Power and Subcarrier Allocation for Multicast Service in OFDMA Decode-and-Forward Relay Systems

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Abstract—Orthogonal Frequency Division Multiplexing Access (OFDMA) relay system is a promising technique for increasing the capacity and the range of wireless communication systems. Dynamic resource allocation for OFDMA relay systems plays an important role in improving the system performance. However, most of the resource allocation algorithms are designed for the unicast traffics, and resource allocation algorithm for multicast traffics in OFDMA relay systems has received little attention. In this paper, the resource allocation problem for the decode-and-forward relay assisted OFDMA multicast system is considered. Our goal is to maximize the sum rate of the multicast system under the constraints of total power limit and exclusive subcarrier usage. The multicast resource allocation problem is formulated as a mixed integer programming problem. The problem is firstly solved by using Lagrangian dual decomposition method, resulting in an asymptotically optimal joint subcarrier and power allocation algorithm with computational complexity being linear with the number of multicast groups and subcarriers. Furthermore, a low complexity suboptimal resource allocation algorithm is presented to further reduce the computational complexity. Simulation results show that both the proposed optimal and suboptimal algorithms can significantly improve the system performance and the proposed suboptimal algorithm can achieve a near optimal solution.

Index Terms—OFDMA, multicast, resource allocation, decode-and-forward.

I. INTRODUCTION

OFDMA system with relay has been considered as a promising technology for the next generation wireless systems due to its resistance to frequency selective fading, flexibility for resource allocation, and ability to provide ubiquitous wide area coverage [1]. On the other hand, wireless multicast is an attractive technique to efficiently deliver bandwidth-intensive traffic loads to multiple destinations simultaneously by taking the advantage of the broadcast channel nature of wireless media [2]. Therefore, OFDMA multicast networks can be used to provide new additional functionalities, including the multimedia applications, real time gaming applications, Internet Protocol Tele Vision (IPTV) applications, and many others. Dynamic radio resource allocation in OFDMA networks aims to efficiently utilize the subcarriers and transmission power among multiple mobile users, and plays an important role in meeting the quality of service (QoS) requirements imposed by the broadband services [3]. Therefore, radio resource management strategies should be designed carefully. Radio resource allocation algorithm for relay-aided OFDMA systems is a challenging task, and has been attracting much interest in recent years [4]. However, most of the literatures dealing with the resource allocation problem in relay-aided OFDMA systems focus on unicast cases [5]-[7], where each user has its individual downlink or uplink traffic flows. Current researches [8]-[10] have considered the multicast radio resource allocation problems in OFDMA networks without relay. In [8], low-complexity suboptimal algorithms are proposed for subcarrier and power allocation to maximize the system throughput with a total transmission power constraint. In [9], a chunk-based suboptimal resource allocation algorithm for the downlink of multicast OFDMA systems is investigated. The proposed algorithm is to maximize the total throughput subject to constraints on total available power and average bit error rate over a chunk under the assumption of uniform power distribution among subcarriers. In [10], a dual decomposition based optimal subcarrier and power allocation for OFDMA cognitive multicast network is carried out. However, to our best knowledge, little research has been done on the resource allocation for multicast service in OFDMA relay systems.

In this paper, the case where the base station serves multiple multicast groups with the assistance of a decode-and-forward relay is considered. The resource allocation problem is formulated as an optimization problem with the objective to maximize the system sum rate under total power and exclusive subcarrier usage constraints. Firstly, the optimization problem is solved by using Lagrangian dual decomposition technique and subgradient update method to obtain an asymptotically optimal joint subcarrier and power allocation algorithm whose complexity is linear with the number of subcarriers and multicast groups. Then, to further reduce the computational complexity, a suboptimal resource allocation algorithm is presented. Finally, it is shown through numerical simulations that the proposed optimal resource allocation algorithm can converge fast, and the proposed suboptimal resource allocation algorithm can provide a near optimal solution. The simulation results also show that both the proposed optimal and suboptimal...
resource allocation algorithms can significantly outperform a fixed subcarrier assignment and water filling power allocation scheme.

