Abstract: In this work multiband orthogonal frequency division multiplexing (OFDM) System is modeled and its performance evaluation is analyzed for ultra wide band (UWB) communication environment. Theoretical derivation for the average pairwise error probability (PEP) and Outage Probability are derived based on Saleh and Valenzuela fading model. In line with this, the impact of joint coding the information across subcarriers in the system performance is investigated. The link budget analysis is also computed and results are obtained in regard to the transmission range for indoor environments. Moreover, average coded bit error rate achieved from convolutional codes over the system is computed and supported with computer simulations for all UWB channel conditions.

Index Terms—Multiband OFDM, Ultrawideband communications, Pairwise error probability, Outage probability, Coding Link budget, S-V channel model

I. INTRODUCTION

Ultra-wideband (UWB) is a high speed technology that has recently attracted considerable interest in research and standardization society, due to its capacity to provide high data rate at a low cost and low power consumption. It has been noted in many literatures that MB-OFDM is the promising candidate for physical layer of short range high data rate UWB Communications [1-3]. Multi band OFDM system allows the data signal to be processed over a much smaller bandwidth reducing overall design complexity as well as enhancing spectral flexibility and worldwide compliance compared to the single band approach [1].

The Analysis and design of a UWB communication system require an accurate channel model and this system is better characterized by Saleh Valenzuela channel model in terms of cluster, ray arrival and decay factors [4]. Uncoded multiband OFDM system cannot gain from the multipath clustering property of UWB channels. Hence to exploit the performance gain from such multipath clustering property captured by S-V model jointly encoding the data across subcarriers is necessary which is demonstrated in this work.

Although the basic principles dealt with in this work are well known in literature, it gives good insight about the behavior of MB-OFDM UWB and leads us to improve the reliability and range of coverage by extending the work to the recently emerging techniques like linear precoding and multiple-input-multiple output (MIMO) OFDM system. By doing so, we can enhance its competitiveness to other leading technologies like IEEE 802.11n standard. Moreover, it is shown that the convolutional coded scheme can achieve better performance in a fading environment. The analysis is made based on the assumption the channel state information is known perfectly at the receiver and there is perfect synchronization between transmitter and receiver.

The paper is organized as follows: In section II, the system model and its description is presented. In section III, theoretical pairwise error probability, outage probability and budget link analysis are derived. In section IV, simulation and analytical results are provided and commented. At last conclusion is drawn in section V.

Note that: all the bold faced variables are vectors.

II. SYSTEM MODEL AND DESCRIPTION

The MB-OFDM UWB system proposed by the IEEE 802.15.3a working group is implemented for a single antenna system with realistic channel estimation using Matlab and Simulink. The block diagram is shown below in Fig.1 followed by its description.
A. Channel Coding

Base code rate of 1/3 convolutional encoder with constraint length K=7 and generator polynomial of \( g_0 = [133] \), \( g_1 = [165] \), \( g_2 = [171] \) is used. To achieve high rate codes for 160Mb/s mode, puncturing rate of 1/2 is used with puncture vector \([1 0 1]\). Decoding is performed using the Viterbi algorithm with hard decision.

B. Interleaver

The bit interleaver used is to increase the robustness against burst errors. It is working in two stages in this model:

- Interleaving across OFDM symbol

\[
S(i) = U \left( \text{Floor} \left( \frac{i}{N_{\text{CBPS}}} \right) + 6\text{Mod} \left( i, N_{\text{CBPS}} \right) \right). \tag{1}
\]

- Tone interleaving across bits within an OFDM symbol

\[
T(i) = S \left( \text{Floor} \left( \frac{i}{N_{\text{Int}}} \right) + 10\text{Mod} \left( i, N_{\text{Int}} \right) \right). \tag{2}
\]

where \( U(i) \) is the input of the symbol interleaver, \( S(i) \) is the output of the symbol interleaver, \( T(i) \) is the output of the tone interleaver for \( i=0,1,\ldots,6 N_{\text{CBPS}}-1 \), \( \text{Floor}(\cdot) \) and \( \text{Mod}(\cdot) \) denote the floor and modulo functions respectively. \( N_{\text{CBPS}} \) is the number of bits per OFDM symbol. \( N_{\text{Int}} = N_{\text{CBPS}} / 10 \).

C. QPSK Mapper

The coded and interleaved bits are mapped to QPSK symbols with Gray labeling.

