New Multi-Dimensional Mapping for MIMO-BICM-ID Systems

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***Abstract*— In this paper, Bit Interleaved Coded Modulation systems with Iterative Decoding (BICM-ID) for Multiple Input Multiple Output (MIMO) channel are considered. Based on minimizing pair-wise error probability, a design criterion is proposed to find the optimal multi-dimensional constellation mapping for a MIMO-BICM-ID system. Using the design criterion and employing the Binary Switching Algorithm, some optimal constellation mappings are found for 2-dimensional and 3-dimensional cases. Mutual information is utilized to evaluate the proposed constellation mappings. It is illustrated that the proposed mapping schemes outperform conventional ones significantly at high Signal to Noise Ratio (SNR) over fading channels.**

***Index Terms*— Constelation/Mapping, MIMO , BICM, Iterative decoding**

I. INTRODUCTION

After introduction of Turbo codes [1], iterative decoding was applied to the bit-interleaved coded modulation systems and is called Bit-Interleaved Coded Modulation systems with Iterative Decoding (BICM-ID). The idea of iteration between decoder and demodulator is studied in [2] to overcome the drawback of conventional BICM systems. In [2], a simple iterative decoding with hard decision feedback is proposed. It has been shown that BICM-ID significantly outperform the conventional *Trellis Coded Modulation* (TCM) over both Gaussian and Rayleigh fading channels with much less complexity. It has been discussed that the performance of BICM-ID is near to turbo TCM (TTCM) while only one *soft-input soft output* (SISO) decoder instead of two is used [1]. In response to increasing demand for using wireless communication systems and more reliable services while the radio spectrum is limited, *Multiple Input Multiple Output* (MIMO) systems was introduced [3]. MIMO systems make use of multiple transmit and receive antennas to improve the data rate by increasing the channel capacity and error performance over fading channels by increasing the diversity [4] [5]. Utilizing multiple antennas at both transmitter and receiver sides allows high data rates as well as reliable communication. Hence MIMO systems are still an interesting topic in both industrial and academic fields. Recently MIMO techniques are adopted in wireless standards like IEEE 802.11n, 3GPP LTE, and mobile WiMAX systems [6]. The potential for wide capacity gains was first addressed in [7]. Data rate over MIMO channels can be increased by various spatial multiplexing schemes [8]. For example, in *V-BLAST* (Vertical-Bell Labs Layered SpaceTime) architecture, independent data streams are sent in parallel over the transmitter antennas [9] [10] [11]. However, spatial multiplexing gain often results in a loss in spatial diversity [12]. Bit interleaved coded modulation schemes (BICM) for MIMO systems have been addressed as an effective mean to achieve high data rates while maintaining high diversity. It was shown in [13] that with a maximum-likelihood (ML) receiver, MIMO-BICM systems outperform spacetime trellis codes in fast fading channels. Employing BICM with iterative decoding (BICM-ID) over MIMO channels is proposed in [14], which improves BER performance significantly. It has been shown that when signal constellation, interleaver and error control code are fixed, signal mapping has a crucial influence on the error performance of a BICM-ID system [15] [16] [17] [18]. Similarly, the role of the signal mapping also applies to the error performance of MIMO-BICM-ID systems. To minimize the BER of the system and maximize the gain of iterative decoding, we make crucial changes to the traditional mappings via using multi-dimensional signal constellation mapping.

The organization of this paper is as follows. System model is discussed in section II. Section III is our main contribution. In this section the design criterion for a multi-dimensional constellation mapping is proposed and a cost function is derived. In section IV, some two and three dimensional mappings are presented. In section V, proposed mappings are evaluated based on mutual information. Section VI is simulation results and discussion. Finally, section VII provides the conclusion.

II. SYSTEM MODEL

In this paper, we consider a MIMO-BICM-ID system which Fig. 1 shows the transmitter scheme of the system. The transmitter is a serial concatenation of the information source, an encoder, a bit-interleaver (π), a serial to parallel convertor and a Multi-Dimensional mapper. The information bits $\mathbf{b}$ are first encoded by an outer convolutional encoder of rate R_c to produce the output coded bit sequence. The coded bits are permuted randomly by the pseudo-random interleaver, the output sequence is $\mathbf{c}$.

For the conventional MIMO-BICM or specifically V-BLAST-BICM systems, a coded and interleaved bit sequence is demultiplexed into N_t sub-sequences. Then

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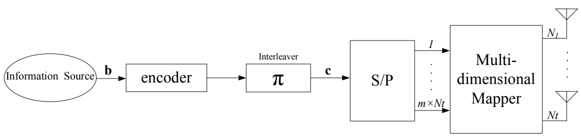

Figure 1. The transmitter of MIMO-BICM-ID.

