Channel Estimation and Interference Cancellation for OFDM Systems Based on Total Least Squares Solution

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***Abstract*[1]—In this paper, we study the effects of interference to channel estimation and present an iterative total least-squares algorithm for channel estimation by using the conjugate gradient method in time-varying multipath fading channels. The Doppler shift of fast-fading channels will generate inter-carrier interference (ICI), hence, degrade the performance of orthogonal frequency-division multiplexing (OFDM) systems. Furthermore, when the length of channel impulse response (CIR) overstep the duration of cyclic prefix (CP), which will cause inter-symbol interference (ISI) and degrade system performance severely. Conventional channel estimation and equalization schemes, if applied to this case of insufficient CP, suffer significant performance degradation. By ISI and ICI analysis, an Interference elimination technique is proposed for channel estimation. After analyzing perturbation of least squares problems, we present total least-squares (TLS) scheme to eliminate the ICI, ISI and noise. We present a short theoretical overview of the TLS problem. In our estimation, residual ISI and ICI are not being handled as a noise. Since the introduction of interference, the operation of matrix becomes perplexed. We implement the Conjugate gradient total least-squares method. Simulation results confirm the performance of Conjugate gradient total least-squares method and our theoretical analysis.**

***Index Terms*—OFDM, Channel Estimation, Total Least-squares, Conjugate gradient total least-squares, ICI, ISI**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has many well known advantages, and it is the most promise as a future high data rate wireless communication system. OFDM divides the available spectrum into a number of overlapping but orthogonal narrowband sub-channels, and hence converts a frequency selective channel into a non-frequency selective channel [1]. OFDM is used in many current wireless communication systems, such as DAB, DVB, WLAN, and WMAN [3, 4].

OFDM is an attractive technique because the ISI introduced by the channel can be mitigated by using a guard interval, rather than the complex equalization schemes that are required for single carrier modulation. However, the ISI may exceed the length of the guard interval due to the very long delay, and such ISI is called residual ISI [2, 13]. Increasing the length of the guard interval to reduce the residual ISI has its limitations because it introduces a bandwidth penalty. An attempt to equalize residual ISI has been suggested in [14], but the technique has high complexity and provides only a limited improvement. Along with ISI, the ICI that arises from the loss of sub-channel Orthogonal in an OFDM block is known to limit the performance of OFDM systems. In fact, basic diversity methods treat ICI as additive white Gaussian noise. However, the receiver should exploit the structure of ICI according to the transmit standard. It was earlier also proposed in the adaptive matrix equalizer [17], although its complexity can be prohibitive for systems with many sub-carriers.

OFDM combines the advantages of high performance and relatively low implementation complexity. However, that requires coherent demodulation that needs to estimate and tracks parameters of fading channel. Usually, the channel estimation is performed by inserting pilot symbols into the transmitted signals. In slow fading channels or time invariant channels, reference [5] proposed an LS channel estimation technique based on the comb-type pilot arrangement. An MMSE channel estimator has been proposed in [6], which makes full use of the channel correlations both in time and frequency domains and can significantly improve the OFDM system performance. To reduce the computational complexity of the MMSE estimator, reference [7] presented a linear MMSE (LMMSE) channel estimation method, which is need to know a channel autocorrelation matrix in the

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frequency domain. However, in fast fading channels, ICI degrades the performance of estimator. To alleviate this, in [8] proposed a new pilot pattern that is the grouped and equipage pilot pattern and the corresponding channel estimation and signal detection to combat the ICI. In [10], a new distributive training sequence based on modified m-sequences was proposed to perform the ICI matrix estimation. The new training sequence can deal with non-circularly ICI matrices, whereas most existing schemes assume that the ICI matrix is circularly. These methods have high complexity since it requires the joint estimation of several adjacent OFDM symbols. In addition, several kinds of blind channel estimation algorithms have been proposed in order to improve transmission efficiency[15-16]. These algorithms are based on the high-order statistical property of received signals,so it cannot be used in a high mobility environment since they require many blocks of data to carry out the estimation procedure.

