Multi Band OFDM System Model and Its Performance Evaluation for UWB Communication

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***Abstract*: In this work multiband orthogonal frequency division multiplexing (OFDM) System is modeled and its performance evaluation is analyzed for ultra wide band (UWB) communication environment. Theoretical derivation for the average pairwise error probability (PEP) and Outage Probability are derived based on Saleh and Valenzuela fading model. In line with this, the impact of joint coding the information across subcarriers in the system performance is investigated. The link budget analysis is also computed and results are obtained in regard to the transmission range for indoor environments. Moreover, average coded bit error rate achieved from convolutional codes over the system is computed and supported with computer simulations for all UWB channel conditions.**

***Index Terms*—Multiband OFDM, Ultrawideband communications, Pairwise error probability, Outage probability, Coding Link budget, S-V channel model**

I. Introduction

Ultra-wideband (UWB) is a high speed technology that has recently attracted considerable interest in research and standardization society, due to its capacity to provide high data rate at a low cost and low power consumption. It has been noted in many literatures that MB-OFDM is the promising candidate for physical layer of short range high data rate UWB Communications [1-3].Multi band OFDM system allows the data signal to be processed over a much smaller bandwidth reducing overall design complexity as well as enhancing spectral flexibility and worldwide compliance compared to the single band approach [1].

The Analysis and design of a UWB communication system require an accurate channel model and this system is better characterized by Saleh Valenzuela channel model in terms of cluster, ray arrival and decay factors [4].Uncoded multiband OFDM system cannot gain from the multipath clustering property of UWB channels. Hence to exploit the performance gain from such multipath clustering property captured by S-V model jointly encoding the data across subcarriers is necessary which is demonstrated in this work.

Although the basic principles dealt with in this work are well known in literature, it gives good insight about the behavior of MB-OFDM UWB and leads us to improve the reliability and range of coverage by extending the work to the recently emerging techniques like linear precoding and multiple-input-multiple output (MIMO) OFDM system. By doing so, we can enhance its competitiveness to other leading technologies like IEEE 802.11n standard. Moreover, it is shown that the convolutionaly coded scheme can achieve better performance in a fading environment. The analysis is made based on the assumption the channel state information is known perfectly at the receiver and there is perfect synchronization between transmitter and receiver.

The paper is organized as follows: In section II, the system model and its description is presented. In section III, theoretical pairwise error probability, outage probability and budget link analysis are derived. In section IV, simulation and analytical results are provided and commented. At last conclusion is drawn in section V.

Note that: all the bold faced variables are vectors.

II. System model and description

The MB-OFDM UWB system proposed by the IEEE 802.15.3a working group is implemented for a single antenna system with realistic channel estimation using Matlab and Simulink. The block diagram is shown below in Fig.1 followed by its description.

Manuscript received July 30, 2012; accepted October 4,2012.

doi: 10.12720/jcm.8.2.91-100

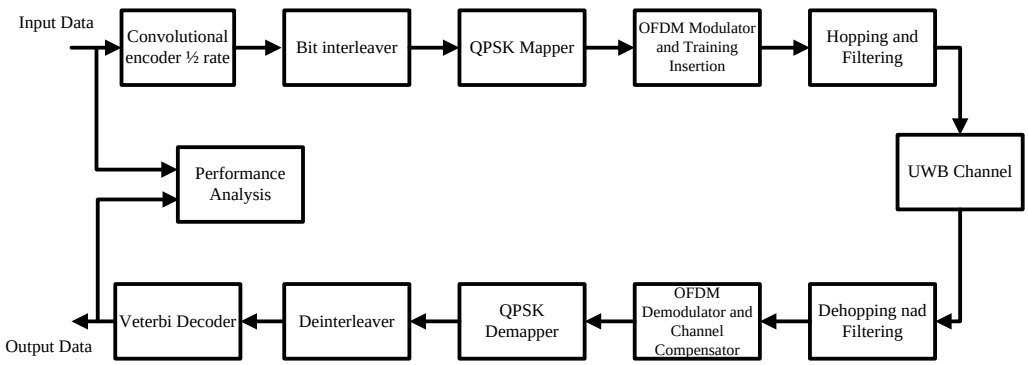


Figure 1. Base band equivalent MB-OFDM UWB system model for 160Mb/s

A. Channel Coding

Base code rate of 1/3 convolutional encoder with constraint length K=7 and generator polynomial of $g_0 = [133_8], g_1 = [165_8], g_2 = [171_8]$ is used. To achieve high rate codes for 160Mb/s mode, puncturing rate of ½ is used with puncture vector [1 0 1].Decoding is performed using the Viterbi algorithm with hard decision

