

Block Diagonal Precoding Based Power Allocation for Coordinated Multi-Point Transmission

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Abstract—This paper studies power allocation in coordinated multi-point (CoMP) transmission of 3GPP LTE-Advanced system with remote radio units (RRUs) power constraints. We apply block diagonal (BD) precoding to downlink transmission, and assume perfect knowledge of downlink channels and transmit messages at each transmit point. We propose a modified water-filling power (MWF) allocation algorithm in order to maximize the downlink sum capacity, at the same time the low complexity is achieved. The interior-point method is also used to solve the optimization problem. Simulations show that interior-point method converges after only a few iterative steps and the system capacity is near-optimal. As for complexity and power efficiency, MWF achieves a good compromise.

Index Terms—CoMP, MU-MIMO, power allocation, optimization problem

I. INTRODUCTION

In conventional cellular networks, intra-cell interference can be eliminated by Orthogonal Frequency Division Multiple Access (OFDMA). However, co-channel interference of adjacent cells is still a main factor to impact the system performance, especially for cell-edge user throughput. In order to meet the requirements of higher cell-edge throughput and spectrum efficiency of next generation broadband wireless communication system, 3GPP LTE-Advanced proposed a multi-cell MIMO technology named coordinated multiple point (CoMP) transmission and reception on August, 2008.

According to 3GPP [1], CoMP is mainly characterized into two categories: Joint Processing (JP) and Coordinated Scheduling/Beamforming (CS/CB). In JP, data intended for a particular UE is jointly processed and transmitted instantaneously from all the coordinated

points, while in CS/CB data is only transmitted from the serving cell. Although CoMP naturally increases system complexity, it yields great capacity improvement and coverage benefits [2-3]. With these CoMP schemes, especially for CoMP JP, efficient power allocation schemes need to be designed to support joint radio resource management among coordinated cells. This is because the CoMP-JP transmission scheme is more likely to be an expansion of the traditional single cell MIMO transmission scheme, in which the system performance is largely influenced by power allocation. However, although there are plenty of literatures that consider power allocation scheme for cooperative multi-cell networks, to the author's best knowledge, relative study on CoMP system is few.

Reference [4-6] studied the power allocation scheme in CoMP system. Reference [4] considered the power allocation problem with carrier aggregation (CA). Reference [5] proposed an iterative algorithm of joint antenna selection and power allocation with the purpose of maximizing the sum rate of coordinated users. Reference [6] proposed a BS scheduling scheme based on threshold judgment and an iterative power allocation approach using NE theory. However, some system models of the literatures mentioned above are not practical. Reference [7] formulated the cooperative multi-cell power allocation as a network utility maximization problem and proposed an iterative algorithm that converges to the global optimum. Reference [8-9] studied the power allocation scheme based on zero-forcing precoding. All these algorithms are based on per-antenna power constraint. But, in our paper, we study the power allocation problem with per-RRU power constraint which fits the situation of CoMP.

In LTE system, eNodeB is composed of a base band unit (BBU) and several remote units (RRUs), each of which covers a cell. In 3GPP RAN1 #59bis meeting on January, 2010, an agreement was reached that CoMP techniques would only be applied in intra-eNodeB

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scenario for Rel-10 [10]. A block diagram of the system is shown in Fig.1.

In the paper, we will focus on power allocation in MU-CoMP-JP system with per-RRU power constraint. We adopt BD precoding transmission scheme, because we think it is a scheme with the greatest potential due to the fact that it has been used in most CoMP-related proposals

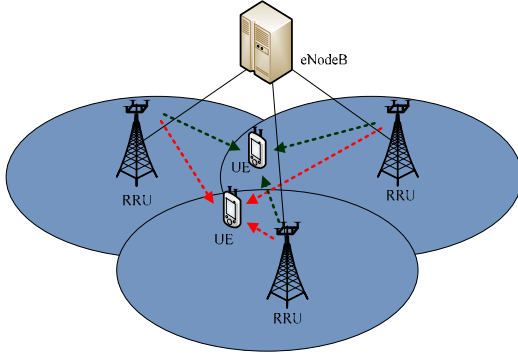


Fig.1 Network Topology of CoMP JP

submitted to 3GPP RAN1. In the paper, we first propose a MWF algorithm. The core is to find the range of height of the water surface by using a simple and efficient iterative algorithm in order to satisfy the RRU power constraint. We then study the power allocation problem from the mathematical aspect of view, and use the barrier method to find a near-optimal solution.