each sub-sequence bit stream is fed to its respective transmitter. At the transmitter each $m = \log_2 M$ bits are mapped independently to one of the signal points chosen from an *M*-ary constellation. Assuming same number of dimensions and transmit antennas, at each time slot *k*, N_t different and independent signals will be sent over N_t transmitter antennas. In contrast to the conventional systems, in the proposed design, the sequence of *N* coded and interleaved bits, $[c_0, c_1, ..., c_{N\text{-}1}]$, is broken down into blocks of $m \times n$ bits, which m is the number of bits per conventional complex symbol and n is the number of dimension. The *k*-th block is denoted as

$$\mathbf{c}_k = [c_{k0}, c_{k1}, ..., c_{k(m\times n-1)}] \tag{1}$$

where c_{ki} is a coded and interleaved bit which gets a value of either 0 or $1 (0 \le i \le m \times n - 1, 1 \le k \le K,$ and $K = \frac{N}{m\times n}$). Now $\mathbf{c}_k$ bits are simultaneously mapped to *n* parallel *M*-ary signal points ($n \times 1$ vector, where $n = N_t$). This will make a bigger constellation ϕ in *n* dimensions, having M^n signal points, where

$$\mathbf{s}_i = \mu(\mathbf{c}_k). \tag{2}$$

$\mu(\cdot)$ denotes the multi-dimensional mapping function, choosing one of the *n*-dimensional signals according to $m\times n$ bits. In the proposed constellation each signal point can be represented as a vector:

$$\mathbf{s}_i = [x_{1,i}, x_{2,i}, ..., x_{n,i}]^T \tag{3}$$

where $x_{p,i}$, represents the *p*-th conventional $M - ary$ constellation point.

The main difference between conventional mapping scheme and our multi-dimensional mapping is that in our scheme, choosing each symbol $x_{p,i} \in \Omega$, $1 \le p \le n$ is a function of $m \times n$ bits while in the conventional scheme it merely depends on m bits. Therefore, conventional mapping scheme can be a special case of our proposed one, when $n = 1$. It is clear that our scheme does not change spectrum efficiency.

We consider the transmission of data frames over a *frequency-nonselective Rayleigh* fading channel with N_t transmitter antennas and N_r receiver antennas. h_{ij} representing the channel path connecting the antenna *i* to the antenna j. It is assumed that h_{ij} is a complex Gaussian distributed coefficient with $E[h_{ij}] = 0$ and $E[|h_{ij}|^2] = 1$, where E[.] is the mathematical expectation.

As a conventional MIMO system, the channel path connecting the antenna i to the antenna j is modeled by a complex Gaussian distributed coefficient h_{ij} with $E[h_{ij}] = 0$ and $E[|h_{ij}|^2] = 1$, where E[.] is the mathematical expectation. The path gains are collected to form an $N_r \times N_t$ *channel matrix* $\mathbf{H}[h_{ij}]$.

At time period *k*, the channel output can be expressed as,

$$\mathbf{y}_k = \mathbf{H}_k.\mathbf{s}_k + \mathbf{n}_k \tag{4}$$

where $\mathbf{y}_k = [y_{k1}, y_{k2}, ..., y_{kN_r}]^T$ is the received vector , $\mathbf{s}_k = [s_{k1}, s_{k2}, ..., s_{kN_t}]^T$ is the transmitted signals vector and $\mathbf{n}_k = [n_{k1}, n_{k2}, ..., n_{kN_r}]^T$ is the additive white Gaussian noise vector with zero mean and variance $\sigma^2 = N_o \mathbf{I}_{N_r}$.

In this paper, it is assumed that the channel is frequency-nonselective Rayleigh fading and the interleaver is sufficiently random so that the neighboring information bits will experience uncorrelated fading. To avoid increasing complexity, we assume equal number of transmitter and receiver antennas (N_t=N_r).

III. Design Criterion

In this section, we derive a criterion for optimal mapping based on *pairwise error probability* (PEP).

Let's $P(S_k \to \hat{S_k})$ express the pairwise error probability, when the decision is made only by observing the received signal during one signal interval. $P(S_k \to \hat{S_k})$ can be written as:

$$P(S_k \to \hat{S_k}) = Q\left(\frac{d}{2\sigma_n}\right) \tag{5}$$

where σ_n^2 is the variance of AWG noise and can be denoted as $\frac{N_o}{2}$. Hence we can rewrite equation 5 as:

$$P(S_k \to \hat{S_k}) = Q\left(\frac{d}{\sqrt{2N_o}}\right) \tag{6}$$

where *d* is the Euclidean distance between S_k and $\hat{S_k}$. In a MIMO system where the channel matrix $\mathbf{H}$ is perfectly known in the receiver, *d* can be expressed as:

$$d = \sqrt{\|\mathbf{H}(S_k - \hat{S_k})\|^2} \tag{7}$$

Hence the pairwise error probability (PEP) in equation (5) can be rewritten as:

$$P(S_k \to \hat{S_k}) = Q\left(\frac{\sqrt{\|\mathbf{H}(S_k - \hat{S_k})\|^2}}{2\sigma_n}\right) = Q\left(\sqrt{\frac{\|\mathbf{H}(S_k - \hat{S_k})\|^2}{2N_o}}\right) \tag{8}$$

