_{AF} < 2 \right) \approx \frac{2 \theta}{P_R \sigma_{sr}^2} + \frac{\gamma_{th}}{P_R \sigma_{rd}^2}. \] (13)

where \( \gamma_{th} \) is the outage threshold and \( P(A) \) denotes the probability of event \( A \).

Similarly, the outage probability of a CAF system can be shown to be
\[ P_{o,CAF} = P_{o,CAF,1} + P_{o,CAF,2} \] (14)

where \( P_{CAF,1} \) is due to \(|y_r| > \sqrt{P_C} \) and \( P_{CAF,2} \) is due to \(|y_r| \leq \sqrt{P_C} \). It is very difficult to derive the exact outage probability of CAF relaying. However, we can derive its upper bound in closed-form. It can be shown that the upper bound of \( P_{CAF,1} \) is given by [8]
\[ P_{CAF,1} = P \left( \gamma_{CAF,1} < \gamma_{th}, |y_r| > \sqrt{P_C} \right) \approx \frac{2 \theta S}{P_C \sigma_{sr}^2} + \frac{\gamma_{th}}{P_C G^2 \sigma_{rd}^2}. \] (15)

The upper bound of \( P_{CAF,2} \) is given by
\[ P_{CAF,2} = P \left( \gamma_{CAF,2} < \gamma_{th}, |y_r| \leq \sqrt{P_C} \right) \approx \frac{2 \theta S}{P_C \sigma_{sr}^2} + \frac{\gamma_{th}}{P_C G^2 \sigma_{rd}^2}. \] (16)

where we have used the approximation \( P(|y_r| < \sqrt{P_C}) \approx \frac{P_C}{P_C \sigma_{sr}^2 + 1} \). The explanation is given as follows. It follows from (3) that \( y_r = \sqrt{P_C} h_{sr} x + N_1 \) is distributed as \( CN(0, P_C \sigma_{sr}^2 + 1) \) since \( h_{sr} \) and \( N_1 \) are zero-mean complex Gaussian random variables. Therefore, \( |y_r|^2 \) is a Chi-square random variable with two degrees of freedom and is exponentially distributed with rate parameter \( 1/(P_S \sigma_{sr}^2 + 1) \). As a result, we have \( P(|y_r| < \sqrt{P_C}) = 1 - \exp(-P_C/2 \sigma_{sr}^2) \approx \frac{P_C}{P_C \sigma_{sr}^2 + 1} \), where we have used \( 1 - \exp(-x) \approx x \) for small \( x \).

The second probability term in (16) can be approximated as [9]
\[ P \left( \frac{P_S G^2 |h_{sr}|^2 |h_{rd}|^2}{2} < \gamma_{th} \right) \approx \frac{2 \theta S}{P_C G^2 \sigma_{sr}^2 \sigma_{rd}^2}. \] (17)

It thus follows that
\[ P_{CAF,2} \approx \frac{2 \theta S}{P_C \sigma_{sr}^2} + \frac{2 \theta S}{P_C G^2 \sigma_{sr}^2 \sigma_{rd}^2} \frac{2 \gamma_{th}}{P_C \sigma_{sr}^2 + 1} \] (18)

where we have used \( \ln(x) \approx \sqrt{x} \) for large \( x \).

In order to achieve the similar performance as the AF relaying protocol, we must choose appropriate \( G \) and \( PC \). For a fair comparison, we set \( P_C G^2 = P_R \) to keep the transmit power at the relay node in the CAF system no more than \( P_R \) in the AF system. In fact, it is easy to show that the average relay transmit power with CAF protocol is less than \( P_R \). In this case, it is obvious that \( P_{CAF,1} < P_{o,AF} \). To have comparable \( P_{o,CAF} \) and \( P_{o,AF} \), one can choose small \( P_C \), or equivalently large \( G \), to mitigate the effect of \( P_{CAF,2} \).

