Interference Alignment: A Building Block of Coordinated Beamforming Transceiver Designs

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Abstract—**Interference alignment (IA) has recently been used for the transceiver design of the K-user constant MIMO interference channel. Since this system is a coordinated beamforming system and there are other design criteria, it is important to evaluate IA transceiver designs in light of other designs. In this paper, relationships between IA and the MMSE, Max Min SINR, Max Sum Capacity, and Min BER transceiver designs are rigorously established for all SNR's and power constraints. A MMSE based transceiver design approach, applicable for general linear equality power constraints, is also proposed—the first such design for this system. It is used, along with some IA transceiver designs, to demonstrate numerically the derived relationships and to perform a comparison of the designs. One of the main conclusions is that IA is a building block of the traditional transceiver designs considered (MMSE, Max Min SINR, etc.) and generally should not be pursued alone.**

Index Terms—**Network MIMO, Coordinated Multi-Point Transmission/Reception, CoMP, Coordinated Beamforming, interference alignment, MMSE, Max Min SINR, Max Sum Capacity, Min BER, per-antenna power constraint**

I. INTRODUCTION

Interference alignment (IA) has been recently proposed for the *K*-user MIMO interference channel. It *completely* nulls out all inter-user interference and can be done in different ways and dimensions, e.g., symbol extensions [1], signal levels [2], signal vector space approach [3-6]. Considered in this paper is the signal vector space approach for constant channels where IA is simply a condition on the precoders and decoders. Loosely speaking, IA is said to be achieved if each pair's data streams see a full rank channel and if there is no inter-user interference after each receiver's decoder (see Section II. for a precise definition). Unfortunately, closed-form solutions for IA precoders and decoders have only been discovered for certain special cases (e.g., [1,Appendix IV] and [3]). For all other cases, one may use iterative algorithms (e.g., [5]'s min leakage algorithm and [6]) which have been designed to search for IA solutions.

IA has received a lot of attention. It has an interesting relationship with the degrees of freedom and capacity (e.g., [1,5,7,8]). In addition, the *K*-user constant MIMO interference channel is actually a Coordinated Beamforming (CBF) [9-10] configuration—CBF transceiver design is an important issue for upcoming cellular systems with Coordinated Multipoint Transmission/Reception (CoMP). Thus, IA needs to be evaluated in light of the more traditional transceiver designs (e.g., MMSE, Max Min SINR, Max SINR, Max Sum Capacity, Min BER). In particular, 1) the relationships (if any) between the physical mechanisms of IA and the more traditional transceiver designs need to be found; 2) the performances of IA and the more traditional transceiver designs need to be compared; 3) IA's proper place among the transceiver designs needs to be determined.

Some works [5,11-21] have touched upon these three issues but, as will be seen, a much more *thorough* treatment is needed. With regard to the first issue, previous remarks in the literature can be split into two categories: those for high SNR or SNR approaching infinity, and those for general SNR's (not necessarily restricted to high ones). For the first category, only a handful of works [11-14] have made remarks. Unfortunately though, some of these remarks are vague, unclear, or without sufficient proof. For instance, [12] states, "In the regime of asymptotically high SNR…it is generally optimal to avoid interference completely…" For the second and far more interesting category, even fewer works [13-15] have made remarks. They however are without sufficient proof and/or possibly wrong. Reference [13] states that both its MMSE and Max SINR

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designs are generalizations of IA. Moreover, it states that "…the max SINR or MMSE metrics are desirable in most environments because they flexibly adapt the solution between interference alignment (high interference power) and SVD precoding (no interference, fixed number of streams)…" To the best of our knowledge though, the IA portions of these two statements are not justified or proved in the paper. Both [14] and [15] claim that their MMSE designs are IA schemes. However, the only reasoning they give for this is based on their BER results. In addition, [14]'s own "percentage of interference leakage versus interference plus desired signal" plot shows relatively high values (10^{-1}) for Eb/N0 = 0dB (granted, only 16 iterations were run). From these works, it is thus clear that the relationships between IA and the more traditional transceiver designs need a more *rigorous* treatment—especially for general SNR's.

With regard to the second issue, multiple works [5,11-19] have showed that their more traditional transceiver designs outperformed certain IA designs at low and moderate SNR's. In addition, [20], using binary power control, showed higher sum rates than [6] when IA was infeasible. However, binary power control basically chooses which subset of the data streams to shut down. And, if a data stream is shut down in their simulations, IA becomes feasible for the remaining data streams. Thus, a performance comparison (if it exists) in which the traditional transceiver design does not shut down data streams is still needed for the infeasible IA case. Such a comparison would help tell whether the IA feasibility limits should be strictly adhered to. It would also help to determine whether IA is a good goal when it is infeasible.

With regard to the third issue, some state that "…IA is a suboptimal strategy at finite SNRs" [11]. Reference [13] states that "…it is unlikely that a direct interference alignment approach is desirable because of its suboptimality in environments where one or more links have little energy relative to the others. Instead, the max SINR or MMSE metrics are desirable in most environments …" All the while, others propose modified IA designs to account for the colored noise, the strength of the desired signal, etc. (e.g., [13,21]). So, at this current point, it is not entirely clear what is IA's proper place among the transceiver designs. In addition, it is not clear whether non IA designs should be confined to IA's feasibility limits.

This paper deals with all three issues in considerable depth. With respect to the first issue, the relationships between IA and the MMSE, Max Min SINR, Max Sum Capacity, and Min BER transceiver designs are rigorously established for all SNR regimes and all power constraints—the previous works [11-15] considered only the per-transmitter power constraint, neglecting the more practical per-antenna power constraint (in practice, each antenna is generally connected to its own power amplifier). In particular, it is *rigorously* shown that IA is one of the building blocks for each of the transceiver designs, i.e., they all seek the optimum (for their respective metrics) balance between preserving the desired data streams, aligning the inter-stream interferences, aligning the inter-user interferences, and knocking out the noises. Furthermore, under the special case of no noise, equivalencies between IA and the other transceiver designs are derived. The balancing is demonstrated numerically using subspace power plots of the Generalized Iterative Approach (GIA), our herein proposed MMSE based transceiver design approach.

With respect to the second issue, a comparison of the GIA and some IA transceiver designs is numerically performed. A feasible and infeasible IA case are run. For the feasible IA case, it is found that the GIA outperforms the implemented IA designs—just like with the previous works. For the infeasible IA case, the GIA does *not* necessarily shut down data streams. It is able to outperform [6] and obtain decent results. With respect to the third issue, it is concluded that IA's proper place is as an important building block of traditional transceiver designs. It should not be pursued by itself in general. Instead, the traditional transceiver designs should be used (they effortlessly do the type of balancing that the modified IA designs seek). It is also concluded that IA's feasibility limits can give some guidelines for choosing the number of cooperating cells in CBF. However, one should be aware that decent performance can be achieved beyond them.

As mentioned before, the herein proposed GIA is a MMSE transceiver design approach for the *K*-user constant MIMO interference channel. [12-17] are MMSE designs as well. Apart from [12], they were all proposed, more or less, at the same time[1]; [12] is earlier than all of them but is much more limited than the GIA. The GIA has some key advantages over the other designs. Unlike [12,17], it can handle multiple data streams per pair. Unlike [12,14-17], it can handle non-diagonal noise covariance matrices—perhaps useful to model "uncoordinated interference" [13]. Unlike [12-14,16], it gives explicit expressions for the Lagrange multipliers. It is not encumbered by complex root or other numerical searches. Lastly, and most importantly, it can handle general linear equality power constraints. [12-16] and [17] only handle the total power constraint and system power constraint, respectively. Only the GIA can individually constrain the power of each antenna.

