Dedicated-Relay vs. User Cooperation in Time-Duplexed Multiaccess Networks

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***Abstract*— The performance of *user cooperation* that results when users forward packets for each other in a multiaccess network is compared to that of *dedicated-relay cooperation* which results from using a dedicated wireless relay when the users do not cooperate. Using the total transmit and processing power consumed at all nodes as a cost metric, the outage probabilities achieved by dynamic decode-and-forward (DDF) and amplify-and-forward (AF) are compared for the two networks. A geometry-inclusive high signal-to-noise ratio (SNR) outage analysis in conjunction with area-averaged numerical simulations shows that in a K-user time-duplexed multiaccess network, user and dedicated-relay cooperation achieve a maximum diversity per user of K and 2, respectively, under both DDF and AF. However, when accounting for energy costs of processing and communication, dedicated-relay cooperation can be more energy efficient than user cooperation, i.e., dedicated-relay cooperation achieves *coding* (SNR) *gains*, particularly in the low SNR regime, that override the diversity advantage of user cooperation.**

I. Introduction

Cooperation results when nodes in a network share their power and bandwidth resources to mutually enhance their transmissions and receptions. Cooperation can be induced in several ways. We compare two approaches to inducing cooperation in a multiaccess channel (MAC) comprised of K sources (users) and one destination. In the first approach, we allow source nodes to forward data for each other and in the second approach, we introduce a dedicated wireless relay node to forward data from the sources assuming cooperation between the source nodes is either undesirable or impossible. We refer to networks employing the former approach as *user cooperative (UC) networks* and those employing the latter as *dedicated-relay cooperative (RC) networks*. Our motivation for this terminology is that although users act as relays for one another in the UC network, they are primarily interested in transmitting their own data, while in contrast, the dedicated-relay node in the RC network is dedicated to relaying packets for the sources.

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There are important differences between UC and RC networks that are not easy to analyze from a communication-theoretic point of view. For example, in UC networks one likely needs economic incentives to induce cooperation amongst users. On the other hand, RC networks incur relay infrastructure costs. While incentives and infrastructure costs are important issues, we use the total transmit and processing power consumed as a cost metric for our comparisons. To this end, ignoring technology-dependent limitations on processing, we model the processing power as a function of the transmission rate, and thereby the transmit signal-to-noise ratio (SNR). We also introduce *processing scale factors* to characterize the ratio of the power costs of processing relative to that for transmission. While the processing (power and chip density) costs for encoding and decoding are complex functions of the specific communication and computing technologies used, the scale factors allow us to parametrize and study the impact of such processing costs.

We motivate the need for this analysis with examples of processing and transmission costs for wireless devices serving three different applications. Consider a Motorola RAZR GSM mobile phone. This device has a maximum transmit power constraint of 1 (2) W in the 900 (1900) MHz band. With a 3.7 V battery rated at 740 mAh it has a capacity of almost 10 kJ of energy resulting in an average talk time of 4 hours. On the other hand, consider the 802.11 wireless local area network (WLAN) interface. An Atheros whitepaper [1] found that typical WLAN interfaces consume 2 to 8 W for active communications. In contrast, the transmit power for this device in the range of 20 to 100 mW is only a small fraction of the processing costs. Finally, consider low-power sensor devices such as the Berkeley motes. The authors in [2] model the energy cost per bit for a reliable 1 Mbps link over a distance d and path-loss exponent α by a transmitter cost of $\mathcal{E}_{tx} = \mathcal{E}_t + \mathcal{E}_{pa} d^\alpha$ where $\mathcal{E}_t = 0.36$ J/MB is the energy dissipated in the transmitter electronics and $\mathcal{E}_{pa} = 8 \times 10^{-5}$ J/m^2/MB scales the required transmit energy per bit. Accounting for the signal processing costs at the receiver as $\mathcal{E}_{rx} = 1.08$ J/MB, they show that for distances less than the transition distance of $d = \sqrt{\mathcal{E}_t/\mathcal{E}_{pa}} = 67$ m, processing energy cost dominates transmission cost and vice-versa. In general, the ratio of processing to transmission power depends on both the device functionality (long distance vs. local links) and the application (high vs. low rate) supported. Thus, accounting for both the transmit and processing power (energy) costs in our comparisons allows us to identify the processing factor regimes where cooperation is energy efficient.

We consider single-antenna half-duplex nodes and constrain

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all transmitting nodes in both networks to time-duplex their transmissions. Thus, in the RC network each source cooperates with the dedicated relay over two-hops where in the first hop the source transmits while the dedicated relay listens and in the second hop both the source and the dedicated relay transmit. For the UC network, for $K > 2$ we consider the cooperative schemes of *two-hop*, where the set of users cooperating with any source transmit in the second hop, and *multi-hop*, where the cooperating users transmit sequentially in time. We assume that transmitters in both networks do not have transmit channel state information (CSI). Under this assumption, we develop geometry-inclusive upper and lower bounds on the outage probabilities of each network for the cooperative strategies of *dynamic* decode-and-forward (DDF) [3] and amplify-and-forward (AF). This allows us to compare the outage performance of each cooperative strategy for the two networks via both the diversity gain and a *coding* (*SNR*) *gain* [4]. Our approach of accounting for the power involved in data processing and transmission enables us to compare the outage performance of the two types of cooperative networks despite differences in the total number of nodes and cooperative strategies.

For single-antenna nodes, the maximum DDF and AF diversity for two-hop relaying is 2 [3] and our geometry-inclusive analysis demonstrates the same. For the two-hop UC network, we show that, if relay selection is allowed, AF achieves a maximum diversity of 2 for all $K \geq 2$. For the same network, we also show here that, except for a *clustered* geometry where the maximum diversity approaches K, DDF also achieves a maximum diversity of 2. On the other hand, when users cooperate using a K-hop scheme, our bounding analysis agrees with the earlier diversity-multiplexing results that both DDF [3] and AF [4] achieve a maximum diversity of K.

The coding gains achieved are in general a function of the transmission parameters and network geometry. In an effort to generalize such results, we present an *area-averaged* numerical comparison. Specifically, we consider a sector of a circular area with the destination at the center, a fixed dedicated-relay position, and the users randomly distributed in the sector which models wireless LAN, cellular, and sensor networks. Our results are summarized by the following observations: i) user cooperation can achieve higher diversity gains than dedicated-relay cooperation but at the expense of increased complexity and ii) dedicated-relay cooperation can achieve larger coding gains when we account for the energy costs of cooperation, thus diminishing the effect of the diversity gains achieved by user cooperation.

This paper is organized as follows. In Section II, we present the network and channel models and develop a power-based cost metric. In Section III, we present outage approximations for DDF and AF strategies for both networks. In Section IV, we present the numerical results. We conclude in Section V.

II. Channel and Network Models

A. Network Model

Our networks consist of K users (source nodes) numbered $1, 2, \ldots, K$ and a destination node d. In the absence of any form of cooperation, this network is modeled as a K-user multiple access channel (MAC). For the RC network there is one additional node, the dedicated-relay node r. We impose a *half-duplex* constraint on every node, i.e., each node can be in one of two modes, *listen* (*L*) or *transmit* (*T*). We write $\mathcal{K} = \{1, 2, \ldots, K\}$ for the set of users and $\mathcal{T} = \mathcal{K} \cup \{r\}$ for the set of transmitters in the RC network. Let $X_{k,i}$ be the transmitted signal (channel input) at node k at time i, $i = 1,\ 2,\ \ldots, n$. We model the wireless multiaccess links under study as additive Gaussian noise channels with fading. For such channels, the received signal (channel output) at node m at time i is

$$Y_{m,i} = \begin{cases} \left(\sum\limits_{k \neq m} H_{m,k,i} X_{k,i} \right) + Z_{m,i} & M_{m,i} = L \\ 0 & M_{m,i} = T \end{cases} \tag{1}$$

where the $Z_{m,i}$ are independent, proper, complex, zero-mean, unit variance Gaussian noise random variables, $M_{m,i}$ is the half-duplex mode at node m, and $H_{m,k,i}$ is the complex fading gain between transmitter k and receiver m at time i. Note that for both networks as well as the MAC, $M_{d,i} = L$, for all i. Further, for the RC network and the MAC, since the sources do not cooperate, and hence do not listen, we have $M_{k,i} = T$, for all i and for all $k \in \mathcal{K}$. We assume that the transmitted signals in both networks are constrained in power as

$$\sum_{i=1}^{n} E\left[|X_{k,i}|^2 \right] \leq nP_k, \quad k \in \mathcal{T} \tag{2}$$

where P_k is the average power constraint at transmitter k. Throughout the sequel we assume that all transmitters use independent Gaussian codebooks with asymptotically large codelengths and the total transmission bandwidth is unity. We also assume that due to lack of transmit CSI, the transmitters do not vary power as a function of channel states. Further, we assume that the modes $M_{k,i}$ are made available to all nodes. We will clarify our motivation for this assumption in the sequel. Finally, we use the usual notation for entropy and mutual information [5] and take all logarithms to the base 2 so that our rate units are bits/channel use. We write random variables (e.g. H_k) with uppercase letters and their realizations (e.g. h_k) with the corresponding lowercase letters and use the notation $C(x) = \log(1+x)$ where the logarithm is to the base 2. Finally, throughout the sequel we use the words "user" and "source" interchangeably.