The rest of the paper is organized as follows. In section II, the system model of coordinated multi-point transmission with BD precoding and problem formulation will be described in detail. In section III, we propose the modified water-filling power allocation algorithm, and then present optimization solution algorithm by using interior-point methods. Simulation results are shown in Section IV. The conclusion follows in section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Network Architecture

We consider an intra-eNodeB CoMP system consisting of M RRUs with N users (UE) served. Each RRU and UE are equipped with n_T and n_R antennas, respectively. Since the system is using BD precoding, the number of antennas should meet the conditions $M \times n_T \geq N \times n_R$. We use $\mathbf{H}_k \in \mathbb{C}^{n_R \times (M \cdot n_T)}$ to represent the downlink channel from all the M cooperative RRUs to the k th user. Let $\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \dots, \mathbf{H}_N^T]^T$ denote the channel of all users. The elements of $\mathbf{H}$ are assumed to be independent zero mean complex Gaussian random variables with unit variance. Thus, matrix $\mathbf{H}$ is full rank, which means each user has n_R independent spatial substreams of data. The received signal of the k th user can be denoted as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{U}_k \mathbf{s}_k + \sum_{i=1, i \neq k}^N \mathbf{H}_k \mathbf{U}_i \mathbf{s}_i + \mathbf{n}_k. \quad (1)$$

where $\mathbf{s}_k \in \mathbb{C}^{(M \cdot n_T) \times 1}$ indicates the transmitted symbols for the k th user. $\mathbf{n}_k \in \mathbb{C}^{n_R \times 1}$ is additive white complex Gaussian noise vector, normalized so that its covariance matrix is the identity matrix. $\tilde{\mathbf{H}}_k = [\mathbf{H}_1^T, \dots, \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, \dots, \mathbf{H}_N^T]^T$ represents all the users' channel except the k th user. $\mathbf{U}_k$ is BD precoding matrix, which should satisfy $\tilde{\mathbf{H}}_k \mathbf{U}_k = \mathbf{0}$. Since $\tilde{\mathbf{H}}_k$ is assumed to be a full rank matrix, we have $\text{rank}(\tilde{\mathbf{H}}_k) = \min(n_R(N-1), n_T M) = n_R(N-1)$. We perform singular value decomposition (SVD) on $\tilde{\mathbf{H}}_k$.

$$\tilde{\mathbf{H}}_k = \tilde{\mathbf{U}}_k \tilde{\Sigma}_k [\tilde{\mathbf{V}}_k^{(1)}, \tilde{\mathbf{V}}_k^{(0)}]^* \quad (2)$$

where $\tilde{\mathbf{V}}_k^{(0)}$ holds the last $(n_T M - n_R(N-1))$ right singular vectors, which forms an orthogonal basis for the null space of $\tilde{\mathbf{H}}_k$. Thus, we have $\tilde{\mathbf{H}}_k \tilde{\mathbf{V}}_k^{(0)} = \mathbf{0}$, which means inter-user interference has been eliminated. We then perform SVD on $\mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)}$ in order to decompose the channel into independent parallel sub-channels.

$$\mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)} = \mathbf{U}_k \Sigma_k [\mathbf{V}_k^{(1)}, \mathbf{V}_k^{(0)}]^* \quad (3)$$

where Σ_k is a $n_R \times n_R$ diagonal matrix, whose element $\lambda_{kj}^{1/2}$ ($j=1, \dots, n_R$) is the singular values of parallel sub-channels. $\mathbf{V}_k^{(1)}$ holds the n_R singular vectors with non-zero singular values. Thus, $\mathbf{U}_k = \mathbf{W}_k \Lambda^{1/2} = \tilde{\mathbf{V}}_k^{(0)} \mathbf{V}_k^{(1)} \Lambda^{1/2}$, where $\Lambda = \text{diag}(P_{k1}, P_{k2}, \dots, P_{kj}, \dots, P_{kn_R})$ indicates the power allocated on the k th user's j th substream. The equivalent channel of user k now can be written as $\mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)} \mathbf{V}_k^{(1)}$ consisting of n_R independent sub-channels, each of which has a gain of $\lambda_{kj}^{1/2}$ ($j=1, \dots, n_R$).