In this paper, we propose an improved channel estimation method for OFDM transmission over the time-varying multipath fading channels using pilot sub-carriers. We consider the channel estimate is not only corrupted by additive white Gaussian noise (AWGN) but also by ICI and ISI. Therefore, the ICI effect due to the high Doppler shift has to be considered. Moreover, when CIR length exceeds the duration of cyclic prefix (CP), it results to ISI，which may degrade system Performance severely. Our aim is to mitigate the BER degradation due to the ICI and the residual ISI that exceeds the length of the guard interval. The interference isn't treated as AWGN, whereas it is seen as the perturbation matrix. After analyzing perturbation of least squares problems, to solve the perturbation equation, we propose that the estimation of channel response can be obtained by the total least squares (TLS) method. It can improve 10-15DB than the least square. Since the introduction of interference, the operation of matrix becomes perplexed. The TLS solution can be obtained through minimizing a Rayleigh quotient function with use of the conjugate gradient (CG) method, which is suitable for large matrix. A closed-form mathematical expression has been derived to express the channel estimation. It has been shown that the proposed channel estimation and data detect can effectively eliminate the ICI effect. Compared with the LS estimation algorithm, the main advantage of the proposed technique is its robustness against ICI and ISI.

The paper is organized as follows. Section II describes the OFDM system and channel models. In Section III, the effect of residual ISI and ICI is present to channel estimation, and given an analytical for perturbation of least squares. The TLS and CGTLS are also introduced in Section III. In Section IV, the effectiveness of the proposed schemes is demonstrated by simulation results. The concluding remarks are drawn in Section V.

Notation

In this paper, the following notations are used. $(\bullet)^T$ and $(\bullet)^H$ represent transpose and complex conjugate transpose (Hermitian), respectively. $A^\dagger$ stands for Moore-Penrose pseudo-inverse of matrix A. $\|A\|_2$, $rank(A)$, $R(A)$ and $N(A)$ represent Spectral function of matrix A, rank of matrix A, the range of matrix A and null space of matrix A. The symbol I denotes an identity matrix with proper size. We use $\langle x, y\rangle$ to denote inner product of x and y vector.

Ⅱ. SYSTEM MODEL

A. Standard CP-OFDM System

Figure 1 shows a functional block diagram of the OFDM system with channel estimation circuit.

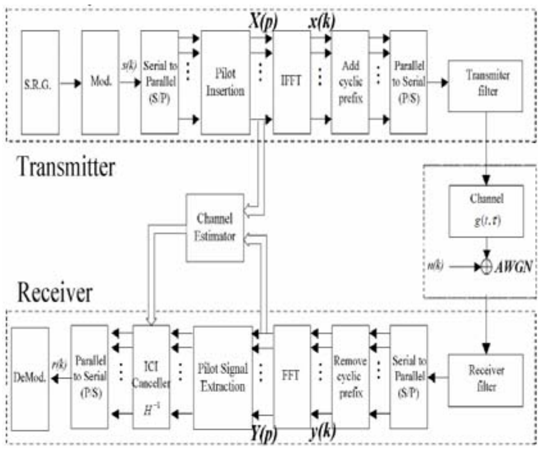

Figure. 1 Simplified block diagram of OFDM system.

We adopt the OFDM system model in [9] without loss of generality. The input binary bits are first fed into a serial-to-parallel (S/P) converter. Each data stream then modulates the corresponding sub-carrier by MPSK or MQAM. The modulation scheme may vary from one sub-carrier to another in order to achieve the maximum capacity or the minimum bit error rate (BER) for a given channel characteristic and total signal power constraint. After inserting pilots, IFFT block is used to transform the data sequence of length $N\{X(p)\}$ into time domain signal $x(k)$ with the following equation:

$$\begin{aligned} x(k) &= IFFT\{X(p)\} \qquad k = 0,1,\cdots,N-1 \\ &= \sum_{p=0}^{N-1} X(p) e^{j2\pi pk/N} \end{aligned} \tag{1}$$

In which N is the number of sub-carriers. An inverse fast Fourier transform (IFFT) can be used as the modulator while a fast Fourier transform (FFT) is used as the demodulator in the OFDM system. The basic idea of OFDM modulation is to partition a wideband signal bandwidth into a set of orthogonal sub-carriers. To maintain orthogonal among sub-carriers, it is necessary to add cyclic prefix (CP) in every symbol. The length of cyclic prefix should cover the duration of channel response. The resultant OFDM symbol is given as follows:

$$x(k)=\begin{cases} x(N+k), & k=-N_{cp},\cdots,-1 \\ x(k), & k=0,1,\cdots,N-1 \end{cases} \quad (2)$$

where N_{cp} denotes the length of the CP. The parallel data $x(k)$ are converted back to a serial data stream, and transmitted over the frequency-selective channel with additive white Gaussian noise (AWGN). The received data are converted back to $y(k)$ after discarding the prefix symbols $y(-N_{cp})$, $\cdots$, $y(-1)$, and applying the FFT and demodulation to the remainder $Y(p)$.