B. RC Network

The RC network has $K + 1$ inputs $X_{k,i}$, $k \in \mathcal{T}$, and two outputs $Y_{r,i}$ and $Y_{d,i}$ [6], [7]. We consider a time-duplexed dedicated-relay cooperative (TD-RC) model where each source transmits over the channel for a period $T_P = 1/K$ of the total time. Further, the transmission period of source k, for all k, is sub-divided into two slots such that the dedicated relay listens in first slot and transmits in the second slot. We denote the time fractions of the first and second slots as θ_k and $\overline{\theta}_k = 1 - \theta_k$, respectively, for user k and note that the duration θ_k of the dedicated relay mode M_r can be different for different k. The time-duplexed two-hop scheme for the RC nework is illustrated in Fig. 1(a) for

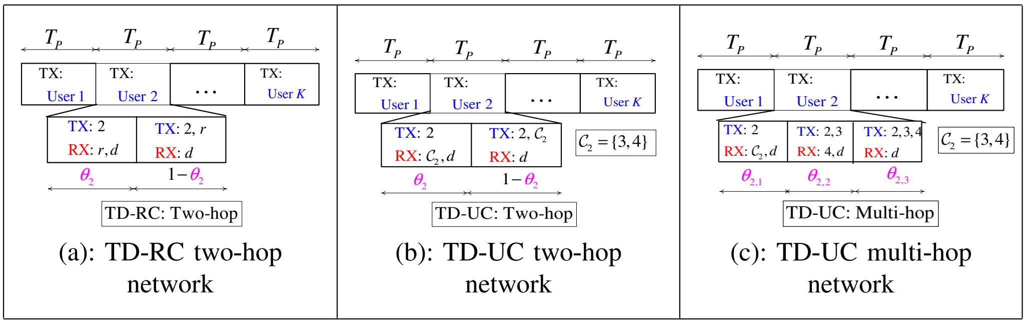

Fig. 1. Time-duplexed transmission schemes for the user cooperative and relay cooperative MACs.

user 2. Time-duplexing simplifies the analysis for each user to that of a single-source relay channel in each period T_P. We assume that the dedicated relay uses negligible resources to communicate its mode transition to the destination. We also assume that, due to the lack of transmit CSI, the transmitters use all available power for transmission subject to (2) in every channel use. Thus, in the k^{th} time period, for all k, user k and the dedicated-relay transmit at power $\overline{P}_k = KP_k$ and $\overline{P}_r = P_r/\overline{\theta}_k$, respectively, where $\overline{\theta}_k = 1 - \theta_k$. Finally, throughout the analysis we assume that P_r is proportional to P_k.

C. UC Network

In a user cooperative (UC) network, there is a combinatorial explosion in the number of ways one can duplex K sources over their half-duplex states. We present two transmission schemes which allow each user to be aided by up to $K-1$ other users. In both schemes the users time-duplex their transmissions, thereby resulting in time-duplexed user cooperation (TD-UC); the two schemes differ in the manner the period T_P is further sub-divided between the transmitting and the cooperating users.

We first consider a *two-hop scheme* such that the period over which user k, for all k, transmits is sub-divided into two slots. In the first slot only user k transmits while in the second slot both user k and the set $\mathcal{C}_k \subseteq \mathcal{K}\backslash\{k\}$ of users that cooperate with user k transmit. This is shown in Fig. 1(b) for user 2 and $\mathcal{C}_2 = \{3,4\}$. We remark that this scheme has the same number of hops as the TD-RC network except now user k can be aided by more than one user in $\mathcal{C}_k$. We write θ_k and $1-\theta_k$ to denote the time fractions associated with the first and second slots of user k.

We also consider a *multi-hop* scheme where the total transmission time for source k is divided into L_k slots, $1 \leq L_k \leq K$, where $L_k = |\mathcal{C}_k| + 1$. Specifically, in each time-slot, except the first slot where only user k transmits, one additional user cooperates in the transmission until all L_k users transmit in slot L_k. When the cooperating users decode their received signals, we assume that the users are ordered in the sense that the new user that cooperates in the l^{th} time fraction is the first user that can decode the message when the l cooperating users are transmitting. We denote the l^{th} time fraction for user k as $\theta_{k,l}$, $l = 1, 2, \ldots, L$ (see Fig. 1(c) for user 2 with $\mathcal{C}_2 = \{3,4\}$). We henceforth refer to the two schemes as two-hop TD-UC and multi-hop TD-UC.

User k transmits at power

$$\overline{P}_k = P_k \cdot K/(N_k+1) \tag{3}$$

where $N_k \leq K-1$ is the total number of users whose messages are forwarded by user k. For the two-hop scheme, in those sub-slots where user k acts as a cooperating node, its transmission power in (3) is scaled by the appropriate $\overline{\theta}_k$. For the multi-hop scheme, let $\pi_k(\cdot)$ be a permutation on $\mathcal{C}_k$ such that user $\pi_k(l)$ begins its transmissions in the fraction $\theta_{k,l}$, for all $l = 2, 3, \ldots, L_k$, and $\pi_k(1) = k$. When user k acts as a cooperating node for user j, $j \neq k$, such that $\pi_j(l) = k$ for some $l > 1$, its power $\overline{P}_k$ in (3) is scaled by the total fraction for which it transmits for user j, i.e., $\sum_{m=l}^{L_j} \theta_{j,m}$. Recall that the modes $M_{k,i}$ are available at all nodes; so we assume that a cooperating node or a dedicated relay uses negligible resources to communicate its transition from one mode to another to the destination and to other cooperating nodes. For AF we assume equal length slots and consider symbol-based two-hop and multi-hop schemes.

It is in general not possible to know *a priori* the number of users N_k whose messages are forwarded by user k. A decentralized scheme is to have all users forward packets for each other. Alternatively, for low mobility environments, the set $\mathcal{C}_k$ for user k can be chosen as the set of proximal users (see, for e.g., [8], [9] and the references therein).

D. Cost Metric: Total Power

We use the total power consumed by all the nodes as a cost metric for comparisons. We model the total power to account for both transmission and processing power costs motivated by the observation that wireless devices operate at different regimes of transmission and processing power requirements which in turn can affect their cooperative (processing and forwarding) capabilities. For instance, in addition to its transmit power a node also consumes processing power to encode and decode its signals. Furthermore, a node that relays consumes additional power in encoding and decoding packets for other nodes. We model these costs by defining encoding and decoding variables η_k and δ_k, respectively, and write the power required to process the transmissions of node j at node k as

$$P_{k,j}^{proc} = P_{k,0}^{proc} + \left(\eta_k I_k^{enc}(j) + \delta_k I_k^{dec}(j)\right) \cdot f(R_j) \quad \text{for all } k \in \mathcal{T}, j \in \mathcal{K} \tag{4}$$

where $P_{k,j}^{proc}$ is the power required by node k to cooperate with user j, $I_k^{enc}(j)$ and $I_k^{dec}(j)$ are indicator functions that are set to 1 if node k encodes and decodes, respectively, for user j, $P_{k,0}^{proc}$ is the minimum processing power at node k which is in general device and protocol dependent, and $f(R_j)$ is a function of the transmission rate R_j in bits/sec at user j (see also [10, (3)-(6)]). For example, a relay node that uses DDF consumes power for overhead, encoding, and decoding costs while a relay node using AF only has overhead costs. The unitless variables η_k and δ_k quantify the ratio of processing to transmission power at user k to encode and decode a bit, respectively. For example, $\eta \ll 1$ and $\delta \ll 1$ for cellular devices in which transmission costs dominate, $\eta \gg 1$ and $\delta \gg 1$ for the wireless LAN cards for which processing costs dominate, and η and δ are determined by the operational parameters for sensor-like devices. In general, the processing cost function f depends on the encoding and decoding schemes used as well as the device functionality. For simplicity, we choose f as

$$f(R_k) = R_k \quad \text{for all } k. \tag{5}$$

This choice fits the scenario where processing power is proportional to device throughput. Finally, we assume that the destination in typical multiaccess networks such as cellular or many-to-one sensor networks has access to an unlimited energy source and ignore its processing costs. We write the total power consumed on average (over all channel uses) at node k, $k \in \mathcal{T}$, as

$$P_{k,tot} = \begin{cases} P_k + P_{k,k}^{proc} + \sum\limits_{j\in\mathcal{K}, j\neq k} I_k(j) P_{k,j}^{proc} & k \in \mathcal{K} \\ P_k + P_{k,k}^{proc} + \sum\limits_{j\in\mathcal{K}} I_k(j) P_{k,j}^{proc} & k = r \end{cases} \tag{6}$$

where $I_k(j)$ is an indicator function that takes the value 1 if node k cooperates with node j. For user k, the first $P_{k,k}^{proc}$ term in (6) accounts for the power used to process its own message while the second summation term accounts for the power node k incurs in cooperating with all other source nodes. Note that at high SNR, i.e., high P_k for all k, the dominating term in (6) is P_k since $P_{k,k}^{proc}$ is usually a constant and R_j increases logarithmically in P_j, for all $k, j \in \mathcal{K}$. The total power consumed by all transmitting nodes in each network is given as

$$P_{tot} = \textstyle\sum_{k\in\mathcal{A}} P_{k,tot}, \quad \mathcal{A} = \begin{cases} \mathcal{K} & \text{TD-MAC or TD-UC} \\ \mathcal{T} & \text{TD-RC} \end{cases} \tag{7}$$

E. Fading Models

We model the fading gains as $H_{m,k,i} = A_{m,k,i} \big/ d_{m,k}^{\gamma/2}$ where $d_{m,k}$ is the distance between the m^{th} receiver and the k^{th} source, γ is the path-loss exponent, and the $A_{m,k,i}$ are jointly independent identically distributed (i.i.d.) zero-mean, unit variance, proper, complex Gaussian random variables. We assume that the fading gain $H_{m,k,i}$ is known only at receiver m. We also assume that $H_{m,k,i}$ remains constant over a coherence interval and changes independently from one coherence interval to another. Further, the coherence interval is assumed large enough to apply information-theoretic quantities such as mutual information. Finally, we also assume that the fading gains are independent of each other and independent of the transmitted signals $X_{k,i}$, for all $k \in \mathcal{T}$ and i.

