Minimization of Channel Impulse Noise using Digital Smear filter

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Abstract— **The paper describes a digital smear-desmear technique (SDT) based on polyphase multilevel sequences of unlimited length with good autocorrelation properties. A design procedure for digital implementation of SDT is defined and sequences with power efficiency higher than 50% are generated. These sequences are applied to the design of digital smear/desmear filters and combined with uncoded and coded ITU-T V.150.1 communication systems. The impulse noise is modeled as a sequence of Poisson arriving delta functions with gaussian amplitudes. The impulse noise parameters are computed from experimental data. Simulation results shows that the SDT filter design method yields a significant improvement in bit error rates for both systems subject to impulse noise, relative to systems with no SDT. The technique also completely removes the error floor caused by impulse noise.**

Index Terms— **Smear, Desmear, Pseudorandom Sequence, Impulse noise, Intersymbol Interference.**

I. INTRODUCTION

Most of the advances in theory and implementation of digital transmission over band limited channels have been made with respect to additive white gaussian noise (AWGN), as the ultimate reliability limitation. With improved equalization, phase jitter tracking, timing recovery and trellis coded modulation (TCM), transmission rates achieved over band limited channels are close to the theoretical limit [1]. However, the required error probabilities for reliable data transmission have not been achieved even at 4.8 kb/s [2] [3]. One of the main impairment on band limited channels, causing burst errors, is impulse noise (IN). In the present-day high speed modems for band limited channels are no measures against impulse noise other than detection [4] [5] [6].

A possible counter-measure to the problem of short impulse noise (less than 10ms) is the smear-desmear technique (SDT) [6]. The SDT in [6] has been implemented in analog technology and the results were not satisfactory due to insufficient quality of analog devices. A digital SDT technique that applies binary sequences of limited length was described in [7], [8]. The design of SDT filters in [7] is based on minimizing intersymbol interference (ISI) and maximizing filter power efficiency. In this design, losses in signal-to-noise ratio (SNR) can be significant since transmit and receive filters are not matched. The purpose of this paper is to derive a more general set of filter design criteria based on minimizing bit error rates and also for practical filter design. As a result, another necessary requirement for minimization of the signal-to-noise ratio (SNR) loss due to mismatching filters is added to the design criteria [7]. In this paper three approaches were applied in the practical filter design. In Design 1, the smearing filters form a pair of matched filters.

The polyphase sequences used in this scheme possess significantly better autocorrelation properties, measured by the merit factor than the binary sequences. Design 2 is proposed for systems where very low values for ISI variance (below -30 dB) are required. Low ISI is achieved by designing the smearing filter to operate as equalizer. The filter sequences are required to have both good autocorrelation and equalization properties. It is shown that polyphase sequences outperform known binary sequences with regard to ISI suppression and mismatching SNR loss. Design 3 can yield ISI as low as Design 2 with reduced system delays. The filter design is based on nonconstant amplitude sequences [9] [10], while the communication system structure is the same as in Design 2. The required filter lengths, for a specified level of ISI, are much smaller than in Design 2.

Simulation results shows that the SDT based on polyphase sequences, yields significant improvement in bit error rates compared to SDT based on binary sequences of the same length. The SDT is attractive on bandwidth limited channels since it does not require bandwidth expansion. The performance improvement is obtained at the cost of an additional delay in the system which can be tolerated in applications of interest. The paper is organized as follows. In section II, we introduce the model of a digital transmission system with smear-desmear filters and discuss the concept of the digital smear-desmear processing. Section III describes the digital SDT in more detail. Section IV defines essential criteria and parameters for SDT design.Section V presents simulation results. Finally, conclusions are summarized in section VI.

II. SYSTEM TRANSMISSION MODEL WITH SDT

A digital communication system with the SDT is depicted in Fig. 1. A binary message sequence generated by

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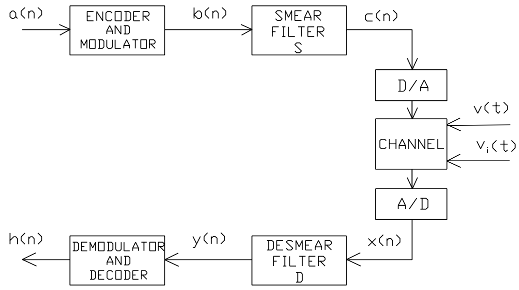

Figure 1. System Transmission model with SDT.

the digital source is mapped into 32-AMPM trellis coded modulation signals and 16-QAM for uncoded signals. The modulator output symbols are then processed by the digital smear filter. In the smear filter the signal is expanded in the time domain over the filter impulse response. This operation results in deliberately introduced intersymbol interference (ISI). The channel is subject to AWGN and impulse noise. Ideal amplitude and phase channel characteristics as well as ideal phase tracking are assumed. In the receiver, the desmear filter performs an inverse operation to the one in the smear filter and thus removes the ISI introduced in the transmitter. Both the smear and desmear filtering are performed in the baseband. After processing by the desmear filter the impulse noise energy is spread out over the filter impulse response length. That results in a significant reduction of the impulse noise effect on the signal. The signal is demodulated by the Viterbi decoder.

