Cross-Layer Design of Combining HARQ with Adaptive Modulation and Coding for Nakagami-m Fading Channels

Cui-Qin Dai, Yu Rao, Li Jiang, Qian-Bin Chen, Qiong Huang
Key Laboratory of Mobile Communication, Chongqing University of Posts and Telecommunications
Chongqing 400065, China
E-mail: daicq@cqupt.edu.cn

Abstract—In this paper, we propose a cross-layer design which combines adaptive modulation and coding (AMC) at the physical layer with a hybrid automatic repeat request (HARQ) protocol at the data link layer to minimize the packet error rate (PER) and maximize the spectral efficiency. By using Markov model to predict the channel condition of next frame according to the second order statistics characteristics of the former frame, we calculate the conditional probability density function and joint probability density function of signal to noise ratio (SNR). Further, we derive the packet error rate and average spectral efficiency in closed-form for transmission over Nakagami-m fading channels. Numerical results show that the proposed scheme can obtain lower PER and higher spectral efficiency compared with traditional HARQ protocols. The cross-layer design scheme combining type-III HARQ with AMC can achieve the highest spectral efficiency and the lowest PER than type-I HARQ and type-II HARQ.

Index Terms—HARQ, AMC, Nakagami-m Fading Channel, Cross-Layer Design, Markov Model

I. INTRODUCTION

Link adaptation technologies [1-2] use the instantaneous channel state information (CSI) to adaptively control the data transmission of wireless channel, maintain constant transmission power to reduce the interference of other users and satisfy different business’ needs, and save resources to improve overall throughput for system. In addition, the adaptive system can easily provide services with different qualities, such as higher information transmission rate, lower packet error rate, and higher spectral efficiency. Link adaptation technologies mainly include adaptive modulation and coding (AMC) [3-5, 8-18] at the physical layer (PHY) and hybrid automatic repeat request (HARQ) [4-6, 8-18] at the data link layer (LL). The AMC schemes can be used to match transmission rates to time-varying channel, in order to obtain maximum throughput and improve spectral efficiency. HARQ is the combination of forward error-correcting coding (FEC) and automatic repeat request (ARQ), it can be used to improve reliability.

In order to improve throughput and reliability, we need to consider the link adaptation (LA) techniques not only at the physical-layer, but also at the upper-protocol-layers such as data-link layer when designing the wireless networks. Cross-layer design which combines AMC at the physical layer with HARQ protocol at the data link layer is discussed in [7], in order to improve reliability and spectral efficiency. In [8], a cross-layer design which combines AMC with a truncated ARQ protocol is analyzed over Nakagami-m fading channels, the packet error rate and delay are calculated to prove the effectiveness of cross-layer design. In [9], analytical expressions of packet error rate, throughput, and average packet delay are derived based on queuing theory, further an optimization problem to maximize the throughput is formulated, and the numerical method is used to solve the optimization problem. In [10], an HARQ scheme with AMC is proposed to reduce packet error rate based on queuing analysis with the effective bandwidth function. A cross-layer link adaptation design for cooperative ARQ is presented in [11], the optimized adaptation solution is used to maximize spectral efficiency and minimize packet error rate. A cross-layer heuristic scheduling policy of multi-user wireless system is proposed in [12], and the analytical expressions of average delay, packet error rate and throughput. In [13], the packet error rate of a HARQ scheme with AMC is analyzed over the Rayleigh channel. The same work in [14] further studies the scheme to maximize the spectral efficiency. In [15], the packet loss rate and average spectral efficiency of cross-layer design over Nakagami-m fading channel are derived. In [16], AMC is used to the transmission of resource allocation message in the system that supporting truncated ARQ, it shows that using AMC to transmit control message is a good way to reduce control overhead. In [17], the joint effects of finite-length queuing and AMC in cross-layer design system are analyzed, and the packet loss rate, the average throughput, and the average spectral efficiency are derived through the discrete Markov chain. Furthermore, in [18], a cross-layer multidimensional discrete-time Markov chain based queuing model is developed, and closed-form expressions of throughput, average packet delay, and packet loss rate are derived, and the optimization problem of maximizing system throughput is formulated. These studies assume that the channel condition at one frame is predicted by the second order statistics characteristics of the former frame, the system is difficult to obtain the real-time channel status information.
In this paper, we proposed a cross-layer design of HARQ at LL with AMC at PHY over Nakagami-m fading channel, the channel condition of next frame is predicted by the second order statistics characteristics of the former frame, and we built status information model to derive the closed form expression of packet error rate and average spectral efficiency. Then, we analyzed the simulation results of type-I, II, III HARQ, and we proved that our proposed scheme is better than the traditional one.