B. Time-Varying Channel Model

At the receiver, transmitted signals will be passed through a wireless fading channel $g(t,\tau)$ .A tapped delay line channel model is expressed as in [3].

$$g(t,\tau)=\sum_{l=0}^{L-1}\alpha_l T_l(t)\delta(\tau-\tau_l) \quad (3)$$

In which, α_l ,*l*= 1, 2, …, *L*, are used to control the energy of *l*-th paths, and *L* is the total number of paths; $T_l(t)$ indicates the time-varying gain of the *l*-th path with normalized power; τ_l is the *l*-th path delay. For a discrete time channel model, assuming the magnitude of the *l*-th path of channel in *q*-th sample time is denoted as $g(q,l)$, then the received signal can be represented as:

$$y(m)=\sum_{l=0}^{L-1}g(m,l)x(m-l)+n(m), m=0,1,\cdots,N-1 \quad (4)$$

Where, $n(m)$ is the additive white Gaussian noise at the *m*-th sample time during an OFDM symbol periods. For convenient representation (3) can be represented in matrix form as follows:

$$y=gx+n\,. \quad (5)$$

Where g is a N by N matrix constructed by $g(q,l)$.

The received signal can then be obtained in the frequency domain as:

$$Y=FgF^H X+W=HX+W\,. \quad (6)$$

Where W is the frequency domain noise vector; H is referred to as the frequency domain channel matrix; F is the Fourier transform in matrix form; F^H denotes applying a Hermitan operation on F. If the CIR is time invariant within an OFDM symbol, H will be a diagonal matrix. When the CIR is time-varying within an OFDM symbol, matrix H will no longer be a diagonal matrix and ICI will occur. In practice, obtaining the matrix H in a time-varying environment is very difficult.

When there is the ICI, the received demodulation symbol in the *k*-th sub-carrier is expressed as：

$$Y(k)=H_k(k)X(k)+\sum_{d=-k,d\neq k}^{N-1-k}H_d(k)X(k+d)+W(k) \quad (7)$$

where $H_k(k)$ is the multiplicative channel gain of the *k*-th sub-carrier, and $H_d(k)$ is the multiplicative ICI gain from the (*k*+*d*)-th sub-carrier to the *k*-th sub-carrier. $H_d(k)$ can be calculated as:

$$H_d(k)=\frac{1}{N}\sum_{l=0}^{L-1}\left(\sum_{n=0}^{N-1}g(n,l)e^{-j\frac{2\pi}{N}dn}\right)e^{-j\frac{2\pi}{N}kl} \quad (8)$$

As was noted by previous work, the ICI term can not be neglected as the maximum Doppler shift increases.

III CHANNEL ESTIMATION

For coherently modulated OFDM system it necessarily knows Channel impulse response. First of all, we give the LS estimation technique as it is needed by many estimation techniques as an initial estimation. In our paper, we assumed that perfect carrier and timing synchronization are obtained an the receiver.

A. Least-Squares Estimation

The received signal Y can be expressed as :

$$Y=HX+W\,. \quad (9)$$

LS channel estimation algorithm is to make the following squared error minimum:

$$\hat{H}=\arg\min(Y-HX)^H(Y-HX)\,. \quad (10)$$

If there is only white Gaussian noise channel, then the equation (8) can be written as:

$$\hat{H}_{LS}=(X^H X)^{-1}X^H Y\,. \quad (11)$$

Simplified:

$$\hat{H}_{LS}=X^{-1}Y\,. \quad (12)$$

In OFDM system, the pilot sub-carrier is first identified by using the transmitted pilots X_p and received pilots Y_p. The least square estimate can be represented by [5]:

$$\hat{H}_{LS}=\left[\frac{Y_1}{X_1},\frac{Y_2}{X_2},\cdots,\frac{Y_{Kp}}{X_{Kp}}\right] \quad (13)$$

Where Kp is the number of pilot sub-carrier.

The greatest advantage of LS estimation algorithm is its simple structure, complexity is low, and channel feature on the pilot sub-carrier can be obtained through an inverse operations and multiplication. However, LS estimation algorithm ignores the effect of noise, so the accuracy of this algorithm is limited; LS algorithm is useful when channel noise is small.