Notations are as follows. All boldface lower case (upper case) letters indicate vectors (matrices). $\mathbf{A}'$, $\mathbf{A}^*$, $\mathbf{A}^{-1}$, $tr(\mathbf{A})$, $<\mathbf{A}>$, $rank(\mathbf{A})$, $\|\mathbf{A}\|_F$, stand for the transpose, conjugate transpose, inverse, trace, expectation, rank, and Frobenius norm of $\mathbf{A}$, respectively. $|a|$ stands for the magnitude of a. $\mathbf{I}_m$ signifies the $m\times m$ identity matrix. $\mathbf{0}_{m\times n}$ signifies the $m\times n$ zero matrix. $\mathbf{e}_j$ signifies the j^{th} column of the identity matrix (the dimension will be apparent from the context). $diag[\ldots]$ denotes the diagonal matrix with elements […] on the main diagonal. $\mathbf{A}*\mathbf{B}$ denotes the Schur product of $\mathbf{A}$ and $\mathbf{B}$ (element-wise product of $\mathbf{A}$ and $\mathbf{B}$). $CN(\mu,\sigma^2)$ denotes a complex normal random variable with mean μ and variance σ^2. $Q(\cdot)$ is the Q-

[1] The GIA was presented in IEEE Sarnoff Symposium 2010, April 2010. See [22,23].

function, the tail probability of the standard normal distribution.

II. FORMULATION AND PRELIMINARIES

The system considered is the K-user constant MIMO interference channel. There are K transmitters and K receivers. The k^{th} transmitter only wants to send data to the k^{th} receiver (k=1,…,K), thus giving K pairs in the system. The k^{th} transmitter and k^{th} receiver have t_k and r_k antennas, respectively.

The source (data) to be transmitted from the k^{th} transmitter to the k^{th} receiver, $\mathbf{s}_k$, is $m_k\times1$ and is characterized by its positive definite source covariance matrix, $\left\langle \mathbf{s}_k\mathbf{s}_k^* \right\rangle = \mathbf{I}_{m_k}$. $\mathbf{s}_k$ is precoded by $\mathbf{F}_k$, the $t_k\times m_k$ precoder, and then transmitted. The channel from the k^{th} transmitter to the l^{th} receiver is $r_l\times t_k$, constant during the transmission, and is denoted by $\mathbf{H}_{lk}$. The received signal vector at the k^{th} receiver is thus given by:

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{F}_k\mathbf{s}_k + \sum_{l=1,l\neq k}^{K} \mathbf{H}_{kl}\mathbf{F}_l\,\mathbf{s}_l + \mathbf{n}_k, \tag{1}$$

where $\mathbf{n}_k$ is the noise vector characterized by its noise covariance matrix, $\mathbf{\Phi}_{\mathbf{n}k} = <\mathbf{n}_k\mathbf{n}_k^*>$. Finally, the k^{th} receiver uses the decoder, $\mathbf{G}_k$, to process its received signal vector, $\mathbf{y}_k$. It is important to note that the sources and noises are all independent of each other and zero-mean. For the d^{th} received data stream at the k^{th} receiver, the desired signal power , the self interference power (from other data streams of the same user), the interference power from other users and the noise power are denoted as $\sigma_{k,d}^D, \sigma_{k,d}^{IS}, \sigma_{k,d}^{IO}$ and $\sigma_{k,d}^N$, respectively and can be expressed as

$$\begin{aligned}\sigma_{k,d}^D &= \left|\mathbf{g}_{k,d}\mathbf{H}_{kk}\mathbf{f}_{k,d}\right|^2,\ \sigma_{k,d}^{IS} = \sum\nolimits_{i=1,i\neq d}^{m_k}\left|\mathbf{g}_{k,d}\mathbf{H}_{kk}\mathbf{f}_{k,i}\right|^2,\\ \sigma_{k,d}^{IO} &= \sum\nolimits_{l=1,l\neq k}^{K}\left\|\mathbf{g}_{k,d}\mathbf{H}_{kl}\mathbf{F}_l\right\|_F^2,\ \sigma_{k,d}^N = \left\|\mathbf{g}_{k,d}\mathbf{\Phi}_{\mathbf{n}k}^{1/2}\right\|_F^2.\end{aligned} \tag{2}$$

Here, $\mathbf{g}_{k,d}$ and $\mathbf{f}_{k,d}$ are the d^{th} rows and columns, respectively, of $\mathbf{G}_k$ and $\mathbf{F}_k$.

Five transceiver designs are considered in this paper. The MMSE design seeks to minimize the sum MSE,

$$\eta = \sum_{k=1}^{K}\eta_k;\ \eta_k = \sum_{d=1}^{m_k}\eta_{kd}. \tag{3}$$

η_k, the MSE for the k^{th} pair, and η_{kd}, the MSE for the d^{th} data stream at the k^{th} receiver, can be expressed as:

$$\eta_k = \left\|\mathbf{G}_k\mathbf{H}_{kk}\mathbf{F}_k - \mathbf{I}_{m_k}\right\|_F^2 + \left\|\mathbf{G}_k\mathbf{\Phi}_{\mathbf{n}k}^{1/2}\right\|_F^2 + \sum_{l=1,l\neq k}^{K}\left\|\mathbf{G}_k\mathbf{H}_{kl}\mathbf{F}_l\right\|_F^2 \tag{4}$$

$$\eta_{kd} = \left|\mathbf{g}_{k,d}\mathbf{H}_{kk}\mathbf{f}_{k,d} - 1\right|^2 + \sigma_{k,d}^{IS} + \sigma_{k,d}^{IO} + \sigma_{k,d}^N. \tag{5}$$

On the other hand, the Max Min SINR design seeks to maximize the smallest SINR (per data stream) in the system. The SINR for the d^{th} data stream of the k^{th} receiver is

$$SINR_{k,d} = \frac{\sigma_{k,d}^D}{\sigma_{k,d}^{IS} + \sigma_{k,d}^{IO} + \sigma_{k,d}^N} \tag{6}$$

As its name indicates, the Max Sum Capacity design seeks to maximize the sum capacity of the system,

$$C_\Sigma = \sum_{k=1}^{K}\sum_{d=1}^{m_k} C_{k,d}; \tag{7}$$

the capacity for the d^{th} data stream of the k^{th} receiver being

$$C_{k,d} = \log_2(1 + SINR_{k,d}). \tag{8}$$

Similarly, the Min BER design considered here seeks to minimize

$$q = \frac{1}{\sum_{i=1}^{K} m_i}\sum_{k=1}^{K}\sum_{d=1}^{m_k} q_{k,d} \tag{9}$$

where

$$q_{k,d} = Q(\sqrt{\beta\cdot SINR_{k,d}}) \tag{10}$$

is approximately proportional to the raw BER of the d^{th} data stream of the k^{th} receiver. β, here, is a positive constant dependent on the modulation coding scheme employed [24]. Finally, the IA design seeks a set of precoders and decoders that achieves IA. IA is said to be achieved if for every k (k=1,2,…,K),

$$rank\left(\mathbf{G}_k\mathbf{H}_{kk}\mathbf{F}_k\right) = m_k, \tag{11}$$

$$\mathbf{G}_k\mathbf{H}_{kl}\mathbf{F}_l = \mathbf{0}_{m_k\times m_l},\quad \forall l\neq k. \tag{12}$$

Here, (12) implies that the inter-user interferences are aligned. Similarly, (11) implies that the intra-user (inter-stream) interferences can, by proper modification of the decoders, be aligned,

$$\mathbf{g}_{k,d}\mathbf{H}_{kk}\mathbf{f}_{k,i} = 0,\ \forall i\neq d,\ \forall d,\ \forall k. \tag{13}$$

In practical implementation, each of the precoders in the above mentioned five designs has to be subject to a power constraint. For the k^{th} transmitter, considered in this paper are the per-antenna power constraint

$$P_{k,j} = \mathbf{e}_j'\left(\mathbf{F}_k\mathbf{F}_k^*\right)\mathbf{e}_j,\ j = 1,\ldots,t_k, \tag{14}$$

and the per-transmitter power constraint

$$P_k = tr\left(\mathbf{F}_k\mathbf{F}_k^*\right). \tag{15}$$

Here, we choose $P_k = \sum_{j=1}^{t_k} P_{k,j}$ and $P_{k,j} > 0$, $\forall j$.