An upper bound for Q-function is used, called Gaussian Tail approximation. It is derived from the chernoff bound [19] and states:

$$Q(x) \le \frac{1}{2}\exp(-\frac{x^2}{2}), \qquad x \ge 0. \tag{9}$$

The use of this bound in equation(8) gives:

$$P(S_k \to \hat{S_k}) \le \frac{1}{2}\exp\left(-\frac{\|\mathbf{H}(S_k - \hat{S_k})\|^2}{4N_o}\right) \tag{10}$$

Assume two codewords $\mathbf{c}$ and $\hat{\mathbf{c}}$ which differ in *d* bits (Hamming distance). Considering ideal interleaving, these *d* different positions will be spread in space and time over d distinct transmission period. Let $P(\mathbf{c} \to \hat{\mathbf{c}})$ denotes error probability of choosing the sequence $\hat{\mathbf{c}}$ instead of transmitted sequence $\mathbf{c}$. This error probability can be achieved by averaging over all symbols defined by constellation and all bit positions(q) as it is determined in [20].

$$P(\mathbf{c} \to \hat{\mathbf{c}}) = \left\{ \frac{1}{q \cdot 2^q} \sum_{i=1}^{q} \sum_{b=0}^{1} \sum_{s_k \in \phi_b^i} \sum_{\hat{s}_k \in \phi_{\bar{b}}^i} P(S_k \to \hat{S_k}) \right\}^d \qquad (11)$$

where $\mathbf{s}_k \in \phi_b^i$ denotes the symbols whose labels have the value $b \in [0,1]$ in their i-th position, similarly $\hat{\mathbf{s}}_k \in \phi_{\bar{b}}^i$ are symbols with $\bar{b}$ in their i-th position.

Obviously the pairwise error probabilities of symbol vectors with a small Euclidean distance dominate the overall error performance. This depicts the playing role of the mapping in error bounds.

Substituting equation (10) in (11) and considering number of bits in the proposed multi-dimensional signal mapping, result in:

$$P(\mathbf{c} \to \hat{\mathbf{c}}) = \left\{ \frac{1}{nm \cdot 2^{nm+1}} \sum_{i=1}^{nm} \sum_{b=0}^{1} \sum_{s_k \in \phi_b^i} \sum_{\hat{s}_k \in \phi_{\bar{b}}^i} \exp\left(-\frac{\|\mathbf{H}(\mathbf{S}_k - \hat{\mathbf{S}}_k)\|^2}{4N_o} \right) \right\}^d \qquad (12)$$

Based on the error probability in equation (12) and considering the influence of Euclidean distance of pairwise symbols, the criterion to choose the best mapping can be defined as optimization of the cost function δ where

$$\delta = E_{\mathbf{H}} \left[\frac{1}{nm \cdot 2^{nm+1}} \sum_{i=1}^{nm} \sum_{b=0}^{1} \sum_{s_k \in \phi_b^i} \sum_{\hat{s}_k \in \phi_{\bar{b}}^i} \exp\left(-\frac{\|\mathbf{H}(\mathbf{S}_k - \hat{\mathbf{S}}_k)\|^2}{4N_o} \right) \right] \qquad (13)$$

and $E_{\mathbf{H}}[\cdot]$ denotes mathematical expectation over channel matrix $\mathbf{H}$. Equation (13) can be rewrite as:

$$\delta = \frac{1}{nm \cdot 2^{nm+1}} \sum_{i=1}^{nm} \sum_{b=0}^{1} \sum_{s_k \in \phi_b^i} \sum_{\hat{s}_k \in \phi_{\bar{b}}^i} E_{\mathbf{H}} \left[\exp\left(-\frac{\|\mathbf{H}(\mathbf{S}_k - \hat{\mathbf{S}}_k)\|^2}{4N_o} \right) \right] \qquad (14)$$

Considering fading situation and using the procedure shown in [5], the criterion function is simplified as:

$$\delta = \frac{1}{nm \cdot 2^{nm+1}} \sum_{i=1}^{nm} \sum_{b=0}^{1} \sum_{\mathbf{s}_k \in \phi_b^i} \sum_{\hat{\mathbf{s}}_k \in \phi_{\bar{b}}^i} \frac{1}{\frac{1}{4N_o} |\mathbf{S}_k - \hat{\mathbf{S}}_k|^2} \qquad (15)$$

It can be observed that δ in equation (15) depends on the channel signal to noise ratio. On the other hand, δ depicts the signal mapping effect on the asymptotic performance of MIMO-BICM-ID systems, the smaller cost function (δ) the lower asymptotic bit error rate performance.

It should be mentioned that, sometimes equation (11) is difficult to manage. In order to minimize the cost function, it is easier to maximize the Euclidean distance between two multi-dimensional constellation points that their indices differ at only one bit.

As it is mentioned, the derived criterion is based on pairwise error probability, in other words, the Euclidean distance between two symbols which are common in all bit positions except one is maximized. Therefore, we are intuitively using the *genie* method that assumes perfect *a priori* information [21].