II. SYSTEM MODEL AND ASSUMPTIONS

In this paper, we consider a point-to-point wireless packet communication system consisting one source node and one destination node. The system model is shown in Fig. 1. In the figure, the source node and the destination node are installed with single antenna respectively. The HARQ protocol with AMC is used over the Nakagami-m fading channel, the channel gain is assumed to be constant during the transmission of one frame, and it varies during the transmission of the next frame.

For the point-to-point wireless communication system, the data unit is assumed to be one frame, and each frame contains a fixed number of symbols. The receiving buffer can accurately detect the received signal-to-noise ratio (SNR) and decide the next modulation scheme, then feed back channel state message to the source node, on the other hand, if the destination node can not decode the received data packet correctly, it feeds back not-acknowledge (NACK) signaling to the source node, otherwise, it feeds back acknowledge (ACK) signaling, and the transmission of all the signaling messages are assumed to be error free, the maximum transmission number is denoted as \( N_{\text{max}} \).

![Fig. 1. System model for point-to-point transmission](image)

The probability density function (PDF) of the received SNR \( \gamma \) over Nakagami-m fading is written as

\[
P_\gamma(\gamma) = \frac{\gamma^{m-1} e^{-\gamma/m}}{\Gamma(m)\Omega^m} \text{exp}\left(-\frac{\gamma}{\Omega}\right),
\]

Where \( \gamma \) is the received instantaneous SNR, \( \Omega = E(\gamma) \) is the received average SNR, \( m \) (\( m \geq 1/2 \)) is Nakagami-m fading parameter, when \( m=1 \), the channel is Rayleigh distribution, when \( m=\infty \), the channel is Gaussian distribution. \( \Gamma(m) = \int_0^\infty e^{-t} t^{m-1} dt \) is the gamma function and \( \Gamma(m,z) = \int_z^\infty e^{-t} t^{m-1} dt \) is the incomplete gamma function.

According to the current channel state information, we establish the Markov model, and select the appropriate modulation mode by assessing the channel state. We consider a variable rate transmission scheme which utilizes a discrete-rate \( M_{\text{QAM}} \) modulation. The channel state are divided into \( N+1 \) regions, at the same time, the received SNR is divided into \( N \) non-overlapping consecutive intervals \( [\Gamma_i, \Gamma_{i+1}) \), \( i=0, \ldots, N \), where \( \Gamma_0 = 0 \) and \( \Gamma_{N+1} = \infty \). When the SNR falls into the interval \( \Gamma_i \), where \( \Gamma_i \in S \), \( S = \{ \text{State}_0, \text{State}_1, \ldots, \text{State}_{N} \} \), the time duration of one frame with mode \( M_i \) is written as

\[
T_i^{(f)} = \frac{L_c}{R_{n_i} R_{s_i}}
\]

where \( R_{n_i} \) (bit/symbol) denotes the transmission rate of mode \( M_i \), and \( R_{s_i} \) denotes the symbol rate per second, \( L_c \) is the length of one packet.