The remainder of this paper is organized as follows. In Section II, the system model is presented and the optimization problem is formulated. In Section III, the optimal joint subcarrier and power allocation algorithm is presented. In Section IV, the low complexity suboptimal resource allocation algorithm is given. In Section V, some numerical examples to verify the performance of the proposed resource allocation algorithms are offered. Finally, Section VI concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The system is composed of one base station (BS), one relay station (RS), and K mobile stations (MSs), as shown in Fig. 1. The K MSs are organized into G multicast groups, and the users in the same group receive the same messages. Similar to [11], it is assumed that there is no direct links between the BS and the MSs because of the distance or obstacles. The communication between the BS and MSs in each multicast group should be assisted by the RS. The BS transmits G traffic flows to the G groups, and each group only receives its own traffic flow which is different from the one received by other groups. Let $U_g, g \in \{1, \ldots, G\}$ denote the set of users which belong to the g-th multicast group, and $|U_g|$ denote the number of users in the g-th group. The RS is assumed to be half-duplex, as a result, the communication between the BS and MSs is divided into two stages. In the first stage, the BS sends one modulated symbol to the RS. In the second stage, the relay decodes the received signals and retransmits them to the members of all multicast groups. It is also assumed that the RS uses the same subcarrier as the one used by the BS in the first phase to forward messages, in other words, the fixed subcarrier pairing strategy is used in this model.

The total bandwidth $BW$ is divided into $N$ equal-bandwidth subcarriers. Let $h_{n}^{RR}, h_{n}^{RM}$ denote the channel coefficients over the first hop between the BS and the RS, and over the second hop between the RS and MS $k$ in subcarrier $n$, respectively. The channels are assumed to remain constant during a two-slot scheduling interval, and vary independently in the next scheduling interval. If subcarrier $n$ is allocated to the multicast group $g$, the transmission rate $R_{g, n}^1$ at the first stage and $R_{g, n}^2$ at the second stage for MS $k$, $k \in U_g$, can be respectively expressed as

$$R_{g, n}^1 = \frac{BW}{2N} \log_2 \left( 1 + \frac{p_{g, n}^{RM} |h_{n}^{RM}|^2}{N_0 BW / N} \right)$$  \hspace{1cm} (1)$$

$$R_{g, n}^2 = \frac{BW}{2N} \log_2 \left( 1 + \frac{p_{g, n}^{RR} |h_{n}^{RR}|^2}{N_0 BW / N} \right)$$  \hspace{1cm} (2)$$

where, $t_{RR}$ represents the path loss between the BS and the RS, $t_{RM}$ represents the path loss between the RS and the MS $k$, $p_{g, n}^{RR}, p_{g, n}^{RM}$ are the transmit powers allocated to subcarrier $n$ for multicast group $g$ at the BS and the RS, respectively. $N_0$ denotes the power spectrum density of the additive white Gaussian noise at the RS and MSs. In this way, for decode-and-forward relay strategy, the maximum achievable average rate $R_{g, n}$ for MS $k, k \in U_g$, in subcarrier $n$ with the help of the RS is

$$R_{g, n} = \min \{ R_{g, n}^1, R_{g, n}^2 \}$$  \hspace{1cm} (3)$$

It is known that the achievable rate in Eq. (3) at the MS $k$ can be maximized when the condition of $R_{g, n}^1 = R_{g, n}^2$ is satisfied. Let $\alpha_{k, n}^{RM} = t_{RR} |h_{n}^{RM}|^2 / (N_0 BW / N)$ and $\beta_{k, n}^{RR} = t_{RR} |h_{n}^{RR}|^2 / (N_0 BW / N)$, and let $p_{g, n}$ denote the total transmission power allocated to the multicast group $g$ on subcarrier $n$. Under the total power constraint of $P_{g, n} = p_{g, n}^{RR} + p_{g, n}^{RM}$ for the multicast group $g$ in subcarrier $n$, to maximize $R_{g, n}$, the transmit power at the BS and RS can be expressed, respectively, as follows