To have an optimum mapping, the cost function defined in equation (15) should be minimized. It is obvious that for a crowded constellation, an exhaustive search to find a mapping that yields the smallest value of the cost function, is impossible due to the complexity. Therefore, Binary Switching Algorithm (BSA) [22] is used to avoid long searches and infeasible complexity.

IV. Some Proposed multi-dimensional mappings

For each multi-dimensional case, the optimum mapping can be obtained by minimizing the cost function δ in equation (15). Such mappings are found for 2-dimensional and 3-dimensional QPSK by computer search using BSA [23]. The proposed mapping for 2-dimensional QPSK is depicted in Fig. 2 followed by table I which shows the index assignment for corresponding constellation.

Fig. 3 shows proposed mapping for 3-dimensional QPSK followed by its index assignment tables.

Table III lists the parameter δ of proposed mappings at $SNR = 10dB$. It is assumed that the average symbol energy is normalized to be one. For a fixed SNR the cost function δ in equation (15) can be interpreted as harmonic mean distance with perfect knowledge of other bits ($\tilde{d_h^2}$). Since $\tilde{d_h^2}$ affects the asymptotic performance of a BICM-ID system, it is expected that a mapping with smaller δ leads to lower asymptotic performance in MIMO-BICM-ID systems. This is shown in section V. In MIMO-BICM-ID systems, It is required to use constellations or mappings with smaller δ to achieve optimum asymptotic performance. Considering the table III, 3-dimensional QPSK has the smaller δ value.

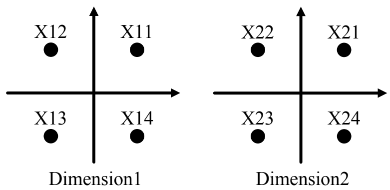

Figure 2. 2-dimensional QPSK constellation scheme

TABLE I.
The proposed mapping for 2-dimensional QPSK scheme

$X_{13}, X_{24} \to 1$	$X_{11}, X_{21} \to 2$	$X_{13}, X_{22} \to 3$	$X_{11}, X_{23} \to 4$
$X_{12}, X_{22} \to 5$	$X_{14}, X_{23} \to 6$	$X_{12}, X_{24} \to 7$	$X_{14}, X_{21} \to 8$
$X_{11}, X_{24} \to 9$	$X_{13}, X_{21} \to 10$	$X_{11}, X_{22} \to 11$	$X_{13}, X_{23} \to 12$
$X_{14}, X_{22} \to 13$	$X_{12}, X_{23} \to 14$	$X_{14}, X_{24} \to 15$	$X_{12}, X_{21} \to 16$

V. Evaluation of Proposed Constellation/Mappings by Mutual Informationl

One of the approaches to evaluate a mapping, is to measure the information about the transmitted signal (s)