III. RELATIONSHIPS BETWEEN IA AND OTHER TRANSCEIVER DESIGNS

A. Under No Noise

When IA is feasible and there is no noise ($\mathbf{\Phi}_{\mathbf{n}k} = \mathbf{0}_{r_k\times r_k}$, $\forall k$), there exist equivalencies between IA and the MMSE, Max Min SINR, Max Sum Capacity, and Min BER designs. The equivalencies for the first two are given below. The equivalencies for the latter two are omitted here due to the fact that they can be derived easily once the Max Min SINR one has been established.

If the minimum η is 0, the MMSE precoders and decoders satisfy the IA conditions (11-12). Conversely, if a set of precoders and decoders satisfy the IA conditions, they can be made (with slight modifications to the decoders if necessary) to make η=0. The proof is as follows: if a set of precoders and decoders, $\{\mathbf{F}_k\}$ and $\{\mathbf{G}_k\}$, make η=0, they make η_k=0 and

$$\mathbf{G}_k\mathbf{y}_k = \mathbf{s}_k \tag{16}$$

for each k. Since right multiplying (16) by $\mathbf{s}_c^*$ and taking the expectation yields

$$\mathbf{G}_k \mathbf{H}_{kc} \mathbf{F}_c = \begin{cases} \mathbf{I}_{m_k} & \text{if } c = k \\ \mathbf{0}_{m_k \times m_c} & \text{if } c \neq k \end{cases} \tag{17}$$

this set of precoders and decoders satisfy the IA conditions. Conversely, if $\{\mathbf{F}_k\}$ and $\{\mathbf{G}_k\}$ satisfy the IA conditions (11-12), form the modified decoders, $(\mathbf{G}_k\mathbf{H}_{kk}\mathbf{F}_k)^{-1}\mathbf{G}_k$. Using $\{\mathbf{F}_k\}$ and these modified decoders results in $(\mathbf{G}_k\mathbf{H}_{kk}\mathbf{F}_k)^{-1}\mathbf{G}_k\, \mathbf{y}_k = \mathbf{s}_k$ and $\eta_k = 0$ for every k. Consequently, the sum MSE is zero.

Similarly, let the precoder, decoder, and channel matrices have elements which are finite in magnitude. If the max min SINR is infinite, the precoders and decoders satisfy the IA conditions (11-12). Conversely, if a set of precoders and decoders satisfy the IA conditions, they can be made (with slight modifications to the decoders if necessary) to make the max min SINR infinite. The proof is as follows: if a set of precoders and decoders, $\{\mathbf{F}_k\}$ and $\{\mathbf{G}_k\}$, results in an infinite max min SINR, the numerators and denominators of each data stream's SINR in (6) are positive and zero, respectively. As such, $\mathbf{G}_k\mathbf{H}_{kk}\mathbf{F}_k$ is a diagonal matrix with positive diagonal elements and $\mathbf{G}_k\mathbf{H}_{kl}\mathbf{F}_l = \mathbf{0}_{m_k \times m_l}$, $\forall l \neq k$, $\forall k$. That is, this set of precoders and decoders satisfy the IA conditions. Conversely, if $\{\mathbf{F}_k\}$ and $\{\mathbf{G}_k\}$ satisfy the IA conditions, form the modified decoders, $(\mathbf{G}_k\mathbf{H}_{kk}\mathbf{F}_k)^{-1}\mathbf{G}_k$. Using $\{\mathbf{F}_k\}$ and these modified decoders results in the numerator and denominator of each data stream's SINR (6) to be equal to one and zero, respectively. Consequently, the max min SINR is infinite.

B. IA-like Behavior of Other Transceiver Designs

The relationships between IA and the transceiver designs are *not* limited to just the zero noise case. Due to the structure of their cost functions, all of the designs have IA-like behaviors even when there is noise present, i.e., all of them seek to align the inter-user interferences (12) and to align the inter-stream interferences (13). Of the designs, first consider the MMSE design. As shown in (5), the MSE of each data stream of each receiver can be split into the MSE due to the desired signal, inter-user interference, and noise. Each of these sub terms is non-negative, thus leading the MMSE design to seek the optimum balance between minimizing them. In other words, for the signal transmitted from the desired user, the MMSE design seeks to force $\mathbf{g}_{k,d}\mathbf{H}_{kk}\mathbf{f}_{k,d}$ equal to 1 and $\mathbf{g}_{k,d}\mathbf{H}_{kk}\mathbf{f}_{k,i}$ equal to 0, $\forall i \neq d$, $\forall d$ (cf. IA condition (11) or (13)). For the inter-user interference at the k^{th} receiver, the MMSE design ultimately seeks to minimize $\left\|\mathbf{G}_k\mathbf{H}_{kl}\mathbf{F}_l\right\|_F^2$, $\forall l \neq k$, i.e., it seeks to force $\mathbf{G}_k\mathbf{H}_{kl}\mathbf{F}_l$ to be equal to the zero matrix for all $l \neq k$ (cf. IA condition (12)). Lastly, for the noise at the k^{th} receiver, the MMSE design seeks to minimize $\langle(\mathbf{G}_k\mathbf{n}_k)^*(\mathbf{G}_k\mathbf{n}_k)\rangle$, i.e., it seeks to knock out the noise using the decoder. In summary, the MMSE design seeks the optimum balance between preserving the desired data streams, *aligning* the inter-stream interferences, *aligning* the inter-user interferences, and knocking out the noises! It does not seek for the noise, inter-stream and inter-user interferences to cancel each other out, nor does it seek for the interference from different interferers/streams to cancel each other out.

It turns out that a very large fraction, $(K-1)/(K+1)$ to be precise, of the terms in η are due to the inter-user interference. As such, these two hypotheses are now posed:

<u>*Hypothesis* 1</u>: Due to the number of terms, the inter-user interference is an important factor in the MSE expression. As K grows, so does the importance of aligning the interferences.

<u>*Hypothesis* 2</u>: If K is large, it may be infeasible for the interferences to sufficiently align. Consequently, the MMSE design may set the precoder and decoder of a pair to zero to avoid blowing up their inter-user interference terms and/or to make it easier to align the remaining interferers.

As many of the details for the other transceiver designs are analogous to that of the MMSE, they will not be included here. One important note to make though is that the results for the Max Sum Capacity and Min BER designs follow readily once the Max Min SINR design is understood due to the dependence of those criteria on $\text{SINR}_{k,d}$, $\forall k$, $\forall d$.