$$p_{g, n}^{RR} = \frac{\alpha_{k, n}^{RM}}{\alpha_{k, n}^{RM} + \beta_{k, n}^{RR}} p_{g, n}$$  \hspace{1cm} (4)$$

$$p_{g, n}^{RM} = \frac{\beta_{k, n}^{RR}}{\alpha_{k, n}^{RM} + \beta_{k, n}^{RR}} p_{g, n}$$  \hspace{1cm} (5)$$

with Eq. (4) or Eq. (5), Eq. (3) can be rewritten as:

$$R_{g, n} = \frac{BW}{2N} \log_2 \left( 1 + \gamma_{k, n} p_{g, n} \right)$$  \hspace{1cm} (6)$$

where $\gamma_{k, n}$ is the equivalent channel gain for user $k$ over subcarrier pair $(n, n)$, which is defined as:

$$\gamma_{k, n} = \frac{\beta_{k, n}^{RR} \alpha_{k, n}^{RM}}{\alpha_{k, n}^{RM} + \beta_{k, n}^{RR}}$$  \hspace{1cm} (7)$$

In the traditional multicast scenario [8], when subcarrier $n$ is allocated to multicast group $g$, the achievable rate over this subcarrier is the minimum one in Eq. (6) which can be expressed as

$$R_{g, n} = \min_{k \in U_g} R_{g, n}$$  \hspace{1cm} (8)$$

Based on Eq. (8), an equivalent channel gain for each multicast group can be defined. The equivalent channel
gain of subcarrier $n$ for multicast group $g$ is given by

$$\gamma_{g,n}^{\text{eq}} = \min_{k \in \mathcal{D}_g} \gamma_{k,n}$$

(9)

Thus, the achievable data rate in subcarrier $n$ allocated to multicast group $g$ is

$$R_{g,n} = \sum_{k=1}^{G} \frac{BW}{2N} \log_2 \left( 1 + \gamma_{g,n}^{\text{eq}} p_{k,n} \right)$$

(10)

The optimal resource allocation problem for the OFDMA multicast system with decode-and-forward relay can be formulated as

$$\max_p R = \sum_{g=1}^{G} \sum_{n=1}^{N} \pi_{g,n} R_{g,n}$$

(11.a)

s.t. $\sum_{g=1}^{G} \sum_{n=1}^{N} \pi_{g,n} P_{g,n} \leq P_T$ ;

(11.b)

$$p_{g,n} \geq 0, \ \forall g,n ;$$

(11.c)

$$\sum_{g=1}^{G} \pi_{g,n} \leq 1, \ \forall n ;$$

(11.d)

$$\pi_{g,n} \in \{0,1\}, \ \forall g,n$$

(11.e)

where $p$ is a vector of transmit power allocation for all multicast groups over the subcarriers, which is given by

$$[p_{1,1},\cdots,p_{1,N},\cdots,p_{G,1},\cdots,p_{G,N}]$$. $\pi$ is a vector of subcarrier allocation indicator variables, which is given by

$$[\pi_{1,1},\cdots,\pi_{1,N},\cdots,\pi_{G,1},\cdots,\pi_{G,N}]$. $\pi_{g,n}$, $\forall g,n$ is the subcarrier allocation indicator, and $\pi_{g,n} = 1$ if subcarrier $n$ is allocated to multicast group $g$, and $\pi_{g,n} = 0$ otherwise. In this paper, it is assumed that a subcarrier is exclusively allocated to one multicast group. As a result, we have the constraints in Eq. (11.d) and (11.e). The constraints in Eq. (11.b) and (11.c) guarantee that the sum transmit power in all subcarrier is smaller than the system available total power, and the transmit power in any subcarrier is nonnegative, respectively.