B. Perturbation of least squares problems description

In this section, we consider the generic perturbed least squares problem and provide an analytical upper bound for perturbation of least squares solutions [18]. Consider a complex data matrix A of dimension $m\times n$ and a complex data vector b of m-dimension. The LS solution is obtained by finding a m-dimensional vector x such that:

$$\min_x \|Ax-b\|^2 \tag{14}$$

The solution to this problem is given by:

$$x_{LS}=A^{\dagger}b \tag{15}$$

where $A^{\dagger}$ is the pseudo-inverse of A. Typically, we may assume that $m\geq n$ and that A is full rank, so that:

$$x_{LS}=(A^H A)^{-1}A^H b \tag{16}$$

Let the $m\times n$ matrix A be perturbed to $\hat{A}=A+\delta A$ and the m-dimension vector to $\hat{b}=b+\delta b$ .the pseudo-inverse solution $x_{LS}=A^{\dagger}b$ is then perturbed to and the residual to $x_{LS}+\delta x_{LS}=\hat{A}^{\dagger}\hat{b}$ and the residual $r=b-Ax$ to $r+\delta r=\hat{b}-\hat{A}x$. Let κ denote the spectral condition number $\kappa=\|A\|_2\|A^{\dagger}\|_2$ and take $\varepsilon=\|\delta A\|_2/\|A\|_2$ and $y=A^{\dagger H}x_{LS}=(AA^H)^{\dagger}b$.

The following theorem gives bounds on $\|\delta x_{LS}\|$.

***Theorem*:** Assume that $rank(\hat{A})=rank(A)$, and $\kappa\varepsilon<1$. Then:

$$\begin{aligned}\|\delta x_{LS}\|_2 \leq \frac{\kappa}{(1-\kappa\varepsilon)\|A\|_2}(\varepsilon\|x_{LS}\|_2\|A\|_2 \\ +\|\delta b\|_2+\varepsilon\kappa\|r\|_2)+\varepsilon\|y\|_2\|A\|_2\end{aligned} \tag{17}$$

NOTE. The last term of (17) vanishes if $rank(\hat{A})=rank(A)=n$, and $\|A^{\dagger}\|_2\|\delta A\|_2<1$.

then :

$$\delta x_{LS}\leq\frac{\kappa}{(1-\kappa\varepsilon)\|A\|_2}(\varepsilon\|x_{LS}\|_2\|A\|_2+\|\delta b\|_2+\kappa\varepsilon\|r\|_2) \tag{18}$$

PROOF. From the decomposition theorem, $x_{LS}=A^{\dagger}b$ and $x_{LS}+\delta x_{LS}=\hat{A}^{\dagger}\hat{b}$, it is seen that:

$$\begin{aligned}\delta x_{LS}&=\hat{A}^{\dagger}\hat{b}-A^{\dagger}b\\ &=(\hat{A}^{\dagger}-A^{\dagger})b+\hat{A}^{\dagger}\delta b\\ &=(\hat{A}^{\dagger}\delta b-\hat{A}^{\dagger}\delta A x_{LS}+\hat{A}^{\dagger}r)-P_{N(\hat{A})}x_{LS}\end{aligned} \tag{19}$$

where the first part belongs to $R(\hat{A})$ and the second to $N(\hat{A})$.

$$\begin{aligned}\|\hat{A}^{\dagger}\delta b\|_2&\leq\|\hat{A}^{\dagger}\|_2\|\delta b\|_2\\ &\leq\frac{\kappa}{1-\kappa\varepsilon}\frac{\|\delta b\|_2}{\|A\|_2}\end{aligned} \tag{20}$$

$$\begin{aligned}\|\hat{A}^{\dagger}r\|_2&=\|\hat{A}^{\dagger}P_{\hat{A}}P_{N(A)}r\|\\ &\leq\|\hat{A}^{\dagger}\|_2\|P_{\hat{A}}P_{N(A)}\|_2\|r\|_2\\ &\leq\|\hat{A}^{\dagger}\|_2\|\delta A\|_2\|r\|_2\\ &\leq\frac{\|\hat{A}^{\dagger}\|_2^2}{1-\kappa\varepsilon}\|\delta A\|_2\|r\|_2\\ &\leq\frac{\kappa^2\varepsilon}{1-\kappa\varepsilon}\frac{\|r\|_2}{\|A\|_2}\end{aligned} \tag{21}$$

$$\begin{aligned}\|\hat{A}^{\dagger}\delta A x_{LS}\|_2&\leq\|\hat{A}^{\dagger}\|_2\|\delta A\|_2\|x_{LS}\|_2\\ &\leq\frac{\|\hat{A}^{\dagger}\|_2}{1-\kappa\varepsilon}\|\delta A\|_2\|x_{LS}\|_2\\ &=\frac{\kappa\varepsilon}{1-\kappa\varepsilon}\|x_{LS}\|_2\end{aligned} \tag{22}$$