III. GEOMETRY-INCLUSIVE OUTAGE ANALYSIS

We first compare the outage performance of the UC and RC networks via a limiting analysis in SNR of the outage probabilities achieved by DDF. We later do the same for both networks for AF. In [4], Laneman develops bounds on the DF and AF outage probabilities for a relay channel where the source and the relay transmit on orthogonal channels to the destination. In [3], the authors introduce a DDF strategy for a half-duplex relay channel where the relay remains in the *listen* mode until it successfully decodes its received signal. Furthermore, the authors show that for both two-hop and multi-hop relay channels, DDF achieves the diversity-multiplexing tradeoff (DMT) performance [11] of an equivalent MIMO channel for small multiplexing gains.

In an effort to quantify both the diversity and the effect of geometry, we present geometry-inclusive upper and lower bounds on the DDF and AF outage probabilities for the TD-RC and the TD-UC networks. We summarize the results here and develop the detailed analyses in the Appendices. Under DDF, a cooperating node (resp. dedicated-relay) in the TD-UC (resp. TD-RC) network can indicate its mode change following successful decoding via a one bit feedback signal that is assumed available to all nodes. This in turn allows user k, for all k, to determine the order of its cooperating DDF users in the multi-hop TD-UC network. For the multi-hop TD-UC network, the difference $\theta_{k,l} - \theta_{k,j}$ for two cooperating users $l, j \in \mathcal{C}_k$ may be smaller than the symbol time in which case user k could choose arbitrarily between the two users. On the other hand, for an AF-based multi-hop TD-UC network, the node order could be determined *a priori* based on the proximity of sources.

A. Dynamic-Decode-and-Forward

1) TD-RC: In general, obtaining a closed form expression for the outage probability of each user is not straightforward. Suppose that $P_r = \lambda \overline{P}_k$ for some constant λ and recall that $\overline{P}_r = P_r / \overline{\theta}_k$. In Appendix II, we develop upper and lower bounds on the DDF outage probability $P_o^{(k)}$ of user k transmitting at a fixed rate R_k, for all k, as (see (36))

$$P_{o,2\times1} \le P_o^{(k)} \le \left[\frac{(2^{R_k/\overline{\theta}_k^*} - 1)^2 \overline{\theta}_k^*}{(2^{R_k} - 1)^2} + \frac{2 d_{r,k}^{\gamma} (2^{R_k/\theta_k^*} - 1)^2}{d_{d,r}^{\gamma} (2^{R_k} - 1)^2} \right] \cdot \frac{(2^R - 1)^2 d_{d,k}^{\gamma} d_{d,r}^{\gamma}}{2\lambda \left(\overline{P}_k\right)^2} + O\left(\left(\overline{P}_k\right)^{-3}\right) \tag{8}$$

where $P_{o,2\times1}$ is the outage probability of a 2×1 *distributed* MIMO channel whose i^{th} transmit antenna is at a distance $d_{d,i}$, $i = k, r$, from the destination and $\theta_k^* \in (0,1)$ is a fraction chosen to upper bound $P_o^{(k)}$. The notation $O(x)$ in (8) means that there is a positive constant M and a real number x_0 such that the $O(x)$ term is upper bounded by $M|x|$ for all $x \geq x_0$. In Appendix II, we show that

$$P_{o,2\times 1} = (2^{R_k}-1)^2 d_{d,k}^{\gamma} d_{d,r}^{\gamma} \Big/ 2\lambda \left(\overline{P}_k\right)^2 + O\left(\left(\overline{P}_k\right)^{-3}\right). \quad (9)$$

Thus, from (8) and (9) we see that for a fixed rate transmission, the maximum diversity (negative exponent of $\overline{P}_k$) achieved by DDF is 2, as predicted by the DMT analysis for DDF in [3, Theorem 4]. Comparing (8) and (9), we further see that the bracketed expressions on the right side of the inequality in (8) upper bounds the coding gains by which $P_o^{(k)}$ differs from the MIMO lower bounds.

2) TD-UC – Two-Hop: The outage analysis for the two-hop TD-RC network can be extended to the two-hop TD-UC network with the observation that θ_k is now determined by the cooperating user that takes the longest to decode. In Appendix II, for sufficiently large power P_k, we bound $P_o^{(k)}$ as (see (38) and (43))

$$P_{o,L_k\times 1} \leq P_o^{(k)} \leq K_2 \cdot \frac{\left(2^{R_k}-1\right)^{L_k} \prod_{j\in\mathcal{S}_k} d_{d,j}^{\gamma}}{(L_k!)\left(\overline{P}_k\right)^{L_k} \prod\limits_{j\in\mathcal{S}_k} \lambda_j} + O\left(\left(\overline{P}_k\right)^{-L_k-1}\right) \quad (10)$$

where $\lambda_j = \overline{P}_j/\overline{P}_k$ for all $j \in \mathcal{S}_k = \mathcal{C}_k \cup \{k\}$, $\theta_k^* \in (0,1)$, $P_{o,L_k\times 1}$ is the outage probability of a $L_k \times 1$ *distributed* MIMO channel whose i^{th} transmit antenna is at a distance $d_{d,i}$, $i = 1,2,\ldots,L_k$, from the destination such that

$$P_{o,L_k\times 1} = \frac{\left(2^{R_k}-1\right)^{L_k}}{(L_k!)\left(\overline{P}_k\right)^{L_k}} \prod_{j\in\mathcal{S}_k} \frac{d_{d,j}^{\gamma}}{\lambda_j} + O\left(\left(\overline{P}_k\right)^{-L_k-1}\right) \quad (11)$$

and

$$K_2 = \left[\frac{\left(2^{R_k/\overline{\theta}_k^*}-1\right)^{L_k}\left(\overline{\theta}_k^*\right)^{L_k-1}}{\left(2^{R_k}-1\right)^{L_k}} + \frac{(2^{R_k/\theta_k^*}-1)^2\left(\sum_{j\in C_k} d_{j,k}^{\gamma}\right)(L_k!)\left(\overline{P}_k\right)^{L_k-2}}{\left(2^{R_k}-1\right)^{L_k}\left(\prod_{j\in\mathcal{C}_k} d_{d,j}^{\gamma}/\lambda_j\right)}\right]. \quad (12)$$

Note that for $L_k = 2$, our analysis simplifies to the outage analysis for the TD-RC network. For $L_k > 2$, comparing (10) and the two terms in square brackets in (12), one obtains a lower bound on the diversity from the first and second terms in (12) as L_k and 2, respectively. In fact, the first term dominates only when

$$\left(\sum_{j\in C_k} d_{j,k}^{\gamma}\right) \leq \frac{\left(2^{R_k/\overline{\theta}_k^*}-1\right)^{L_k-2}}{(L_k!)\left(\overline{P}_k\right)^{L_k-2}} \cdot \frac{\left(\prod_{j\in\mathcal{S}_k} d_{d,j}^{\gamma}/\lambda_j\right)}{d_{d,k}^{\gamma}}. \quad (13)$$

For a given P_k, for all k, achieving the maximum diversity L_k requires that user k and its cooperating users in $\mathcal{C}_k$ are clustered close enough to satisfy (13). Thus, the maximum DDF diversity for a two-hop cooperative network does not exceed that of TD-RC except when user k and its cooperating users are *clustered*, i.e., the inter-node distances satisfy (13). We illustrate this distance-dependent behavior in Section IV.