A. Transmitter Model

Let **b** = [b(0),$\cdot\cdot\cdot, b(n), \cdot\cdot\cdot$, b(m)] denote a complex symbol sequence at the output of the modulator in fig. 1. The smear filter is represented by a sequence of tap coefficients, denoted by **s** = [s(0),$\cdots, s(i), \cdots$, s(*N*)] where $s(i)$ is the ith tap coefficient and (*N*+1) is the number of taps. The output sequence **c** is obtained by convolving the sequence **b** and the smear filter sequence s. We assume that the filter gain denoted by A_s, is normalized to unity. That is,

$$A_s = \sum_{j=0}^{N} s(j)s^*(j) = 1 \tag{1}$$

where * denotes complex conjugate. The output signal c(*n*) has a gaussian distribution with a zero mean and the variance *P*.

B. Channel Model

The input symbol to the desmear filter at time *n* is given by:

$$x(n) = c(n) + v(n) + v_i(n) \tag{2}$$

where $v(n)$ is a sample of zero mean complex AWGN with the variance σ^2, and $v_i(n)$ is a sample of the channel impulse noise $v_i(t)$ with the variance σ_i^2. The impulse noise event times are represented by a Poisson process. The impulse noise, denoted by $v_i(t)$, can be written in the form [5]

$$v_i(t) = \sum_{k=-\infty}^{\infty} z_i(k)\delta(t - t_k) \tag{3}$$

where t_i represent the impulse noise event times and z_i is the impulse noise amplitudes. The Poisson random process t_i has the intensity of λ events/s and z_i is a Gaussian process with a zero mean and the variance σ_i^2 . The parameters λ and σ_i^2 are obtained from experimental data [11], [12]. Most of the symbols received are not corrupted by impulse noise. Due to the nonstationary character of impulse noise we define the signal to impulse noise power ratio over one symbol interval as

$$SNR_{in} = 10\log(\frac{P}{\sigma_i^2}) \tag{4}$$

where *P* represents the signal power. We assume that the average time interval between two consecutive impulse noise events of $v_i(t)$ is larger than the smear/desmear filters impulse response length consisting of *N* symbol intervals. That means that $\lambda T_s N << 1$, where T_s is the symbol interval.

C. Receiver Model

The desmear filter is represented by a coefficient sequence, denoted by d = [d(0), d(1),$\cdot\cdot\cdot$, d(N)]. To avoid a trivial solution in filter design for the desmear filter coefficient set we include the following power constraint

$$A_{sd} = \sum_{j=0}^{N} s(j)d^*(N-j) = 1 \tag{5}$$

where A_{sd} is the gain of the smear-desmear filter pair. The gain A_d of the desmear filter is given by

$$A_d = \sum_{j=0}^{N} d(j)d^*(j) \tag{6}$$

Combining (1) and (5) we obtain that $A_d \geq 1$ where equality is satisfied if and only if $s(j) = d^*(N-j), j = 0, 1, 2, \cdot\cdot\cdot, N$. The output symbol of the desmear filter signal *y*(*n*) can be represented by:

$$y(n) = b(n) + b_{isi}(n) + \text{v}(n) + \text{v}_s(n) \tag{7}$$

The total channel distortion at time *n* is given by the sum of the residual ISI, $b_{isi}(n)$, the additive white gaussian noise after the desmearing filter, v(n), with a zero mean and the variance $\sigma_{\text{V}}^2 = A_d\sigma^2$ and the impulse noise, v_s, smeared over *N* symbol intervals. The impulse noise v_s with variance in the *(n + j)*th symbol interval is given by:

$$\sigma_s^2(n+j) = E(\text{v}_s(n+j)\text{v}_i^*(n+j)) = \sigma_i^2|d(j)|^2 \tag{8}$$

The residual ISI, $b_{isi}(n)$, is given by the sum of *N* independent random variables. Typically, *N* is larger than

10 and according to the central limit theorem, b_{isi} has a gaussian distribution. The sum of the three independent gaussian variables b_{isi}, v and v_i is another gaussian variable

$$e(n) = y(n) - b(n) = b_{isi}(n) + v(n) + v_s(n) \quad (9)$$

with the variance

$$\sigma_e^2 = E(|b_{isi}(n)|^2) + E(|v(n)|^2) + E(|v_s(n)|^2) \quad (10)$$

The variance σ_e^2 can be upperbounded by

$$\sigma_e^2 = \sigma_V^2 + \sigma_{isi}^2 + \sigma_s^2(n+j) \leq \sigma_V^2 + \sigma_{isi}^2 + max\, \sigma_i^2 \quad (11)$$