$$\begin{aligned}\|P_{N(\hat{A})}x_{LS}\|_2&=\|(I-\hat{A}^H\hat{A}^{H\dagger})A^{\dagger}AA^{\dagger}b\|_2\\ &=\|(I-\hat{A}^H\hat{A}^{H\dagger})A^H A^{\dagger H}b\|_2\\ &\leq\|\delta A\|_2\|A^{\dagger H}x_{LS}\|_2\\ &=\|\delta A\|_2\|y\|_2\end{aligned} \tag{23}$$

And arrive at (17) from the expression for δx_{LS}.

When $rank(\hat{A})=rank(A)=n$, $P_{N(\hat{A})}=0$,we can get the (18).

When $\delta A=0$ and δb is a zero mean white Gaussian noise vector, x_{LS} has zero bias. It also is the maximum-likelihood estimator. When $\delta A\neq 0$, x_{LS} will in general be biased, and will exhibit an increased MSE. It will not be equivalent to a canonical optimal estimator like the MLE. It suffers from perturbation of noise errors and increased covariance due to the accumulation of interference in $A^H A$.

C. Interference analysis of channel

This section provides a brief review of interference analysis of channel. By adding cyclic prefix (CP) to the beginning of each OFDM symbol, inter-symbol interference (ISI) and inter-carrier interference (ICI) are mitigated [2]. However, there exist the residual ISI induced by multipath components whose delays exceed CP due to shortening of CP for the purpose of increasing spectral efficiency or unforeseen channel behaviors. The resultant ISI and ICI deteriorate the system performance [14]. For more elaborate introduction to OFDM, the reader may refer to [12], [13], [19] and [20], wherein numerous further references are found.

At the receiver, CP is removed after serial to parallel conversion. The received time-domain signal vector $y_i = [y_{i,1}, y_{i,2}, \cdots, y_{i,N-1}]'$ is

$$\begin{aligned} y_i &= Cx_i - C_T x_i + C_s x_{i-1} + n_i \\ &= CQ^H X_i - C_T Q^H X_i + C_s Q^H X_{i-1} + n_i \end{aligned} \quad (24)$$

where Q is the N-point DFT matrix whose element is

$$[Q]_{l,m} = \frac{1}{\sqrt{N}} e^{j\frac{2\pi}{n}(l-1)(m-1)}, \quad 1 \le l, m \le N . \quad (25)$$

and C, C_T and C_s are N by N matrices that can be formed as in(24), (25) and (26).

$$C = \begin{bmatrix} g_0 & 0 & \cdots & 0 & g_L & g_{L-1} & \cdots & g_1 \\ g_1 & g_0 & 0 & \cdots & 0 & g_L & \cdots & g_2 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ g_{L-1} & g_{L-2} & 0 & g_0 & 0 & \cdots & 0 & g_L \\ g_L & g_{L-1} & \cdots & \cdots & g_0 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & g_L & g_{L-1} & \ddots & \ddots & g_0 \end{bmatrix} \quad (26)$$

$$C_T = \begin{bmatrix} 0 & \cdots & g_L & \cdots & \cdots & g_{N_{cp}+1} & 0 & \cdots & 0 \\ 0 & \vdots & 0 & g_L & \cdots & g_{N_{cp}+2} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & g_L & 0 & \cdots & \vdots \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \quad (27)$$

$$C_s = \begin{bmatrix} 0 & \cdots & 0 & g_L & \cdots & \cdots & g_{N_{cp}+1} \\ 0 & \cdots & 0 & 0 & g_L & \cdots & g_{N_{cp}+2} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & g_L \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & 0 \end{bmatrix} \quad (28)$$

The first term of the right hand in (24) represents the desired signals, while the second term and third term represent ICI and ISI, respectively. If the length N_{cp} of the CP is not smaller than L, C_T and C_s are zero matrices, and thus neither ICI nor ISI exists. However, if the CP is insufficient, the residual ISI of length M equal to L minus N_{cp} induces the ICI within the current symbol and the ISI from previous symbol. When CP is insufficient, ISI and ICI will degrade the system performance seriously, the estimation scheme must be taken to counteract these interferences. Assuming that the previous symbol X_{i-1} has been detected correctly, ISI can be easily removed with decision-feedback method.