3) TD-UC – Multi-Hop: Recall that $\pi_k(\cdot)$ is a permutation on $\mathcal{C}_k$ such that user $\pi_k(l)$ begins its transmissions in the fraction $\Theta_{k,l}$, for all $l = 2,3,\ldots,L_k$, and $\pi_k(1) = k$. Unlike the two-hop case where Θ_k is dictated by the node with the worst receive SNR, the fraction $\Theta_{k,l}$, for $l = 1,2,\ldots,L_k-1$, is the smallest fraction that ensures that at least one cooperating node, denoted as $\pi_k(l+1)$, decodes the message from user k. In general, developing closed form expressions for $P_o^{(k)}$ is not straightforward. In Appendix III, we lower bound $P_o^{(k)}$ by the MIMO outage probability, $P_{o,L_k\times 1}$ and use the CDF of $\Theta_{k,l}$, for all l, to upper bound $P_o^{(k)}$ for any $0 < \theta_{k,l}^* < 1$, for all l, as (see (56))

$$P_o^{(k)} \leq \frac{\left(2^{R_k}-1\right)^{L_k}}{(L_k!)\left(\overline{P}_k\right)^{L_k}} \left(\prod_{j=1}^{L_k} \frac{d_{d,\pi_k(j)}^{\gamma}}{\lambda_{\pi_k(j)}}\right) \cdot [K_c + K_d] + O\left(\left(\overline{P}_k\right)^{-L_k-1}\right) \quad (14)$$

where the constants K_c and K_d are given by (57) in Appendix III. Our analysis shows that DDF achieves a maximum diversity of L_k for a L_k-hop TD-UC network.

B. Amplify-and-Forward

A cooperating node or a dedicated relay can amplify its received signal and forward it to the destination; the resulting AF strategy is appropriate for nodes with limited processing capabilities. We present the outage bounds for two-hop TD-RC and TD-UC and L_k-hop TD-UC networks. We assume $\theta_k = 1/2$ and $\theta_{k,l} = 1/L_k$, $l = 1,2,\ldots,L_k$, for the two-hop and L_k-hop schemes, respectively.

1) TD-RC and TD-UC – Two-hop: We first consider a two-hop AF protocol where only user k transmits in the first fraction and both user k and its cooperating users (TD-UC) or dedicated relay (TD-RC) transmit in the second fraction. User k transmits with a different codebook in the first and second fractions. The other users transmit $X_{l,2} = c_l Y_{l,1}$ where $X_{l,2}$ represents a symbol in the second time fraction, c_l is a complex constant, and $Y_{l,1}$ represents a symbol in the first time fraction. The outage analysis for the two-hop TD-RC network, i.e., $|\mathcal{C}_k| = 1$, is the same as that developed for the half-duplex relay channel in [12]. For the TD-UC two-hop network, i.e., $L_k \geq 2$, in which all $L_k - 1$ cooperating nodes amplify and forward their received signals in the second fraction, the destination receives a signal from user k in the first fraction and receives a sum of signals from user k and the amplified signals from the $L_k - 1$ cooperating users. The resulting outage $P_o^{(k)}$ is given as

$$P_o^{(k)} = \Pr\left(\frac{1}{2}C(G) < R_k\right) \quad (15)$$

where

$$G = |H_{d,k}|^2 \overline{P}_k \left(1 + \frac{1}{c_s^2}\right) + \frac{\overline{P}_k}{c_s^2} \left|\sum_{j\in\mathcal{C}_k} c_j H_{d,j} H_{j,k}\right|^2, \quad (16)$$

the pre-log factor of $1/2$ is a result of $\theta_k = 1/2$, $|c_j| = (2\overline{P}_j / \big/ |H_{j,k}|^2 \overline{P}_k + 1)^{1/2}$, and $c_s^2 = 1 + \sum_{j\in\mathcal{C}_k} |c_j H_{d,j}|^2$.

We can lower bound $P_o^{(k)}$ by the outage probability of a $L_k \times 1$ MIMO channel where all the relaying antennas transmit the same signal, i.e.,

$$P_o^{(k)} \geq \Pr\left(C\left(|H_{d,k}|^2 \overline{P}_k + \overline{P}_k \left|\sum_{j\in\mathcal{C}_k} H_{d,j}\right|^2\right) < R_k\right) \tag{17}$$

$$= \frac{\left(2^{R_k}-1\right)^2 d_{d,k}^{\gamma}}{2\left(\overline{P}_k\right)^2 \left(\sum_{j\in\mathcal{C}_k} 1/d_{d,j}^{\gamma}\right)} + O\left(\left(\overline{P}_k\right)^{-3}\right). \tag{18}$$

Thus, the maximum diversity of two-hop AF is bounded by 2. Further, since AF achieves a maximum diversity of 2 with one cooperating node or dedicated relay [4], allowing selection of one cooperating node with the smallest outage allows us to achieve diversity 2. Finally, using the fact that $P_o^{(k)}$ for a non-orthogonal relay channel is at most that of the orthogonal relay channel, we bound $P_o^{(k)}$ using the bound developed for the orthogonal case in [4] as

$$P_o^{(k)} \leq \frac{\left(2^{2R_k}-1\right)^2 d_{d,k}^{\gamma} \max_{j\in\mathcal{C}_k}\left(d_{j,k}^{\gamma}+d_{d,j}^{\gamma}\right)}{2\left(\overline{P}_k\right)^2}. \tag{19}$$

Thus, the maximum diversity achievable by a two-hop AF scheme is at most 2 and is independent of the number of cooperating users in $\mathcal{C}_k$.

2) TD-UC – Multi-hop: We consider an L_k-hop cooperative AF protocol where only user k and user $\pi_k(l)$, $l = 1, 2, \ldots, L_k$, transmit in the l^{th} fraction, i.e., user $\pi_k(l)$ forwards in the fraction $\theta_{k,l}$ a scaled version of the signal it receives from user k in the first fraction. User k transmits with a different codebook in each fraction. Note that $\pi_k(1) = k$ and $\theta_{k,l} = 1/L_k$ for all l. We write the received signal, $Y_{d,l}$, at the destination in the l^{th} fraction as

$$Y_{d,l} = \begin{cases} H_{d,k}X_{k,l} + Z_{d,l} & l = 1 \\ H_{d,k}X_{k,l} + H_{d,\pi_k(l)}X_{\pi_k(l),l} + Z'_{d,l} & l = 2,\ldots,L_k \end{cases} \tag{20}$$

where the signal transmitted by user $\pi_k(l)$ in the l^{th} fraction is $X_{\pi_k(l),l} = c_{\pi_k(l)}Y_{\pi_k(l),1}$ such that $c_{\pi_k(l)}$is as given for the two-hop case earlier with $\mathcal{C}_k = \{\pi_k(l)\}$. We can write (20) compactly as $\underline{Y}_d = \mathbf{H}\underline{X}_k + \underline{Z}$, where the L_k entries of $\underline{Y}_d$ and $\underline{X}_k$ are related by (20) and $\mathbf{H}$ is the resulting channel gains matrix. The destination decodes after collecting the received signals from all L_k fractions. Choosing $X_{k,l}$, for all l, as independent Gaussian signals, we have $P_o^{(k)} = \Pr\left(\log\left|I + \overline{P}_k\mathbf{H}\mathbf{H}^{\dagger}\right| < L_kR_k\right)$ where $\mathbf{H}^{\dagger}$ is the conjugate transpose of $\mathbf{H}$. We lower bound $P_o^{(k)}$ with the outage probability of a $L_k \times 1$ MIMO channel in (11). On the other hand, one can upper bound $P_o^{(k)}$ by the outage probability of an orthogonal AF protocol in which user k and its cooperating users transmit on orthogonal channels, i.e., only user $\pi_k(l)$ transmits in the fraction $\theta_{k,l}$, as developed in [4]. Thus, we have

$$P_{out} \leq \frac{\left(2^{L_kR_k}-1\right)^{L_k} d_{d,k}^{\gamma} \prod_{j\in\mathcal{C}_k}\left(d_{d,j}^{\gamma}+d_{j,k}^{\gamma}\right)}{L_k!\left(\overline{P}_k\right)^{L_k}}. \tag{21}$$

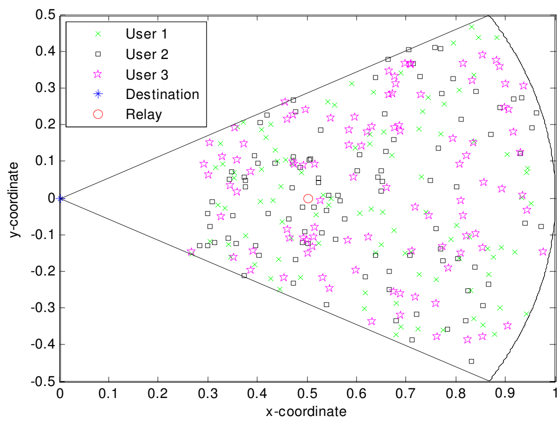

Fig. 2. Sector of a circle with the destination at the origin and 100 randomly chosen locations for a three-user MAC.

Comparing (11) and (21), we see that the L_k-hop AF scheme can achieve a maximum diversity of L_k in the high SNR regime at the expense of user k repeating the signal L_k times.