We use a finite discrete-time markov chain to describe the fading channel, as is illustrated Fig.2, no data is sent when \( \gamma \in [0, \Gamma_1) \), which corresponds to the outage mode, when the channel is in \( \text{State}_i, i=1, \ldots, N \), the corresponding SNR is \( \gamma \in [\Gamma_i, \Gamma_{i+1}) \), and \( M_i \) is selected as the transmission mode. The probability for selecting mode \( M_i \) is derived as

\[
P_i = \frac{\int_{\Gamma_i}^{\Gamma_{i+1}} P_{\gamma}(\gamma) d\gamma}{\int_{\Gamma_0}^{\Gamma_{N+1}} P_{\gamma}(\gamma) d\gamma} = \frac{\int_{\Gamma_i}^{\Gamma_{i+1}} P_{\gamma}(\gamma) d\gamma}{\int_{\Gamma_0}^{\Gamma_{N+1}} P_{\gamma}(\gamma) d\gamma} = \frac{\Gamma_{i+1}(m, m\Gamma_{i+1}) - \Gamma_{i+1}(m, m\Gamma_i)}{\Gamma(m)},
\]

where \( i = 0, \ldots, N \), \( \Omega \) is the average received SNR, and \( \Omega \geq 0 \).

Each state transition probability is given by

\[
\begin{align*}
P_{ij} &= \sum_{j=1}^{N} P_{ij} \quad i = 0, \ldots, N - 1, \\
\end{align*}
\]

where \( N(\Gamma_i) [19] \) is the level of passing rate of the received signal in the threshold range of SNR.

![Fig. 2. State transition diagram](image)
$$\text{PER}(M_t, \gamma) = \begin{cases} a_{M,t} \exp(-g_{M,t} \gamma), & 0 \leq \gamma \leq \Gamma_{pM_t} \\ 0, & \gamma > \Gamma_{pM_t} \end{cases},$$  
(5)

where $a_{M,t}$, $g_{M,t}$ and $\Gamma_{pM_t}$ are dependent on the AMC modes, $\text{PER}(M_t, \gamma)$ represents the instantaneous packet error rate. In order to ensure the continuity of the function, the mode threshold $\Gamma_{pM_t}$ is defined as

$$a_{M,t} \exp(-g_{M,t} \Gamma_{pM_t}) = 1,$$  
(6)

When the mode $M_t$ is selected, the expression of $\text{PER}$ for type-I HARQ is derived as

$$\text{PER}(M_t, \gamma) = a_{M,t,0} \exp(-g_{M,t} \gamma),$$  
(7)

According to (6), the intervals of different transmission modes for type-I HARQ can be figured out

$$\gamma = \frac{1}{g_{M,t,0}} \ln\left(\frac{a_{M,t,0}}{\Gamma_{\text{loss}/N_{\max}}}ight),$$  
(8)

Similarly, the expressions of $\text{PER}$ for the type-II and type-III HARQ are derived as

$$\text{PER}(M_t, \gamma, N_{\max}) = \frac{a_{M,t,0} \Gamma_{\text{loss}/N_{\max}}}{\Gamma_{pM_t}},$$  
(9)

The intervals of different transmission modes for the type-II and type-III HARQ can be figured out

$$\gamma = \frac{1}{g_{M,t}(N_{\max})} \ln\left(\frac{a_{M,t}(N_{\max})}{\Gamma_{\text{loss}/N_{\max}}}ight),$$  
(10)

and

$$g_{M,t}(N_{\max}) = g_{M,t,0} + \ldots + g_{M,t, N_{\max}-1}/N_{\max},$$  
(11)

$$a_{M,t}(N_{\max}) = g_{M,t,0} \ldots g_{M,t, N_{\max}-1},$$  
(12)

where $a_{M,t}$ and $g_{M,t}$ for type-II and type-III HARQ can be found in table 1 and table 2, the index $t$ represents the number of retransmission, $t=0,1,2,3, M_t$ represents the AMC mode, $t=1,2,3,4,5,6$. $N_{\max}$ can be determined by dividing the maximum allowable system delay over the round trip delay required for each transmission. If a packet is still erroneous after $N_{\max}$ transmissions, it will be dropped, and packet loss will be declared.