D. Total Least-Squares Estimation

When there is ISI and ICI, The expression in (7) can be rewritten as:

$$(H+E)X = (Y+r) \quad (29)$$

Where E is the perturbation of matrix which is caused by the interference, and r is observe noise. We consider the problem:

$$\begin{aligned} &\min_{E,r,X} \|E\|^2 + \|r\|^2 \\ &subject \quad to : (Y+r) \in Range(H+E) \end{aligned} \quad (30)$$

The Lagrangian of problem (32) is given by

$$\Phi(E,r,\lambda) = \|E\|^2 + \|r\|^2 + 2\lambda^T((H+E) - Y - r) \quad (31)$$

Note that problem (31) is a linearly constrained convex problem with respect to the disturbance variables E and r .we conclude that (E,r) is an optimal solution of (31) if and only if there exists λ such that

$$\frac{\partial\Phi(E,r,\lambda)}{\partial E} = 2E + 2\lambda X^T = 0 . \quad (32)$$

$$\frac{\partial\Phi(E,r,\lambda)}{\partial r} = 2r - 2\lambda = 0 . \quad (33)$$

From (32) and (33) we have

$$r = \lambda . \quad (34)$$

$$E = -\lambda X^T . \quad (35)$$

Combining (34) and (35) we obtain:

$$(H - \lambda X^T)X = (Y+r) , \quad (36)$$

So,

$$r = \frac{HX - Y}{1 + \|X\|^2} . \quad (37)$$

From (35), (36) and (37) we have:

$$E = -\lambda \frac{(HX-Y)^T X^T}{1 + \|X\|^2} \quad (38)$$

For a given optimal solution X to the simplified TLS problem (30), the optimal pair (E,r) to the original TLS problem is given by (37) and (38).

The TLS solution is obtained by perturbing H and Y, in order to correct for perturbation present on H and Y, while simultaneously keeping the sum of the squares of the norms of the perturbations at a minimum.

We name this algorithm as convention total least-squares, and it can be abbreviated as TLS.

E. Conjugate gradient total least-squares

The problem described by equation (29) can be restated as:

$$(G+D)z = 0 . \quad (39)$$

where $G = [H \mid Y]$ is augmented matrix, $D = [E \mid r]$ is perturbation matrix, vector $z = [x, -1]^T$. Solving the homogeneous equation (39) the general least squares (TLS) can be expressed as the constraint optimization problem:

$$\begin{aligned} &\min \|G\|_F^2 \\ &subject \quad to: (Y + r) \in Range(H + E) \end{aligned} \quad . (40)$$

It has been shown that the constrained minimization problem in requirements (40) is equivalent to the following Rayleigh quotient minimization problem:

$$\min F(q) = \frac{q^H G^H G q}{q^H q} \quad (41)$$

which in turn is equivalent to finding the eigenvector q associated with the smallest eigenvector of $G^H G$.

In most reported studies the TLS problem is solved by SVD [21]. In general, the SVD calculation needs $O(N^3)$ multiplications. Therefore the SVD-based method is not suitable for large-scale systems. In this perturbation equation considered here, our goal is to find an scheme that can be fit for very large matrices. Toward this goal the conjugate gradient CG method can be applied here. The conjugate gradient CG method usually requires $O(N^2)$ multiplications per iteration, but the conjugate gradient CG method converges very fast and is particularly used to handle large matrices.

As stated above, the TLS solution can be obtained by minimizing the Rayleigh quotient in requirement (41). Let $q(k)$ represent the solution at the *k*-th iteration. The conjugate gradient CG algorithm updates q by successive approximation:

$$q(k+1) = q(k) + \alpha(k) p(k) \quad (42)$$

where $\alpha(k)$ is chosen to reach the minimum of $F(q)$ in the direction $p(k)$. $\alpha(k)$ is satisfied with the equation (45):

$$D[\alpha(k)]^2 + B[\alpha(k)] + C = 0 \quad (43)$$

Among the two possible solutions of equation (43), the one that yields the smaller value of $F(q)$ is given by:

$$\alpha(k) = (-B + \sqrt{B^2 - 4CD}) / 2D \quad (44)$$

Where:

