

Impact of CSI on Radio Resource Management Techniques for the OFDMA Downlink

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Abstract—Adaptive resource allocation can drastically increase the throughput of an Orthogonal Frequency Division Multiple Access (OFDMA) system when the Channel State Information (CSI) is accurately known. Unfortunately, in practice, perfect CSI is rarely possible. In this paper, we consider adaptive sub-carrier assignment for downlink multiuser OFDMA systems, where the transmitter has no knowledge of the instantaneous channel realizations. The problem we address is maximizing the sum-capacity of the system subject to user Quality of Service (QoS) requirements. A heuristic algorithm presented in [1] is modified in order to provide an enhanced sub-optimal solution. Numerical results show that resources can be adaptively allocated using statistical CSI (SCSI) and that such an approach allows for an important number of user QoS requirements to be met. Comparisons between the instantaneous CSI (ICSI) and SCSI based resource allocation schemes demonstrate that their performance difference is highly dependent on the number of active users present in the cell, the QoS constraint, and the transmit power.

I. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) is based on Orthogonal Frequency Division Multiplex (OFDM); and thus, inherits its key benefits while allowing for multiuser diversity to be exploited [2]. This leads to more efficient radio resource management (RRM) as spectrum can be allocated to users with better channel conditions. For these reasons, RRM solutions for OFDMA systems have attracted significant interest. The research in this area can be broadly divided into two categories, namely margin-adaptive and rate-adaptive. Margin adaptation is the minimization of the transmit power subject to minimum Quality of Service (QoS) requirements for each user [3]. Examples of such work are [4] and [5]. Rate adaptation is the maximization of the data-rates subject to QoS constraints [3]. An example of rate adaptation is presented in [1]. The solution to these problems depends on the availability of accurate Channel State Information (CSI) at the transmitter. There are a number of reasons that lead to unavailable user CSI at the transmitter. Under significant user mobility, the small coherence time makes channel estimation procedures less accurate. Other reasons that contribute towards unavailable instantaneous CSI are prediction errors as well as feedback/processing delays. Therefore, in some cases, it is more reasonable to send back channel distribution information. We refer to knowledge of the channel distribution at the transmitter as statistical CSI (SCSI). Under SCSI based resource allocation, users only need

to feed back the mean of the channel SNR distribution. This leads to fewer wireless resources such as transmit power and bandwidth being consumed for feedback purposes.

In this paper, we solve a rate-adaptation problem for users whose instantaneous channel realizations are unavailable at the transmitter but perfectly known by the receiver. The optimum data-rate with which each sub-carrier can be loaded is computed by using a relationship between the average user signal-to-noise ratio (SNR) and the Lambert-W function. To further enhance the performance of the system, a well-known heuristic algorithm presented in [1] is extended. Using this approach, it is shown that a significant number of user QoS constraints can be met. However, comparisons between the instantaneous CSI (ICSI) and SCSI based RRM schemes show that the lack of accurate CSI causes a significant degradation on the overall system performance. The incurred losses heavily depend on the number of active users present in the cell, the QoS constraint, and the transmit power.

II. SYSTEM MODEL

A downlink OFDM system with K users and N sub-carriers is considered. Each sub-carrier n has a total bandwidth equal to B . The k_{th} user's minimum bit-rate is denoted by R_k . Resource allocation is performed for each sub-carrier, and sub-carriers cannot be shared between users. An assignment indicator c_{kn} is defined for the k_{th} user and the n_{th} sub-carrier. Therefore, $c_{kn} = 1$ when carrier n is allocated to user k and 0 otherwise. When the instantaneous channel conditions are unknown by the transmitter but known at the receiver side, the capacity of each sub-carrier is viewed as a random variable and is

$$C(\nu) = B \log_2(1 + \nu \frac{P_t}{N_o}), \quad (1)$$

where ν is exponentially distributed as Rayleigh fading is considered. Here, P_t is the transmit power and N_o is the noise spectral density. Under these conditions, there is a non-zero probability that the actual channel conditions cannot support an assigned rate ρ . This value is given as [6]

$$\begin{aligned} P_{out} &= \Pr(\frac{C(\nu)}{B} < \rho) \\ &= 1 - \exp[-(\bar{\gamma}^{-1})(2^\rho - 1)]. \end{aligned} \quad (2)$$