Where σ_{isi}^2, the variance of the ISI and $\max \sigma_i^2$ is the maximum impulse noise variance.Thus the total channel distortion can be considered as an equivalent gaussian process with the variance σ_e^2. The main objective of the system design is to minimize the bit error probability. At high signal-to-noise ratio, the bit error probability can be estimated by

$$P_b \approx \frac{N_s}{n_b} Q(\frac{d}{2\sigma_e}) \quad (12)$$

where N_s is the average number of the nearest neighbours in the signal set, n_b is the number of bits in a symbol and d is the minimum Euclidean distance in the signal set. The ultimate performance limit of a communication system is determined by gaussian noise, the only disturbance in the channel. The performance of a real communication system with impulse noise and ISI caused by smearing filters is compared to an ideal system with AWGN only. The measure of the real system performance loss relative to the ideal system is defined as the ratio of the ideal and the real system signal-to-noise ratios given by

$$L = 10 \log_{10} \frac{\sigma_e^2}{\sigma^2} [dB] \quad (13)$$

Ideally, smearing filter parameters should be selected to obtain a zero performance loss. In practical filter design, the objective is to minimize the performance loss or, in other words, minimize the variance of the equivalent gaussian process σ_e^2 .

D. THE SMEARING FILTER DESIGN CRITERIA

The smear and desmear filters are implemented as digital filters. The initial criterion in the filter design is minimization of the performance loss given by Eq. (13). Hence, the performance loss is required to be below a specified threshold T.

$$L_s \leq T_s \quad (14)$$

The performance loss directly depends on the equivalent gaussian process variance, σ_e^2. The variance σ_e^2 is given by the sum of the AWGN variance σ_V^2, the maximum smeared impulse noise variance $\max_j \sigma_s^2(N+j)$ and the residual intersymbol interference variance σ_{isi}^2 as expressed in Eq. (11). The minimization of σ_e^2 can be done by independent minimization of each of the three components in the sum. However, minimization of individual components can cause significant degradation of the others. For example, minimization of the residual intersymbol interference variance can cause a considerable increase in the AWGN variance due to mismatched smearing filters. Since the system performance in the absence of impulse noise is of paramount importance, the priority is given to the minimization of the AWGN variance. The smearing filter design criteria can be summarized as follows:

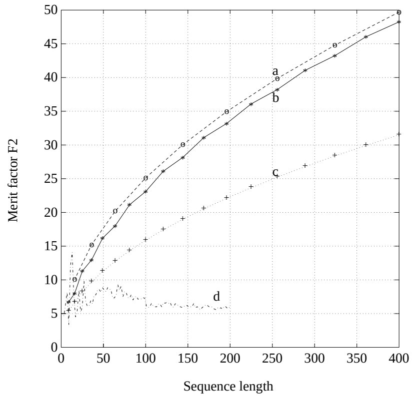

Figure 2. Merit factor F_2 for sequences with constant amplitude: a) P2 sequence, b) Frank and P1 sequence, c) P3 and P4 sequences, d) Binary sequence.

1) *Criterion I: Minimize the AWGN variance :*
2) *Criterion II: Minimize the residual ISI variance :*
3) *Criterion III: Minimize the impulse noise variance :*

E. Criterion I

In order to minimize the AWGN variance the smear and the desmear filters should satisfy the matching condition in the form of

$$s(j) = d^*(N-j) \qquad j = 0, 1, 2, \cdots, N \quad (15)$$

or equivalently, A_d should be equal to 1. If the matching condition expressed by (15) cannot be satisfied, we define a measure of mismatching between the smear and desmear filters, called the mismatching loss in [6].

$$L_m = 10 \log_{10} \frac{\sigma_V^2}{\sigma^2} = 10 \log A_{sd} \, [dB] \quad (16)$$

In filter design we require that the mismatching loss L_m, is less than a specified threshold T_m, therefore $L_m \leq T_m$. The mismatching loss is caused by an increase of the AWGN variance relative to the ideal system with matched filters. In practical filter design we require that the mismatching loss is smaller than 0.3dB. But mismatched loss requirement depends on the application.The proposed scheme requirement is suitable for 64QAM application but will cause a devastating effect for 256QAM, because they are more susceptible to noise corruption than 64QAM. Hence mismatched loss has to be less than 0.3.

$$L_m \leq 0.3dB \tag{17}$$

F. Criterion II

To obtain the minimum ISI variance, the overall transfer function of the smear filter, the channel and the desmear filter should be flat. Since we assumed that the channel does not introduce ISI, the above condition is satisfied if the convolution of sequences **s** and **d**, denoted by C(k), has values z(0) = 1, and z(k) = 0, $k \neq 0$. where convolution C(k) is defined as

$$C(k) = \sum_{j=0}^{N} s(j)d(k+N-j) \quad k = N+1, \cdot\cdot, -1, 0, 1 \tag{18}$$

If condition (15) is satisfied then Eq.(18) simplifies to

$$C(k) = R(k) = \sum_{j=0}^{N} d^*(j)d(k+j) \tag{19}$$

where R(k) is the autocorrelation function of the sequence **d** . Practical difficulties make a zero ISI an unattainable objective in filter design. Typically, a certain amount of residual ISI after the desmearing filter is tolerated. It is measured by the variance of the residual ISI given by

$$\sigma_{isi}^2 = \sum_{k=N+1}^{N-1} |C(k)|^2 - |C(0)|^2 \tag{20}$$

The signal to noise ratio loss caused by the ISI is defined as

$$L_s = 10\log\frac{\sigma^2 + \sigma_{isi}^2}{\sigma^2}\,[dB] \tag{21}$$

Equivalently,

$$L_s = 10\log\left(1 + \frac{SNR}{F_2}\right)\,[dB] \tag{22}$$

where SNR is the signal to AWGN power ratio defined as $SNR = \frac{|C(0)|^2}{\sigma^2}$ and $F_2 = \frac{|C(0)|^2}{\sigma_{isi}^2}$ is the merit factor defined in [7].