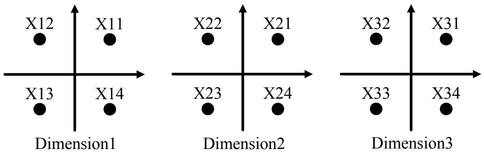

Figure 3. 3-dimensional QPSK constellation scheme

TABLE II.
THE PROPOSED MAPPING FOR 3-DIMENSIONAL QPSK SCHEME

$x_{14}, x_{21}, x_{32} \rightarrow 1$	$x_{12}, x_{23}, x_{32} \rightarrow 2$	$x_{13}, x_{23}, x_{31} \rightarrow 3$
$x_{11}, x_{21}, x_{34} \rightarrow 4$	$x_{13}, x_{24}, x_{33} \rightarrow 5$	$x_{14}, x_{22}, x_{31} \rightarrow 6$
$x_{11}, x_{22}, x_{34} \rightarrow 7$	$x_{13}, x_{24}, x_{32} \rightarrow 8$	$x_{13}, x_{22}, x_{34} \rightarrow 9$
$x_{11}, x_{24}, x_{34} \rightarrow 10$	$x_{11}, x_{24}, x_{32} \rightarrow 11$	$x_{13}, x_{22}, x_{32} \rightarrow 12$
$x_{11}, x_{23}, x_{31} \rightarrow 13$	$x_{13}, x_{21}, x_{33} \rightarrow 14$	$x_{13}, x_{22}, x_{33} \rightarrow 15$
$x_{11}, x_{24}, x_{31} \rightarrow 16$	$x_{11}, x_{23}, x_{34} \rightarrow 17$	$x_{13}, x_{21}, x_{31} \rightarrow 18$
$x_{12}, x_{21}, x_{33} \rightarrow 19$	$x_{14}, x_{23}, x_{32} \rightarrow 20$	$x_{12}, x_{22}, x_{31} \rightarrow 21$
$x_{11}, x_{24}, x_{33} \rightarrow 22$	$x_{14}, x_{24}, x_{32} \rightarrow 23$	$x_{12}, x_{22}, x_{34} \rightarrow 24$
$x_{12}, x_{24}, x_{32} \rightarrow 25$	$x_{14}, x_{22}, x_{33} \rightarrow 26$	$x_{14}, x_{22}, x_{34} \rightarrow 27$
$x_{12}, x_{24}, x_{31} \rightarrow 28$	$x_{14}, x_{21}, x_{33} \rightarrow 29$	$x_{12}, x_{23}, x_{31} \rightarrow 30$
$x_{12}, x_{21}, x_{31} \rightarrow 31$	$x_{14}, x_{23}, x_{33} \rightarrow 32$	$x_{12}, x_{23}, x_{34} \rightarrow 33$
$x_{14}, x_{21}, x_{34} \rightarrow 34$	$x_{11}, x_{21}, x_{32} \rightarrow 35$	$x_{12}, x_{23}, x_{33} \rightarrow 36$
$x_{14}, x_{22}, x_{32} \rightarrow 37$	$x_{12}, x_{24}, x_{33} \rightarrow 38$	$x_{13}, x_{24}, x_{34} \rightarrow 39$
$x_{11}, x_{22}, x_{31} \rightarrow 40$	$x_{11}, x_{21}, x_{33} \rightarrow 41$	$x_{13}, x_{23}, x_{32} \rightarrow 42$
$x_{13}, x_{23}, x_{33} \rightarrow 43$	$x_{11}, x_{21}, x_{31} \rightarrow 44$	$x_{13}, x_{24}, x_{31} \rightarrow 45$
$x_{11}, x_{22}, x_{33} \rightarrow 46$	$x_{11}, x_{23}, x_{32} \rightarrow 47$	$x_{13}, x_{21}, x_{34} \rightarrow 48$
$x_{13}, x_{21}, x_{32} \rightarrow 49$	$x_{11}, x_{23}, x_{33} \rightarrow 50$	$x_{13}, x_{23}, x_{34} \rightarrow 51$
$x_{14}, x_{21}, x_{31} \rightarrow 52$	$x_{12}, x_{24}, x_{34} \rightarrow 53$	$x_{13}, x_{22}, x_{31} \rightarrow 54$
$x_{11}, x_{22}, x_{32} \rightarrow 55$	$x_{14}, x_{24}, x_{33} \rightarrow 56$	$x_{14}, x_{23}, x_{31} \rightarrow 57$
$x_{12}, x_{21}, x_{34} \rightarrow 58$	$s_{12}, s_{21}, s_{32} \rightarrow 59$	$s_{14}, s_{23}, s_{34} \rightarrow 60$
$x_{12}, x_{22}, x_{33} \rightarrow 61$	$x_{14}, x_{24}, x_{31} \rightarrow 62$	$s_{14}, s_{24}, s_{34} \rightarrow 63$
$s_{12}, s_{22}, s_{32} \rightarrow 64$		

provided by the received signal ($\mathbf{r}$) (channel observations). This is expressed in [24]:

$$I(X;Y) = \oint \oint p(X,Y) \cdot \log \frac{p(X,Y)}{p(X) \cdot p(Y)} d_X d_Y \quad (16)$$

Consider an M-ary constellation ψ where $M = 2^m$ and channel relationship in equation (4). The average symbol-wise mutual information in the multi-dimensional case, where each constellation signal represents $n \times m$ bits, can be expressed as:

$$I(\mathbf{s};\mathbf{r}) = \frac{1}{2^{n \times m}} \sum_{k=1}^{n \times m} \int_{-\infty}^{+\infty} p(\mathbf{H}) \int_{-\infty}^{+\infty} p(\mathbf{r}|\mathbf{s}_k) \cdot \log_2 \frac{p(\mathbf{r}|\mathbf{s}_k)}{p(\mathbf{r})} d\mathbf{r} d\mathbf{H} \quad (17)$$

where

$$p(\mathbf{r}) = \frac{1}{2^{n \times m}} \sum_{k=1}^{2^{n \times m}} p(\mathbf{r}|\mathbf{s}_k) \quad (18)$$

In the equation (17) integration over $\mathbf{r}$ and $\mathbf{H}$ are two-fold integrals. On the other hand, $p(\mathbf{r}|\mathbf{s}_k)$ is easily found using equation (4),

TABLE III.
IMPROVEMENT OF PROPOSED MAPPINGS OVER CONVENTIONAL ONES

Constellation/Mapping Type	δ	BER Improvement
2-dimensional QPSK	8.33e-02	3.5dB
3-dimensional QPSK	4.53e-02	2.4dB

$$p(\mathbf{r}|\mathbf{s}_k) = \frac{1}{(2\pi\sigma^2)^N} \cdot \exp\left(-\frac{\|\mathbf{r} - \mathbf{H} \cdot \mathbf{s}_k\|^2}{2\sigma^2}\right) \quad (19)$$