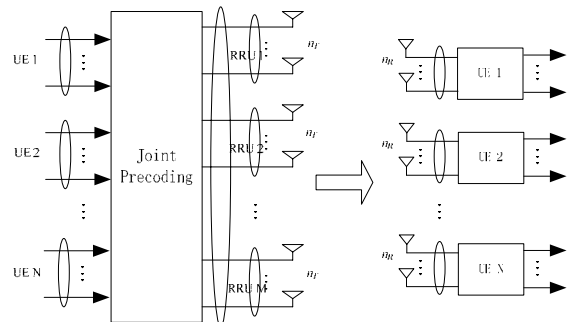


Fig.2 System Structure of Downlink CoMP-JP

B. Problem Formulation

BD precoding enables the system to transmit multiple data streams to each user while removing the inter-user interference. The whole system channel can now be seen as $N \times n_R$ independent parallel sub-channels. The rate of the j th sub-channel of the k th user is given by

$$R_{kj} = \log_2(1 + \lambda_{kj} P_{kj}). \quad (4)$$

Define w_{kj}^{il} as an element of $\mathbf{W}_k$ representing a mapping relation between P_{kj} and the transmit power of the i th RRU's l th antenna. The total transmitting power of the i th RRU can be written as

$$P_{RRU_i} = \sum_{k=1}^N \sum_{j=1}^{n_R} \sum_{l=1}^{n_T} |w_{kj}^{il}|^2 P_{kj} \quad (\forall i = 1, \dots, M). \quad (5)$$

Let $P_{MAX_PER_RRU}$ be the power constraint of each RRU. Therefore, our optimization problem to maximize the sum rate with per-RRU power constraint can be formulated as

$$(OP1) \max_{P_{kj}} \sum_{k=1}^N \sum_{j=1}^{n_R} \log_2(1 + \lambda_{kj} P_{kj}).$$

s.t.

$$c1: P_{kj} \geq 0 \quad (\forall k = 1, \dots, N \quad \forall j = 1, \dots, n_R)$$

$$c2: P_{RRU_i} \leq P_{MAX_PER_RRU} \quad (\forall i = 1, \dots, M)$$

It is easy to notice that the objective function and constraints are all convex over the solution set. Thus, it is a convex optimization problem [11].

III. POWER ALLOCATION ALGORITHMS

A. Modified Water-filling Power Allocation Algorithm (MWF)

Since the entire system has been decomposed into $N \times n_R$ independent sub-channels, one way of solving the optimal power allocation problem is using the conventional water-filling strategy. If P is the total volume of water filled into a vessel, the depth of water at each sub-channel is the power allocated to it, and $1/\lambda_{kj}$ traces out the bottom of the vessel [12].

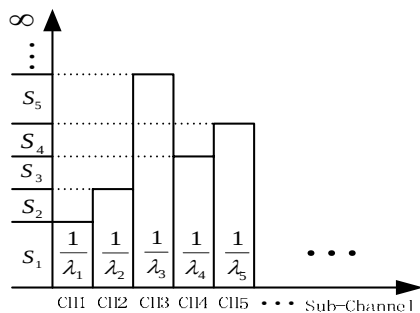


Fig.3 Intervals Divided by the Bottom of the Vessel

Here, we can not use the water-filling strategy directly because of the per-RRU power constraint. Let's define $\mu \in [0, +\infty)$ as the height of the water surface. The bottom of the vessel naturally divides $[0, +\infty)$ into $N \times n_R$ left-closed right-open intervals denoted by $S_1, S_2, \dots, S_{N \times n_R}$ as shown in Fig.3. In MWF, we firstly determine which interval μ is in by using a simple iterative algorithm. Then we obtain μ by analyzing a set of linear functions. Our iterative algorithm works as follows.