D. OFDM Modulator

In the transmitter, the data sequence is partitioned in to blocks as \( X = [x(0), x(1), \ldots, x(N-1)]^T \) and the block of data will be mapped to an Nx1 matrix. The QPSK symbols are spread in time by repeating the same information over two OFDM symbols in different sub-bands in order to get an added spreading gain. Each OFDM symbol has 100 data sub-carriers, 12 pilot sub-carriers, and 10 guard sub-carriers. Therefore, the number of total sub-carriers used is 122. Passing through the 128-point inverse fast Fourier transform (IFFT) operation, the time domain signals are generated. The duration for the OFDM symbol is \( T_c = 242.42\text{ ns} \) (128 subcarriers). After that , a cyclic prefix (CP) which is used to eliminate the Inter symbol Interference (ISI) is prepended to the OFDM symbol and the guard interval (GI) which ensures a smooth transition between two adjacent OFDM symbols is appended. Duration for CP is \( T_c = 60.61\text{ ns} \) (32 subcarriers) and duration for GI is \( T_c = 9.47\text{ ns} \) (5 subcarriers).

E. Frequency Hopping and Filtering

The radio frequency (RF) of transmitted OFDM signals are up sampled and filtered with low pass filter then hopped among the three 528 MHz frequency bands with center frequencies at 3.432, 3.960 or 4.488 GHz. The channelization is based on a set of time frequency code. Fig. 2 shows a time-frequency representation with time frequency code \{1 3 2 1 3 2\} for the lowest three bands.

F. Channel Model

In order to capture the behavior of UWB channels in terms of cluster and ray arrival rate, cluster and ray decay factors, the Saleh and Valenzuela model is adopted as...
stated earlier in part I[4]. The impulse response of the multipath channel is described as:

$$h(t) = X \sum_{c=1}^{C} \sum_{k=1}^{K} \alpha_{ck} \delta(t - T_c - \tau_{ck})$$  \hspace{1cm} (3)$$

Where $\alpha_{ck}$ is the gain of the $k$-th ray of the $c$-th cluster, $T_c$ is the time of arrival of the $c$-th cluster, and $\tau_{ck}$ is the delay of the $k$-th ray in the $c$-th cluster relative to the cluster arrival time and $X$ represents the lognormal shadowing. The distributions of the cluster and ray arrivals within each cluster are modeled by Poisson process.

The multipath gain coefficient $\alpha_{ck}$ is defined as

$$\alpha_{ck} = p_{ck} \rho_{c} \beta_{ck}.$$  \hspace{1cm} (4)$$

where $p_{ck}$ is a discrete random variable to model signal inversion caused by reflections which assumes values $\pm 1$ with equal probability, $\rho_{c}$ represents the fading related to the $c$-th cluster and $\beta_{ck}$ models the fading of the $k$-th ray of the $c$-th cluster. The small scale fading coefficient $\rho_{c} \beta_{ck}$ is modeled as random variable with lognormal distribution which is defined as

$$20 \log_{10} (\rho_{c} \beta_{ck}) \approx \text{Normal} \left( \mu_{c,k}, \sigma_{c,1}^2 + \sigma_{c,2}^2 \right).$$  \hspace{1cm} (5)$$

where $\sigma_{c,1}^2$ and $\sigma_{c,2}^2$ denotes the fading contributed by the cluster and the ray respectively. The power decay profile for different propagation paths can be described as

$$\sigma_{ck} = E \left[ \left| \alpha_{ck} \right|^2 \right] = E \left\{ \rho_{c}^2 \beta_{ck}^2 \right\} = \sigma_{0,0} \exp \left( -\frac{T_c}{\Gamma} - \frac{\tau_{ck}}{\gamma} \right).$$  \hspace{1cm} (6)$$

where $\sigma_{0,0}$ is the mean energy of the first path of the first cluster, $\Gamma$ is the cluster decay factor, and $\gamma$ is the ray decay factor and they are constants. From the above two expressions, we can obtain

$$10 \log_{10} \left( \sigma_{0,0} \right) - 10 \frac{T_c}{\Gamma} - 10 \frac{\tau_{ck}}{\gamma} = \left( \sigma_{c,1}^2 + \sigma_{c,2}^2 \right) \log_{10} e^{10}.$$  \hspace{1cm} (7)$$

For each realization, the total energy described by $\alpha_{ck}$ is normalized to unity

$$\sum_{c=0}^{C} \sum_{k=0}^{K} \alpha_{ck}^2 = 1.$$  \hspace{1cm} (8)$$

The large scale fading coefficient $X$ is modeled as a lognormal random variable

$$20 \log_{10} X \approx \text{Normal} \left( 0, \sigma^2 \right).$$  \hspace{1cm} (9)$$

The list of parameters for the four different environmental scenarios is provided in[4-5].