For example, if we choose \( \gamma_{th}/P_C \sigma_{sr}^2 \leq \frac{1}{10} \) and \( \gamma_{th}/P_C G^2 \sigma_{sr}^2 \sigma_{rd}^2 \leq \frac{1}{10} \), or equivalently,
\[ G \geq \max \left( \frac{20 P_R}{P_S \sigma_{sr}^2}, \frac{1}{P_C \sigma_{sr}^2}, \frac{20 P_C}{P_S \sigma_{sr}^2} \right). \] (19)

we will have \( P_{CAF,2} < \frac{1}{10} P_{o,AF} \). It leads to \( P_{o,CAF} = P_{CAF,1} + P_{CAF,2} < 1.1 P_{o,AF} \). Since (15) and (16) are loose upper bounds, we may expect that the CAF system actually has very close performance to the AF system.

Using (19), the minimum possible \( G \) values under various channel conditions are listed in Table I. It is interesting to find that we can choose a relatively small amplification gain (e.g., \( G \leq 10 \)) in most cases.
IV. SIMULATIONS AND DISCUSSIONS

We performed some experiments to compare the proposed CAF relaying protocol with the AF relaying protocol. The source-destination distance is normalized to be 1 and we let \( d \) denote the S-R distance. In all the simulations, we set \( \alpha = 4 \) and BPSK or QPSK modulation is used accordingly. Note that general MPSK and CPM modulations can also be employed. All the results below are obtained using Monte Carlo simulations, averaging over a large number of samples of channels.

We first consider a fixed relay location of \( d = 0.5 \) and vary the source-relay power ratio \( (P_S/P_R) \). According to Table I, we choose \( G = 1 \) for \( P_S/P_R = 10 \) and \( P_S/P_R = 4 \), \( G = 3 \) for \( P_S/P_R = 1 \), and \( G = 8 \) for \( P_S/P_R = 1/6 \). Figure 2 compares the SER performance of different relaying protocols versus total transmit power \( (P_T) \), which is defined as \( P_T = P_S + P_R \) with BPSK modulation. From the figure we can see although the CAF relaying has significantly reduced complexity at the relay node (no channel estimation needed) and is much simpler to implement in practice (fixed-gain amplification), it has almost the same performance as the AF relaying.

In the case when the relay has no CSI at all, an alternative fixed gain AF relaying scheme with relay function \( Y_R = \sqrt{P_R/(P_S\sigma^2_{sr})} \cdot Y_s \) is usually used. With this scheme, a long-term power constraint is satisfied at the relay node for ergodic fading channels. Figure 3 compares the CAF scheme with the traditional AF relaying and the fixed gain AF relaying, where QPSK modulation is adopted. We can see that the CAF and traditional AF schemes still have very close performance. At low average SNRs, the fixed gain AF relaying has slightly better performance than the CAF and traditional AF schemes, while both the CAF and traditional AF schemes outperform the fixed gain AF relaying scheme at medium to large average SNRs.

Next, we fix the source-relay power ratio to \( P_S/P_R = 1 \) and vary the relay location. Again, according to Table I, we choose \( G = 1 \) for \( d = 0.1 \) and \( d = 0.3 \), \( G = 3 \) for \( d = 0.5 \), and \( G = 10 \) for \( d = 0.8 \). From Figure 4, where BPSK modulation is adopted, we observe that the CAF relaying achieves the similar performance as the traditional AF relaying as long as a proper amplification gain is chosen. Note that the CAF relaying actually consumes less relay power than the traditional AF relaying.

The advantages of the proposed CAF relaying protocol are manifold: 1) the hard clipper bypasses the need of channel estimation to normalize the received signals at the relay node; 2) only a simple amplifier with a fixed amplification gain \( G \) is needed at the relay node; 3) the maximum power of the output signals after the hard clipper never exceeds \( P_C \), which ensures the amplifier to work in its linear range and thus there is no need to worry about saturation as discussed in [10]; 4) With relatively small amplification gains \( (G \leq 10) \), the CAF protocol achieves almost the same performance as the traditional AF protocol.

V. CONCLUSION

In this paper, we proposed a simple relaying strategy, namely clip-amplify-forward, which requires no channel estimation at the relay node. Moreover, only a simple amplifier with a constant amplification gain is needed at the relay node. The proposed relaying protocol can
work with constant-amplitude modulation schemes such as PSK and CPM. Simulation results show that with appropriate values of clipping threshold and amplification gain, the proposed CAF relaying protocol has very close performance to the AF relaying protocol under various channel conditions.

REFERENCES


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