A useful measure for resource allocation purposes is the goodput which is defined [7] as the average successfully

transmitted rate. For user k and sub-carrier n it is expressed as

$$G_{k,n} = \rho_{k,n}(1 - P_{out}(k, n)), \quad (3)$$

Each sub-carrier can be optimally loaded by selecting the value of $\rho_{k,n}$ which maximizes $G_{k,n}$. This value is

$$\rho_{kn} = \frac{W(\gamma_{kn})}{\ln(2)}, \quad (4)$$

where W denotes the Lambert- W function, the solution to the transcendental equation $W(x)e^{W(x)} = x$. A derivation of (4) is presented in Appendix A. Using (4), the maximum goodput user k can achieve on sub-carrier n is

$$G_{k,n} = \frac{W(\gamma_{kn})}{\ln(2)} (\exp[-(\gamma_{kn}^{-1})(2^{\frac{W(\gamma_{kn})}{\ln(2)}} - 1)]). \quad (5)$$

When the transmitter knows the ICSI there are no outages, and each sub-carrier is loaded with a bit-rate equal to the Shannon capacity.

III. PROBLEM FORMULATION

The objective of this problem is to maximize the sum-goodput of the OFDMA downlink under minimum user data-rate requirement constraints. Equal power allocation across all sub-carriers is assumed as this reduces the complexity of the problems and minimally decreases the data throughput of a multiuser OFDM system [8]. This is due to the nature of OFDMA systems, where sub-carriers are commonly assigned to the users with the best channel gains. For the SCSi based scheme, the problem can be mathematically formulated as follows:

$$P1: \quad \max_{c_{kn}} \sum_{k=1}^K \sum_{n=1}^N c_{kn} G_{k,n} B \quad (6)$$

Subject to :

$$C1: \quad \sum_{n=1}^N c_{kn} G_{k,n} B \geq R_k, \forall k$$

$$C2: \quad \text{If } c_{k'n} = 1, \text{ then } c_{kn} = 0 \quad \forall k \neq k'.$$

Note that the first constraint, C1, ensures that the QoS requirement is met for all users k , while the second constraint ensures that a single carrier is not shared between different users. The equivalent problem can be formulated for the case of ICSI by replacing the goodput with the Shannon capacity in P1. That is:

$$P2: \quad \max_{c_{kn}} \sum_{k=1}^K \sum_{n=1}^N c_{kn} \log_2(1 + \frac{\nu P_t}{N_o}) B \quad (7)$$

Subject to :

$$C1: \quad \sum_{n=1}^N c_{kn} \log_2(1 + \frac{\nu P_t}{N_o}) B \geq r_k, \forall k$$

$$C2: \quad \text{If } c_{k'n} = 1, \text{ then } c_{kn} = 0 \quad \forall k \neq k'.$$

A. Complexity of the Problem

In P1, both the goodput and the Shannon capacity can be treated as constants. Therefore, P1 is converted into an integer linear programming problem which is one of the earliest members of the NP-hard class [9]. There are now KN integer variables and $K + N$ constraints, where the number of sub-carriers is high (i.e $N = 1024$ used in our simulations). As the complexity of the problem grows exponentially with KN and $K + N$ [10], it cannot be solved by using standard integer linear programming methods such as Branch and Bound. Thus, a heuristic needs to be developed. In [1], an algorithm which exhibited excellent sub-optimal properties was proposed to solve a problem of the same nature. However, its use may lead to carriers being allocated to users who are unable to meet their QoS constraints. In this paper, we use an extended version of that algorithm in order to mitigate this problem.

B. Heuristic Subcarrier-Bit Allocation Algorithm

The algorithm used in [1] first allocates sub-carriers to the users who can transmit the highest amount of data on them. As this process does not guarantee fairness, they are then re-allocated to the users whose constraints have not been met by using a cost function. This cost function ensures that any reallocations cause a small reduction in the overall sum-capacity of the system, and that the running time of the algorithm is minimized. Any remaining sub-carriers are then assigned to the users with the better channel gains. Here, we extend the algorithm as when carriers are allocated to users who do not meet C1, their requests will be rejected, and any carriers allocated to them will be wasted. Consider $X \subseteq \{1, 2, \dots, K\}$ to be the set of users who have not had their QoS constraints satisfied following the execution of the algorithm proposed in [1]. The cardinality of this set equals l . It is expected that a number of these users will still have been allocated some sub-carriers. The data-rates allocated to these l users by [1] are given in vector $\mathbf{y} = [y_1, \dots, y_l]$ whereas their associated QoS constraints are $\mathbf{z} = [t_1, \dots, t_l]$. Furthermore, it is assumed that $W \subseteq \{1, 2, \dots, N\}$ is the set of sub-carriers that have been allocated to these l users through [1]. In order to improve the overall performance, the following extension is proposed:

Algorithm 1 Proposed extension

- 1: Initialize : $W \subseteq \{1, 2, \dots, N\}$, $X \subseteq \{1, 2, \dots, K\}$, y, z
 - 2: $\forall$ users $u \in X$
 - 3: Calculate $r(u) = z(u) - y(u)$
 - 4: $u^* = \arg \min_u r(u)$ //find the user u^* closest to meeting his QoS requirement
 - 5: $y_{u^*} = y_{u^*} + \rho_{u^*} B$ // give sub-carrier $i \in W$ to u^*
 - 6: $W = W - i$ // remove sub-carrier i from W
 - 7: **if** $y_{u^*} > t_{u^*}$ **then**
 - 8: $X = X - \{u^*\}$ // remove u^* from X
 - 9: **end if**
-

In Algorithm 1, ρ_{u^*} is the optimum goodput user u^* can achieve on carrier i . The use of this extension enables any

unused sub-carriers to be assigned to the users closest to meeting their data-rate requirements. Therefore, the number of utilized sub-carriers increases, and a higher overall system performance can be achieved. Moreover, the number of satisfied users grows.

C. Sub-Optimal Properties of Heuristic Algorithm

The sub-optimal properties of the heuristic algorithm are demonstrated in Fig. 1. The results of the algorithm are compared with the optimal results obtained through a brute-force search. The algorithm efficiency is defined as the ratio of the goodput achieved through the use of the sub-optimal algorithm to the goodput that can be achieved through a brute-force search.

Due to the long computational time, only eight sub-carriers and three users are considered. The QoS constraint is set to 4000 bits. Fig.1 shows that the algorithm exhibits excellent sub-optimal properties when SCSi is used to perform RRM.

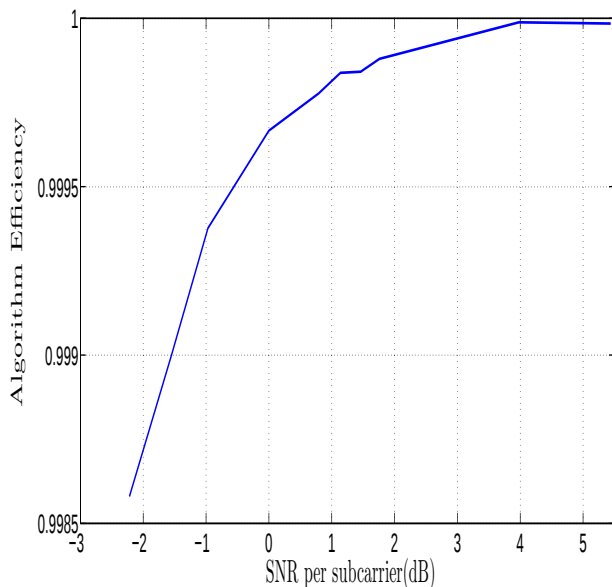


Fig. 1. Performance comparison between brute-force search and sub-optimal algorithm used to perform resource allocation

D. Impact of Extension to Original Algorithm

In order to present the importance of the final part of the algorithm, a throughput gain factor is defined as the ratio of the goodput achieved with the original version to the goodput that can be achieved using the extended version. Assume that there are 4 users in a cell and 8 sub-carriers that can be assigned to them. Fig.2 shows that when the QoS constraint is high and SCSi is used for resource allocation, an important increase in throughput occurs. As the user QoS requirements grow, it becomes increasingly difficult for the SCSi based scheme to meet these demands. Therefore, a higher number of unsatisfied users will be realized. With the original algorithm, any sub-carriers allocated to them would be discarded.