G. Channel Estimation and Compensation

The received signal after removing the cyclic prefix can be computed as

$$y(n) = x(n) H(n) + z(n).$$  \hspace{1cm} (10)$$

where $x(n)$ is the transmitted data symbol in the $n$th subcarrier, $z(n)$ is the additive noise component in the $n$th subcarrier, $H(n)$ is the channel response in the $n$th subcarrier and can be computed as:

$$H(n) = \sum_{i=0}^{P-1} h(i) \exp(-j 2 \pi n i / N),$$  \hspace{1cm} (11)$$

for $n = 0, 1, ..., N - 1$.

$x(n)$ can be easily estimated from

$$\hat{x}(n) = \frac{y(n)}{H(n)}, \quad n = 0, 1, ..., N - 1.$$  \hspace{1cm} (12)$$

The block type channel estimation with LS estimator is implemented in our model.LS estimators have low complexity even though they suffer from high mean square error [6].

III. PERFORMANCE EVALUATION

A. Average Pair wise Error Probability

Let the data matrix $X$ at the transmitter side is a concatenation of $P = [N/M]$ data matrices as:

$$X = \begin{bmatrix} X_0^T & X_1^T & \ldots & X_{(P-1)}^T \end{bmatrix}$$

where $X_p$ is an $M \times 1$ data matrix defined as

$$X_p = [x_p(0) \ x_p(1) \ldots x_p(M-1)]^T$$

with $x_p(n) \triangleq x(pM + n)$; $p = 0, 1, \ldots P-1$. $0_{n \times n}$ stands for an $m \times n$ all-zero matrix. The data matrices $X_p$ are designed independently for different $p$’s. The mean square error is defined as:

$$\eta = \left\| A_p H_p \right\|^2 = \left( \Lambda_p H_p \right)^\dagger \left( \Lambda_p H_p \right).$$  \hspace{1cm} (13)$$

which is in quadratic form as in[7] and $E[ \Lambda_p H_p^\dagger ] = 0$. It is chi squared distributed.

$A_p = f(X_p) - f(\hat{X}_p)$ is a signal difference matrix in which $f(X_p)$ is a diagonal matrix and $H_p$ is the channel matrix. Note that $(\cdot)^\dagger$ represents a conjugate transpose operation.
\[ Y_p = f(X_p)H_p + Z_p. \] (14)

Using the definition of quadratic forms in [7], we can approximate variable \( \eta \) as

\[ \eta \approx \sum_{n=1}^{M} \varepsilon_n(\Phi) \| \varphi_n \|^2. \] (15)

\[ \sum_{n=1}^{M} \| \varphi_n \|^2 \] is chi square distributed with the probability density function given by

\[ f_{\chi^2}(x) = \frac{1}{(K-1)!} x^{K-1} e^{-x} \]

where \( K \) is the number of terms in the sum, i.e., the chi square distribution has \( 2K \) degrees of freedom, \( \varphi_n \) are iid complex standard Gaussian random variables, (22) can be written as

\[ \Phi = \left[ \begin{array}{c} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_M \end{array} \right] \]

\[ = \left[ \begin{array}{c} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_M \end{array} \right]^{\dagger} \left[ \begin{array}{c} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_M \end{array} \right] \]

where the \( M \times M \) correlation matrix which is given by

\[ R_M = \begin{pmatrix} 1 & R(1) & \cdots & R(M-1) \\ R(1) & 1 & \cdots & R(M-2) \\ \vdots & \vdots & \ddots & \vdots \\ R(M-1) & R(M-2) & \cdots & 1 \end{pmatrix}_{M \times M} \]

The diagonal elements of the autocorrelation matrix can be calculated as:

\[ R(n,n) = E[|H(n)|^2] = \sum_{c=0}^{C} \sum_{k=0}^{K} E[|\alpha_{ck}|^2] = 1. \] (18)

and the off-diagonal elements for \( n \neq n' \)

\[ R(n,n') = E[|H(n)H(n')^\dagger|] = \sum_{c=0}^{C} \sum_{k=0}^{K} E[|\alpha_{ck}|^2] e^{-j2\pi(n-n')\Delta f(T_c+\tau_c)} \] (19)

\[ \cong R(n-n') \]