Initialize $\alpha = N \times n_R + 1$.

Repeat

$$\alpha = \alpha - 1, P = \min(S_\alpha).$$

Use water-filling algorithm to allocate power among sub-channels, and update P_{kj} .

Calculate P_{RRU_i} according to (5).

$$\text{Until } P_{RRU_i} \leq P_{MAX_PER_RRU} \quad (\forall i = 1, \dots, M).$$

Thus, we know $\mu \in S_\alpha$. And we can determine the value of P_{kj} according to (6).

$$P_{kj} = \begin{cases} \mu - 1/\lambda_{kj} & \text{when } \mu \geq 1/\lambda_{kj} \\ 0 & \text{when } \mu < 1/\lambda_{kj} \end{cases} \quad (6)$$

From (5) and (6), P_{RRU_i} can be written as a function of μ . That is to say, we now have M linear functions of one variable, each of which can be expressed as $P_{BS_i} = a_i \mu + b_i$ (a_i and b_i are coefficients). For each $P_{RRU_i} = P_{MAX_PER_RRU}$ ($\forall i = 1, \dots, M$), we can get a corresponding μ_i . The water surface μ equals $\min(\mu_i)$. Finally, we calculate P_{kj} according to (6).

B. Interior-point Methods

Considering (OP1) is a convex optimization problem, we solve the problem by a particular interior-point algorithm, the barrier method. The main idea of the method is to approximately formulate the inequality constrained problem as an unconstrained problem by an indicator function. We construct the indicator function as

$$B(P_{kj}) = \sum_{k=1}^N \sum_{j=1}^{n_R} \ln(P_{kj}) + \sum_{i=1}^M \ln(P_{MAX_PER_RRU} - \sum_{k=1}^N \sum_{j=1}^{n_R} \sum_{l=1}^{n_T} |w_{kj}^{il}|^2 P_{kj}) \quad (7)$$

Hence, the barrier function is defined as:

$$G(P_{kj}, r) = - \sum_{k=1}^N \sum_{j=1}^{n_R} \log_2(1 + \frac{\lambda_{kj} P_{kj}}{\sigma^2}) - r B(P_{kj}). \quad (8)$$

Since $r \in \mathbb{R}^+$ is very small, $G(P_{kj}, r)$ is an approximation of the original objective function. As r decreases, the approximation becomes more accurate. Also, when P_{kj} tends to the boundary of the feasible set S , $G(P_{kj}, r)$ increases to ∞ . Thus, $B(P_{kj})$ builds a barrier at the boundary of S . (OP1) can now be simplified to the following equivalent problem (OP2) which can be easily solved.

$$(OP2) \min G(P_{kj}, r)$$

$$\text{s.t. } P_{kj} \in \text{int } S$$

Let P_{kj}^* indicate the optimal solution. The barrier method can be described as follows.

Initialize $\tilde{P}_{kj}$, r , coefficient $\beta \in (0,1)$, tolerance $\varepsilon > 0$.

Repeat

Calculate P_{kj}^* by minimizing (9), starting at $\tilde{P}_{kj}$.

if $|rB(P_{kj}^*)| < \varepsilon$

Break; We find the optimal solution.

else

Update $\tilde{P}_{kj} = P_{kj}^*$ and $r = \beta r$;

end

IV. SIMULATION RESULTS

In this section, we perform computer simulations to evaluate the performance of the proposed intra-eNodeB power allocation algorithms and to compare it with other traditional method. We consider one single eNodeB with 3 RRUs. There are 2 users served at the same time. Each RRU has 4 antennas, and each user is equipped with 2 antennas. The channel $\mathbf{H}$ is assumed to be quasi-static flat Rayleigh fading and Rice fading with Rice factor equal to 10. Also, ideal channel estimation is used.

In the simulation, we compare MWF and interior-point methods with equal power allocation strategy. The initial values of parameters of interior-point algorithm are set as $\gamma = 1$, $\beta = 1/3$ and $\varepsilon = 10^{-3}$. We implement the algorithms under different power constraints of RRU.