IV. DEVELOPMENTS FOR MMSE

In this section, the GIA is briefly explained for the per-antenna and per-transmitter power constraints (other linear equality power constraints can be handled similarly). To account for the power constraints, the method of Lagrange multipliers is used to form the augmented cost function,

$$\xi = \eta + \sum_{k=1}^{K} tr\left(\mathbf{\Lambda}_k \left(\mathbf{F}_k\mathbf{F}_k^* - \mathbf{P}_k\right)\right). \tag{18}$$

Here, $\mathbf{\Lambda}_k$ is an unknown diagonal matrix representing the Lagrange multipliers and $\mathbf{P}_k$ is a diagonal matrix related to the desired power(s). For the per-antenna power constraint,

$$\mathbf{\Lambda}_k = diag[\lambda_{k,1},\ldots,\lambda_{k,t_k}],\ \mathbf{P}_k = diag[P_{k,1},\ldots,P_{k,t_k}],\ \forall k. \tag{19a}$$

For the per-transmitter power constraint,

$$\mathbf{\Lambda}_k = \lambda_k \mathbf{I}_{t_k},\ \mathbf{P}_k = \mathbf{I}_{t_k} P_k / t_k,\ \forall k. \tag{19b}$$

Once (18) is set up, use the usual completing the square to obtain the MMSE decoder

$$\mathbf{G}_k = \mathbf{F}_k^*\mathbf{H}_{kk}^*\mathbf{M}_k,\ \mathbf{M}_k = \left[\sum_{l=1}^{K} \mathbf{H}_{kl}\mathbf{F}_l\mathbf{F}_l^*\mathbf{H}_{kl}^* + \mathbf{\Phi}_{\mathbf{n}k}\right]^{-1}. \tag{20}$$

In addition, use the technique of variation on (18) (replace $\mathbf{F}_k$ in (18) by $\mathbf{F}_k + \varepsilon\mathbf{\Delta}$, take the derivative with respect to ε, set $\varepsilon = 0$, etc.) to obtain the MMSE precoder

$$\mathbf{F}_k = \left[\sum_{l=1}^{K} \mathbf{H}_{lk}^*\mathbf{G}_l^*\mathbf{G}_l\mathbf{H}_{lk} + \mathbf{\Lambda}_k\right]^{-1} \mathbf{H}_{kk}^*\mathbf{G}_k^*. \tag{21}$$

To obtain the Lagrange multipliers, first substitute (20) into (18) to obtain ξ_1, a reduced augmented cost function, i.e., a cost function without the decoders. Then, use the technique of variation on ξ_1 to obtain

$$\mathbf{F}_k^*\mathbf{\Lambda}_k = \mathbf{B}_k, \tag{22}$$

where

$$\mathbf{B}_k = \mathbf{F}_k^* \mathbf{H}_{kk}^* \mathbf{M}_k \mathbf{H}_{kk} - \mathbf{F}_k^* \left(\sum_{l=1}^{K} \mathbf{H}_{lk}^* \mathbf{M}_l \mathbf{H}_{ll} \mathbf{F}_l \mathbf{F}_l^* \mathbf{H}_{ll}^* \mathbf{M}_l \mathbf{H}_{lk} \right). \tag{23}$$

Finally, left multiply (22) by $\mathbf{F}_k$ and apply the appropriate power constraint to obtain

$$\mathbf{\Lambda}_k = \mathbf{P}_k^{-1} \left\{ \mathbf{I}_{t_k} * \left(\mathbf{F}_k \mathbf{B}_k \right) \right\} \tag{24a}$$

for the per-antenna power constraint case and

$$\lambda_k = P_k^{-1} tr\left(\mathbf{F}_k \mathbf{B}_k \right) \tag{24b}$$

for the per-transmitter power constraint case.

Due to the interdependent nature of these expressions, the GIA iterates between them in search for the MMSE decoders and MMSE precoders. There are three steps in each iteration of the GIA:

<u>Step 1</u>: Given $\{\mathbf{F}_k\}$, obtain $\{\mathbf{G}_k\}$ by (20).

<u>Step 2</u>: Given $\{\mathbf{F}_k\}$, obtain $\{\mathbf{\Lambda}_k\}$ by (24a) or (24b).

<u>Step 3</u>: Given $\{\mathbf{G}_k\}$ and $\{\mathbf{\Lambda}_k\}$, obtain $\{\mathbf{F}_k\}$ by (21).

Of course, this iterative procedure is not unique. For example, take the conjugate transpose of (22) and right multiply it by (22) to obtain

$$\mathbf{\Lambda}_k^* \mathbf{F}_k \mathbf{F}_k^* \mathbf{\Lambda}_k = \mathbf{B}_k^* \mathbf{B}_k . \tag{25a}$$

Take the Schur product of (25a) with $\mathbf{I}_{t_k}$, and apply the per-antenna power constraint to obtain

$$\mathbf{P}_k \begin{bmatrix} \left|\lambda_{k,1}\right|^2 & & \\ & \ddots & \\ & & \left|\lambda_{k,t_k}\right|^2 \end{bmatrix} = \mathbf{I}_{t_k} * \left(\mathbf{B}_k^* \mathbf{B}_k \right). \tag{25b}$$

Finally, requiring all of the Lagrange multipliers to be non-negative, one obtains

$$\mathbf{\Lambda}_k = \left\{ \mathbf{P}_k^{-1} \left[\mathbf{I}_{t_k} * \left(\mathbf{B}_k^* \mathbf{B}_k \right) \right] \right\}^{1/2} . \tag{25c}$$

Using (25c) instead of (24a) would give another iterative procedure.

V. DEVELOPMENTS FOR IA

To perform the numerical comparison between the GIA and IA based approaches, the following are proposed.

A. Satisfying the Power Constraints

Implementation-wise, a IA solution can only be used if it satisfies the given power constraints. As such, the following two remarks are important (their proofs are left to the reader).

<u>Remark 1</u>: If the IA conditions (11-12) hold for a set of precoders and decoders, $\{\mathbf{F}_k\}$ and $\{\mathbf{G}_k\}$, scaling the columns of the precoders with non-zero scalars will not cause the IA conditions to be violated.

<u>Remark 2</u>: Given a set of precoders and decoders, $\{\mathbf{F}_k\}$ and $\{\mathbf{G}_k\}$, which satisfy the IA conditions (11-12), a set of precoders and decoders can be found to satisfy (11-12) and the per-transmitter power constraint (15). A set can also be found to satisfy (11-12) and the relaxed per-antenna power constraint (14), i.e., the constraint obtained by replacing the "=" in (14) by "≤".

B. Joint IA/MMSE Approaches

It is of great importance that further processing be done after obtaining a set of precoders and decoders, $\{\mathbf{F}_k\}$ and $\{\mathbf{G}_k\}$, which satisfy the IA conditions (11-12). The reason being is that satisfying the IA conditions will *only* guarantee that the system has been decoupled into K SU-MIMO systems with full rank equivalent channels ($\mathbf{G}_k\mathbf{H}_{kk}\mathbf{F}_k$, $\forall k$). Thus, $\forall k$, let the modified precoder and decoder be

$$\mathbf{F}_k = \mathbf{F}_{k,L}\mathbf{F}_{k,R}, \tag{26}$$

$$\mathbf{G}_k = \mathbf{G}_{k,L}\mathbf{G}_{k,R}, \tag{27}$$

where $\mathbf{F}_{k,L}$ and $\mathbf{G}_{k,R}$ are the original IA satisfying matrices. The goal of this subsection is thus to design $\{\mathbf{F}_{k,R}\}$ and $\{\mathbf{G}_{k,L}\}$ to minimize the MSE at each receiver while satisfying any one of the given power constraints. There are multiple possible ways to proceed; one way being to derive an iterative approach similar to that in Section IV. The approach employed in the following numerical results (simply called MMSE post processing) is $\forall k$, to let $\mathbf{F}_{k,R} = \mathbf{I}_{m_k}$ and to let $\mathbf{G}_{k,L}$ be the MMSE decoder. Note, such a modified precoder and decoder still satisfy the IA conditions.