The residual ISI is considered as an additional gaussian process introducing a certain SNR loss at the receiver. Clearly, this loss should be minimized. It should be maintained below a specified threshold T_s, thus $L_s \leq T_s$. The loss T_s of 0.3dB is considered to be acceptable. From a practical point of view it is much easier to use a quantity called normalized ISI level, denoted by L_{isi}, defined as $L_{isi} = -10\log F_2$[dB]. In practical filter design parameter L_{isi} is more convenient. For systems employing multilevel modulation schemes we assumed that the SNR is at least as high as 20dB, to meet the requirement (14), we require

$$L_{isi} \leq -30dB \tag{23}$$

G. Criterion III

Criterion III consists of minimization of the maximum smeared impulse noise variance. To be consistent with the already established design criteria [7], we introduce the merit factor defined as the ratio of the maximum impulse noise variance and the maximum smeared impulse noise variance in a single symbol interval

$$F_2 = \frac{\sigma_i^2(n)}{\sigma_s^2(n+j)} = \frac{1}{\max_j |d(j)|^2} \tag{24}$$

in dB $L_{F_2} = 10\,logF_2$ [dB]. Where σ_i^2 is the maximum impulse noise variance and $\sigma_s^2(N+j)$ is the maximum smeared impulse noise. In practical filter design we required that

$$L_{F_2} \leq T_{F_2} = 20[dB] \tag{25}$$

The impulse noise variance before spreading can be as large as the signal variance [11]. That is, occasional impulse hits can reach well above the signal level. Impulse noise spreading should reduce the impulse noise variance to the level of the AWGN variance. A further spreading is not effective [13] [14]. Since multilevel modulation systems shown in figure 5.18 in [15] and figure 9.17 in [16] required the minimum SNR of 20 dB. Clearly the merit factor F_2, as defined by Eq. (24), should be as large as possible. It shows how much the impulse noise variance in a single symbol interval has been reduced by smearing. A filter with a larger length can produce a larger merit factor F_2. It is convenient to introduce a measure for filter smearing efficiency which does not depend on filter length in [17], [18]. The power efficiency of a sequence **d**, denoted by η, is defined by

$$\eta = \frac{\sum_{j=0}^{N} |d_j|^2}{(N+1)\max_j |d_j|^2} \tag{26}$$

The power efficiency has its maximum value of 1, for constant amplitude sequences, while it is less than 1 for nonconstant amplitude sequences. Combining Eqs. (6), (24) and (26) we obtain the following expression for the merit factor F_2

$$F_2 = \frac{\eta(N+1)}{A_d} \tag{27}$$

In order to maximize the merit factor F_2, sequences should have the power efficiency as large as possible. For a finite desmear filter length *N*, subject to constraints (1) and (5), it has been shown in [7] that the maximum value for F_2 is bounded by *N* + 1, i.e. $F_2 \leq N+1$. The equality is satisfied if and only if

$$|d_j|^2 = \frac{1}{N+1} \quad and \quad s(j) = d^*(N-j) \tag{28}$$

That is, the optimum merit factor F_2 is achieved only when the smearing filters form a matched filter pair and both of them are represented by sequences with constant amplitude.

III. PRACTICAL FILTER DESIGN

The optimum values for the three filter design criteria cannot be achieved simultaneously. There are three approaches in practical filter design. In Design 1 we search for sequences with constant amplitude and good autocorrelation properties. The smear/desmear filter pair consists of matched filters. This design satisfies the requirements for optimum values of Criteria I and III, as defined by Equation. (28).The value of Criterion II clearly depends on the autocorrelation properties of the filter sequences.The constant amplitude sequences with the best known autocorrelation properties are polyphase sequences [14]. The smearing filter design based on these sequences achieves an improvement of 7.78 dB in merit factor F_2 relative to the design based on binary sequences for the sequence length of 200. This type of filter design based on polyphase sequences can produce filters with ISI level, L_{isi}, of -15 dB with sequence lengths of 200, while filters designed in [7] cannot achieve σ^2_{isi} less than -8 dB. Design 2 is suitable for systems where very low values for ISI level ($\leq -30dB$) are required. The improvement in suppresion of intersymbol interference is achieved by sequence equalization. That is, the smearing filter is designed as an equalizer and therefore the smear/desmear filter pair is not matched. While the smearing filters in this design will produce low ISI (Criterion II), the mismatched filters will result in an increased AWGN variance (Criterion I) and smeared impulse noise variance (Criterion III).