**Table 1. Transmission modes for type-I and type-II HARQ**

<table>
<thead>
<tr>
<th>Type/III</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
<th>Mode 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indices</td>
<td>Rb</td>
<td>GB8K</td>
<td>QPSK</td>
<td>16QAM</td>
<td>64QAM</td>
<td>64QAM</td>
</tr>
<tr>
<td>$a_{t,0}$</td>
<td>0.15259</td>
<td>0.150</td>
<td>0.153</td>
<td>0.300</td>
<td>0.300</td>
<td>0.45</td>
</tr>
<tr>
<td>$a_{t,1}$</td>
<td>0.0959</td>
<td>0.0959</td>
<td>0.0959</td>
<td>0.0959</td>
<td>0.0959</td>
<td>0.0959</td>
</tr>
<tr>
<td>$a_{t,2}$</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
</tr>
<tr>
<td>$a_{t,3}$</td>
<td>0.0423</td>
<td>0.0423</td>
<td>0.0423</td>
<td>0.0423</td>
<td>0.0423</td>
<td>0.0423</td>
</tr>
<tr>
<td>$a_{t,4}$</td>
<td>0.0102</td>
<td>0.0102</td>
<td>0.0102</td>
<td>0.0102</td>
<td>0.0102</td>
<td>0.0102</td>
</tr>
<tr>
<td>$a_{t,5}$</td>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

As the wireless channel is time-varying, we can not directly calculate the packet error rate based on the current state of the received information, therefore, we use the channel state which is predicted by the second order statistics characteristics of the former frame. The packet error rate of the next state is derived as

$$\text{PER} = \int_0^\infty \text{PER}(M_t, \gamma) P_{\gamma_t/\gamma_t}(\gamma_t | \gamma) d\gamma_t,$$  
(13)

The conditional probability density function of $\gamma$ is given by

$$P_{\gamma_t/\gamma_t}(\gamma_t | \gamma) = P_{\gamma_t}(\gamma_t | \gamma) \frac{\gamma_t}{\Gamma_t},$$  
(14)

The joint probability density function of $\gamma$ is given by

$$P_{\gamma_t,\gamma}(\gamma_t, \gamma) = P_{\gamma_t}(\gamma_t) P_{\gamma}(\gamma) \sum_{m=0}^\infty a_m(m)_a \delta^{2a}$$

$$\times L_a^{(m-1)}(m \gamma_t) \frac{(m)_a}{\Omega^2} \times L_a^{(m-1)}(m \gamma) \frac{(m)_a}{\Omega^2}$$  
(15)

where $a$, $\delta$ can be found in [20], $L_a^{(m-1)}(x)$ is the generalized Laguerre polynomial, which is formulated as

$$L_a^{(m-1)}(x) = \sum_{j=0}^m (-1)^j \frac{(a+j)!}{j!} \frac{x^j}{j!},$$  
(16)

and the Pochhammer symbol is given by

$$(x)_n = x(x-1)(x-2) \ldots (x-n+1) = \frac{x!}{(x-n)!}.$$  
(17)

Putting (14) and (15) into the expression (13), we can rewrite the $\text{PER}$ as

$$\text{PER} = \int_0^\infty \text{PER}(M_t, \gamma) P_{\gamma_t}(\gamma_t) \sum_{m=0}^\infty a_m(m)_a \delta^{2a}$$

$$\times \frac{L_a^{(m-1)}(m \gamma_t)}{\Omega^2} \times \frac{L_a^{(m-1)}(m \gamma)}{\Omega^2} d\gamma_t.$$  
(18)