Letting \( n-n' = m \) Combining (6) and (19), we can obtain

\[ R(m) = \sum_{c=0}^{C} \sum_{k=0}^{K} E[|\alpha_{ck}|^2] e^{-j2\pi m\Delta f(T_c+\tau_c)} \]

\[ = \sum_{c=0}^{C} \sum_{k=0}^{K} \sigma_{0,0}^2 E[e^{-j\frac{\pi}{4}2\pi m\Delta f}] = \sum_{c=0}^{C} \sum_{k=0}^{K} \sigma_{0,0}^2 D_{ck}(m) \] (20)

denoting the inter-arrival time between clusters j and j-1 as \( b_j \) and inter-arrival time between rays i and i-1 as \( g_{c,i} \). As mentioned earlier in part II, the clusters and ray arrivals within each cluster in S-V channel are modeled by Poisson processes. The cluster delay \( b_j \) and ray delay \( g_{c,i} \) can be modeled as IID exponential random variables with parameter \( \Lambda \) and \( \lambda \) respectively. The c-th cluster delay can be expressed as \( T_c = \sum_{j=0}^{b_j} b_j \) and similarly the k-th delay within cluster C can be specified as \( \tau_{ck} = \sum_{i=0}^{g_{c,i}} g_{c,i} \). From (20), the expression for \( D_{ck}(m) \) can be written as

\[ D_{ck}(m) = \exp \left( -T_c \left[ \frac{1}{\gamma} + j2\pi m\Delta f \right] \right) \exp \left( -\tau_{ck} \left[ \frac{1}{\gamma} + j2\pi m\Delta f \right] \right) \] (21)

Since the rays and clusters are statistically independent, (21) can be written as

\[ D_{ck}(m) = E \left[ \prod_{j=0}^{c} e^{-b_j \left[ \frac{1}{\gamma} + j2\pi m\Delta f \right]} \right] \times E \left[ \prod_{i=0}^{k} e^{-g_{c,i} \left[ \frac{1}{\gamma} + j2\pi m\Delta f \right]} \right] \] (22)

Since \( b_j \) and \( g_{c,i} \) are iid random variables, (22) can be computed as
\[ D_{\alpha}(m) = \left( E \left[ \exp(-b \left( \frac{1}{\Gamma} + j2\pi m\Delta f \right)) \right] \right)^k \]
\[ \times \left( E \left[ \exp(-g \left( \frac{1}{\gamma} + j2\pi m\Delta f \right)) \right] \right)^k \]
\[ = \left( \int_0^\infty e^{ab} \lambda e^{-\lambda b} \left( \frac{\lambda b}{(n)!} \right) db \right)^c \] . \quad (23)
\[ \times \left( \int_0^\infty e^{bg} \lambda e^{-\lambda g} \left( \frac{\lambda g}{(n)!} \right) dg \right)^k \]
\[ = \left( \frac{\Lambda}{\Lambda - \alpha} \right)^c \left( \frac{\lambda}{\lambda - \beta} \right)^k \]

where
\[ \alpha = -\left( \frac{1}{\Gamma} + j2\pi m\Delta f \right) \]
\[ \beta = -\left( \frac{1}{\gamma} + j2\pi m\Delta f \right) \]
and \( n \) stands for number of cluster or ray arrivals. (20) can be written as:
\[ R(m) = \sigma_{0,0} \sum_{c=0}^{C} \left( \frac{\Lambda}{\Lambda + \frac{1}{\Gamma} + j2\pi m\Delta f} \right)^c \]
\[ \times \sum_{k=0}^{K} \left( \frac{\lambda}{\lambda + \frac{1}{\gamma} + j2\pi m\Delta f} \right)^k . \quad (24) \]

Using the knowledge power series convergence, for large number of clusters
\[ \lim_{c \to \infty} \left( \frac{\Lambda}{\Lambda + \frac{1}{\Gamma} + j2\pi m\Delta f} \right)^c \]
\[ = \frac{1}{1 - \frac{\Lambda}{\Lambda + \frac{1}{\Gamma} + j2\pi m\Delta f}} . \quad (25) \]

for \( 0 < \frac{\Lambda}{\Lambda + \frac{1}{\Gamma} + j2\pi m\Delta f} < 1 \)
\[ = \frac{\Lambda + \frac{1}{\Gamma} + j2\pi m\Delta f}{\frac{1}{\Gamma} + j2\pi m\Delta f} \]

By Ostrowski’s theorem[7] the eigen values of matrix \( \Phi \) in (16) can be given by
\[ \varepsilon_n(\Phi) = \xi_n \varepsilon_n(\mathbf{R}_M) \quad n = 1, 2, \ldots, M . \quad (27) \]
where \( \xi \) is a nonnegative real number that satisfies:
\[ \varepsilon_M(\Lambda_\rho \Lambda_p^\dagger) \leq \xi_n \leq \varepsilon_1(\Lambda_\rho \Lambda_p^\dagger) \]

for \( n = 1, 2, \ldots, M \). \quad (28)