IV. ILLUSTRATION OF RESULTS

We consider a planar geometry with the users distributed randomly in a sector of a circle of unit radius and angle $\pi/3$. We place the destination at the center of the circle and place the dedicated relay at $(0.5, 0)$ as shown in Fig. 2. The K users are distributed randomly over the sector excluding an area of radius 0.3 around the destination. We consider 100 such random placements and for each such random placement, we compute the outage probabilities P_{out} for the TD-RC, the TD-UC, and the TD-MAC network as an average over the outages of all the time-duplexed users in each network. Finally, we also average P_{out} over the 100 random node placements. We consider a three-user MAC. We assume that all three users have the same transmit power constraint, i.e., $P_k = P_1$ for all k. For the dedicated relay we choose $P_r = f_r \cdot P_1$ where $f_r \in \{0.5, 1\}$. We set the path loss exponent $\gamma = 4$ and the processing factors $\eta_k = \delta_k = \eta$ for all k. We plot P_{out} as a function of P_{tot} for $\eta = 0.01$, 0.5, and 1 thereby modeling three different regimes of processing to transmit power ratios. We consider a symmetric transmission rate, i.e., all users transmit at $R = 0.25$ bits/channel use. We first plot P_{out} as a function of the transmit SNR P_1 in dB obtained by normalizing P_1 by the unit variance noise. We also plot P_{out} as a function of P_{tot} in dB where P_{tot} is given by (6) and (7). For user cooperation, we plot the outage for both the two-hop and three-hop schemes.

A. Outage Probability: DDF

We compare the DDF outage probability in Figs. 3 and 4. The plots validate our analytical results that DDF does not achieve the maximum diversity gains of 3 for the two hop TD-UC network (denoted Coop. 2-hop in plots). For the three-hop TD-UC network (denoted Coop. 3-hop), the slope of P_{out} approaches 3. Further, relative to DDF performance for the TD-RC network, DDF for this 3-hop TD-UC network achieves

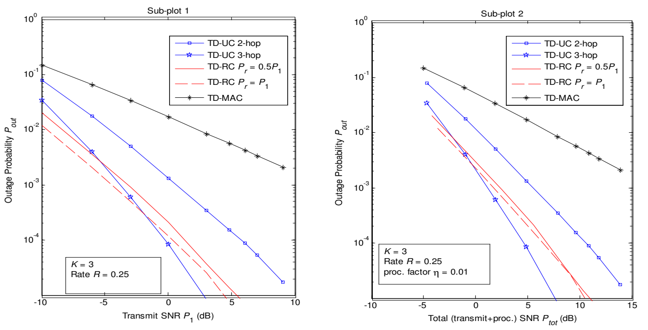

Fig. 3. DDF P_{out} vs. P_1 (sub-plot 1) and vs. P_{tot} (sub-plot 2) for $\eta = 0.01$ and $K = 3$.

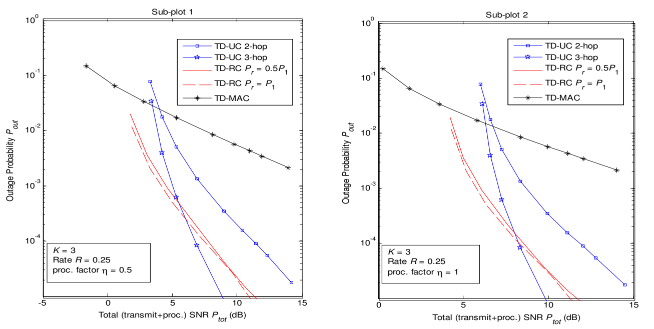

Fig. 4. DDF P_{out} vs. P_{tot} (dB) for $\eta = 0.5$ (sub-plot 1) and $\eta = 1$ (sub-plot 2) for $K = 3$.

coding gains only as the SNR increases. This difference persists when the energy costs of cooperation are accounted for in sub-plot 2 and Fig. 4 by plotting P_{out} as a function of P_{tot}. This difference in SNR gains between UC and RC is due to the fact that UC increases spatial diversity at the expense of requiring users to consume power for cooperative transmissions. Observe that with increasing η, the outage curves are translated to the right. In fact, for a fixed R, the processing costs increase with increasing η, and thus, we expect the SNR gains from cooperation to diminish relative to TD-MAC, particularly in lower SNR regimes.

B. Outage Probability: AF

In Figs. 5 and 6 we plot the two user AF outage probability for all three networks. As predicted, both TD-RC and TD-UC networks achieve a maximum diversity of 2 for the two-hop scheme. The TD-UC three-hop scheme has a maximum diversity approaching 3. However, it achieves coding gains relative to the TD-RC network only as the SNR increases. These gains are a result of the model chosen for front-end processing and amplification costs, and thus, the total processing power will scale proportionate to the number of users that a node relays for. Observe that AF outperforms DDF for large processing factors η because AF requires little processing.

The numerical analysis can be extended to arbitrary dedicated relay positions. In general, the choice of the dedicated-relay position is a tradeoff between cooperating with as many

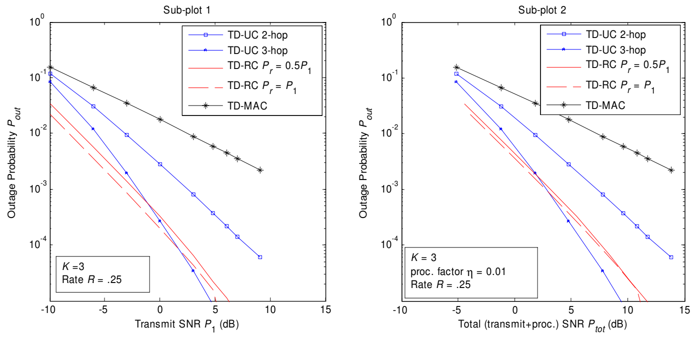

Fig. 5. AF P_{out} vs. P_1 (sub-plot 1) and vs. P_{tot} (sub-plot 2) for $\eta = 1$ (sub-plot 2) and $K = 3$.

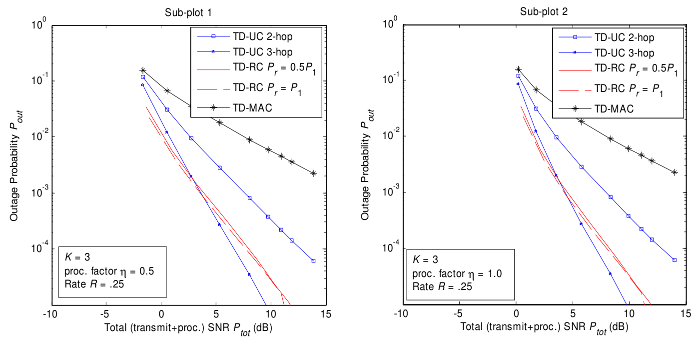

Fig. 6. AF P_{out} vs. P_{tot} for $\eta = 0.5$ (sub-plot 1) and $\eta = 1$ (sub-plot 2) and $K = 3$.

users as possible and being, on average, closer to the users than the destination is. Placing the dedicated relay at the symmetric location $(0.5, 0)$ seems to be a reasonable tradeoff.

V. Concluding Remarks

We compared the outage performance of user and dedicated-relay cooperation in a time-duplexed multiaccess network using the total transmit and processing power as a cost metric. We presented geometry-inclusive upper and lower bounds on the outage probability of DDF and AF to facilitate comparisons of diversity and coding gains achieved by the two cooperative approaches. Using area-averaged numerical results that account for the costs of cooperation, we demonstrated that the TD-RC network achieves SNR gains that diminish the diversity advantage of the TD-UC network.

In conclusion, we see that user cooperation is desirable only if the processing costs associated with achieving the maximum diversity gains are not prohibitive, i.e., in the regime where user cooperation achieves positive coding gains relative to the dedicated-relay cooperative and non-cooperative networks. The simple processing cost model presented here captures the effect of transmit rate on processing power. One can also tailor this model to explicitly include delay, complexity, and device-specific processing costs.

APPENDIX I
DISTRIBUTION OF WEIGHTED SUM OF EXPONENTIAL RANDOM VARIABLES

Consider a collection of i.i.d. unit mean exponential random variables E_l, $l = 1, 2, \ldots, L$. We denote a weighted sum of E_l, for all l, as $H = \sum_{l=1}^{L} c_l E_l$ where $c_l > 0$ and $c_m \neq c_k$ for all l and $m \neq k$. The following lemma summarizes the probability distribution of H [13, p. 11].

Lemma 1: The random variable H has a distribution given as

$$p_H(h) = \begin{cases} \sum_{l=1}^{L} \frac{C_l}{c_l} e^{-h/c_l} & h \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

where the constants C_l, for all l, are

$$C_l = \begin{cases} 1 & L = 1 \\ \frac{(-c_l)^{L-1}}{\prod_{j=1, j\neq l}^{L}(c_j - c_l)} & L > 1. \end{cases} \quad (23)$$

The cumulative distribution function of H is

$$F_H(\eta) = \sum_{l=1}^{L} C_l \left(1 - e^{-\eta/c_l}\right) \quad (24)$$

and the first non-zero term in the Taylor series expansion of $F_H(\eta)$ about $\eta = 0$ is $\eta^L \Big/ \left(L! \prod_{l=1}^{L} c_l\right)$.