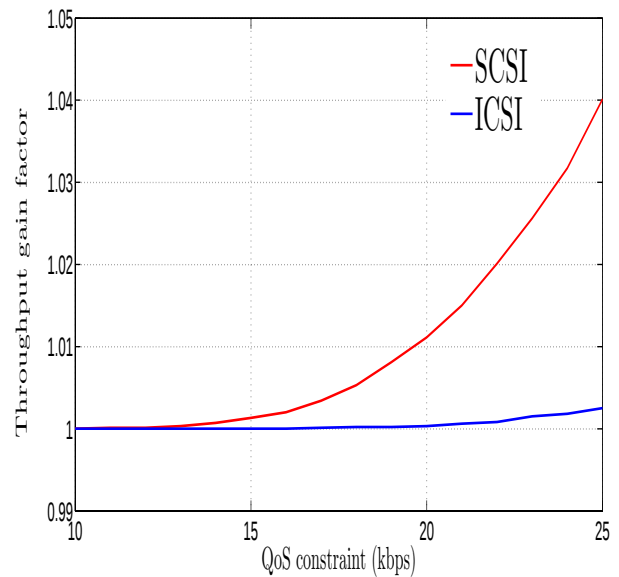


Fig. 2. Impact of the proposed extension to the original algorithm

IV. SIMULATION RESULTS

In this section, the significance of accurate CSI on RRM techniques for OFDMA systems is presented. In order to measure the impact of ICSI knowledge, a capacity gain factor is defined. This is the ratio of the optimum sum-capacity of a system where the user ICSI is known to the optimum sum-goodput of that same system when only the user SCSi is available. In the first subsection, the effect of a varying transmit power on the performance difference between SCSi and ICSI based adaptive RRM is presented. Then, the significance of a varying QoS constraint is analyzed. The simulation parameters used are listed in Table I. The environment is assumed to be variable, which is modeled by a fast fading with independently fading Rayleigh processes, whose power delay profile is described by the ITU Vehicular A model. The performances are evaluated using simulations over 10,000 instances of independent channel realizations. When averaging over a large number of channel realizations it is possible to accurately compare the ICSI and SCSi resource allocation schemes.

TABLE I
TABLE I: SIMULATION PARAMETERS USED

Parameter	Value
Number of sub-carriers	1024
Tx Power	$8 \rightarrow 35mW$ per sub-carrier
Noise power density	$10^{-10} W/Hz$
Channel Model	ITU Vehicular A
Bandwidth	10MHz
QoS constraint	1Mbps, $0 \rightarrow 1.6Mbps$

A. Impact of Transmit Power

It is worthwhile to investigate the significance ICSI knowledge has on the probability of satisfying the user QoS con-

straints. In Fig. 3, we notice that an important number of QoS constraints can be met by using SCSl. However, when the instantaneous channel realizations are known, nearly all of the user data-rate requirements are satisfied. A further comparison

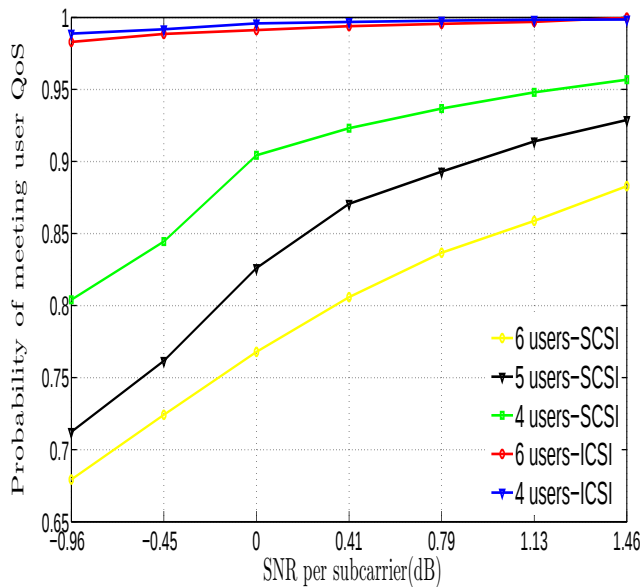


Fig. 3. Probability of user QoS requirements being met versus SNR per subcarrier for the SCSl and ICSl based resource allocation schemes

between the SCSl and ICSl based resource allocation schemes is made in Fig. 4. Here, it is observed that approximately 6dB more power per sub-carrier is required when the transmitter does not know the ICSl.