By definition, the moment generating function (MGF) is
\[ \mu_n(t) = E \left[ e^{t \varepsilon_n(\Phi)} \right] . \quad (29) \]

Combining (29) and (15), we can obtain
\[ \mu_n(t) = E \left[ \exp \left( t \sum_{n=1}^{M} \varepsilon_n(\Phi) \phi_n^\dagger \right) \right] . \quad (30) \]

and computed as \( \mu_n(s) = \int_0^\infty f_X(x)e^{sx}dx \) where
\( f_X(x) \) is the probability density function of random variable \( X \). After some algebraic operation on (30) we shall obtain the result as
\[ \mu_n(s) = \prod_{n=1}^{M} (1 - s\varepsilon_n(\Phi))^{-1} . \quad (31) \]

Using (31) and (27) we can obtain expression:
\[ \mu_n(s) = \prod_{n=1}^{M} (1 - s\xi_n \varepsilon_n(\mathbf{R}_M))^{-1} . \quad (32) \]

As in [9], we have the average PEP defined
\[ P_e = E \left[ Q \left( \sqrt{\delta/2} \left\| \mathbf{H}_p \right\| \right) \right] . \quad (33) \]

Applying the Gaussian error function to (33) and combining it with (32) we can obtain the average PEP of the system as:
\[ P_e \approx \frac{1}{\pi} \int_{0}^{\pi/2} \prod_{n=1}^{M} \left( 1 + \frac{\delta \varepsilon_n}{4\sin^2 \theta} \varepsilon_n(\mathbf{R}_M) \right)^{-1} d\theta . \quad (34) \]

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For case of no jointly coding over subcarriers (that is for M=1), (34) can be simplified to

$$P_e = \frac{1}{2} \left(1 - \frac{1}{\sqrt{1 + \frac{1}{\delta}}} \right).$$  \hspace{0.5cm} (35)$$

which is a similar result to narrowband communications under Rayleigh fading. In the case of jointly coding over subcarriers, for M=2 and assuming the information is repeated over two OFDM symbols, (34) can be approximated as

$$P_e \approx \frac{1}{\pi} \left(\frac{8\pi}{6k + 48} - \frac{\pi}{2\sqrt{k}} \left(\frac{32 + 5k}{24 + 3k} - \sqrt{\frac{k}{k + j_4} - \sqrt{k - j_4}}\right)\right).$$ \hspace{0.5cm} (36)$$

where

$$k \approx \begin{cases} 0.0924\delta^2 \zeta^2 & \text{for CM1} \\ 0.3026\delta^2 \zeta^2 & \text{for CM4} \end{cases}$$

and $\zeta = |x - \bar{x}|^2$ and $\delta = E_s/N_o$ is signal to noise ratio. (35) and (36) are used to trace the theoretical error probability curve for MB-OFDM UWB system for all UWB channel models. Assuming the channel-state information (CSI) is known perfectly at the receiver, the receiver performs maximum likelihood decoding by choosing the codeword $X_p$ that minimizes the square Euclidean distance between the hypothesized and actual received signal matrices, that is,

$$X_p = \arg \min_{X_p} \|Y_h - f(X_p)H_p\|^2.$$ \hspace{0.5cm} For M=2,

Eigen values of the autocorrelation matrix are $1 + |R(1)|$ and $1 - |R(1)|$, where the correlation matrix element is computed as

$$[R(1)] = \frac{\Lambda + \frac{1}{\Gamma}}{\sqrt{\left(\frac{1}{\Gamma}\right)^2 + (2\pi\Delta f)^2}} + \frac{(2\pi\Delta f)^2}{\sqrt{\left(\frac{\lambda + \frac{1}{\gamma}}{\Gamma}\right)^2 + (2\pi\Delta f)^2}} + \frac{(2\pi\Delta f)^2}{\sqrt{\left(\frac{1}{\gamma}\right)^2 + (2\pi\Delta f)^2}}.$$  \hspace{0.5cm} (37)$$