APPENDIX II
DDF OUTAGE BOUNDS

A. Two-Hop Relay Cooperative Network

For a DDF dedicated relay, the listen fraction is the random variable (see [3, (13), pp. 4157])

$$\Theta_k = \min\left(1, R_k \Big/ \log\left(1 + \frac{|A_{r,k}|^2 \overline{P}_k}{d_{r,k}^{\gamma}}\right)\right). \quad (25)$$

Θ_k is a mixed (discrete and continuous) random variable with a cumulative distribution function (CDF) given as

$$F_{\Theta_k}^{(r)}(\theta_k) = \begin{cases} 0 & \theta_k \leq 0 \\ \exp\left[-\frac{(2^{R_k/\theta_k} - 1)d_{r,k}^{\gamma}}{\overline{P}_k}\right] & 0 < \theta_k < 1 \\ 1 & \theta_k = 1. \end{cases} \quad (26)$$

The mutual information collected at the destination over both the *listen* and *transmit* fractions is (see [3, Appendix D]) $I_2^{DF} = \Theta_k G_1 + \overline{\Theta}_k G_2$ where $\overline{\Theta}_k = (1 - \Theta_k)$, $\overline{P}_k = K P_k$, $\overline{P}_r = P_r / \overline{\Theta}_k$, $G_1 = C(|H_{d,k}|^2 \overline{P}_k)$, and $G_2 = C(|H_{d,k}|^2 \overline{P}_k + |H_{d,r}|^2 \overline{P}_r)$. The outage probability for user k transmitting at a fixed rate R_k is then given as $P_o^{(k)} = \Pr\left(I_2^{DF} < R_k\right)$. From (25), $\Theta_k = 0$ only for $d_{r,k} = 0$, i.e., only when user k and the dedicated relay are co-located, and for this case $P_o^{(k)}$ simplifies to the 2×1 MIMO channel outage probability given by

$$P_{o,2\times1} = \frac{(2^{R_k} - 1)^2 d_{d,k}^{\gamma} d_{d,r}^{\gamma}}{2\lambda \left(\overline{P}_k\right)^2} + O\left(\left(\overline{P}_k\right)^{-3}\right). \quad (27)$$

where we let P_r and $\overline{P}_k$ scale such that $P_r / \overline{P}_k = \lambda$ is a positive constant. Using (24), we have $P_{o,2\times1}$ is a lower bound on $P_o^{(k)}$ because $G_2 \geq G_1$. On the other hand, for any θ_k, $P_o^{(k)}(\theta_k)$ can be upper bounded as

$$P_o^{(k)}(\theta_k) \leq \Pr\left(\theta_k G_1 < R_k\right) = P_{o,1}^{(k)}(\theta_k), \text{ and} \quad (28a)$$

$$P_o^{(k)}(\theta_k) \leq \Pr\left(\overline{\theta}_k G_2 < R_k\right) = P_{o,2}^{(k)}(\theta_k) \quad (28b)$$

Thus, we have

$$P_o^{(k)} = \mathbb{E} P_o^{(k)}(\Theta_k) \leq \mathbb{E}\min(P_{o,1}^{(k)}(\Theta_k), P_{o,2}^{(k)}(\Theta_k)) = P_{UB}^{(k)} \quad (29)$$

Let $\eta = 2^{R_k/\overline{\theta}_k} - 1$, $c_1 = \overline{P}_k \Big/ d_{d,k}^{\gamma}$, and $c_2 = \overline{P}_r \Big/ d_{d,r}^{\gamma}$. From (23), we have $C_1 = c_1/(c_1 - c_2)$ and $C_2 = c_2/(c_2 - c_1)$. Using Lemma 1, we have

$$P_{o,1}^{(k)}(\theta_k) = \Pr\left(G_1 < \frac{R_k}{\theta_k}\right) = 1 - \exp\left[\frac{-(2^{R_k/\theta_k} - 1)d_{d,k}^{\gamma}}{\overline{P}_k}\right] \quad (30a)$$

$$\leq \frac{(2^{R_k/\theta_k} - 1)d_{d,k}^{\gamma}}{\overline{P}_k} \quad (30b)$$

$$P_{o,2}^{(k)}(\theta_k) = \Pr\left(G_2 < \frac{R_k}{\overline{\theta}_k}\right) = \sum_{l=1}^{2} C_l \left(1 - e^{-\eta/c_l}\right) \quad (31a)$$

$$= \frac{(2^{R_k/\overline{\theta}_k} - 1)^2 \overline{\theta}_k d_{d,k}^{\gamma} d_{d,r}^{\gamma}}{2\lambda \left(\overline{P}_k\right)^2} + O\left(\left(\overline{P}_k\right)^{-3}\right) \quad (31b)$$

where the bound in (31) follows from expanding and simplifying the exponential functions. From (31), we see that for a fixed $\overline{P}_k$ and $d_{j,k}$ for all j, k, the minimum in (29) is dominated by $P_{o,2}^{(k)}(\theta_k)$ for small θ_k and by $P_{o,1}^{(k)}(\theta_k)$ as θ_k approaches 1. Finally, we have $P_{o,2\times1} = P_{o,2}^{(k)}(\theta_k = 0)$. In general, $P_{UB}^{(k)}$ is not easy to evaluate analytically. Since we are interested in the achievable diversity, we develop a bound on $P_{UB}^{(k)}$ for a fixed R_k. We have, for any θ_k^*, $0 < \theta_k^* < 1$,

$$P_{UB}^{(k)} = \int_0^1 P_{\Theta_k}(\theta_k) \min\left(P_{o,1}^{(k)}(\theta_k), P_{o,2}^{(k)}(\theta_k)\right) d\theta_k \quad (32)$$

$$\leq \int_0^{\theta_k^*} P_{\Theta_k}(\theta_k) P_{o,2}^{(k)}(\theta_k) d\theta_k + \int_{\theta_k^*}^1 P_{\Theta_k}(\theta_k) P_{o,1}^{(k)}(\theta_k) d\theta_k \quad (33)$$

$$\leq F_{\Theta_k}(\theta_k^*) P_{o,2}^{(k)}(\theta_k^*) + (1 - F_{\Theta_k}(\theta_k^*)) P_{o,1}^{(k)}(\theta_k^*) \quad (34)$$

$$\leq P_{o,2}^{(k)}(\theta_k^*) + \frac{(2^{R_k/\theta_k^*} - 1)d_{r,k}^{\gamma}}{\overline{P}_k} \cdot P_{o,1}^{(k)}(\theta_k^*) \quad (35)$$

$$\leq \left[\frac{(2^{R_k/\overline{\theta}_k^*} - 1)^2 \overline{\theta}_k^*}{(2^{R_k} - 1)^2} + \frac{2d_{r,k}^{\gamma}(2^{R_k/\theta_k^*} - 1)^2 \lambda}{d_{d,r}^{\gamma}(2^{R_k} - 1)^2}\right] \cdot \frac{(2^{R_k} - 1)^2 d_{d,k}^{\gamma} d_{d,r}^{\gamma}}{2\lambda \left(\overline{P}_k\right)^2} + O\left(\left(\overline{P}_k\right)^{-3}\right) \quad (36)$$

where the equality in (33) holds when $P_{o,2}^{(k)}(\theta_k) < P_{o,1}^{(k)}(\theta_k)$ for $\theta_k < \theta_k^*$ and vice-versa, and (34) follows because $P_{o,1}^{(k)}(\theta_k)$ and $P_{o,2}^{(k)}(\theta_k)$ decrease and increase, respectively, with θ_k and (35) follows from using (26) to bound $1 - F_{\Theta_k}(\theta_k^*)$. Finally, we note that for any fixed $0 < \theta_k^* < 1$, for fixed inter-node distances, the term in square brackets in (36) is a multiplicative constant separating the upper bound (36) and the lower bound (27) on $P_o^{(k)}$.

B. Two-hop User Cooperative Network

The above analysis extends to the two-hop TD-UC network. Recall that a DDF cooperating node remains in the *listen* mode until it successfully decodes its received signal from the source. Thus, for the two-hop TD-UC network, the *listen* fraction for each cooperating node j, for all $j \in \mathcal{C}_k$, is given by (25) with the substition $r = j$. Further, since the *listen* fraction Θ_k is now the largest among all j, from (25) we have

$$\Theta_k = \min\left(1, \max_{j\in\mathcal{C}_k}\left\{R_k \Big/ C\left(|A_{j,k}|^2\,\overline{P}_k\Big/ d_{j,k}^{\gamma}\right)\right\}\right) \tag{37}$$