where $\delta^2 = \frac{N_o}{2}$ (double-sided noise power spectral density) and N is the number of dimensions. In our system of interest which is a specific case, N is equal to N_t (number of transmitter and receiver antennas). Bit-wise mutual information is a technique to measure the information about the each of the bits in a symbol label by observing the received symbol. This mutual information can be defined in two ways: 1- Bit-wise mutual information without any knowledge about other bits I_0. 2- Bit-wise mutual information with perfect knowledge about other bits I_1. Consider a MIMO communication system and let $[c_1, c_2, ..., c_{n \times m}]$ denote the $n \times m$ bits mapped to one symbol in the constellation ϕ. The average bit-wise mutual information without any knowledge about other bits can be expressed as:

$$I_0 = \frac{1}{n \times m} \sum_{k=1}^{n \times m} I(c_k; \mathbf{r}) \quad (20)$$

where

$$I(c_k; \mathbf{r}) = \frac{1}{2} \sum_{b=0}^{1} I(c_k = b; \mathbf{r}) \quad (21)$$

and $I(c_k = b; \mathbf{r})$ is calculated as:

$$I(c_k = b; \mathbf{r}) = \int_{-\infty}^{+\infty} p(\mathbf{H}) \int_{-\infty}^{+\infty} \left(\frac{1}{2^{n \times (m-1)}} \sum_{\mathbf{s}_j \in \phi_b^k} p(\mathbf{r}|\mathbf{s}_j) \right) \times \log_2 \frac{\left(\frac{1}{2^{n \times (m-1)}} \sum_{\mathbf{s}_j \in \phi_b^k} p(\mathbf{r}|\mathbf{s}_j) \right)}{\frac{1}{2} \cdot \frac{1}{2^{n \times (m-1)}} \left[\sum_{\mathbf{s}_j \in \phi_b^k} p(\mathbf{r}|\mathbf{s}_j) + \sum_{\mathbf{s}_j \in \phi_{\bar{b}}^i} p(\mathbf{r}|\mathbf{s}_j) \right]} d\mathbf{r} d\mathbf{H} \quad (22)$$

$$= \frac{1}{2^{n \times (m-1)}} \sum_{\mathbf{s}_j \in \phi_b^k} \int_{-\infty}^{+\infty} p(\mathbf{H}) \int_{-\infty}^{+\infty} p(\mathbf{r}|\mathbf{s}_j) \times \log_2 \frac{2 \cdot \sum_{\mathbf{s}_j \in \phi_b^k} p(\mathbf{r}|\mathbf{s}_j)}{\sum_{j=1}^{2^{n \times m}} p(\mathbf{r}|\mathbf{s}_j)} d\mathbf{r} d\mathbf{H} \quad (23)$$

In equation (22), $\mathbf{s}_j \in \phi_b^k$ denotes the symbols whose labels have the value $b \in [0, 1]$ in their k-th position. For the AWGN channel, the integration over the probability density function p($\mathbf{H}$) of the channel state matrix in equation (23), can be omitted.

I_1 is the average bit-wise mutual information with perfect knowledge about the $n \times m - 1$ other bits. The expression for I_1 can be obtained as follows.

$$I_1 = \frac{1}{n \times m} \sum_{k=1}^{n \times m} I(c_k; \mathbf{r} | \text{all other } n \times m - 1 \text{ bits are known}) \quad (24)$$

Let $[c_1, c_2, ..., c_{n\times m}]$ denote the $n \times m$ bits mapped to one symbol in constellation ϕ. We define:

$$\underline{\mathbf{C_k}} = [c_1, c_2, ..., c_{k-1}, c_{k+1}, ..., c_{n\times m}] \tag{25}$$

and therefore (24) can be rewritten as:,

$$I_1 = \frac{1}{n \times m} \sum_{k=1}^{n\times m} I(c_k; \mathbf{r}|\underline{\mathbf{C_k}}). \tag{26}$$

$I(c_k; \mathbf{r}|\underline{\mathbf{C_k}})$ can be expressed as:

$$I(c_k; \mathbf{r}|\underline{\mathbf{C_k}}) = \frac{1}{2^{n\times m}} \sum_{b=0}^{1} \sum_{\forall \mathbf{c}_k} I(c_k = b; \mathbf{r}|\underline{\mathbf{C_k}} = \mathbf{c}_k) \tag{27}$$

where $\mathbf{c}_k = [b_1, b_2, ..., b_{k-1}, b_{k+1}, b_{n\times m}]$ while b_j gets the values of either 0 or 1. Regarding the mentioned assumptions we evaluate $I(c_k = b; \mathbf{r}|\underline{\mathbf{C_k}} = \mathbf{c}_k)$ by numerical integration:

$$I(c_k = b; \mathbf{r}|\underline{\mathbf{C_k}} = \mathbf{c}_k) = \int_{-\infty}^{+\infty} p(\mathbf{H}) \int_{-\infty}^{+\infty} p(\mathbf{r}|c_k = b, \underline{\mathbf{C_k}} = \mathbf{c}_k) \times \log_2 \frac{2 \cdot p(\mathbf{r}|c_k = b, \underline{\mathbf{C_k}} = \mathbf{c}_k)}{p(\mathbf{r}|c_k = 0, \underline{\mathbf{C_k}} = \mathbf{c}_k) + p(\mathbf{r}|c_k = 1, \underline{\mathbf{C_k}} = \mathbf{c}_k)} d\mathbf{H} d\mathbf{r} \tag{28}$$