$$D = P_b(k) P_c(k) - P_a(k) P_d(k) \quad (45)$$

$$B = P_b(k) - \lambda(k) P_d(k) \quad (46)$$

$$C = P_a(k) - \lambda(k) P_c(k) \quad (47)$$

$$\lambda(k) = \langle Gq(k), Gq(k) \rangle \quad (48)$$

$$P_a(k) = \langle Gq(k), Gp(k) \rangle \quad (49)$$

$$P_b(k) = \langle Gp(k), Gp(k) \rangle \quad (50)$$

$$P_c(k) = \langle p(k), q(k) \rangle \quad (51)$$

$$P_d(k) = \langle p(k), p(k) \rangle \quad (52)$$

At time $k+1$ the new search direction is chosen as:

$$p(k+1) = r(k+1) + \beta(k) p(k) \quad (53)$$

and the residue $k+1$ is given by:

$$r(k+1) = \lambda(k) q(k+1) - G^H G q(k+1) \quad (54)$$

the $\beta(k)$ is given as:

$$\beta(k) = \frac{\langle r(k+1), r(k+1) \rangle}{\langle r(k), r(k) \rangle} \quad (55)$$

to make the direction vectors $p(k)$, $G^H G$-conjugate, i.e.

$$\langle G^H G p(k), p(k+1) \rangle = 0 \quad (56)$$

The convergence criterion that we use is:

$$\left| \frac{\lambda(k+1) - \lambda(k)}{\lambda(k)} \right| < 10^{-6} \sim 10^{-7}. \quad (57)$$

The channel frequency response $\hat{H}_{LS}$ on the pilot sub-carriers can be obtained using the Least Square (LS) method described above section Ⅲ. An IFFT transform in time domain, we have channel impulse response estimation $\hat{h}_{LS}$. It is used to construct the circular matrix and interference matrix. The initial solution $q(0)$ is set to a vector in which all elements except the last come from $\hat{h}_{LS}$ and the last element is set to 1.

We name this algorithm is conjugate gradient total least-squares, and it can be abbreviated as CGTLS.

Ⅳ. SIMULATION RESULT

A. Simulation Parameter

In this section, we will investigate performance of the TLS channel estimation algorithm in the time-varying multipath Rayleigh fading channel. In order to verify the performance of this algorithm, we use MATLAB to do simulation and compare it to LS algorithm. Let us briefly refer to main parameters in the OFDM system that we consider. The system uses 16QAM modulation mode, channel model is the Rayleigh fading channel with Doppler frequency shift, maximum Doppler frequency shift is 132Hz, the number of sub-carriers is 128, and guard interval is 7.8125 kHz. In our simulations, we assume that the channel in an OFDM symbol is changes slowly. Furthermore, we have assumed the 5-tap channel model and having path delays of 0, 2, 4, 8 and 12 μs, respectively. In order to verify the robustness of the CGTLS channel estimation, the CP length and RMS delay are mutative.

B. Simulations Results

First, to compare the original LS channel estimation with the convention total least-squares by simulation to verify the bit error rate (BER). The simulation results are shown in Figure 2. Figure 2 shows the performances of

LS and TLS when CP=16. Because the treatment of perturbed matrix is different between TLS and CGTLS, the performance of CGTLS is showed the next figure.

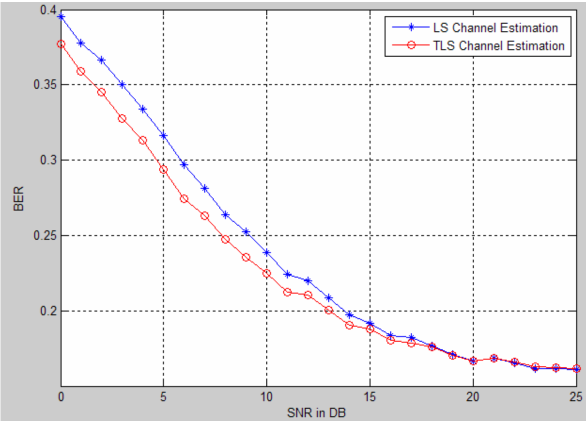

Figure.. 2 Bit error rate comparisons between LS and TLS

Figure 2 show the performance BER of versus the SNRs between LS and TLS channel estimation schemes. LS channel estimator presents poor performance in low SNRs. Also, as is illustrated in this figure 2, in low SNRs, where noise is the dominant factor degrading channel estimation performance, the convention total least-squares scheme can eliminate noise much more effectively because it reduces the effect of noise. With the increased SNR, external interference and noise will be degraded, so both of estimation schemes have an approximately performance. TLS scheme can work even better than LS scheme, because the TLS is susceptible to the presence of noise and interference.