B. Interleaver

The bit interleaver used is to increase the robustness against burst errors. It is working in two stages in this model

- Interleaving across OFDM symbol

$$S(i) = U\left\{ Floor\left(\frac{i}{N_{CBPS}}\right) + 6Mod\left(i, N_{CBPS}\right)\right\}. \quad (1)$$

- Tone interleaving across bits within an OFDM symbol

$$T(i) = S\left\{ Floor\left(\frac{i}{N_{T\text{int}}}\right) + 10Mod\left(i, N_{T\text{int}}\right)\right\}. \quad (2)$$

where $U(i)$ is the input of the symbol interleaver, $S(i)$ is the output of the symbol interleaver , $T(i)$ is the output of the tone interleaver for i=0,1……6 N_{CBPS} -1,Floor(.) and Mod(.) denote the floor and modulo functions respectively. N_{CBPS} is the number of bits per OFDM symbol. $N_{T\text{int}} = N_{CBPS}/10$.

C. QPSK Mapper

The coded and interleaved bits are mapped to QPSK symbols with Gray labeling.

D. OFDM Modulator

In the transmitter, the data sequence is partitioned in to blocks as $\mathbf{X} = [x(0)\, x(1) \ldots x(\mathrm{N}-1)]^{\mathbf{T}}$ and the block of data will be mapped to an Nx1 matrix. The QPSK symbols are spread in time by repeating the same information over two OFDM symbols in different sub bands in order to get an added spreading gain. Each OFDM symbol has 100 data sub-carriers, 12 pilot sub-carriers, and 10 guard sub-carriers. Therefore, the number of total sub-carriers used is 122. Passing through the 128-point inverse fast Fourier transform (IFFT) operation, the time domain signals are generated. The duration for the OFDM symbol is T_s=242.42ns(128 subcarriers).After that , a cyclic prefix(CP) which is used to eliminate the Inter symbol Interference(ISI) is preappended to the OFDM symbol and the guard interval(GI) which ensures a smooth transition between two adjacent OFDM symbols is appended. Duration for CP is T_c =60.61ns (32 subcarriers) and duration for GI is T_g=9.47ns (5 subcarriers)

E. Frequency Hopping and Filtering

The radio frequency (RF) of transmitted OFDM signals are up sampled and filtered with low pass filter then hopped among the three 528 MHz frequency bands with center frequencies at 3.432, 3.960 or 4.488 GHz. The channelization is based on a set of time frequency code. Fig. 2 shows a time-frequency representation with time frequency code {1 3 2 1 3 2}for the lowest three bands

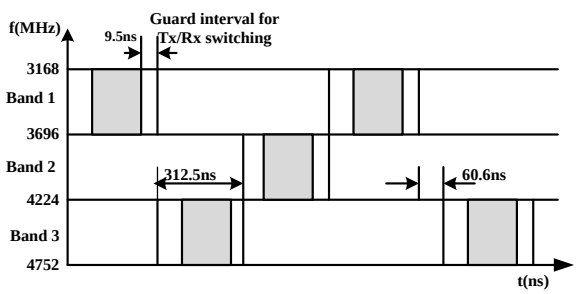


Figure 2. Time Frequency Representation of MB-OFDM signal

F. Channel Model

In order to capture the behavior of UWB channels in terms of cluster and ray arrival rate, cluster and ray decay factors, the Saleh and Valenzuela model is adopted as

stated earlier in part I[4].The impulse response of the multipath channel is described as:

$$h(t)=X\sum_{c=1}^{C}\sum_{k=1}^{K}\alpha_{ck}\delta(t-T_c-\tau_{ck}) \quad (3)$$