with

$$p(\mathbf{r}|c_k = b, \underline{\mathbf{C_k}} = \mathbf{c}_k) = p(\mathbf{r}|\mathbf{s}_j), \quad \mathbf{s}_j = \mu(c_1, c_2, ..., c_{k-1}, b, c_{k+1}, ..., c_{n\times m}) \tag{29}$$

where $\mu(\cdot)$ is the mapping scheme and $p(\mathbf{r}|\mathbf{s}_j)$ is defined in equation (19). It is explicit that in calculation of $I(c_k = b; \mathbf{r}|\underline{\mathbf{C_k}} = \mathbf{c}_k)$ we deal only with two symbols, chosen from constellation ϕ. These symbols differ only in their k-th position.

As it has been mentioned, in calculation of the average bit-wise mutual information I_0, it is assumed that there is no information about the other bits. Hence I_0 is an important parameter for the first iteration of the systems employing BICM-ID where no information is available to feedback from the decoder. So I_0 is usually more important for BICM systems. When information between the channel decoder and demodulator starts iterating then I_0 is not important any more. Eventually, if a perfect information about all other bits is available at the demodulator, the average mutual information with perfect knowledge about the other bits (I_1) is dominant.

The sum of bit-wise mutual information, for a fixed constellation, is always a constant value (I), independent of the applied mapping [25]. Therefore, there is a trade-off between the bit-wise mutual information. It has been found that, for a constant constellation, usually a mapping with maximum I_1 induces minimum I_0 and vice versa [26] [27]. To have a good performance with no iteration, a mapping should be chosen which maximizes I_0. On the other hand, a mapping with larger I_0 and consequently smaller mutual information with *a priori* knowledge (I_1), can not gain the error performance of the system through the iterations. Gray mapping is a good example of such a mapping. In the same way, the error performance of a system employing a mapping with large I_1 and very small I_0 does not improve through the iterations. This is due to early crossing of the transfer function which stops the iterative process quickly [28]. A compromise solution is to find a mapping with big enough I_0 to make the iterations work, while maximizing I_1 to achieve good performance after number of iterations. According to the design criterion defined in the previous section, we expect to improve the mutual information while there is a perfect (reliable enough) information about the other bits.

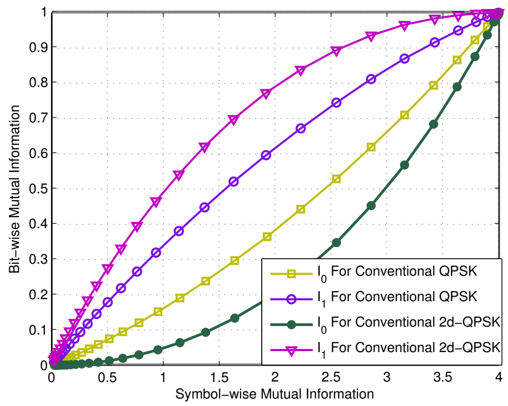

Figure 4. The average bit-wise mutual information versus symbol-wise mutual information for 2-dimensional QPSK

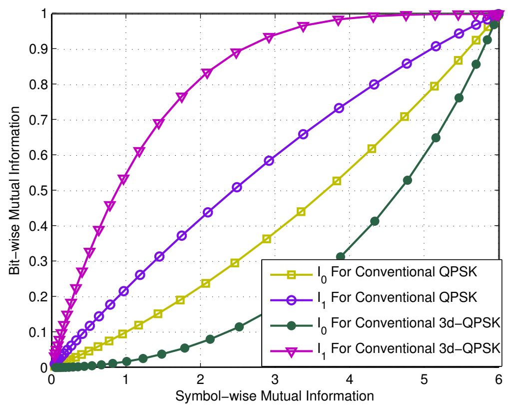

Figure 5. The average bit-wise mutual information versus symbol-wise mutual information for 3-dimensional QPSK

Figs. 4 and 5 show simulation results for the proposed constellations/mappings and the conventional ones, where average bit-wise mutual information versus symbol-wise mutual information is plotted. Although I_0 has not been considered in designing the criterion, it can be observed that I_0 is still large enough to make the iterations work while I_1 is maximized. As it has been discussed, there is always a trade-off between I_1 and I_0, maximizing I_1 results in minimizing I_0.

It should be mentioned that, in this paper the set-partitioning mapping is considered as the conventional mapping.