VI. NUMERICAL RESULTS

The *goal* of this section is to compare and analyze the MMSE and IA based transceiver designs when IA is feasible and when it is not. To this end, a system where IA is feasible (denoted Case 1) and a system where IA is infeasible (denoted Case 2) are considered. In both cases, each node has two antennas and only one data stream is transmitted per pair, i.e., $t_k = r_k = 2$, $m_k = 1$, $\forall k$. What makes IA feasible in Case 1 and infeasible in Case 2 is that the number of pairs $K = 3$ in the former while $K = 4$ in the latter [3,7]. Also to this end: the GIA is run in both cases, the closed-form IA design in [3] with MMSE post processing is run in Case 1, and the IA seeking design from [6] with MMSE post processing is run in Case 2. Since the closed-form IA design in [3] generally yields two sets of IA transceivers, the two solutions are denoted here as IA1 and IA2. Furthermore, since the IA seeking design [6] is iterative, it is referred to here as Iterative IA (IIA).

For both cases, the elements of the channel matrices are independently identically distributed $CN(0,2)$ random variables and each data stream consists of uncoded BPSK modulated symbols. Both per-antenna and per-transmitter power constraints (14-15) are considered. No matter which constraint, the power specified at a transmitter is the same. This is done by choosing $P_{k,1} = P_{k,2} = 0.5P_k$. Also regardless of the constraint, all the transmitters have the same power specified ($P_1 = P_2$). With P_1 and P_2 held constant for the entire simulation and the noise covariance matrix $\mathbf{\Phi}_{\mathbf{n}k} \triangleq \varepsilon\, \mathbf{I}_2$, $\forall k$, the (transmit) signal to noise ratio, SNR $\triangleq 10\log_{10}(P_k/\varepsilon)$. If a solution is not found to satisfy its power constraint, a backing off of the power is allowed. That is, a "≤" is allowed instead of the "=" in the power constraint.

The following plots (Figs. 1-12) and observations are for a typical channel realization. In Figs. 1-3, the sum

MSE, system BER, and sum capacity of both feasibility cases and both power constraints are plotted versus the SNR. A few observations can be made. Firstly, no matter for which power constraint, which feasibility case, the superiority of the GIA can be seen in all three metrics here. Actually, the GIA, though designed for the MSE metric, always has similar or superior system BER's and sum capacities to the IA based transceiver designs in all of the channel realizations considered. Secondly, in the feasible IA case (Case 1), there is generally a merging of the performances of the GIA and the better of the IA1 and IA2 (happens to be IA2 for this channel realization) at high SNR. Thirdly, as evident here, there can be a *significant* difference in performance between IA solutions; between IA1 and IA2, there is an approximate 4 dB, 7 dB, and 3dB difference at a sum MSE of 0.5, system BER of 10^{-2}, and sum capacity of 10, respectively, for the per-antenna power constraint (the differences are even more drastic here for the per-transmitter power constraint). Obviously, an IA solution can be extremely suboptimal in any of these three metrics. Fourthly, comparing the two cases, the superiority of Case 1 over Case 2 can be seen in all three metrics. Fifthly, the GIA still has decent system BER's when IA is infeasible (Case 2). Lastly, the sum MSE results under the per-antenna and per-transmitter power constraint are very similar for the GIA. In other words, the performance achieved by the MMSE based design under the more flexible per-transmitter power constraint can also be achieved, at least approximately, under the more practical per-antenna power constraint.

To investigate the physics behind these approaches, it is insightful to study the powers of the various components of the received signals (e.g., to see what kind of balance or weighting of the various components the approaches chose). In order to do so, some definitions and preliminary results are needed. For the k^{th} receiver, assuming that $\mathbf{G}_k \neq \mathbf{0}_{1\times r_k}$, denote the desired signal subspace (the space spanned by $\mathbf{G}_k$') as M_k and the orthogonal complement of M_k as $M_k^{\perp}$. Since the projection of an arbitrary vector $\mathbf{x}$ onto M_k is $\mathbf{G}_k^*(\mathbf{G}_k\mathbf{G}_k^*)^{-1}\mathbf{G}_k\mathbf{x}$, the power of the part of $\mathbf{x}$ that will not be knocked out by the decoder $\mathbf{G}_k$ is $P_{M_k} = (\mathbf{G}_k\mathbf{G}_k^*)^{-1} tr\{\mathbf{G}_k\mathbf{Q}\mathbf{G}_k^*\}$ where $\mathbf{Q} = \langle\mathbf{x}\mathbf{x}^*\rangle$. By nature of the orthogonal complement, the power of the part of $\mathbf{x}$ that will be knocked out by the decoder is $P_{M_k^{\perp}} = tr\{\mathbf{Q}\} - P_{M_k}$. In the following, the superscript D, IO, or N will be added to P_{M_k} and $P_{M_k^{\perp}}$ to denote whether $\mathbf{x}$ is the desired signal, inter-user interference, or noise, respectively, at receiver k. Similarly, the superscript IO+N is added when $\mathbf{x}$ is the inter-user interference plus noise vector. Interestingly, $P_{M_k}^{IO+N} = P_{M_k}^{IO} + P_{M_k}^{N}$ and $P_{M_k^{\perp}}^{IO+N} = P_{M_k^{\perp}}^{IO} + P_{M_k^{\perp}}^{N}$.

Since $P_{M_k}^{D} = \left(\mathbf{G}_k\mathbf{G}_k^*\right)^{-1}\sigma_{k,1}^{D}$, $P_{M_k}^{IO} = \left(\mathbf{G}_k\mathbf{G}_k^*\right)^{-1}\sigma_{k,1}^{IO}$, and $P_{M_k}^{N} = \left(\mathbf{G}_k\mathbf{G}_k^*\right)^{-1}\sigma_{k,1}^{N}$,

$$
\begin{aligned}
SINR_{k,1} &= \frac{P_{M_k}^{D}}{P_{M_k}^{IO} + P_{M_k}^{N}} = \frac{P_{M_k}^{D}}{P_{M_k}^{IO+N}} \\
&= \frac{\tilde{P}_{M_k}^{D}}{\tilde{P}_{M_k}^{IO} + \tilde{P}_{M_k}^{N}} = \frac{1}{\tilde{P}_{M_k}^{IO+N}}.
\end{aligned}
\tag{28}
$$

Here, the ~ over the P indicates normalization by $P_{M_k}^{D}$, i.e., $\tilde{P}_{M_k}^{D} = P_{M_k}^{D} / P_{M_k}^{D}$ =1, $\tilde{P}_{M_k}^{IO} = P_{M_k}^{IO} / P_{M_k}^{D}$, etc. Obviously, if $P_{M_k}^{D}$ =0, this normalization and the last equality in (31) are invalid. Lastly, consider the effect of MMSE post processing on $P_{M_k}^{D}$, $P_{M_k^{\perp}}^{D}$, etc. Partitioning the decoder as in (27), some simple calculations yield that P_{M_k} =$(\mathbf{G}_{k,R}\mathbf{G}_{k,R}^*)^{-1} tr\{\mathbf{G}_{k,R}\mathbf{Q}\mathbf{G}_{k,R}^*\}$ granted that $\mathbf{G}_{k,L}\neq 0$ and $\mathbf{G}_{k,R}\neq \mathbf{0}_{1\times r_k}$, i.e., P_{M_k} and $P_{M_k^{\perp}}$ are independent of MMSE post processing. On a similar note, $P_{M_k}^{N}$ =ε provided that $\mathbf{G}_k\neq \mathbf{0}_{1\times r_k}$.