B. Average Coded bit error probability and Link budget Analysis

The average coded bit error probability is defined as the bit error rate (BER) performance after the convolutional decoder. The convolutional encoder with CC(3,1,7) is used for the data rate of 160Mbps. Its free distance, $d_{\text{free}}$ is 15 and the sum of the hamming weight of all input sequences $N_b$ is 7. Assuming each coded bit has an equal probability of being wrongly decoded, the average coded bit error probability is bounded by:

$$P_{bc} \leq \frac{\partial T(W, I)}{\partial I} \left[1 - W = 1 \text{ for } k = 1 \text{ to } 160 \text{ } \text{for } W ight].$$ \hspace{0.5cm} (38)$$

where $T(W, I)$ is the transfer function of the convolutional encoder[9]. $P_{uc}$ denotes uncoded bit error probability. (38) can be approximated to:

$$P_{bc} \approx N_b \left[4P_{uc}(1 - P_{uc})\right]^{d_{\text{free}}/2} \approx N_b \left(1 + \frac{E_u}{N_0}\right)^{-d_{\text{free}}/2}.$$ \hspace{0.5cm} (39)$$

Where $E_u/N_0$ is signal to noise ratio in terms of average energy per bit and average white noise power. (39) can be used to trace the theoretical bit error rate for coded MB-OFDM UWB system. Since the S-V channel model does not take into account the issue of link budget, in real situation it needs to be considered. Link budget is a consideration of all the gains and losses in a transmission system. The link budget will take the form of the equation:

$$\text{Received power(dBm)} = \text{Transmitted Power(dBm)} + \text{Gains(dB)} + \text{Losses(dB)}.$$ \hspace{0.5cm} (40)$$

The signal to noise ratio at the receiver side can be computed as

$$\frac{E_u}{N_0} = P_{uc} - 20\log_{10} \left(\frac{4\pi f_c d}{c}\right) - (k_b Temp + 10\log_{10} (R_b)) - Q \text{ in dB}.$$ \hspace{0.5cm} (41)$$

Based on (11) we can derive result for $E \left[|H(n)|^2\right]$.
\[ H(n) = \sum_{i=0}^{N-1} h(i) \exp(-j2\pi ni/N) \]
\[ = \sum_{i=-\infty}^{\infty} h(i) \exp(-j2\pi ni/N) \]
\[ = \sum_{i=-\infty}^{\infty} \sum_{c=0}^{C} \sum_{k=0}^{K} X_{\alpha_c} p(iT - T_c - \tau_{ck}) \]
\[ \times \exp(-j2\pi \left( \frac{ni}{N} + f_{center}(T_c + \tau_{ck}) \right)) \]

Assuming the length of the cyclic prefix is longer than the channel impulse response, that is \( h(i) = 0 \) if \( i \neq 0, 1, \ldots, P - 1 \). Since

\[ E\{\alpha_{\alpha_c}\} = 0 \Rightarrow E\{H(n)\} = 0 \]

Let \( p(t) \) is the pulse shape used for transmission and supposed to have a bandwidth of \( 1/2T \) and satisfies the Nyquist criterion. Its Fourier transform is given by

\[ P(f) = \begin{cases} T \exp(-j2\pi k_0 T), & \frac{-1}{2T} \leq f \leq \frac{1}{2T} \\ 0, & \text{otherwise} \end{cases} \]

For some integer \( k_0 \), using sampling theorem, we can write

\[ \sum_{i=-\infty}^{\infty} p(iT - T_c - \tau_{ck}) e^{-j2\pi ni/N} \]
\[ = \begin{cases} e^{-j2\pi k_0 (T_c + \tau_{ck})/NT}, & 0 \leq n \leq \frac{N}{2} - 1 \\ e^{-j2\pi (n - N)(k_0 T_c + \tau_{ck})/NT}, & \frac{N}{2} \leq n \leq N - 1 \end{cases} \]

Let

\[ n = \begin{cases} n, & 0 \leq n \leq \frac{N}{2} - 1 \\ n - N, & \frac{N}{2} \leq n \leq N - 1 \end{cases} \]

Then (44) can be expressed as

\[ \sum_{i=-\infty}^{\infty} p(iT - T_c - \tau_{ck}) \exp(-j2\pi ni/N) \]
\[ = \exp(-j2\pi \tilde{n}(k_0 T_c + T_c + \tau_{ck})/NT) \]