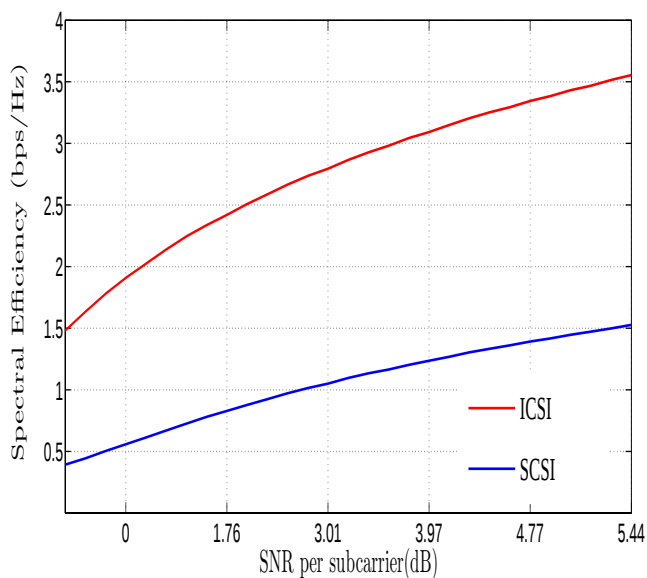


Fig. 4. Maximized sum-capacity versus average SNR per subcarrier

Fig. 5 presents the variation of the capacity gain factor

with SNR per subcarrier for 4 users without QoS constraints. For comparison purposes, a curve corresponding to the same amount of users each requiring 1Mbps is also given. As the SNR per subcarrier increases, the performance difference between the two cases is reduced because more QoS constraints can be met using SCSl. Also, the actual value of the capacity gain factor decreases as the power grows. In Appendix B, we show that as the SNR approaches infinity, the value of the capacity gain factor will be equal to one. In general the figures show that the performance difference between the ICSl and SCSl based resource allocation schemes is small. This is attributed to the Lambert-W approach which optimally loads the subcarriers in a Rayleigh environment. Optimal loading of subcarriers will lead to an improved performance when the channel gains are distributed according to a different p.d.f (i.e Ricean). However, closed form expressions that relate the optimum goodput with the data rate need to be developed.

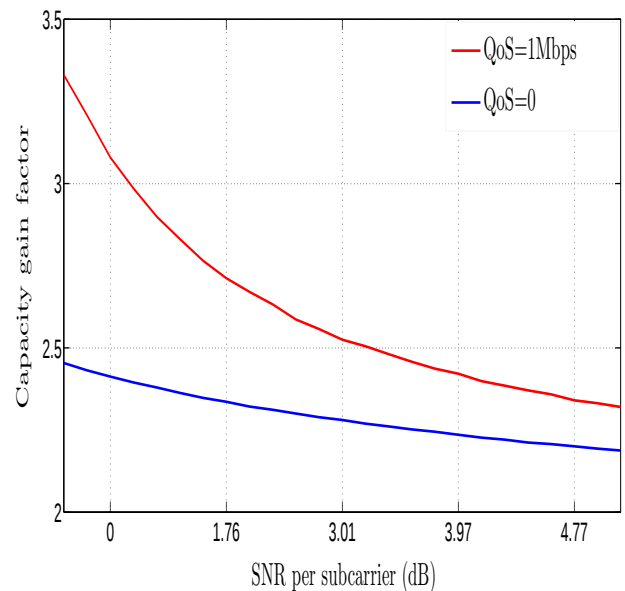


Fig. 5. Impact of SNR per subcarrier on the capacity gain factor

B. Impact of QoS constraint

The user QoS constraint in P1 plays an important role on the performance difference between the SCSl and ICSl based resource allocation schemes. Fig. 6 shows the variation of the capacity gain factor with the user QoS requirement. The transmit power equals 10mW (SNR per subcarrier is equal to 0dB). A close observation of this figure indicates that the value of this factor grows with the QoS constraint. As the user demands increase, the number of sub-carrier reallocation operations required to satisfy users whose instantaneous channel realizations are unavailable grows. This process has a negative impact on the optimum sum-goodput of the SCSl based scheme. Moreover, multiuser diversity has a strong effect on the results of Fig. 6. When the users' ICSl is known by the transmitter, it is easier for multiuser diversity to be