Note that the optimization problem in Eq. (11) is a mixed integer programming problem which is NP-hard. If the problem is solved by considering its dual problem, the duality gap between the primal problem and its dual problem will not be zero due to the non-convexity of the primal problem. However, it has been shown that when the number of subcarriers in a multicarrier communication system is sufficiently large, the duality gap tends to zero [12]. Therefore, the Lagrange dual decomposition method can be used to design an asymptotically optimal resource allocation algorithm for the optimization problem in Eq. (11), as shown in section III.

### III. Optimal Subcarrier and Power Allocation via Lagrangian Dual Decomposition Method

The Lagrangian function of problem Eq. (11) can be formulated by relaxing the total power constraint as follows

$$L(p,\lambda) = \sum_{g=1}^{G} \sum_{n=1}^{N} R_{g,n} + \lambda \left( P_T - \sum_{g=1}^{G} \sum_{n=1}^{N} p_{g,n} \right)$$

(12)

where $\lambda$ is the Lagrange multiplier.

$$L_g(p,\lambda) = \sum_{n=1}^{N} (R_{g,n} - \lambda p_{g,n})$$

(13)

which is the per-subcarrier Lagrangian function. The constraint in Eq. (11.d) is not taken into account in Eq. (12), and this constraint will take effect when solving per-subcarrier optimization problem with dual decomposition method.

Thus, the dual problem is defined as:

$$\min_{\lambda} D(\lambda) \ \text{s.t.} \ \lambda \geq 0$$

(14.a)

where the dual objective function is given by

$$D(\lambda) = \max_{p} L(p,\lambda)$$

(14.b)

$$= \max_{p} \left( \sum_{g=1}^{G} \sum_{n=1}^{N} (R_{g,n} - \lambda p_{g,n}) + \lambda P_T \right)$$

(14.c)

$$= \lambda P_T + \max_{p} \left( \sum_{n=1}^{N} \max_{g} \left( R_{g,n} - \lambda p_{g,n} \right) \right)$$

(14.d)

where Eq. (14.c) follows the fact that the per-subcarrier Lagrangian functions are independent of each other. Eq. (14.d) can be obtained when considering the subcarrier exclusive allocation constraints in Eq. (11.d), and we allocate the subcarrier $n$ to group $g(n)$ which achieves the maximal value of the term $(R_{g,n} - \lambda p_{g,n})$ in (14.d).

For a given $\lambda$, when subcarrier $n$ is exclusively allocated to multicast group $g$, the optimal transmit power $p_{g,n}^*$ can be obtained by using Karush-Kuhn-Tucker (KKT) conditions, and is given by

$$p_{g,n}^* = \left[ \frac{U_g BW - \frac{1}{\gamma_{g,n}^{\text{eq}}}}{2N\lambda} \right]^{+}$$

(15)

where $[x]^+ = \max[x,0]$ , $g \in \{1,\cdots,G\}$ and $n \in \{1,\cdots,N\}$. Then, the group index $g^*(n)$ can be given by

$$g^*(n) = \arg \max_{g \in \{1,\cdots,G\}} \left( \frac{U_g BW}{2N} \log_2 \left( 1 + \gamma_{g,n}^{\text{eq}} p_{g,n}^* \right) - \lambda p_{g,n}^* \right)$$

(16)

And the subcarrier allocation indicator variables can be determined by

$$\pi_{g,n}^* = \begin{cases} 1, & g = g^*(n) \\ 0, & \text{otherwise} \end{cases} \ \forall n$$

(17)

Consequently, the transmit power can be updated by
\[ p_{g,n}^* = \pi_{g,n}^* p_{g,n}^*, \quad \forall g,n \]  

Based on Eq. (15) and (16), the dual objective function in Eq. (13) can be determined. The dual problem is always convex, therefore, gradient update method can be used to find the optimal value of the dual variable \( \lambda \). However, due to the exclusive subcarrier allocation constraints, the dual objective function may not be differentiable, thus, the subgradient update method [12] should be used to solve Eq. (13) instead of gradient update method, that is

\[ \lambda^{t+1} = \left[ \lambda^t - \delta^t \left( P_T - \sum_{n=1}^{G} \sum_{g=1}^{N} p_{g,n}^* \right) \right] \]  

where \( t \) is the iteration number and \( \delta^t > 0 \) is the update step size at the \( t \)-th iteration. According to [13], the diminishing rules \( \delta^t = \xi / \sqrt{t} \) for some constant \( \xi \) can guarantee the convergence of the subgradient update method.