The average $\text{PER}$ for transmission mode $n$ can be derived as

$$\text{PER} = \frac{1}{\gamma_n} \int_0^\infty \text{PER} \ P_{\gamma}(\gamma) d\gamma,$$  
(19)

Putting (18) into (19), we can obtain
The average number of transmission can be derived as:

\[
N(N_{\text{max}}; \bar{P}) = 1 + \bar{P} + \bar{P}^2 + \ldots + \bar{P}^{N_{\text{max}}-1} = \frac{1 - \bar{P}^{N_{\text{max}}}}{1 - \bar{P}},
\]

where \(\bar{P}\) is the average PER, it can be gotten in (21).

As a result, the average spectral efficiency cab be formulated as

\[
\overline{S}_e(N_{\text{max}}) = \begin{cases} 
0, & i = 0 \\
\frac{1}{N(N_{\text{max}}; \bar{P})} \sum_{n=1}^{N_{\text{max}}} R_n \pi_i, & \text{other}
\end{cases}
\]

\(\text{State}_0\) is the outage mode, so we only consider the situation \(i = 1, \ldots, N\), when analyzing the spectral efficiency.

IV. PERFORMANCE ANALYSIS

In this section, we present numerical results and make comparisons about PER and spectral efficiency of the proposed cross-layer design schemes with AMC and three types of HARQ. For the convenience of illustration, we assume the related parameters of AMC mode are accurate.

Fig.3, Fig.4 and Fig.5 illustrate the PER for different cross-layer design schemes combining AMC with type-I, II, III HARQ, the PER for traditional HARQ is also provided as comparison. For all the three schemes, we set \(N_{\text{max}} = 4\) and \(m = 1\). From the figure, we can find that the PER of the proposed cross-layer scheme is lower than the one of the traditional HARQ scheme. When the PER of the system is fixed on \(10^{-5}\), the SNR of the traditional type-I, II and III HARQ schemes is 10.94dB, 7.41dB and 6.61dB respectively, while the SNR of the proposed cross-layer scheme can be reduced to 8.67dB, 6.72dB and 5.67dB.

Fig. 6 depicts the PER of type-I, II, III HARQ with \(N_{\text{max}} = 4\), \(m = 1\). We can see that the type-III HARQ has the lowest PER, and the PER of type-II HARQ is lower than the one of type-I HARQ in the case of the same SNR. Because the retransmission data of type-III HARQ not only contains the redundant bits, but also contains the original bits, as a result, the type-III HARQ can decode data message by self. The differences of PER among three types of HARQ in all modes depend on the transmission mode. Comparing with mode 4 and 5, we can see that the differences of PER among type-I HARQ protocols in mode 1, 2 and 3 are smaller, the reason is that mode 4 and 5 use the QAM modulation, while mode 1, 2 and 3 use BPSK or QPSK modulation. With the same binary number and the average power, QAM has stronger anti-interference ability than BPSK or QPSK.
type-III HARQ has the highest one, type-I HARQ has the lowest one, and type-II HARQ has a compromise one between them. The main reason is the differences among the three types of HARQ. The type-I HARQ uses only one code with a fixed coding rate. For a data message, both the information bits and all redundancy bits are transmitted at every transmission attempt. The type-II HARQ uses correction code with low rate. At the first transmission, the data message contains data bits and none or a few parity bits for error correction. The incremental blocks containing redundancy bits are transmitted according to retransmission requests. The destination combines the transmitted and retransmitted blocks together to form a more powerful error correction code to recover the information. Based on type-II HARQ, the redundancy bits of type-III HARQ can self-decoded, and it can adjust the coding rate according to the channel status, in order to achieve the optimal information transmission rates.