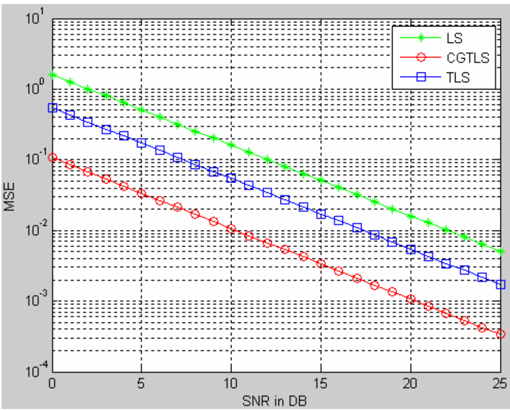

Figure. 3 MSE of channel estimates with three different channel estimations

The MSE of the proposed TLS and CGTLS channel estimator in section III was simulated. The MSE curve of the three channel estimators were plotted in Figure 3 when CP=0. From the comparison, we can see that CGTLS offers a huge improvement in the MSE and TLS algorithm has also significantly increased in the MSE. Performance of TLS method depends on the optimal pairs (E, r) . It can not match with the actual perturbation. The CGTLS based on produced by mechanism of ISI and ICI, and integrated noise, so it has the best performance in three algorithms. From the theoretical analysis, total least square usually can improve 10-15 DB than the least square.

Finally, we investigate the performance of the CGTLS scheme. Figure 4 and Figure 5 show the BER performance of the OFDM system when the RMS delay spreads are 200 ns and 400 ns, respectively. In order to verify the impact of interference, we give simulation results with four different lengths of cyclic prefix. It is 4, 8, 12 and 16, respectively.

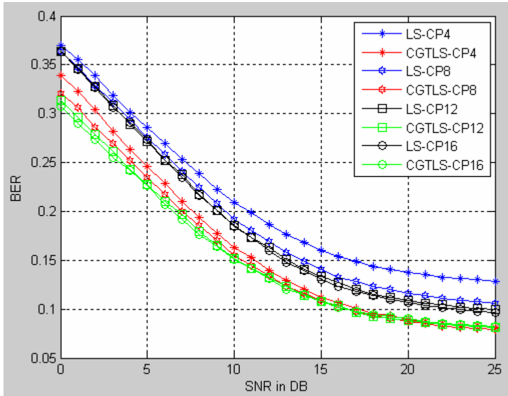

Figure. 4 Comparison of the BER between LS and CGTLS in rms = 200 ns with different cyclic prefix. (CP=4, 8, 12, 16)

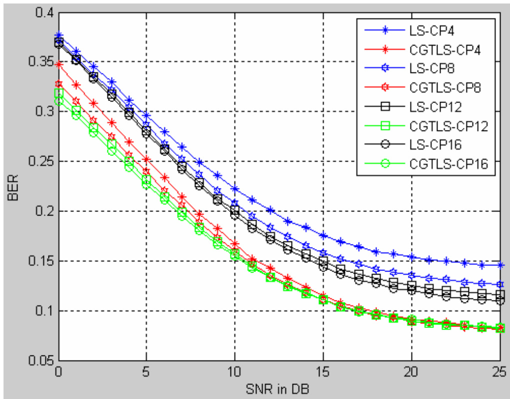

Figure. 5 Comparison of the BER between LS and CGTLS in rms = 400 ns with different cyclic prefix. (CP=4, 8, 12, 16)

When cyclic prefix is insufficient, especially at short cyclic prefix; whereas, we can observe that the BER performance of LS was seriously disturbed by the system introduced ISI and ICI. The CGTLS scheme is robust to the length of cyclic prefix especially in the higher SNRs. Because in the low SNRs, the effect of noise is master, in the higher residual ISI and ICI become evident. From the result of simulation, the CGTLS channel estimation is superior to the LS channel estimation in the same SNR. From the comparison, we can see that CGTLS can improve the channel estimation performance both residual ISI and ICI.

V CONCLUSION

In this paper, we have considered the improvement of channel estimation with residual ISI and ICI cancellation.

We have given an effective TLS algorithm and CGTLS for channel estimation of OFDM systems over time varying multipath channels.

As the structure of LS channel estimation algorithm is simple, the performance is very general when the channel environment is poor. Based on interference analysis of channel, residual ISI and ICI are not being handled as a noise. By applying TLS criterion, we successfully reduce the effect of error and interference. The major disadvantages of the presented channel method include iteration operation. Algorithm effectiveness is verified by theoretical analysis and simulation and the frequency of utilization of OFDM systems is also improved.

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