\[ E\{[H(n)]^2\} \]
\[ = \left\{ \sum_{i=-\infty}^{\infty} \sum_{c=0}^{C} \sum_{k=0}^{K} X_{\alpha_c} p(iT - T_c - \tau_{ck}) \right\} \]
\[ \times e^{-j2\pi \left( \frac{ni}{N} + f_{center}(T_c + \tau_{ck}) \right)} \]
\[ = E\{X^2\} E\left\{ \sum_{c=0}^{C} \sum_{k=0}^{K} \left[ \alpha_c e^{-j2\pi f_{center}(T_c + \tau_{ck})} \right] \right\} \]
\[ \times e^{-j2\pi \left( \frac{ni}{N} + f_{center}(T_c + \tau_{ck}) \right)} \]
\[ = E\{X^2\} \times E\left\{ \sum_{c=0}^{C} \sum_{k=0}^{K} \alpha_c \right\} = E\{X^2\} \times 1 \]
\[ = \left\{ \int_{-\infty}^{\infty} \left(\frac{X}{\sigma_x^2}\right)^2 \exp(-X^2/2\sigma^2) dx \right\} \]
\[ \times \left\{ \frac{1}{\sqrt{2\pi \sigma_x^2}} \right\} \]
\[ = \frac{1}{\sqrt{2\pi \sigma_x^2}} \int_{-\infty}^{\infty} \exp\left(-\frac{X^2}{2\sigma_x^2}\right) dx \]
\[ = e^{-\sigma_x^2/2} = e^{0.0265\sigma_x^2} = 1.2693 \]

For all channel model parameters.

Now (41) can be interpreted as : \( 10 \log_{10} \left\{ E\{[H(n)]^2\} \right\} \) which is fading gain captured by S-V channel model. It should be noted that \( k_B \) is Boltzman’s constant in (J/k), \( Temp \) is temperature in Kelvin, \( R_b \) is the bit rate in bps, \( c \) and \( d \) are the speed of light and distance of the transmission respectively, \( f_g \) is the geometric average of the upper frequency, \( f_{\text{max}} \) and the lower frequency, \( f_{\text{min}} \) in MHz of the transmission spectrum. \( Q \) represents the noise figure of the antenna which is 6.6dB and implementation loss 2.5dB[10]. It should be noted that the mathematical expressions \( k_B Temp + 10 \log_{10}(R_b) \) is average noise power per bit and \( P_{\text{tx}} \leq -41.25 + 10 \log_{10}(f_{\text{max}} - f_{\text{min}}) \) is the average transmitted power in dBm.
C. Outage Probability

The outage probability is defined as the probability that the combined SNR, \( \nu \), value gets below a specified threshold, \( \nu_o \):

\[
P_{\text{out}} = P \left( \nu \leq \nu_o \right) = \int_0^{\nu_o} p_{\nu} (x) dx. \tag{47}
\]

where \( p_{\nu} (x) \) represents the PDF of \( \nu \). The combined SNR for the jointly encoded each data matrix \( X_n \) is defined as:

\[
\nu = \frac{E_s \left[ X_n H_p \right]^2}{E \left[ Z_p \right]^2} = \frac{\delta}{M} \nu. \tag{48}
\]

where \( \delta \) is SNR defined earlier in (36) and

\[
\nu = \sum_{n=0}^{M-1} |H_p(n)|^2.
\]

The MGF of \( \nu \) can be obtained as:

\[
\mathcal{M}_\nu (s) = \prod_{n=1}^{M} \frac{1}{1 - s \mathcal{E}_n \left( R_M \right)}.
\]

The probability density function (PDF) of \( \nu \) can be computed by finding the inverse Laplace transform of (49)

\[
p_\nu (x) = \mathcal{L}^{-1} \left\{ \mathcal{M}_\nu (s) \right\} = \mathcal{L}^{-1} \left\{ \sum_{n=1}^{M} \frac{B_n}{1 - s \mathcal{E}_n \left( R_M \right)} \right\} = \sum_{n=1}^{M} B_n \mathcal{L}^{-1} \left\{ \frac{1}{1 - s \mathcal{E}_n \left( R_M \right)} \right\} \tag{50}
\]

Using (47) and (50), we can obtain the outage probability relation for a multiband OFDM system for any UWB channel model parameters as:

\[
P_{\text{out}} = P \left( \nu \leq \frac{M \nu_o}{\delta} \right) = \int_0^{M \nu_o/\delta} \left( \sum_{n=1}^{M} B_n \mathcal{E}_n \left( R_M \right) e^{-x M/\delta \mathcal{E}_n \left( R_M \right)} \right) \tag{51}
\]

where, \( B_n = \prod_{n=1}^{M} \frac{\mathcal{E}_n \left( R_M \right)}{\mathcal{E}_n \left( R_M \right)} \) that can be obtained from techniques of partial fractions. Example for CM1 and M=2, \( B_1 \) and \( B_2 \) can be computed as 1.0075 and -0.0075 respectively and the outage probability can be expressed as

\[
P_{\text{out}} = 1 - 0.5 \left( 1 + |R|^{-1} \right) e^{-2 \mathcal{E}_n \left( R_M \right)/\delta} \tag{52}
\]