Combining Eq. (15) to Eq. (19), the algorithm to find the optimal subcarrier assignment and power allocation can be designed. The pseudocode of the proposed asymptotically optimal resource allocation (AORA) algorithm is outlined in Algorithm A.

**ALGORITHM A: Asymptotically OPTIMAL RESOURCE ALLOCATION ALGORITHM (AORA)**

**Step 1** Find \( \gamma_{g,n}^{\text{opt}} \), for \( g \in \{ 1, \cdots, G \} \), and \( n \in \{ 1, \cdots, N \} \)

**Step 2** Initialize \( \lambda^t \), and let \( t = 1 \)

**Step 3** for \( n = 1 \) to \( N \) do

- Compute \( p_{g,n}^* \), \( \forall g \), using \( \lambda^t \) in (15)
- Compute \( \pi_{g,n}^* \), \( \forall g \), using \( p_{g,n}^* \) in (17)
- Update \( p_{g,n}^* \) using \( \pi_{g,n}^* \) in (18)

end for

**Step 4** Update \( \lambda^{t+1} \) using \( \lambda^t \), \( p_{g,n}^* \) in (19), and \( t = t + 1 \)

**Step 5** Repeat step 3 to 4 until convergence

In step 3, the subcarrier assignment and optimal transit power allocation are performed with a given dual variable. In step 4, the dual variable is updated by means of subgradient method. Step 3 to 4 should be repeated until convergence.

Let \( T \) be the number of iteration required for the subgradient method in step 4 of the joint subcarrier and power allocation algorithm to converge. In each iteration, it requires \( NG \) comparisons, and \( T \) times of iteration are needed for convergence. In step 1, it requires \( NG \) comparisons to obtain \( \gamma_{g,n}^{\text{opt}} \), \( \forall g,n \). Therefore, the total computational complexity of the presented algorithm is \( O(NG + NGT) \). As pointed in [12], \( T \) is a polynomial function of \( N \). So, compared with the computational complexity of the exhaustive search for the optimal solution, e.g., \( O(G^N) \), the presented algorithm can significantly reduce the computation burden.

**IV. LOW COMPLEXITY SUBOPTIMAL RESOURCE ALLOCATION ALGORITHM**

The joint subcarrier and power allocation algorithm presented in section III can provide optimal solution for the system with a larger number of OFDM subcarriers. However, the convergence of the dual decomposition based algorithm is sensitive to the initial value of the Lagrangian multiplier and subgradient update step size, and the algorithm may have a slow convergence behavior. In order to further improve the efficiency of the resource allocation algorithm, inspired by [8] we propose a two-step suboptimal resource allocation algorithm with low complexity for the relay assisted OFDMA multicast system in this section.

The idea of the suboptimal algorithm is to first assign subcarriers to multicast groups in a greedy manner to maximize the system throughput with the assumption of equal transmission power distribution on every subcarrier. After that, the algorithm performs water filling power allocation among the subcarriers to improve the system capacity further.