With these definitions and preliminary results in mind, consider Figs. 4-6 where the normalized subspace powers are plotted for the three transceiver designs (GIA, IA1, IA2) in the feasible IA case (Case 1) subject to the per-antenna power constraint—the plots for the per-transmitter power constraint case share the same trends with it and are thus omitted. For the k^{th} receiver, $\tilde{P}_{M_k}^{D}$ (always equal to 1) and $\tilde{P}_{M_k}^{IO+N}$ are plotted for the GIA, IA1, IA2. But, $\tilde{P}_{M_k}^{IO}$, $\tilde{P}_{M_k^{\perp}}^{IO}$, and $\tilde{P}_{M_k}^{N}$ are plotted only for the GIA. Before comparing the subspace powers for the GIA and the IA based approaches, consider them for each approach individually. For IA1 and IA2, $\tilde{P}_{M_k}^{IO}$=0 and $\tilde{P}_{M_k}^{IO+N} = \tilde{P}_{M_k}^{N} = \varepsilon / P_{M_k}^{D}$ because IA is feasible in Case 1. Furthermore, $\tilde{P}_{M_k}^{IO+N}$ for IA1 and IA2 are always straight lines versus the SNR with the same slope. For the GIA, most of the interference is in $M_k^{\perp}$. As the SNR increases, the ratio $\tilde{P}_{M_k^{\perp}}^{IO} / (\tilde{P}_{M_k}^{IO} + \tilde{P}_{M_k^{\perp}}^{IO})$ is high and generally increases. Moreover, $\tilde{P}_{M_k}^{IO}$ generally decreases, i.e., the interference suppression relative to the desired signal generally improves with SNR.

Now, comparing the GIA, IA1, and IA2, a number of observations are in order. Firstly, no matter the SNR, the GIA generally has a better (or similar) set of SINR's than both the IA1 and IA2. For this channel realization, this is obvious for IA2 but not so obvious for IA1 at high SNR. IA1 has a better SINR than the GIA at receiver 3 but a much poorer one than the GIA at receiver 1. This is undesirable for IA1 since, as is well known, the BER at low SINR's can be very poor and the user with the worst SINR dominates the system BER. Secondly, all of the SINR's of IA2 are bounded below by those of the GIA here. This is because IA2, in enforcing IA, knocks out too much of the desired signals. If it allows some interference to pass through the decoder as the GIA does, it may obtain better SINR's. By using (28) and by noting that $P_{M_k}^N = \varepsilon$ for both approaches, it is clear that the higher SINR's of the GIA are precisely because of its higher $P_{M_k}^D$'s. Lastly, it is observed that the SINR's of the better IA based solution (IA2 for this channel realization) generally merge (or are merging) with those of the GIA at higher SNR's.

Since $\tilde{P}_{M_k}^{IO} = \sigma_{k,1}^{IO} / \sigma_{k,1}^{D}$ is plotted in Figs. 4-6 and not $\sigma_{k,1}^{IO}$ alone, $\sigma_{k,1}^{IO}$ is plotted for the GIA alone (still under the per-antenna power constraint) in Fig. 7. As can be clearly seen, $\sigma_{k,1}^{IO}$, which can be thought of as a measure of the total deviation of the $\mathbf{g}_{k,d}\mathbf{H}_{kl}\mathbf{F}_l$ scalars ($\forall l \neq k$) from zero, is small and is generally decreasing as SNR increases. That is, the GIA has enforced IA condition (12) to a pleasing degree and enforces it more and more rigorously as the SNR increases.

The preceding discussion of subspace powers and IA-like behavior was for the feasible IA case (i.e., Case 1). Now, consider Figs. 8-11 where the normalized subspace powers are plotted for the four receivers in the infeasible IA case (Case 2) subject to the per-antenna power constraint—again, the plots for the per-transmitter power constraint case share the same trends with it and are thus omitted. This time, since IIA cannot perform precise IA, $\tilde{P}_{M_k}^{D}$ (always equal to 1), $\tilde{P}_{M_k}^{IO}$, $\tilde{P}_{M_k^\perp}^{IO}$, $\tilde{P}_{M_k}^{N}$, and $\tilde{P}_{M_k}^{IO+N}$, $\forall k$, are plotted for both the GIA and IIA.

Before comparing the subspace powers for the GIA and the IIA, consider them individually. Since P_{M_k} and $P_{M_k^\perp}$ are independent of MMSE *post processing* (i.e., $\mathbf{G}_{k,L}$ in the partitioning of (27)), $P_{M_k}^{D}$, $P_{M_k^\perp}^{D}$, $P_{M_k}^{IO}$, and $P_{M_k^\perp}^{IO}$ for IIA are constant as SNR increases. Consequently, $\tilde{P}_{M_k}^{IO+N}$ asymptotically approaches $\tilde{P}_{M_k}^{IO}$ at high SNR's as can be clearly seen in Figs. 8-11. For the GIA, as in the feasible IA case, most of the interference is in $M_k^\perp$ and $\tilde{P}_{M_k}^{IO}$ and $\tilde{P}_{M_k}^{N}$ generally decrease as the SNR increases. Sometimes though, unlike the feasible IA case, there is a receiver whose $\tilde{P}_{M_k}^{IO}$ remains flat or slightly increases as SNR increases—even under such a case, $\tilde{P}_{M_k}^{IO+N}$ is still observed to be decreasing.

Now, comparing the GIA and IIA, a number of observations are in order. Firstly, neither the GIA nor the IIA shut down any data streams (observe that $\tilde{P}_{M_k}^{D} = 1$ in the plots). Secondly, no matter the SNR, the GIA generally has a better set of SINR's than IIA. In particular, the worst SINR of IIA is lower than the worst of the GIA. This is undesirable for IIA since, as is well known, the BER at low SINR's can be very poor and the user with the worst SINR dominates the system BER. Lastly, at high SNR, the GIA is sometimes seen to have lower or approximately the same $\tilde{P}_{M_k}^{IO}$, $\forall k$, as IIA. This is interesting since, as IA is the main goal of IIA, one might expect it to always have better, or at least the same, interference suppression than the GIA.

In Fig. 12, $\sigma_{k,1}^{IO}$ is plotted for the GIA and IIA (both still under the per-antenna power constraint). Counter intuitively, $\sigma_{k,1}^{IO}$ for IIA is increasing for every k as the SNR increases; with less noise, one would expect better IA-like behavior. But, realizing that IIA's

$$\mathbf{G}_{k,L} = \mathbf{F}_k^* \mathbf{H}_{kk}^* \mathbf{G}_{k,R}^* \Bigg/ \left(\sum_{l=1}^{K} \left| \mathbf{G}_{k,R} \mathbf{H}_{kl} \mathbf{F}_l \right|^2 + \varepsilon \left\| \mathbf{G}_{k,R} \right\|_F^2 \right) \quad (29)$$

(granted that $\mathbf{G}_{k,R} \neq \mathbf{0}_{1\times r_k}$),

$$\sigma_{k,1}^{IO} = \left| \mathbf{G}_{k,L} \right|^2 \sum_{l=1,l\neq k}^{K} \left| \mathbf{G}_{k,R} \mathbf{H}_{kl} \mathbf{F}_l \right|^2, \quad (30)$$

and that all terms other than ε in (29-30) remain constant versus SNR, it becomes obvious that the observed increasing behavior is correct. Comparing $\sigma_{k,1}^{IO}$ for the GIA and IIA, it can be seen that their values are similar at higher SNR. This implies that they have a similar degree of IA-like behavior. Recalling that IA is the main goal of IIA, this is quite impressive.

One last note: recall that in Hypothesis 2 of Section III., it is hypothesized that the MMSE design may shut off a pair(s) when K is large. Actually, this is observed in the numerical simulations. It is observed that the GIA shuts off one pair for one channel realization in Case 2 (K=4) and all of the channel realizations in Case 3 (this case has K=5, $t_k = r_k = 2$, $m_k = 1$, $\forall k$, and is not mentioned above). It is interesting that this transition from no pairs (in Case 1) to one pair (in Case 3) being shut off does not exactly coincide with the feasibility of IA (Case 1 feasible; Case 2 infeasible; Case 3 infeasible). That is, it is interesting that IA being infeasible does not necessarily result in pair(s) being shut off by the GIA.