Fig. 7 depicts the PER of three types of HARQ with AMC in mode 3, \( N_{\text{max}} = 1, 2, 3, 4 \). We can find that when \( N_{\text{max}} = 1 \), the PER of type-I, II, III HARQ keeps in the same line, because the data message can be received successfully at the first transmission. When \( N_{\text{max}} > 1 \), the type-III HARQ has the lowest PER, while the type-I HARQ has the highest PER. When the SNR is fixed to \( 8 \)dB, the PER of type-III HARQ is \( 2.5023 \times 10^{-5}, 1.8794 \times 10^{-7} \) and \( 3.9883 \times 10^{-8} \) corresponding to \( N_{\text{max}} = 2, 3, 4 \). Obviously, the increment of \( N_{\text{max}} \) can lead to the decrement of the PER, which is the same with type-I and type-II HARQ.

Fig. 8 depicts the PER of three types of HARQ with AMC in mode 3, \( N_{\text{max}} = 4 \) and \( m = 1, 5, 10 \). From the figure, we can see that the increment of \( m \) could lead to the decrement of the PER at the same SNR. For the same type of HARQ, when the channel parameter \( m \to \infty \), the channel condition approximates to AWGN channel.

Fig. 9 plots the average spectral efficiency of type-I, II, III HARQ with \( N_{\text{max}} = 2 \) and \( m = 1 \). We can find that comparing the spectral efficiency of three types of HARQ, type-III HARQ has the highest one, type-I HARQ has the lowest one, and type-II HARQ has a compromise one between them. The main reason is the differences among the three types of HARQ. The type-I HARQ uses only one code with a fixed coding rate. For a data message, both the information bits and all redundancy bits are transmitted at every transmission attempt. The type-II HARQ uses correction code with low rate. At the first transmission, the data message contains data bits and none or a few parity bits for error correction. The incremental blocks containing redundancy bits are transmitted according to retransmission requests. The destination combines the transmitted and retransmitted blocks together to form a more powerful error correction code to recover the information. Based on type-II HARQ, the redundancy bits of type-III HARQ can self-decoded, and it can adjust the coding rate according to the channel status, in order to achieve the optimal information transmission rates.

Fig. 10 depicts the spectral efficiency of type-I and II HARQ with AMC, \( N_{\text{max}} = 2, 3, 4 \) and \( m \equiv 1 \). When \( N_{\text{max}} = 2, 3, 4 \), the optimal spectral efficiency of type-I HARQ is \( 1.9063, 2.0276 \) and \( 2.0908 \), respectively. While the optimal spectral efficiency of type-II HARQ is \( 2.0065, 2.1816 \) and \( 2.2505 \), respectively. Comparing with type-I HARQ, the spectral efficiency of type-II HARQ is higher at the same SNR. As we know, the higher modulation corresponds to the higher spectral efficiency. One reason is that \( P = P_{\text{loss}}^{1/N_{\text{max}}} \) increases with the increment of \( N_{\text{max}} \). The other reason is that the type-II HARQ can combine the received data message to decode, which makes the decrement of the requirement of PER. For the PER at the PHY is constant, and each transmission rate is fixed at different retransmissions, the performance improves not obviously. So we could not
merely depend on the increment of \(N_{\text{max}}\) to get a high spectral efficiency. In low SNR, the system is in the outage mode, and the spectral efficiency becomes smaller. When SNR is enough small, the spectral efficiency achieve to zero.

V. CONCLUSIONS

In order to satisfy the PER constraints, as well as to maximize the spectral efficiency, we have analyzed the performance of type-I, II, III HARQ with AMC over Nakagami-m fading channel. Using the Markov chain, we performed the simulation results to evaluate the performance of type-I, II, III HARQ with AMC.

Our results indicated that the PER of the proposed cross-layer scheme was lower than the traditional one. And the application of the type-III HARQ could achieve highest spectral efficiency and lowest PER. The type-I HARQ had the lowest spectral efficiency and the highest PER. Meanwhile the type-II HARQ was a compromise between them. In addition, the increment of the maximum number of transmission could lead to the decrement of the requirement of the PER and the increment of the spectral efficiency

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