where the transmit power $\overline{P}_k$, for all $k \in \mathcal{K}$, satisfies (2) and is given by (3). Let $F_{\Theta_k}^{(j)}(\theta_k)$ be the CDF $F_{\Theta_k}^{(r)}(\theta_k)$ in (26) with the index r replaced by j. From the independence of $A_{j,k}$ for all $j \in \mathcal{C}_k$, the CDF of Θ_k is $F_{\Theta_k}(\theta_k) = \prod_{j\in\mathcal{C}_k} F_{\Theta_k}^{(j)}(\theta_k)$ which in turn is given by $F_{\Theta_k}^{(r)}(\theta_k)$ evaluated at $d_{r,k} = \sum_{j\in C_k} d_{j,k}^{\gamma}$. The destination collects information from the transmissions of user k and all its cooperating nodes in $\mathcal{C}_k$ over both the *transmit* and *listen* fractions. The resulting mutual information achieved by user k at the destination is (see [14]) $I_{2,DF}^{c}(\Theta_k) = \Theta_k G_1 + \overline{\Theta}_k G_2$ where $\overline{\Theta}_k = 1 - \Theta_k$, $G_1 = C(|H_{d,k}|^2\,\overline{P}_k)$, and $G_2 = C(|H_{d,k}|^2\,\overline{P}_k + \sum_{j\in\mathcal{C}_k}|H_{d,j}|^2\,\overline{P}_j/\overline{\Theta}_k)$.The DDF outage probability for user k transmitting at a fixed rate R_k in a two-hop TD-UC network is thus given as $P_o^{(k)} = \Pr\left(I_{2,DF}^{c} < R_k\right)$. Analogously to the two-hop TD-RC analysis earlier, we can lower bound $P_o^{(k)}$ by the outage probability, $P_{o,L_k\times 1}$, of a $L_k \times 1$ distributed MIMO channel using (24), and scaling $\overline{P}_j$ and $\overline{P}_k$ such that $\overline{P}_j/\overline{P}_k = \lambda_j$ is a constant, for all j, as

$$P_{o,L_k\times 1} = \frac{(2^{R_k}-1)^{L_k} d_{d,k}^{\gamma}}{(L_k!)\left(\overline{P}_k\right)^{L_k}}\cdot\left(\prod_{j\in\mathcal{S}_k}\frac{d_{d,j}^{\gamma}}{\lambda_j}\right) + O\left(\left(\overline{P}_k\right)^{-L_k-1}\right) \tag{38}$$

where we enumerate the $(L_k - 1)$ cooperative nodes in $\mathcal{C}_k$ as $l = 2, 3, \ldots, L_k$, and write $\mathcal{S}_k = \{k\}\cup\mathcal{C}_k$. Let $\eta = 2^{R_k/\overline{\theta}_k} - 1$, $c_1 = \overline{P}_k \Big/ d_{d,k}^{\gamma}$, and $c_l = \overline{P}_l \Big/ d_{d,l}^{\gamma}\overline{\theta}_k$, $l = 2, 3, \ldots, L_k$, where the $\overline{\theta}_k$ in c_l is due to the definition of $\overline{P}_l$ in (3). The C_l, for all $l = 1, 2, \ldots, L_k$, are given by (23). For a fixed R_k, we upper bound $P_o^{(k)}$ using (28) as

$$P_o^{(k)} = \mathbb{E}P_o^{(k)}(\Theta_k) \le \mathbb{E}\min(P_{o,1}^{(k)}(\Theta_k), P_{o,2}^{(k)}(\Theta_k)) = P_{UB}^{(k)}. \tag{39}$$

We upper bound $P_{o,1}^{(k)}(\theta_k)$ using (30) and compute

$$P_{o,2}^{(k)}(\theta_k) = \frac{\left(2^{R_k/\overline{\theta}_k}-1\right)^{L_k}\left(\overline{\theta}_k\right)^{L_k-1}}{(L_k!)\left(\overline{P}_k\right)^{L_k}}\left(\prod_{j\in\mathcal{S}_k}\frac{d_{d,j}^{\gamma}}{\lambda_j}\right) + O\left(\left(\overline{P}_k\right)^{-L_k-1}\right). \tag{40}$$

Analogous to the steps in (32)-(36) for the TD-RC case, for any θ_k^*, $0 < \theta_k^* < 1$, we can upper bound $P_{UB}^{(k)}$ by

$$P_{UB}^{(k)} \le F_{\Theta_k}(\theta_k^*)\,P_{o,2}^{(k)}(\theta_k^*) + (1 - F_{\Theta_k}(\theta_k^*))\,P_{o,1}^{(k)}(\theta_k^*) \tag{41}$$

$$\le P_{o,2}^{(k)}(\theta_k^*) + \frac{(2^{R_k/\theta_k^*}-1)\left(\sum_{j\in C_k} d_{j,k}^{\gamma}\right)}{\overline{P}_k}\cdot P_{o,1}^{(k)}(\theta_k^*) \tag{42}$$

which simplifies to the expression

$$\left[\frac{(2^{R_k/\overline{\theta}_k^*}-1)^{L_k}\left(\overline{\theta}_k^*\right)^{L_k-1}}{(2^{R_k}-1)^{L_k}} + \frac{(L_k!)\left(\sum_{j\in\mathcal{C}_k} d_{j,k}^{\gamma}\right)\left(\overline{P}_k\right)^{L_k-2}(2^{R_k/\theta_k^*}-1)^2}{\left(\prod_{j\in\mathcal{C}_k} d_{d,j}^{\gamma}/\lambda_j\right)(2^{R_k}-1)^{L_k}}\right]$$
$$\cdot\left[\frac{(2^{R_k}-1)^{L_k}}{(L_k!)\left(\overline{P}_k\right)^{L_k}}\left(\prod_{j\in\mathcal{S}_k}\frac{d_{d,j}^{\gamma}}{\lambda_j}\right)\right] + O\left(\left(\overline{P}_k\right)^{-L_k-1}\right) \tag{43}$$

APPENDIX III
MULTI-HOP COOPERATIVE NETWORK – DDF OUTAGE ANALYSIS

The DDF outage probability of user k transmitting at a fixed rate R_k in a multi-hop user cooperative network is $P_o^{(k)} = \Pr\left(I_{2,DF}^{c} < R_k\right)$ where $I_{2,DF}^{c}(\Theta_k) = \sum_{l=1}^{L_k}\Theta_{k,l}G_l$. The function G_l is given by

$$G_l = C\left(\sum_{j=1}^{l}\left|H_{d,\pi_k(j)}\right|^2\frac{\overline{P}_{\pi_k(j)}}{\overline{\Theta}_{k,j}}\right) \quad l = 1, 2, \ldots L_k, \tag{44}$$

where $\overline{P}_k$ is given by (3) and

$$\Theta_{k,l}^{sum} \triangleq \sum_{j=1}^{l-1}\Theta_{k,j}, \quad \text{for } l = 1, 2, \ldots, L_k, \text{ and} \tag{45}$$

$$\overline{\Theta}_{k,l}^{sum} \triangleq 1 - \Theta_{k,l}^{sum} \tag{46}$$

with $\overline{\Theta}_{k,L_k}^{sum} = \Theta_{k,L_k}$ and $\Theta_{k,-1} = 0$ such that $\overline{\Theta}_{k,1}^{sum} = 1$. Recall that $\pi_k(\cdot)$ is a permutation on $\mathcal{C}_k$ such that user $\pi_k(l)$ begins its transmissions in the fraction $\Theta_{k,l}$, for all $l = 2, 3, \ldots, L_k$. Furthermore, $\pi_k(1) = k$ and we write $\pi_k(i:j) = \{\pi_k(i), \pi_k(i+1), \ldots, \pi_k(j)\}$.

We write $\underline{\Theta}_k$ to denote a $(L_k - 1)$-length random vector with entries $\Theta_{k,l}$, $l = 1, 2, \ldots, L_k - 1$, and $\lambda_{\pi_k(j)} = \overline{P}_{\pi_k(j)}/\overline{P}_k$ for all $\pi_k(j) \in \mathcal{C}_k$. Further, we write $\underline{\Theta}_k^{(l)}$ to denote the vector of the first l entries of $\underline{\Theta}_k$. The fraction $\Theta_{k,l}$, $l = 1, 2, \ldots, L_k - 1$, is the smallest value such that at least one new node, denoted as $\pi_k(l+1)$, decodes the message from user k. The analysis for this problem seems difficult; so we replace it by analyzing a simpler strategy where node $\pi_k(l+1)$ collects energy only in fraction $\Theta_{k,l}$ from the transmissions of user k as well as the users in $\pi_k(1:l)$. For this strategy, we have

$$\Theta_{k,l} = \min\left\{\overline{\Theta}_{k,l}^{sum}, \min_{\pi_k(l+1)\in\mathcal{C}_k\setminus\pi_k(1:l)} f(\pi_k(l+1))\right\} \tag{47}$$

where $f(\pi_k(l+1))$ is given by

$$\frac{R_k}{C\left(\sum_{m=1}^{l} \left|A_{\pi_k(l+1),\pi_k(m)}\right|^2 \overline{P}_{\pi_k(m)} \Big/ d^{\gamma}_{\pi_k(l+1),\pi_k(m)}\right)}. \tag{48}$$

Applying Lemma 1, the CDF of $\Theta_{k,l}$ conditioned on $\underline{\Theta}_k^{l-1} = \underline{\theta}_k^{l-1}$ simplifies to

$$F_{\Theta_{k,l}|\underline{\Theta}_{k,1}^{l-1}}(\theta_{k,l}|\underline{\theta}_k^{l-1}) = \begin{cases} 0 & \theta_{k,l} \le 0 \\ 1-\tilde{F} & 0 < \theta_{k,l} < \overline{\theta}_{k,l}^{sum} \\ 1 & \theta_{k,l} = \overline{\theta}_{k,l}^{sum}. \end{cases} \tag{49}$$

where

$$\tilde{F} = \prod_{j \in \mathcal{C}_k \setminus \pi_k(2:l)} \left[F_{H_{j,l}^{sum}}(2^{R_k/\theta_{k,l}} - 1)\right] \tag{50}$$

and from (47), $H_{j,l}^{sum} \triangleq \sum_{m=1}^{l} c_m \left|A_{j,\pi_k(m)}\right|^2$ with $c_m = \lambda_{\pi_k(m)} \overline{P}_k \big/ d^{\gamma}_{j,\pi_k(m)}$ for all $m = 1, 2, \ldots, l$, and $\overline{\theta}_{k,l}$ is given by (45). The dominant term of each $F_{H_{j,l}^{sum}}$ is proportional to $\left(\overline{P}_k\right)^{-l}$, and thus, the dominant term of $1 - F_{\Theta_{k,l}|\underline{\Theta}_k^{l-1}}$ is proportional to $\left(\overline{P}_k\right)^{-l(L_k - l)}$.