In the suboptimal algorithm, after the subcarrier allocation variables \( \pi_{g,n} \) for all \( g \) and \( n \) are determined, the optimization problem in Eq. (11) can be rewritten as

\[ \max_p R = \sum_{g=1}^{G} \sum_{n=1}^{N} \pi_{g,n} R_{g,n} \]  

s.t. \[ \sum_{n=1}^{N} \pi_{g,n} p_{g,n} \leq P_T \] \( \pi_{g,n} \geq 0, \quad \forall g,n \)  

The optimal power allocation problem in Eq. (20) is a convex programming problem, and it can be solved by using Lagrangian multiplier technique and KKT condition. The resultant optimal power allocated to the \( k \) user in subcarrier \( n \) is

\[ p_{g,n} = \pi_{g,n} \left( \frac{|U_n| \cdot BW}{2NA} - \frac{1}{\gamma_{g,n}^{\text{opt}}} \right)^{\lambda} \]  

where \( \lambda \) is the Lagrangian multiplier, which can be determined by using

\[ \sum_{g=1}^{G} \sum_{n=1}^{N} p_{g,n} = P_T \]  

It can be noted that the power allocation across the subcarriers in Eq. (21) follows the standard water filling scheme, except that the water level may be not a constant for subcarriers.

The outline of the low complexity suboptimal resource allocation (LCSRA) algorithm is summarized in algorithm B. It has a computational complexity of \( O(NG) \), therefore, the suboptimal scheme is more efficient than the optimal scheme presented in Section III.

**Algorithm B: Low Complexity Suboptimal Resource Allocation Algorithm**

The joint subcarrier and power allocation algorithm presented in section III can provide optimal solution for the system with a larger number of OFDM subcarriers. However, the convergence of the dual decomposition based algorithm is sensitive to the initial value of the Lagrangian multiplier and subgradient update step size, and the algorithm may have a slow convergence behavior. In order to further improve the efficiency of the resource allocation algorithm, inspired by [8] we propose a two-step suboptimal resource allocation algorithm with low complexity for the relay assisted OFDMA multicast system in this section.

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s.t. \[ \sum_{n=1}^{N} \pi_{g,n} p_{g,n} \leq P_T \] \( \pi_{g,n} \geq 0, \quad \forall g,n \)  

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where \( \lambda \) is the Lagrangian multiplier, which can be determined by using

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The outline of the low complexity suboptimal resource allocation (LCSRA) algorithm is summarized in algorithm B. It has a computational complexity of \( O(NG) \), therefore, the suboptimal scheme is more efficient than the optimal scheme presented in Section III.
Allocation Algorithm (LCSRA)

Step 1 Initialization
1: Find $\gamma_{\log}^{eq}$, and let $\pi_{g,n} = 0, \forall g, n$
2: Set the equal power distribution as: $p_{g,n} = P_t/N, \forall g, n$

Step 2 Greedy Subcarrier Allocation
1: for $n=1$ to $N$ do
2: Find the best group index $g'(n)$ for subcarrier $n$ using
$$g'(n) = \arg \max_{g \in \{1,...,G\}} \left\{ \frac{U_g}{2N} \log_2 (1 + \gamma_{g,n}^{eq} p_{g,n}) \right\}$$
3: Let $\pi_{g'(n),n} = 1$
4: end for

Step 3 Power Allocation
1: Calculate power allocation using Eq. (21) and (22)
2: Calculate the sum rate of the system
$$R = \sum_{g=1}^{G} \sum_{n=1}^{N} R_{g,n} = \sum_{g=1}^{G} \left\{ \frac{U_g}{2N} \log_2 (1 + \gamma_{g,n}^{eq} p_{g,n}) \right\}$$

V. SIMULATION RESULTS

In this section, we present the simulation results to show the performance of the proposed subcarrier and power allocation algorithms. We consider an OFDMA decode-and-forward relay system with a radius of 1 km. The total bandwidth $BW$ is 5 MHz, which is evenly divided into 128 subcarriers. The total available transmission power at the base station and the relay is 20 W. The RS is fixed with a distance of 2/3 km to the BS, and all the MSs are uniformly distributed around the RS with distances ranging from 0.03 km to 0.3 km to the RS. The distance-dependent path loss model used in the simulation is given by
$$L = 128.1 + 37.6 \log_{10}(d) \text{ dB}$$
where $d$ is the distance between two stations in km. The shadowing component follows a lognormal distribution with a mean value of 0 dB and standard deviation of 8 dB. For the small scale fading effect, the wireless channel is modeled as a frequency selective fading channel consisting of six independent Rayleigh multipaths. All the multipath components independently experience identical Rayleigh fading, and the corresponding power delay profile is exponentially decaying with $e^{-2t}$, where $l$ is the multipath index.