Where α_{ck} is the gain of the k-th ray of the c-th cluster, T_c is the time of arrival of the c-th cluster, and τ_{ck} is the delay of the k-th ray in the c-th cluster relative to the cluster arrival time and X represents the lognormal shadowing. The distributions of the cluster and ray arrivals within each cluster are modeled by Poisson process. The multipath gain coefficient α_{ck} is defined as

$$\alpha_{ck} = p_{ck}\rho_c\beta_{ck}. \quad (4)$$

where p_{ck} is a discrete random variable to model signal inversion caused by reflections which assumes values ± 1 with equal probability, ρ_c represents the fading related to the c-th cluster and β_{ck} models the fading of the *k*-th ray of the *c*-th cluster. The small scale fading coefficient $\rho_c\beta_{ck}$ is modeled as random variable with lognormal distribution which is defined as

$$20\log_{10}{}^{(\rho_c\beta_{ck})} \approx Normal\left(\mu_{ck}, \sigma_1^2+\sigma_2^2\right). \quad (5)$$

where σ_1^2 and σ_2^2 denotes the fading contributed by the cluster and the ray respectively. The power decay profile for different propagation paths can be described as

$$\sigma_{ck} = E\left[\left|\alpha_{ck}\right|^2\right] = E\left\{\rho_c^2\beta_{ck}^2\right\} = \sigma_{0,0}\exp\left(-\frac{T_c}{\Gamma}-\frac{\tau_{ck}}{\gamma}\right). \quad (6)$$

where $\sigma_{0,0}$ is the mean energy of the first path of the first cluster, Γ is the cluster decay factor, and γ is the ray decay factor and they are constants. From the above two expressions, we can obtain

$$\mu_{ck} = \frac{10\log_e\left(\sigma_{0,0}\right)-10\frac{T_c}{\Gamma}-10\frac{\tau_k}{\gamma}}{\log_e{}^{10}} - \frac{\left(\sigma_1^2+\sigma_2^2\right)\log_e{}^{10}}{20}. \quad (7)$$

For each realization, the total energy described by α_{ck} is normalized to unity

$$\sum_{c=0}^{C}\sum_{k=0}^{K}\alpha_{ck}^2 = 1. \quad (8)$$

The large scale fading coefficient X is modeled as a lognormal random variable

$$20\log_{10} X \approx Normal\left(0, \sigma_x^2\right). \quad (9)$$

The list of parameters for the four different environmental scenarios is provided in[4-5].

G. Channel Estimation and Compensation

The received signal after removing the cyclic prefix can be computed as

$$y(n) = x(n)H(n) + z(n). \quad (10)$$

where $x(n)$ is the transmitted data symbol in the nth subcarrier, $z(n)$ is the additive noise component in the nth subcarrier, $H(n)$ is the channel response in the nth subcarrier and can be computed as:

$$H(n) = \sum_{i=0}^{P-1} h(i)\exp(-j2\pi ni/N), \quad for \quad n = 0,1,\ldots,N-1. \quad (11)$$

$x(n)$ can be easily estimated from

$$\hat{x}(n) = \frac{y(n)}{H(n)}, \quad n = 0,1,\ldots.N-1. \quad (12)$$

The block type channel estimation with LS estimator is implemented in our model.LS estimators have low complexity even though they suffer from high mean square error [6]

III. Performance Evaluation

A. Average Pair wise Error Probabiltiy

Let the data matrix X at the transmitter side is a concatenation of $P = \left[N/M\right]$ data matrices as: $\mathbf{X} = \left[\mathbf{X}_0^T\ \mathbf{X}_1^T\ \ldots.\ \mathbf{X}_{(P-1)}^T\ 0_{(N-PM)\times 1}\right]^T$ where $\mathbf{X}_P$ is an $M\times 1$ data matrix defined as $\mathbf{X}_P = \left[x_p(0)\ x_p(1)\ldots.\ x_p(M-1)\right]^T$ with $x_p(n) \triangleq x\left(pM+n\right); p = 0,1,\cdots P-1$, $\mathbf{0}_{m\times n}$ stands for an $m \times n$ all-zero matrix. The data matrices X_p are designed independently for different p's. Let the variable η is defined as:

$$\eta = \left\|\mathbf{\Delta}_p\mathbf{H}_p\right\|^2 = \left(\mathbf{\Delta}_p\mathbf{H}_p\right)^{\dagger}\left(\mathbf{\Delta}_p\mathbf{H}_p\right). \quad (13)$$

which is in quadratic form as in[7] and $E\left[\mathbf{\Delta}_p\mathbf{H}_p\right] = 0$.It is chi squared distributed. $\mathbf{\Delta}_p = f(\mathbf{X}_p) - f(\widehat{\mathbf{X}}_p)$ is a signal difference matrix in which $f(\mathbf{X}_p)$ is a diagonal matrix and $\mathbf{H}_p$ is the channel matrix. Note that $(\cdot)^{\dagger}$ represents a conjugate transpose operation. (10) can be written in vector form as:

$$\mathbf{Y}_p = f(\mathbf{X}_p)\mathbf{H}_p + \mathbf{Z}_p . \quad (14)$$

Using the definition of quadratic forms in [7], we can approximate variable η as

$$\eta \approx \sum_{n=1}^{M} \varepsilon_n(\mathbf{\Phi})|\varphi_n|^2 . \quad (15)$$