Therefore, the three design criteria should be monitored and adjusted simultaneously. It should be noted that the smear/desmear filters in this design might introduce a significant system delay of several sequence lengths. Design 3 relies on sequences with nonconstant amplitude. This design approach can achieve ISI as low as in Design 2, with an additional advantage of a lower system delay. In this method Criteria I and II are satisfied at the expense of the filter power efficiency η. Sequences with nonconstant amplitude will result in a non-optimum value for the impulse noise variance (Eq. (28)) for the given desmear filter length. A possible choice is Huffman sequences which have good autocorelation properties [17] or sequences presented in [19] which have both good autocorrelation properties and power efficiency.

A. Design 1

In Design 1 we focus on constant amplitude polyphase sequences with good autocorrelation properties. Polyphase sequences with constant amplitude, known as Frank and P1-P4 sequences [14], have better autocorrelation properties than M-sequences. It is important to note that polyphase sequences also are resilient to carrier phase and timing instabilities. In the sequel, we will discuss the properties of these sequences with respect to SDT applications.

1) Constant Amplitude Polyphase Sequences: : For a polyphase Frank sequence of length $N = L^2$, the phase of a sequence element is $\phi(k,l) = 2\pi/L(k-1)(l-1)$ and sequence elements are $d[k + L(l-1)] = \exp(j\phi(k,l))$, where $k = 1, \cdot\cdot\cdot, L, l = 1, \cdot\cdot\cdot, L$. The phases of P1 and P2 sequence elements are given by $\phi(k,l) = -\pi/L[L-(2k-1)][(k-1)L+(l-1)]$ and $\phi(k,l) = \pi/2L(L+1+2k)(l-1)$, respectively. It is important to observe that both P1 and P2 sequences are available only for square integer lengths, i.e N = ...36, 49, 64, ... P2 sequences are further restricted to even lengths only. Odd length P2 sequences possess rather bad autocorrelation properties . Sequences P3 and P4 are defined for any integer length. Phases of their elements are $\phi(k) = \pi/N(k-1)^2$ and $\phi(k) = \pi/4N(2k-1)^2 - \pi/4(2k-1) for 1 \leq k \leq N$. The most important property of constant amplitude polyphase sequences, relevant to SDT applications, is that the mainlobe to sidelobe power ratio is a monotonically increasing function of the sequence length. This property makes them much more effective in suppressing ISI then binary sequences [7], [8]. In addition, these sequences have constant amplitude and consequently, the optimum Criterion III (28). The method for generating binary sequences with high F_2 involves computer search and sequence elimination, which for large values of sequence lengths become prohibitively time consuming . On the other hand, polyphase sequences are generated analytically.

B. Design 2

A distinguishing property of this design method is a very low ISI level ($L_{isi} \leq -30dB$) achieved by sequence equalization. On the other hand, mismatched filters inevitably introduce a certain level of SNR loss [19]. This SNR loss can be maintained below a specified value by choosing a proper sequence for the desmearing filter. The sequence should have both, good autocorrelation properties measured by F_2 and good equalization properties. In the sequel a simple criterion to estimate the equalization properties of a sequence is discussed.

1) Zero Forcing Sequence Equalization : For a sequence $s_j, j = 0, 1, \cdots N$, we define zero forcing equalizer or inverse filter [4] as a digital filter with a Kronecker delta sequence response to the sequence **s**. The Z - transform of the zero forcing equalizer is given by

$$D(z) = \frac{1}{\sum_{j=0}^{N} s_j z^{-j}} \tag{29}$$

A necessary requirement for the filter existence is that the Z-transform, *S(z)*, does not have zeros on the unit circle [4]. Where *S(z)* is the Z-transform of the sequence s_j . If the sequence power is normalized to unity (Eq. (1)), the SNR loss of the inverse filter, denoted by L_{ZF} , is given by the ratio

$$L_{ZF} = 10 \log \frac{1}{\frac{1}{2\pi}\int_{-\pi}^{\pi} |D(e^{j\omega})|^2 d\omega} \tag{30}$$

The quantity $D(e^{j\omega})$ can be evaluated at a closely spaced set of points by the use of fast Fourier transform, and the integral can thus be calculated to a close approximation. The L_{ZF} loss shows the difference in SNR between a zero forcing equalizer and a matched filter. This quantity is an upper bound on the SNR loss in all other equalization methods [4]. Therefore, the L_{ZF} can be used in sequence search as an indicator of their performance with respect to the equalization SNR loss. It is worth noting that the equalization performance should be evaluated by both L_{ZF} and F_2 parameters, since a good merit factor F_2 does not guarantee a low L_{ZF} loss.