Fig. 2 illustrates the evolution of the dual variable $\lambda$ in the dual decomposition based joint subcarrier and power allocation algorithm under different noise power densities. The simulation parameters are configured as: $G=4$, $|U_g|=|U|=4$, $|U|=|U|=5$, $N_0 \in \{-155 \text{dBm/Hz} , -165 \text{dBm/Hz}, -175 \text{dBm/Hz}\}$. It can be seen that the proposed resource allocation algorithm can converge with a small iteration number. In our simulation, it is found that the diminishing iteration step size should be selected carefully so that the algorithm can reliably converge under different noise power density.

![Figure 2](image.png)

Figure 2. Dual variable $\lambda$ versus number of iterations with different noise power density for $G=4, |U_g|=|U|=4, |U|=|U|=5$

To evaluate the proposed asymptotically optimal and suboptimal resource allocation algorithms, a fixed subcarrier assignment scheme is introduced as a benchmark in the simulation. In the fixed subcarrier assignment scheme which is denoted as FSA, the subcarrier pairs are evenly allocated to each multicast group and remain unchanged, and only the transmission power is allocated among subcarriers by using water filling method. Fig. 3 shows the system sum rate versus the noise power density, and the results are obtained through average over 1000 times of channel realizations. The number of multicast group $G=4$, and $|U_g|=|U|=4$, $|U|=|U|=5$. As expected, the system sum rate decreases with the increasing of the noise power density. We further observe that the proposed asymptotically optimal joint subcarrier and power allocation algorithm (AORA) outperforms the other schemes, but the performance gaps between the proposed asymptotically optimal algorithm and suboptimal scheme (LCSRA) is diminutive, especially when the noise power density is small. In other words, the proposed suboptimal resource allocation algorithm can achieve a near optimal solution. Since the multiuser diversity is not exploited in the fixed subcarrier assignment scheme, its system sum rate is significantly lower than that of the proposed asymptotically optimal and suboptimal algorithms.

![Figure 3](image.png)

Figure 3. Comparison of system sum rate versus noise variance for $G=4, |U_g|=|U|=4, |U|=|U|=5$

Fig. 4 shows the system sum rate comparison of AORA, LCSRA and FSA for the number of multicast groups ranging from 1 to 8. The number of MSs in each multicast group is fixed with 4, and the noise power
density $N_0$ is assumed to be $-155$dBm/Hz. It is shown that the performance of the AORA and LCSRA significantly outperform that of the FSA. The performance of the LCSRA algorithm can approach that of the AORA algorithm when the number of multicast groups is small, whereas the performance gap becomes obvious with a large number of multicast groups.

![Figure 4. System sum rate verse different group numbers for $|U_i|\neq |U_j|$](image)

VI. CONCLUSION

This paper has investigated the resource allocation problem for multicast service in an OFDMA decode-and-forward relay system. The subcarrier and power allocation is formulated as a constraint optimization problem which maximizes the sum rate of the multicast system while satisfying the total power constraint and exclusive subcarrier usage among multicast groups. The optimization problem is first solved by using the Lagrangian dual decomposition technique to obtain an asymptotically optimal resource allocation algorithm. In order to further improve the efficiency of the resource allocation, a low complexity suboptimal algorithm is presented. Simulation results show that the iteration number of the asymptotically optimal resource allocation algorithm is small, and the proposed suboptimal algorithm can achieve a near optimal solution, and outperforms the fixed subcarrier assignment and water filling power allocation scheme under various signal to noise ratio.

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