exploited. However, when SCSi is used to perform RRM, the benefits of multiuser diversity outweigh the drawbacks of sub-carrier reallocation only when the QoS constraint is low (i.e. $< 0.1\text{Mbps}$ in our simulations). It is important to further investigate the significance of multiuser diversity on these results. In Fig. 7, we notice that when the user QoS requirement is equal to 0.1Mbps and the SCSi based RRM scheme is used, the overall system throughput increases with the number of users. However, when the user ICSI is unknown, it becomes increasingly difficult to exploit multiuser diversity as the QoS requirements grow. In this case, the effects of sub-carrier reallocation counteract multiuser diversity even when a relatively low number of active users are present in the cell. On the other hand, using ICSI based RRM enables multiuser diversity to be utilized much more efficiently when the QoS demands are high.

V. CONCLUSION

In this work, adaptive resource allocation has been performed for users whose instantaneous channel realizations are unavailable at the transmitter but known by the receiver. Sub-carriers were optimally loaded by using a relationship between the average SNR and the Lambert-W function. To further enhance the overall spectral efficiency, a well-known sub-optimal algorithm was extended. Using the proposed approach, numerical results showed that a significant number of user QoS requirements could be met using SCSi. However, an important loss in performance was observed when a performance comparison between the SCSi and ICSI based resource allocation scheme was made. This degradation was dependent on the number of active users present in the cell, the QoS constraint, and the transmit power.

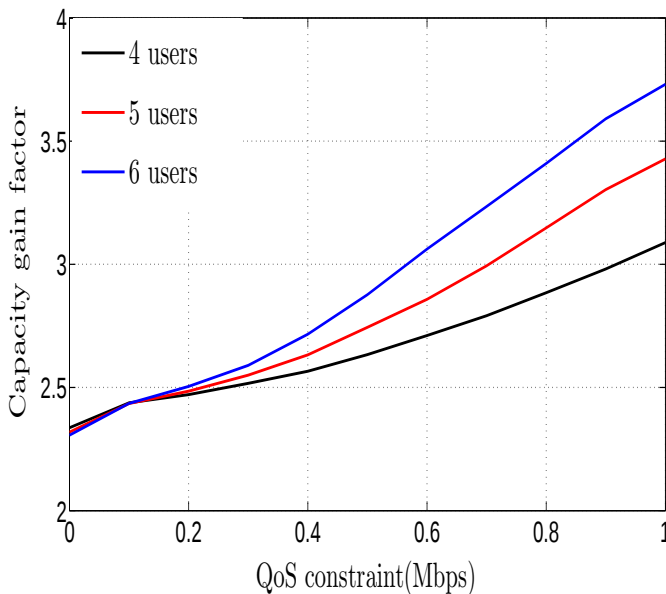


Fig. 6. Capacity gain factor dependence on QoS constraints

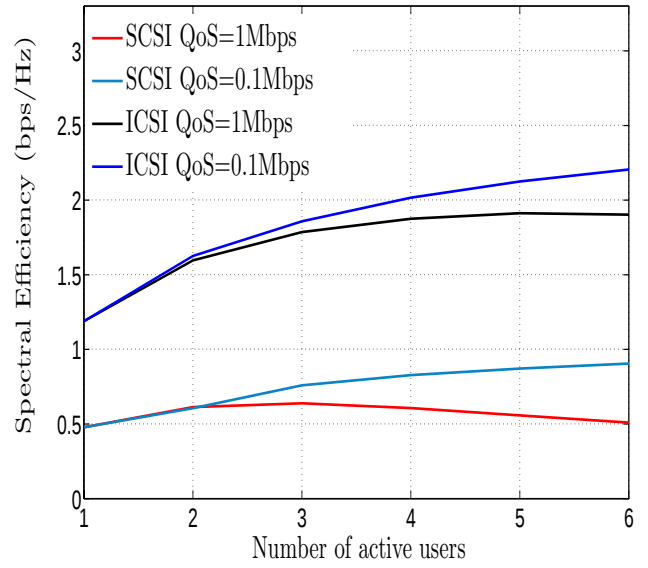


Fig. 7. Effect of QoS constraint on multiuser diversity

VI. APPENDIX A PROOF OF (4)

The goodput is written as

$$G(\rho) = \rho(\exp[-(\bar{\gamma}^{-1})(2^\rho - 1)]) \quad (8)$$

In order to find the value of ρ that yields the maximum values of G derivative of G with respect to ρ is set to zero. If we set $g(\rho) = -(\bar{\gamma}^{-1})(2^\rho - 1)$ this can be written as:

$$\frac{dG}{d\rho} = \exp(g(\rho)) + \rho \exp(g(\rho)) * \ln(2)g(\rho) = 0 \quad (9)$$