2) Sequences with Good Equalization Properties : Fig. 2 shows the merit factor F_2 binary sequences and the polyphase sequence for length 200 and 400 respectively . The main lobe to side lobe power ratio has a floor for binary sequences, while it increases monotonically with the sequence length for polyphase sequences. It has been observed that this ratio is proportional to the square root of the sequence length. Fig. 3 and 4 illustrate the smearing filter lengths needed to obtain a specified level of ISI suppression. The results are presented for short and long Frank and binary sequences of comparative lengths. As a rule of thumb, binary sequences require considerably larger filter lengths relative to corresponding Frank sequences. Both figures indicate that filter lengths of binary sequences are almost doubled relative to the Frank sequences. Fig. 3 shows the required filter lengths for the P2 (36) sequence. Although this sequence has very good autocorrelation properties (F_2 = 15.22) it requires very long filters, relative to the binary and Frank sequences, in order to achieve a low level of ISI suppression. This property of P2 sequences is a consequence of their infinite zero forcing equalization loss. Fig. 5 shows the L_{ZF} loss for binary sequences obtained by limited search [9] based on the merit factor F_2 only. The L_{ZF} loss of selected polyphase sequences are indicated in fig. 5 for comparison. It can be observed from Fig. 5 that many binary sequences (about 60%) have a small L_{ZF} loss. However, most sequences with lengths above 100 have an excessively high zero forcing equalization loss which makes them unsuitable for equalization. Finally, a simple example is singled out to illustrate that the merit factor F_2 and the L_{ZF} loss are not closely related and that both of them have to be used in evaluating sequence equalization properties. Polyphase sequence P2 (36) has the merit factor F_2 of 15.22 which is superior to the best known binary Barker(13) sequence with F_2 of 14.083. However, the L_{ZF} loss for P2 (36) is infinite, while Barker (13) sequence has the lowest known value for L_{ZF} of 0.21 dB.

3) Evaluation of Design 2 in Communication Systems : To evaluate the performance of filters obtained by Design 2, a number of sequences have been selected. They are used to design smearing mismatched filters in a real system where the receive filter operates as an MMS equalizer. The MMS equalization method has been chosen because it provides the best trade-off in reducing the effects of residual ISI and gaussian noise. The principle of MMS equalization can be summarized as follows. Let *u* filter pair defined by sequence $\mathbf{s} = (s(0), \cdots, s(K))$ at the transmitter and $\mathbf{d} = (d(0), \cdots, d(N))$ at the receiver. Note that the lengths of the smear and desmear filters are in general different due to various equalization requirements for the ISI level. If the output of the smearing filter is sequence **s**, the output **c** of the desmearing filter **d** can be expressed in matrix form as **c** = **A**.**s** where **A** is a $[(N+K+1) \times (K+1)]$ Toeplitz matrix defined by the first row $(d(0), 0_1, \cdots 0_K)$ and first column $(d(0), \cdots d(N), 0_{N+1}, \cdots, 0_{N+K})^T$. If a desired desmearing filter response to sequence **s** is a sequence **z** = *(z(0), z(1), ...z (N + K))* then the mean squared error between the actual filter response denoted by **c**, and the desired filter response denoted by **z**, is given by

$$\varepsilon = \sum_{j=0}^{N+K-1} |c(j) - z(j)|^2 \tag{31}$$

The filter sequence **s** which minimizes the mean squared error (31) is given by [3]

$$\mathbf{s} = (\mathbf{A}^H.\mathbf{A})^{-1}.\mathbf{A}^H.\mathbf{z} \tag{32}$$

where $()^H$ denotes a transposed and conjugated matrix. Matrix $\mathbf{A}^H.\mathbf{A}$ is a $(K+1) \times (K+1)$ correlation matrix of sequence **s**. Note that $\mathbf{A}^H.\mathbf{A}$ is a Toeplitz matrix whose inverse can be calculated by the Levinson-Durbin algorithm [3]. We define the desired desmearing filter response **z** as a sequence with no ISI, with elements $z(L)$ = 1, and L is the largest integer $\leq (N+K)/2$ and $z(j) = 0, j \neq L$. It is important to note that though for some sequences the zero forcing equalizer (Eq. (29)) might not exist, the minimum mean square approximation, defined by Eq. (32), always exists.

Note that the selected binary sequences were chosen to optimize the merit factor F_2 as proposed in [9]. In general, it has been observed that binary sequences have the SNR loss in MMS equalizers, L_m, close to the SNR loss for zero forcing equalizers, L_{ZF} .

C. Design 3

In system with sequence equalization the requirements for Criteria I and II, expressed by equation (17) and (23), respectively, cannot be met simultaneously due to prohibitively large filter lengths. To improve the filter performance for practical sequence lengths, we propose Design 3 based on equalization and non-constant amplitude sequences. In this design approach we slightly sacrifice the filter power efficiency, η, (Eq. 26) relative to its maximum value obtained in Design 2 in order to satisfy Criteria I and II. Huffman sequences [17] are nonconstant amplitude sequences with best known ISI and power efficiency properties. A thorough examination of results presented in [17] reveals that the power efficiency of Huffman sequences does not exceed 0.43. A drawback of Huffman sequences is that good power efficiency is not guaranteed and it usually ranges between 0.3 and 0.4 [18]. We propose to use nonconstant amplitude sequences,

Filter length N+1	ISI level L_{ISI}[dB]	SNR loss L_m[dB]	Power efficiency η
256	-30.18	0.0012	0.54
484	-30.54	0.001	0.56