VII. CONCLUSION

This work gives important insights into the bridge between the information theoretic IA and classical MIMO beamforming transceiver designs. It does this for the K-user constant MIMO interference channel by *rigorously* establishing the relationships between the IA and the MMSE, Max Min SINR, Max Sum Capacity, and Min BER transceiver designs. For the no noise scenario, it is proved that there are equivalencies between them and the IA transceiver design. For any noise covariance matrix, it is proved that IA is actually one of the building blocks for each of the transceiver designs; they all seek the optimum (for their respective metrics) balance between preserving the desired data streams, aligning the inter-stream interferences, aligning the inter-user interferences, and knocking out the noises. This work also proposes the GIA to perform the MMSE transceiver design of the K-user constant MIMO interference channel subject to general linear equality power constraints.

In addition to proposing the above relationships and the GIA, this work uses them to draw some conclusions about IA's proper place among the transceiver designs. It is observed that the GIA, with its IA-like behavior, is able to outperform [6] and produce decent results in a system where IA is infeasible. It is also observed that the GIA is able to provide superior or similar performance to [3] in a system where IA is feasible. Thus, it is concluded that IA's proper place is as a building block of the transceiver designs. It generally should not be pursued alone. Instead, the traditional transceiver designs should be used since they already balance IA and the other building blocks. Though IA is not pursued alone, its feasibility limits (e.g., [3,7]) may still be useful in deciding the number of cooperating cells in CBF. Moreover, keeping in mind that it is one of the building blocks of transceiver designs may be able to aid in the development of practical CBF transceiver designs in the near future.

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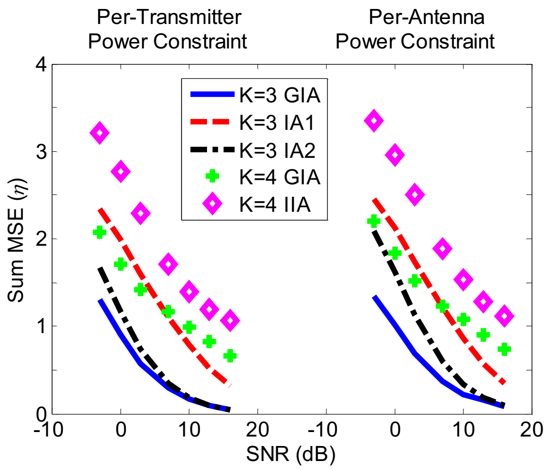

Figure 1. Sum MSE, η, for the Approaches in the Feasible and Infeasible IA Cases.

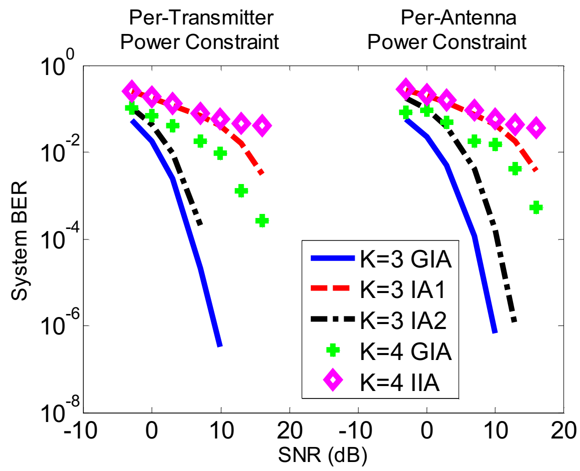

Figure 2. System BER for the Approaches in the Feasible and Infeasible IA Cases.

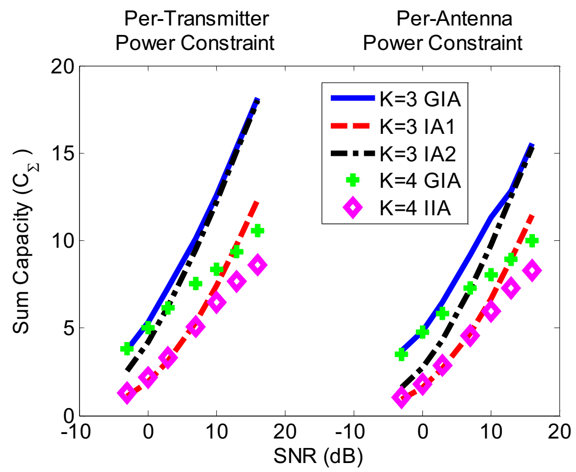

Figure 3. Sum Capacity, C_Σ, for the Approaches in the Feasible and Infeasible IA Cases.

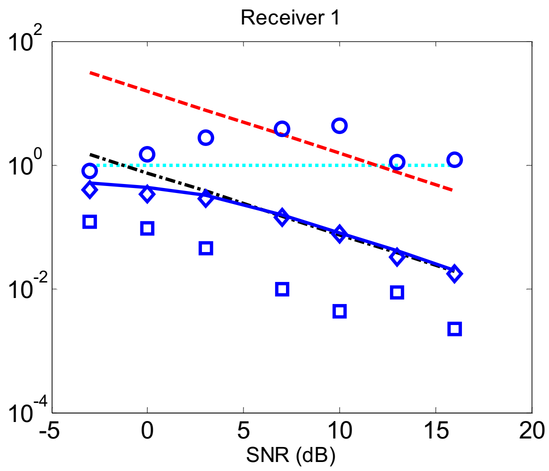

Figure 4. Normalized Subspace Powers for Receiver 1: Feasible IA Case under the per-antenna power constraint. (Legend: the cyan dotted line corresponds to $\tilde{P}^{D}_{M_1}$; the red dashed, black dash-dotted, and blue solid lines correspond to $\tilde{P}^{IO+N}_{M_1}$ of the IA1, IA2, and GIA, respectively; the blue circle, square, and diamond markers correspond to, respectively, $\tilde{P}^{IO}_{M_1^{\perp}}$, $\tilde{P}^{IO}_{M_1}$, and $\tilde{P}^{N}_{M_1}$ of the GIA).

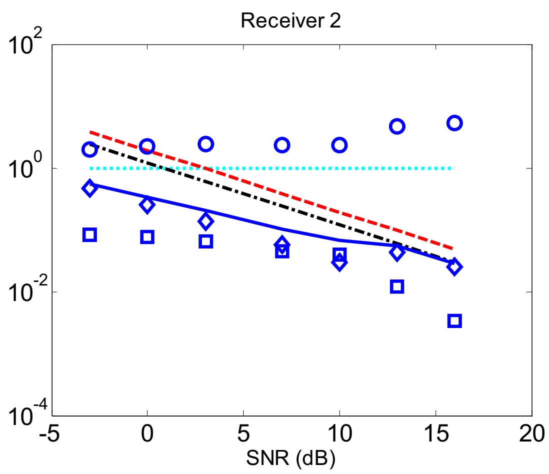

Figure 5. Normalized Subspace Powers for Receiver 2: Feasible IA Case under the per-antenna power constraint. (Legend is analogous to that of Fig. 4)

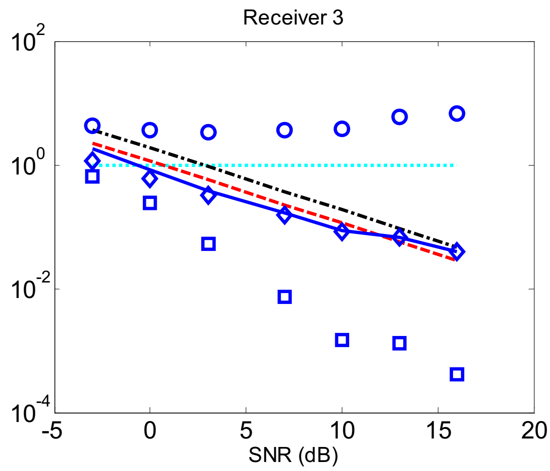

Figure 6. Normalized Subspace Powers for Receiver 3: Feasible IA Case under the per-antenna power constraint. (Legend is analogous to that of Fig. 4)