This reduces to

$$1 + \ln(2)\rho g(\rho) = 0 \quad (10)$$

By replacing $g(\rho)$ with its original value we obtain

$$\frac{\bar{\gamma}}{(2^\rho) \ln(2)\rho} = 1 \quad (11)$$

Using $2^\rho = \exp(\ln(2)\rho)$ this can be written as

$$\frac{\bar{\gamma}}{(\exp(\ln(2)\rho)) \ln(2)\rho} = 1 \quad (12)$$

By setting $y = \ln(2)\rho$ (12) can be written in the form $y \exp(y) = \bar{\gamma}$. By applying the Lambert W function we get $y = W(\bar{\gamma})$. Replacing with the original value of y will result in (4).

VII. APPENDIX B CALCULATION OF THE CAPACITY GAIN FACTOR LIMIT

In this appendix, we prove that when the SNR approaches infinity, the capacity gain factor becomes 1. After averaging over a large number of channel realizations, the values of $\bar{\gamma}$

and γ can be considered equal. The limit can therefore be written as:

$$\lim_{\gamma \rightarrow \infty} \exp \left[-(\gamma^{-1})(2^{\frac{W(\gamma)}{\ln(2)}} - 1) \right] \left(\frac{W(\gamma)}{\ln(2)} \right) \left(\frac{1}{\log_2(1 + \gamma)} \right)$$

Using the Lambert function identity, this can be written as:

$$\lim_{\gamma \rightarrow \infty} \exp \left[-\frac{\exp^{W(\gamma)} - 1}{\gamma} \right] \left(\frac{W(\gamma)}{\ln(\gamma)} \right) \left(\frac{\ln(\gamma)}{\ln(1 + \gamma)} \right)$$

Simple algebraic manipulations and application of Del'Hospital's rule to the second and third factor yield:

$$\lim_{\gamma \rightarrow \infty} \exp \left[-\frac{1}{W(\gamma)} - \frac{1}{\gamma} \right] \left(\frac{W(\gamma)}{1 + W(\gamma)} \right) \left(\frac{\gamma + 1}{\gamma} \right) = 1$$

REFERENCES

- [1] Y. Zhang and K.B. Letaief, 'Multi-user Adaptive Sub-carrier and bit allocation with adaptive cell selection for OFDM systems', in *IEEE Trans. Wireless Commun.*, vol. 3, No. 5, pp. 1566-1575, September 2004.
- [2] S. Pietrzyk, OFDMA for Broadband Wireless Access, Artech House, London, UK, 2006.
- [3] B.Evans and I.B Wong, Resource Allocation in Multiuser Multicarrier Systems, Springer Science and Media, 2008.
- [4] D.Kivanc and G.Liu, 'Computationally efficient bandwidth allocation and power control for OFDMA', in *IEEE Trans. Wireless Commun.*, vol.2, No 6, pp 1150-1158, November 2006.
- [5] J.Jang and K.B.Lee, Transmit Power Adaptation for Multiuser OFDM Systems, in *IEEE Journal Selected Areas on Communications*, vol.21, No. 10, pp 171- 178, February 2003.
- [6] E.Biglieri, J.Proakis, and S.Shamai, 'Fading Channels Information Theoretic and Communication Aspects' *Information Theory, IEEE Transactions on*, vol. 44, No 6, pp 1747-1758, October 1998.
- [7] S. Stefanatos and N. Demetriou, 'Downlink OFDMA Resource Allocation under partial Channel State Information', in *Proc.IEEE ICC'09*, pp. 1-5, June 2009.
- [8] W. Rhee and J.M. Cioffi, 'Increase in capacity of multi-user OFDM system using dynamic sub-channel allocation', in *Proc. IEEE Veh. Technol. Conf.*, pp. 1085-1089, Spring 2000.
- [9] G.Sierksma, Linear and Integer Programming. New York: Marcel Dekker, 1996.
- [10] G. Reklaitis, A. Ravindran, and K. M. Ragsdell, Engineering Optimization, Methods, and Applications. New York: Wiley, 1983.

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