$\sum_{n=1}^{M}|\varphi_n|^2$ is chi square distributed with the probability density function given by $f_{|\varphi_n|^2}(x) = \frac{1}{(K-1)!}x^{K-1}e^{-x}$ where K is the number of terms in the sum .i.e the chi square distribution has $2K$ degress of freedom, φ_n are iid complex standard Gaussian random variables with zero mean and unit variance[8], and $\varepsilon_n(\mathbf{\Phi})$ are the eigen-values of the matrix

$$\begin{aligned}\mathbf{\Phi} = E\left[\left\|\mathbf{\Delta}_p\mathbf{H}_p\right\|^2\right] &= E\left[\left(\mathbf{\Delta}_p\mathbf{H}_p\right)^{\dagger}\mathbf{\Delta}_p\mathbf{H}_p\right] \\ &= \mathbf{\Delta}_p E\left[\mathbf{H}_p\mathbf{H}_p^{\dagger}\right]\mathbf{\Delta}_p^{\dagger} \\ &= \mathbf{\Delta}_p\mathbf{R}_M\mathbf{\Delta}_p^{\dagger}\end{aligned} . \quad (16)$$

where the $M \times M$ correlation matrix which is given by

$$\mathbf{R}_M = \begin{pmatrix} 1 & R(1)^* & \cdots & R(M-1)^* \\ R(1) & 1 & \cdots & R(M-2)^* \\ \vdots & \vdots & \ddots & \vdots \\ R(M-1) & R(M-2) & \cdots & 1 \end{pmatrix}_{M\times M} . \quad (17)$$

The diagonal elements of the autocorrelation matrix can be calculated as:

$$R(n,n) = E\left[|H(n)|^2\right] = \sum_{c=0}^{C}\sum_{k=0}^{K} E\left[|\alpha_{ck}|^2\right] = 1. \quad (18)$$

and the off-diagonal elements for $n \neq n'$

$$\begin{aligned} R(n,n') &= E\left[\left|H(n)H(n')^*\right|\right] \\ &= \sum_{c=0}^{C}\sum_{k=0}^{K} E\left[E\left[|\alpha_{ck}|^2\right]\mathrm{e}^{-j2\pi(n-n')\Delta f(T_c+\tau_{ck})}\right]. \\ &\triangleq R(n-n') \end{aligned} \quad (19)$$

Letting $n-n'=m$ Combining (6) and (19),we can obtain.

$$\begin{aligned} R(m) &= \sum_{c=0}^{C}\sum_{k=0}^{K} E\left[E\left[|\alpha_{ck}|^2\right]\mathrm{e}^{-j2\pi m\Delta f(T_c+\tau_{ck})}\right] \\ &= \sum_{c=0}^{C}\sum_{k=0}^{K}\sigma_{0,0}E\left[\mathrm{e}^{-T_c\left(\frac{1}{\Gamma}+j2\pi m\Delta f\right)-\tau_{ck}\left(\frac{1}{\gamma}+j2\pi m\Delta f\right)}\right]. \\ &= \sum_{c=0}^{C}\sum_{k=0}^{K}\sigma_{0,0}D_{ck}(m) \end{aligned} \quad (20)$$

denoting the inter-arrival time between clusters j and j-1 as b_j and inter-arrival time between rays i and i-1 as $g_{c,i}$. As mentioned earlier in part II, the clusters and ray arrivals within each cluster in S-V channel are modeled by Poisson processes. The cluster delay b_j and ray delay $g_{c,i}$ can be modeled as IID exponential random variables with parameter Λ and λ respectively. The c-th cluster delay can be expressed as $T_c = \sum_{j=0}^{c} b_j$ and similarly the k-th delay within cluster C can be specified as $\tau_{ck} = \sum_{i=0}^{k} g_{ci}$ .From (20), the expression for $D_{ck}(m)$ can be written as