VI. Simulation Results and Discussion

In this section, the error performance of the proposed system is shown. Many multi-dimensional constellations/mappings included the proposed ones in the table III are tested. These constellations/mappings are applied to the MIMO system proposed in [29]. At the transmitter, a simple 8 state, rate $\frac{1}{2}$ convolutional code with $g_1 = [1101]$ and $g_2 = [1111]$ is used. A block of 5000-bits is fed to the bit-interleaver. A MIMO system is considered without any special space-time coding. The MIMO system benefits from spatial multiplexing or in other words uses a V-BLAST method to assure high data rate communications. At the receiver side, two soft-input soft-output blocks, the inner demodulator/demapper and the outer decoder, are utilized. These blocks exchange their information iteratively. This process stops when the convergence is achieved. The optimal algorithm for the outer soft-input soft-output decoder is derived by Bahl *et al* in [30] which is known as BCJR decoder algorithm. The inner detection is based on the List Sphere Decoder (LSD) in order reduce the complexity of the Maximum-Likelihood (ML) detectors. It is assumed that the channel is frequency-nonselective Rayleigh fading and state information or the channel matrix $\mathbf{H}$ is perfectly known at the receiver. The radius for LSD is considered to be fixed. Therefore, the list size may be variable.

2-dimensional QPSK is applied to a 2×2 MIMO system. The comparison between the bit-error rate of the first and fifth iterations of the conventional mapping and the proposed one is illustrated in Fig. 6. As it has been mentioned the set-partitioning mapping is considered as the conventional mapping.

Fig. 6 shows that at very low SNR and the first iteration, the conventional mapping performs slightly better than the proposed one. This is expected since the proposed mapping does not minimizing the bit error rates when non-iterated decoding is used. This applies to all constellations/mappings which are illustrated. It should be mentioned that the comparisons between the proposed constellations/mappings and the conventional ones are made at bit error rate of 10^{-5}. At the end, 3-dimensional QPSK is applied to a 3×3 MIMO system, Fig. 7 presents the result. Simulation results illustrate that utilizing the proposed mappings significantly improve the error performance of the MIMO-BICM-ID systems over fading channels. This is summarized in table III.

As it is mentioned in section IV, the lower value for δ should result in better performance. Simulations show that lower value of δ can be achieved by using 3-dimensional QPSK and results in lower BER.

It should be indicated that the complexity of the overall system is almost the same as the conventional MIMO structure using iterative approach. This is due to the use of BSA in obtaining the modified mapping and employing low complexity LSD as the inner decoder.

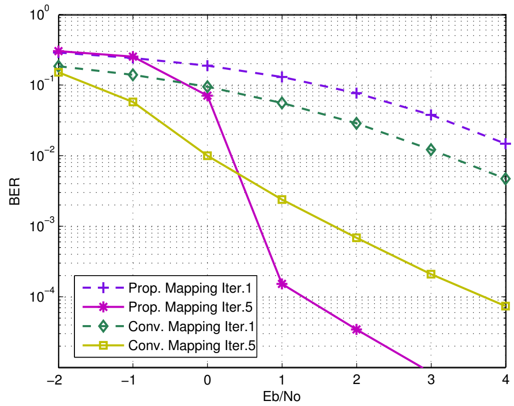

Figure 6. BER comparison between the proposed 2-dimesional QPSK and conventional QPSK schemes, using 2×2 MIMO system over fading channel

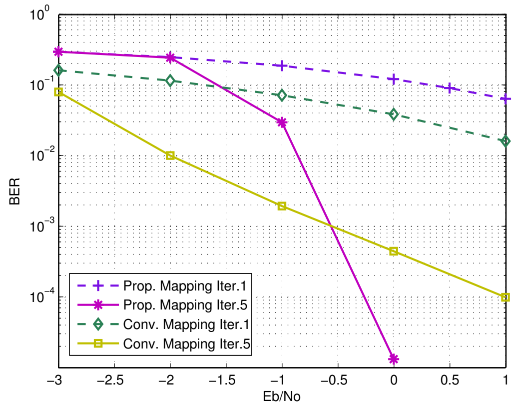

Figure 7. BER comparison between the proposed 3-dimesional QPSK and conventional QPSK schemes, using 3×3 MIMO system over fading channel

VII. Conclusion

In this paper, a design criterion is proposed to find the optimal constellation/mapping for MIMO-BICM-ID systems. Some optimal constellations/mappings for 2-dimensional and 3-dimensional cases are designed using the derived criterion and Binary Switching Algorithm.

A measurement based on the mutual information is developed to evaluate the proposed constellations/mappings. The Monte-Carlo numerical results show that the proposed constellations/mappings sacrifice bit-wise mutual information without *a priori* information but gain significantly when *a priori* information feedback is perfect. This suggests that the proposed schemes can achieve better performance if applied to a MIMO-BICM-ID system, as it is expected.

To manage the computational complexity at the receiver, an iterative receiver based on List Sphere Decoder is employed. The MIMO transceiver with the proposed constellations/mappings can achieve the promised error performance. Simulations were carried out and results show that the proposed mapping schemes outperform the conventional ones significantly at higher signal to noise ratio.

System simulations were performed specifically for 2-dimensional QPSK and 3-dimensional QPSK constellations/mappings. Results show an improvement of 3.5 dB, and 2.4 dB compared to conventional constellations/mappings over fading channels, respectively.

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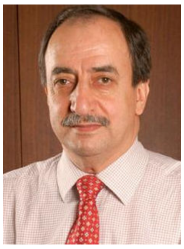

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