It is shown in Fig.4 and Fig. 5 that CoMP has a gain over the traditional single cell MU-MIMO transmission scheme. This is because in CoMP-JP strategy there are more transmit antennas and the inter-cell interference is eliminated. The sum rate of 2 users of barrier method outperforms those of the other two algorithms under various RRU power constraints, since the barrier method approaches to the optimal solution on every iterative step. Besides, MWF has more obvious rate gain than equal power allocation when the power constraint of RRU is small, which is consistent with the property of traditional water-filling algorithm.

As for power efficiency, Fig.6 plots the data rate per watt. We can see that MWF has the highest power

efficiency while the performance of the barrier method is relatively poor. That means in order to obtain the same data rate, the barrier method may require more transmit power.

Besides, the iterative number of MWF is related to the number of sub-channels, which is very small in practical system. However, the iterative number of barrier method is always kept in about 12 times in our simulation. Thus, we can see that MWF has much lower complexity and higher efficiency.

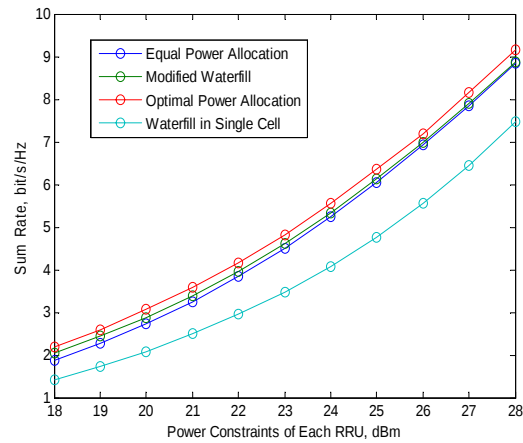


Fig.4 Sum Rate of the 2 users under Different Power Constraints, Rayleigh Fading

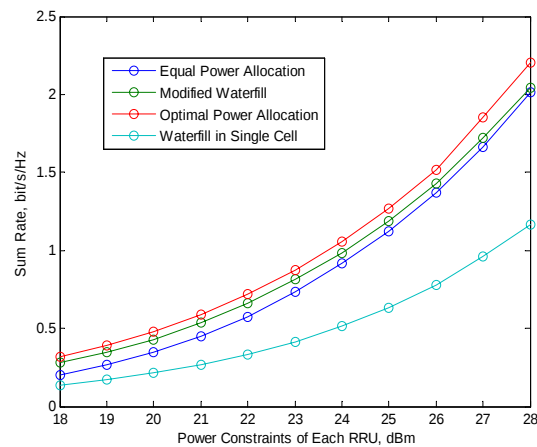


Fig. 5 Sum Rate of the 2 users under Different Power Constraints, Rice Fading

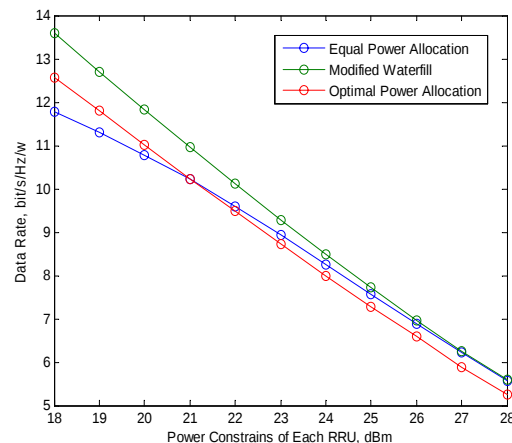


Fig.6 Bit Rate per Watt

V. CONCLUSION

In this paper, we formulate the optimal power allocation problem with per-RRU power constraint in multi-user CoMP-JP network to achieve the maximal sum rate of the system. We first propose a MWF algorithm. The algorithm roughly determines the range of μ , before obtaining the sub-optimal solution through solving a set of equations. Since the power allocation problem is convex optimization, we then use barrier method to obtain the optimal solution. We construct the barrier function to transform the original equality constrained problem to an unconstrained problem which can be easily solved. Simulation results show that the barrier method can get the near-optimal sum data rate, while using relatively more transmit power. Considering the algorithm complexity and power efficiency, MWF is a good compromise alternative.

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