$$\begin{aligned} &D_{ck}(m) \\ &= E\left\{\exp\left(-T_c\left(\frac{1}{\Gamma}+j2\pi m\Delta f\right)\right.\right. \\ &\qquad \left.\left.-\tau_{ck}\left(\frac{1}{\gamma}+j2\pi m\Delta f\right)\right)\right\} \\ &= E\left\{\exp\left(\sum_{j=0}^{c}-b_j\left(\frac{1}{\Gamma}+j2\pi m\Delta f\right)\right)\right. \\ &\qquad \left.\times\exp\left(\sum_{i=0}^{k}-g_{ci}\left(\frac{1}{\gamma}+j2\pi m\Delta f\right)\right)\right\} \end{aligned} . \quad (21)$$

Since the rays and clusters are statistically independent, (21) can be written as

$$\begin{aligned} D_{ck}(m) = E&\left[\prod_{j=0}^{c}\mathrm{e}^{-b_j\left(\frac{1}{\Gamma}+j2\pi m\Delta f\right)}\right] \\ &\times E\left[\prod_{i=0}^{k}\mathrm{e}^{-g_{ci}\left(\frac{1}{\gamma}+j2\pi m\Delta f\right)}\right] \end{aligned} . \quad (22)$$

Since b_j and $g_{c,i}$ are iid random variables, (22) can be computed as

$$D_{ck}(m)=\left(E\left[\exp(-b\left(\frac{1}{\Gamma}+j2\pi m\Delta f\right)\right]\right)^{c}$$
$$\times\left(E\left[\exp(-g\left(\frac{1}{\gamma}+j2\pi m\Delta f\right)\right]\right)^{k}$$
$$=\left(\int_0^\infty \mathrm{e}^{\alpha b}.\lambda e^{-\lambda b}\frac{(\lambda b)^n}{(n)!}db\right)^{c} \quad . \quad (23)$$
$$\times\left(\int_0^\infty \mathrm{e}^{\beta g}.\lambda e^{-\lambda g}\frac{(\lambda g)^n}{(n)!}dg\right)^{k}$$
$$=\left(\frac{\Lambda}{\Lambda-\alpha}\right)^{c}\left(\frac{\lambda}{\lambda-\beta}\right)^{k}$$

where

$\alpha=-(\frac{1}{\Gamma}+j2\pi m\Delta f)$, $\beta=-(\frac{1}{\gamma}+j2\pi m\Delta f)$ and n stands for number of cluster or ray arrivals. (20) can be written as:

$$R(m)=\sigma_{0,0}\sum_{c=0}^{C}\left(\frac{\Lambda}{\Lambda+\frac{1}{\Gamma}+j2\pi m\Delta f}\right)^{c}$$
$$\times\sum_{k=0}^{K}\left(\frac{\lambda}{\lambda+\frac{1}{\gamma}+j2\pi m\Delta f}\right)^{k} \quad . \quad (24)$$

Using the knowledge power series convergence, for large number of clusters

$$\lim_{c\to\infty}\left(\frac{\Lambda}{\Lambda+\frac{1}{\Gamma}+j2\pi m\Delta f}\right)^{c}$$
$$=\frac{1}{1-\frac{\Lambda}{\Lambda+\frac{1}{\Gamma}+j2\pi m\Delta f}} \quad . \quad (25)$$

$$for\ 0<\frac{\Lambda}{\Lambda+\frac{1}{\Gamma}+j2\pi m\Delta f}<1$$
$$=\frac{\Lambda+\frac{1}{\Gamma}+j2\pi m\Delta f}{\frac{1}{\Gamma}+j2\pi m\Delta f}$$

Similarly the same result will be obtained for large rays. Hence to find the off - diagonal elements of the correlation matrix we can write the expression in (24) as:

$$R(m)=\sigma_{0,0}\times\frac{\Lambda+\frac{1}{\Gamma}+j2\pi m\Delta f}{\frac{1}{\Gamma}+j2\pi m\Delta f}$$
$$\times\frac{\lambda+\frac{1}{\gamma}+j2\pi m\Delta f}{\frac{1}{\gamma}+j2\pi m\Delta f} \quad . \quad (26)$$