TABLE I.
THE SMEARING FILTER PARAMETERS FOR THE UNCODED AND CODED SYSTEMS

Design type	Seq. length	ISI level	SNR loss	IN merit factor	System delay
	N+1	L_{ISI}[dB]	L_m[dB]	F[dB]	N+K+1
Design 2	195	-30.0	2.3	20.6	2500
	196	-30.0	0.3	22.6	1500
Design 3	256	-30.18	0.0012	21.4	511
	484	-30.54	0.001	24.3	967

TABLE II.
THE SMEARING FILTER DESIGN METHOD COMPARISON

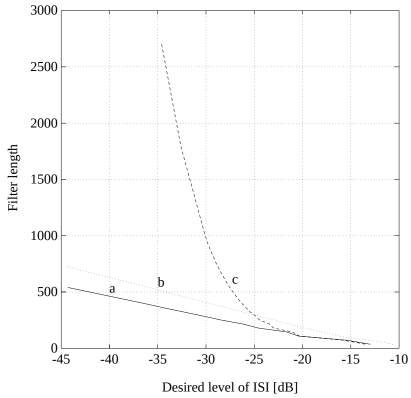

Figure 3. The smearing filter length, K + 1 for systems with short sequence equalization: a) Frank sequence of length 36, b) Binary sequence of length 33, c) P2 sequence of length 36.

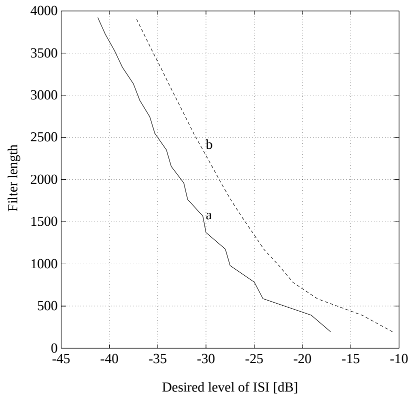

Figure 4. The smearing filter length, K + 1 for systems with long sequence equalization: a) Frank sequence of length 196, b) Binary sequence of length 195.

generated by a method presented in [20] [21] [22] [23] which are superior to Huffman sequences with regard to power efficiency.

The design method can be summarized as follows. Step 1 Choose a Frank sequence with good zero forcing equalization loss ($L_{ZF} \leq 1dB$) as an input **d** sequence in equation (32). Step 2 Calculate a filter sequence **s** by the MMS algorithm (Eq.(32)). It is assumed that the filter and input sequences have the same length. Step 3 Normalize the sequence **s** to satisfy $\mathbf{ss}^*$ = 1. Step 4 Compute the ISI level, L_{isi}, and SNR loss, L_m. Test whether they satisfy conditions (17) and (23). If the answer is positive, stop the procedure, otherwise use sequence s as the input vector and repeat Step 2. This method was used to generate a series of sequences that satisfy Criteria I-III. The sequence properties are illustrated in Fig. (6-8). The results are shown for two sequence lengths of 256 and 484. These lengths were selected to achieve the specified impulse noise suppression in (Eq. (25))for the uncoded 16QAM and coded 32AMPM for ITU.T V.150.1 data transmission systems.The results shows that increasing the number of iterations in the sequence design procedure generally results in a lower SNR loss (Fig.6) and ISI level (Fig.7). However, the power efficiency, η, is also reduced compared to the constant amplitude sequences, and its variations as a function of the number of iterations are shown in Fig. 8. Criteria I-III was met after 16 and 20 iterations for sequence lengths of 484 and 256, respectively.

The sequence parameters are listed in Table 1. Table 2 compares Design 2 and Design 3 techniques. The two design methods are compared on the basis of values for Criteria I-III. Clearly, design 3 offers lower values for the SNR loss and for the same values of ISI and smeared impulse noise variances introduces a lower system delay.

IV. SIMULATION RESULTS

The simulation results in are for coded and uncoded ITU-T V.150.1 data communication systems. Fig. [9-10] shows simulation results for 32-AMPM trellis coded modulation signals and 16 QAM for uncoded systems in the presence of impulse noise (IN). Parameters of IN are: $\lambda = 10^{-3}$ events/s, $SNR_{in} = 0$[dB]. The average time interval between two consecutive impulse noise is 0.0008s. In a communication system subject to impulse noise, it is highly likely that all affected symbols will be incorrect, resulting in error bursts with the bit error rates close to 0.5. Typical error rate curves exhibit an error floor which cannot be eliminated by increasing SNR. The results indicate that the SDT offers a significant reduction in the SNR required to achieve the same bit error rate as in a system with no SDT for both coded and uncoded systems. The coding gain of the coded system relative to the reference uncoded system is 2.5 dB at the BER at 10^{-5}, which is almost the same as the coding gain on gaussian channels. Also, the SDT completely removes the error floor in both systems. In the above example for the filter impulse response of length *N*=256 and the power efficiency η = 0.54, the theoretical SD gain is F=22 dB. In most cases a gain of this order is sufficient to suppress the influence of IN on the bit error rate. The real SD gain is reduced due to error clustering caused by impulse noise spreading.