This is used to trace the theoretical outage probability

IV. NUMERICAL AND SIMULATION RESULTS

Fig.3a shows the bit error rate performance of 160Mbps is 1.5dB lower than that of 200Mbps for the same bit error rate of \( 10^{-5} \). This is because in MB-OFDM system with the lowest data rate has the lowest Bit Error rate, since it uses the lowest coding rate. Fig.3b shows the merit obtained from frequency hopping. Frequency hopping from one band to another using time frequency code offers a system gain of about 3.5dB at bit error rate of \( 10^{-1} \) for the data rate of 160 Mbps. This gain is due to the frequency diversity achieved by use of a three times larger frequency band (diversity gain). From Fig.3c we can observe that the theoretical BER performance and simulated result for an uncoded system is the same for all UWB channels. However the simulation over the AWGN channel for an uncoded scenario shows better performance due to the absence of multipath fading. The performance over all channels for coded case is very satisfactory. As in [11] result for CM2 and CM3 above 8dB and CM4 and CM1 above 9dB with convolutional coding scheme shows acceptable performance.
Figure 3 Simulation result for probability of error versus signal to noise ratio a) Simulation result for 200 and 160 Mbps speed mode b) effect of frequency hopping on performance c) performance comparison of all channel models

Fig. 4 shows the performance of the MB-OFDM UWB system in terms of signal to noise ratio, Bit error rate versus transmission distance when Average Tx power = -9dBm, $f_{\text{max}}$=4.8GHz, $f_{\text{min}}$=3.1GHz and $R_b=160$Mbps. The bit error rate performance for the uncoded system which is shown in Fig.4b is not satisfactory even for shorter distances. But the performance is improved when coding is introduced in the system as seen in Fig.4c. For instance the performance at 11.8 meters which corresponds to a bit error rate of $10^{-6}$ is satisfactory [11]. But for distance more than 11.8 meters, the performance deteriorates and this makes the MB-OFDM UWB system limited to short distances.

From Fig.5 we observe that for no coding (M=1) across subcarriers the theoretical performance of UWB system are similar for all UWB channel models and there is no performance gain from the multipath clustering property of UWB channels. Fig.5a shows that the performance of the MB-OFDM system with information jointly encoded across two subcarriers (M=2) is better than the system with no coding over subcarriers. The performance obtained under CM4 is superior to that of CM1 because CM4 has less correlation in the frequency response among different subcarriers. The theoretical outage probability performance shown in Fig.5b follows the same behavior as the average symbol error rate (PEP).
In this paper, we have modeled, described the MB-OFDM UWB system and provided numerical derivations about its performance in accordance with the S-V channel model. Moreover, the behavior of the system is examined in terms of transmission distance capability and its performance improvement when the data is jointly coded across subcarriers that are without using any channel coding schemes. The diversity gain of CM4 is better than CM1 when there is jointly encoding the information across subcarriers and this is due to the presence of less correlation in the frequency response among different subcarriers. It should be noted that the diversity gain can be improved not only by increasing the number of jointly encoded subcarriers but also increasing the number of jointly encoded OFDM symbols, or the number of antennas. Nevertheless, increasing the number of jointly encoded subcarriers leads to the loss in coding gain as in [1]. We have observed that MB-OFDM UWB system gains much from channel coding schemes and jointly encoding the data over subcarriers. However, its performance can be enhanced further using MIMO techniques, Linear precoding and Dynamic Resource allocation. Hence more research and improvements on the existing UWB systems are required to increase the system range, transmission rate and robustness. For instance combining MIMO with Linear precoding[12] can make the MB-OFDM system more robust and can provide up to a data rate of 1 Gb/s with a reasonable additional system complexity [13].

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Yilma T. Desta received B.Sc degree in Electricity/Electronics Technology in 1999 from Adama University, Ethiopia and MEng in Telecommunication Engineering from Dublin City University, Dublin in 2002 and is currently working toward the Ph.D degree in Communication Engineering at Harbin Engineering University, China. His research interest include OFDM Channel Estimation and Multiple input and Multiple Output(MIMO) for wireless communication systems.

Jiang Tao received B.Sc degree in Electrical Engineering in 1994, his M.Sc degree in information and signal processing in 1999 and Ph.D degree in communication and information systems in 2002 from Harbin Engineering University, China. He worked at Harbin Institute of Technology, China as a post-doctoral fellow in 2003 and worked also at the National University of Singapore as a research fellow in 2004. Currently he is a professor in Harbin Engineering University, China. His research interests are UWB systems, computational electromagnetics, microwave engineering, radio wave propagation, navigation, EMC and other applications for wireless communication.