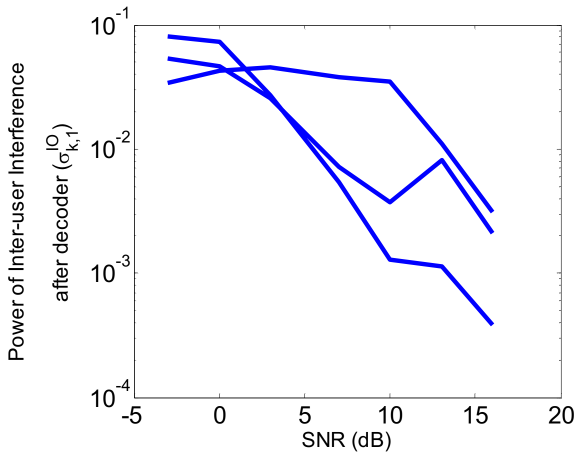

Figure 7. Power of Inter-user Interference after Decoder, $\sigma^{IO}_{k,1}$: Feasible IA Case under the per-antenna power constraint. (The 3 curves correspond to GIA's $\sigma^{IO}_{1,1}$, $\sigma^{IO}_{2,1}$, $\sigma^{IO}_{3,1}$)

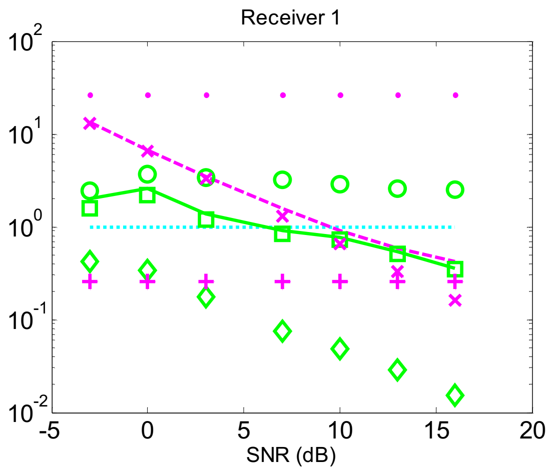

Figure 8. Normalized Subspace Powers for Receiver 1: Infeasible IA Case under the per-antenna power constraint. (Legend: the cyan dotted line corresponds to $\tilde{P}^{D}_{M_1}$; the magneta dashed and green solid lines correspond to the $\tilde{P}^{IO+N}_{M_1}$ of IIA and GIA, respectively; the magneta dot and green circle markers correspond to the $\tilde{P}^{IO}_{M_1^{\perp}}$ of IIA and GIA, respectively; the magneta '+' and green square markers correspond to the $\tilde{P}^{IO}_{M_1}$ of IIA and GIA, respectively; the magneta 'x' and green diamond markers correspond to the $\tilde{P}^{N}_{M_1}$ of IIA and GIA, respectively)

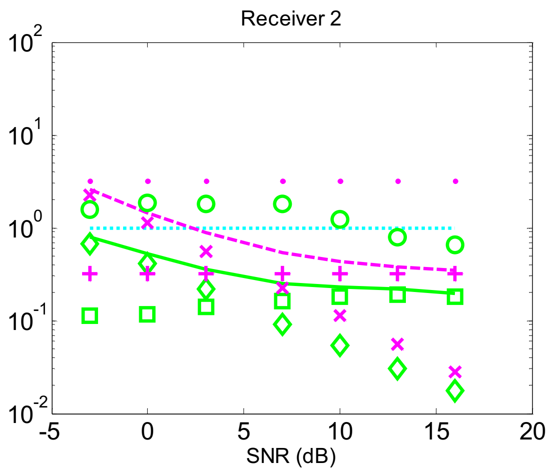

Figure 9. Normalized Subspace Powers for Receiver 2: Infeasible IA Case under the per-antenna power constraint. (Legend is analogous to that of Fig. 8)

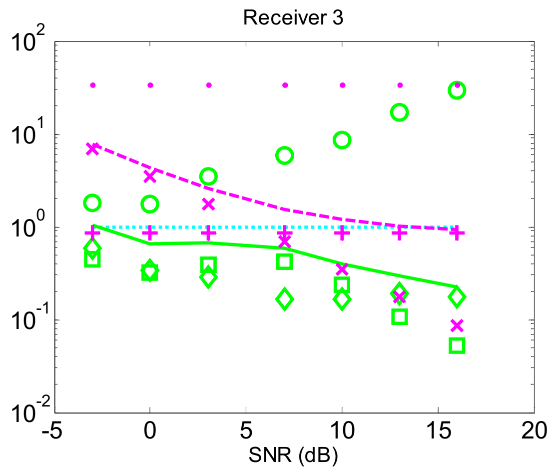

Figure 10. Normalized Subspace Powers for Receiver 3: Infeasible IA Case under the per-antenna power constraint. (Legend is analogous to that of Fig. 8)

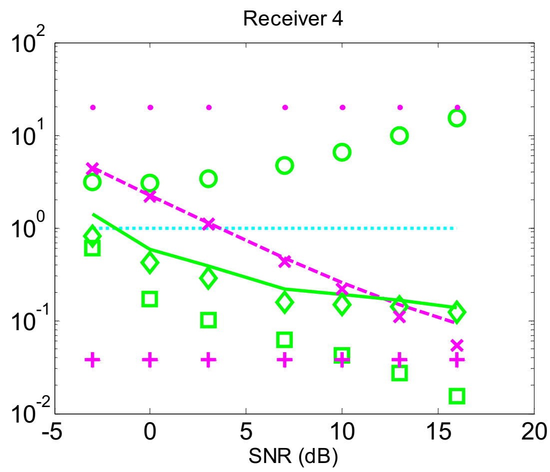

Figure 11. Normalized Subspace Powers for Receiver 4: Infeasible IA Case under the per-antenna power constraint. (Legend is analogous to that of Fig. 8)

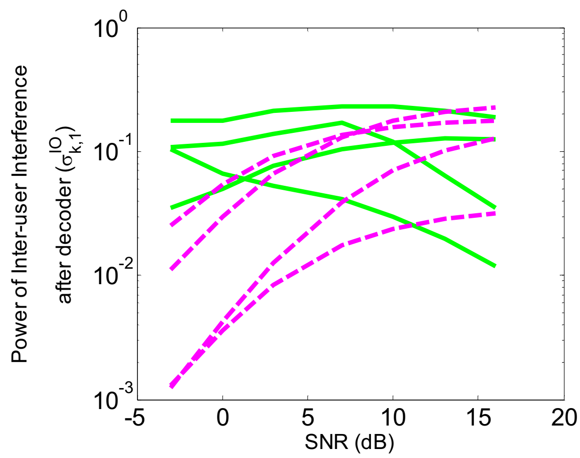

Figure 12. Power of Inter-user Interference after Decoder, $\sigma^{IO}_{k,1}$: Infeasible IA Case under the per-antenna power constraint. (The 4 magenta dashed lines correspond to IIA's $\sigma^{IO}_{1,1}$,…, $\sigma^{IO}_{4,1}$; the 4 green solid lines correspond to GIA's $\sigma^{IO}_{1,1}$,…, $\sigma^{IO}_{4,1}$)