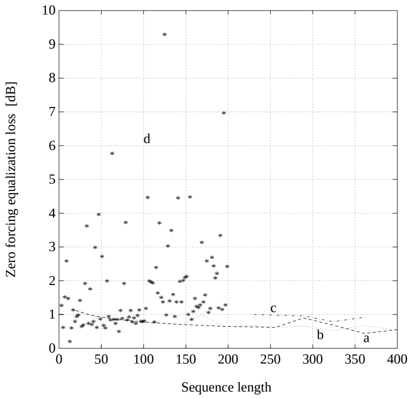

Figure 5. The zero forcing equalization loss for sequence with constant amplitude: a) Frank sequence, b) P1 sequence c) P3 and P4 sequence d) Binary sequence.

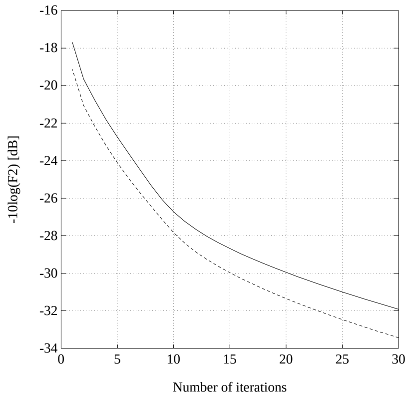

Figure 7. Desired level of residual ISI, L_{ISI} in terms of the number of iterations in design 3.Solid line: Sequence of length 256, Dash line: Sequence of length 484.

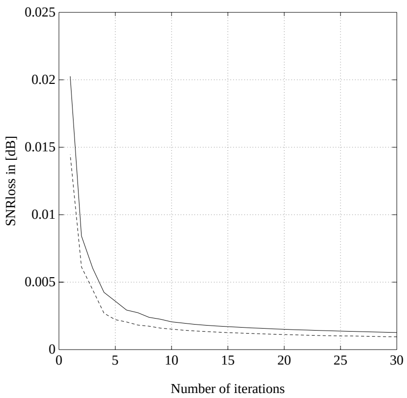

Figure 6. The SNR loss, Lm for the system employing sequences with nonconstant amplitude obtained by design 3. Solid line: Sequence series of length 256, Dash line: Sequence series of length 484.

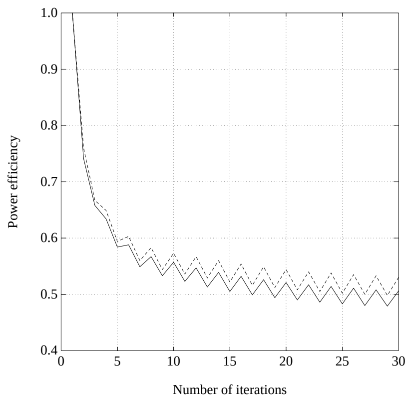

Figure 8. Sequence efficiency, η in design 3.Solid line: Sequence of length 256, Dash line: Sequence of length 484.

V. CONCLUSION

The paper presents a digital smear-desmear technique (SDT) applied to data transmission over band limited channels. A generalized set of filter design criteria based on minimizing the average bit error probability is introduced. The design criteria were applied to practical filter design and used in digital implementation of the SDT. Polyphase sequences that meet the design requirements were generated. The SDT is simulated and combined with uncoded and coded communication systems for high data transmission. Simulation results show that the SDT yields a significant improvement in bit error rates for both systems, subject to impulse noise, relative to the systems with no SDT and the systems with filters based on binary sequences of corresponding length. The technique also completely removes the error floor caused by impulse noise.

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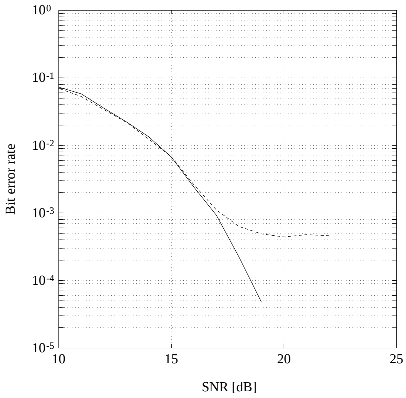

Figure 9. Bit error rate for uncoded system in the presence of impulse noise: Solid line: with SDT, Dash line: without SDT

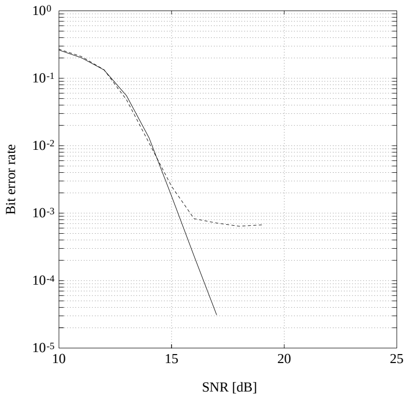

Figure 10. Bit error rate for coded system in the presence of impulse noise: Solid line: with SDT, Dash line: without SDT

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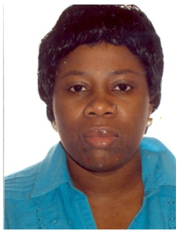

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