For a fixed R_k, we lower bound $P_o^{(k)}$ by the outage probability $P_{o,L_k \times 1}$ of a $L_k \times 1$ distributed MIMO channel in (38). Generalizing the analyses in Appendix II, we upper bound $P_o^{(k)}$ as

$$P_o^{(k)} \le \mathbb{E} \min_{l \in \mathcal{K}} (P_{o,l}^{(k)}(\underline{\Theta}_k)) = P_{UB}^{(k)} \tag{51}$$

where we use Lemma 1 to write

$$P_{o,l}^{(k)}(\underline{\theta}_k) \triangleq \Pr\left(G_l < \frac{R_k}{\theta_{k,l}}\right) \tag{52}$$

$$= \frac{\left(2^{R_k/\theta_{k,l}} - 1\right)^l}{(l!) \left(\overline{P}_k\right)^l} \left(\prod_{j=1}^{l} \frac{d^{\gamma}_{d,\pi_k(j)} \overline{\theta}_{k,j}^{sum}}{\lambda_{\pi_k(j)}}\right) + O\left(\left(\overline{P}_k\right)^{-l-1}\right). \tag{53}$$

The probability $P_{UB}^{(k)}$ is given as (see (45))

$$P_{UB}^{(k)} = \int_{\theta_{k,1}=0}^{1} \cdots \int_{\theta_{k,L_k-1}=0}^{\overline{\theta}_{k,L_k-1}^{sum}} P_{\underline{\Theta}_k}\left(\underline{\theta}_k\right) \min_{l \in \mathcal{K}} (P_{o,l}^{(k)}(\theta_{k,l})) d\theta_k. \tag{54}$$

For any $0 < \theta_{k,l}^* < \overline{\theta}_{k,l}^*$, $1 \le l < L_k$, the integral in (54) over the $(L_k - 1)$-dimensional hyper-cube can be written as a sum of $2^{L_k - 1}$ integrals, each spanning $(L_k - 1)$-dimensions, such that there are $\binom{L_k - 1}{j}$ integrals for which j of the $(L_k - 1)$ $\theta_{k,l}$ parameters range from 0 to $\theta_{k,l}^*$, $j = 0, 1, \ldots, L_k - 1$ while the remaining range from $\theta_{k,l}^*$ to 1. Thus, we upper bound $P_{UB}^{(k)}$ in (54) by

$$\int_0^{\theta_{k,1}^*} \int_0^{\overline{\theta}_{k,2}^{sum}} \cdots \int_0^{\overline{\theta}_{k,L_k-1}^{sum}} P_{\underline{\Theta}_k}\left(\underline{\theta}_k\right) P_{o,L_k}^{(k)}(\underline{\theta}_k) d\underline{\theta}_k + \int_{\theta_{k,1}^*}^{1} \int_0^{\overline{\theta}_{k,2}^{sum}} \cdots \int_0^{\overline{\theta}_{k,L_k-1}^{sum}} P_{\underline{\Theta}_k}\left(\underline{\theta}_k\right) P_{o,1}^{(k)}(\underline{\theta}_k) d\underline{\theta}_k \tag{55}$$

where the dominant outage terms for $\theta_{k,1} \le \theta_{k,1}^*$ and $\theta_{k,1} > \theta_{k,1}^*$ are bounded by $P_{o,L_k}^{(k)}(\underline{\theta}_k)$ and $P_{o,1}^{(k)}(\underline{\theta}_k)$, respectively. Furthermore, using the monotonic properties of $P_{o,l}^{(k)}$, the first term in (55) is bounded by $P_{o,L_k}^{(k)}(\underline{\theta}_k^*)$ and the second term is bounded by $\left(1 - F_{\Theta_{k,1}}\left(\theta_{k,1}^*\right)\right) P_{o,1}^{(k)}(\underline{\theta}_k^*)$. From (49) and (52), using the fact that $P_{o,1}^{(k)}(\underline{\theta}_k^*)$ has the smallest absolute exponents of $\overline{P}_k$, namely 1, and $\left(1 - F_{\Theta_{k,1}}\left(\theta_{k,1}^*\right)\right) P_{o,1}^{(k)}(\underline{\theta}_k^*)$ scales as $\left(\overline{P}_k\right)^{-L_k}$, $P_{UB}^{(k)}$ can be upper bounded by

$$\begin{aligned} &P_{o,L_k}^{(k)}(\underline{\theta}_k^*) + \left(1 - F_{\Theta_{k,1}}\left(\theta_{k,1}^*\right)\right) P_{o,1}^{(k)}(\underline{\theta}_k^*) \\ &\quad \le \frac{\left(2^{R_k} - 1\right)^{L_k}}{(L_k!) \left(\overline{P}_k\right)^{L_k}} \left(\prod_{j=1}^{L_k} \frac{d^{\gamma}_{d,\pi_k(j)}}{\lambda_{\pi_k(j)}}\right) [K_c + K_d] \\ &\qquad + O\left(\left(\overline{P}_k\right)^{-L_k-1}\right) \end{aligned} \tag{56}$$

$$\text{where } K_c = \frac{\left(2^{R_k/\theta_{k,L_k}^*} - 1\right)^{L_k} \left(\prod_{j=1}^{L_k} \left(\overline{\theta}_{k,j}^{sum}\right)^*\right)}{\left(2^{R_k} - 1\right)^{L_k}}, \text{ and} \tag{57}$$

$$K_d = \frac{\left(2^{R_k/\theta_{k,1}^*} - 1\right)^{L_k} (L_k!)}{\left(2^{R_k} - 1\right)^{L_k}} \cdot \prod_{j=2}^{L_k} \frac{d^{\gamma}_{\pi_k(j),\pi_k(1)}}{\lambda_{\pi_k(j)}}. \tag{58}$$

Combining (56) and (38), the maximum achievable diversity is L_k.

REFERENCES

[1] Atheros Communications, “Power consumption and energy efficiency comparisons of wlan products,” www.atheros.com/pt/whitepapers/atheros_power_whitepaper.pdf.

[2] W. Heinzelman, A. Chandrakasan, and H. Balakrishnan, “Energy-efficient communication protocols for wireless microsensor networks,” in *Proc. 33rd Hawaii Intl. Conf. Systems and Sciences*, Maui, HA, Jan. 2000, pp. 1–10.

[3] K. Azarian, H. El Gamal, and P. Schniter, “On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels,” *IEEE Trans. Inform. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.

[4] J. N. Laneman, “Network coding gain of cooperative diversity,” in *Proc. IEEE Military Comm. Conf. (MILCOM)*, Monterey, CA, Nov 2004, pp. 3714–3722.

[5] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[6] G. Kramer, M. Gastpar, and P. Gupta, “Cooperative strategies and capacity theorems for relay networks,” *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3027–3063, Sept. 2005.

[7] L. Sankaranarayanan, G. Kramer, and N. B. Mandayam, “Cooperation vs. hierarchy: An information-theoretic comparison,” in *Proc. IEEE Int. Symp. Inf. Theory*, Adelaide, Australia, Sept. 2005, pp. 411–415.

[8] A. Nosratinia and T. E. Hunter, “Grouping and partner selection in cooperative wireless networks,” *IEEE JSAC Cooperative Communications and Networking*, vol. 25, no. 2, pp. 369–378, Feb. 2007.

[9] S. Kadloor and R. Adve, “Relay selection and power allocation in cooperative cellular networks,” Mar. 2009, arxiv.org e-print 0903.2426.

[10] M. Bhardwaj, T. Garnett, and A. P. Chandrakasan, “Upper bounds on the lifetime of sensor networks,” in *Proc. 2001 IEEE Intl. Conf. Commun.*, Helsinki, Finland, June 2001.

[11] L. Zheng and D. N. C. Tse, “Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels,” *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[12] R. U. Nabar, H. Bölcskei, and F. W. Kneubühler, “Fading relay channels: Performance limits and space-time signal design,” *IEEE JSAC*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[13] D. R. Cox, *Renewal Theory*. London: Methuen’s Monographs on Applied Probability and Statistics, 1967.

[14] G. Kramer, “Models and theory for relay channels with receive constraints,” in *42nd Annual Allerton Conf. on Commun., Control, and Computing*, Monticello, IL, Sept. 2004.