By Ostrowski's theorem[7] the eigen values of matrix $\mathbf{\Phi}$ in (16) can be given by

$$\varepsilon_n\left(\mathbf{\Phi}\right)=\zeta_n\varepsilon_n\left(\mathbf{R}_M\right) \quad n=1,2,....,M \ . \quad (27)$$

where ζ is a nonnegative real number that satisfies:

$$\varepsilon_M\left(\mathbf{\Delta}_p\mathbf{\Delta}_p^{\dagger}\right)\le\zeta_n\le\varepsilon_1\left(\mathbf{\Delta}_p\mathbf{\Delta}_p^{\dagger}\right) \quad . \quad (28)$$
$$for \quad n=1,2,...,M$$

By definition, the moment generating function (MGF) is

$$\mathfrak{M}_\eta(t)=E\left[e^{t.\eta}\right]. \quad (29)$$

Combining (29) and (15), we can obtain

$$\mathfrak{M}_\eta(t)=E\left[\exp\left(t\sum_{n=1}^{M}\varepsilon_n\left(\Phi\right)\left|\varphi_n\right|^2\right)\right]. \quad (30)$$

and computed as $\mathfrak{M}_x(s)=\int_0^\infty f_x(x)e^{sx}dx$ where $f_x(x)$ is the probability density function of random variable X. After some algebraic operation on (30) we shall obtain the result as

$$\mathfrak{M}_\eta(s)=\prod_{n=1}^{M}(1-s\varepsilon_n(\mathbf{\Phi}))^{-1} \ . \quad (31)$$

Using (31) and (27) we can obtain expression:

$$\mathfrak{M}_\eta(s)=\prod_{n=1}^{M}(1-s\varsigma_n\varepsilon_n(\mathbf{R}_M))^{-1} \ . \quad (32)$$

As in [9], we have the average PEP defined

$$P_e=E\left[Q\left(\sqrt{(\delta/2)\left\|\mathbf{\Delta}_p\mathbf{H}_p\right\|^2}\right)\right]. \quad (33)$$

Applying the Gaussian error function to (33) and combining it with (32) we can obtain the average PEP of the system as:

$$P_e\approx\frac{1}{\pi}\int_0^{\pi/2}\prod_{n=1}^{M}\left(1+\frac{\delta\zeta_n}{4\sin^2\theta}\varepsilon_n\left(\mathbf{R}_M\right)\right)^{-1}d\theta \ . \quad (34)$$

For case of no jointly coding over subcarriers (that is for M=1), (34) can be simplified to

$$P_e = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+\frac{1}{\delta}}}\right). \quad (35)$$

which is a similar result to narrowband communications under Rayleigh fading. In the case of jointly coding over subcarriers,for M=2 and assuming the information is repeated over two OFDM symbols,(34) can be approximated as

$$P_e \approx \frac{1}{\pi}\left(\left(\frac{8\pi}{6k+48} - \frac{\pi}{2\sqrt{k}}\cdot\left(\frac{32+5k}{24+3k}\right)\left(\sqrt{\frac{\sqrt{k}}{\sqrt{k}+j4}} - \sqrt{\frac{\sqrt{k}}{\sqrt{k}-j4}}\right)\right)\right). \quad (36)$$

$$k \approx \begin{cases} 0.0924\delta^2\zeta^2 & for \quad CM1 \\ 0.3026\delta^2\zeta^2 & for \quad CM4 \end{cases}$$

where $\zeta = |x - \hat{x}|^2$ and $\delta = E_s / N_o$ is signal to noise ratio. (35) and (36) are used to trace the theoretical error probability curve for MB-OFDM UWB system for all UWB channel models.Assuming the channel-state information (CSI) is known perfectly at the receiver, the receiver performs maximum likelihood decoding by choosing the codeword $\hat{\mathbf{X}}_p$ that minimizes the square Euclidean distance between the hypothesized and actual received signal matrices, that is, $\hat{\mathbf{X}}_p = \arg\min_{\mathbf{X}_p} \|\mathbf{Y}_p - f(\mathbf{X}_p)\mathbf{H}_p\|^2$. For M=2, the Eigen values of the autocorrelation matrix are $1+|R(1)|$ and $1-|R(1)|$.where the correlation matrix element is computed as

$$|R(1)| = \sigma_{0,0} \frac{\sqrt{\left(\Lambda + \frac{1}{\Gamma}\right)^2 + (2\pi\Delta f)^2} \times \sqrt{\left(\lambda + \frac{1}{\gamma}\right)^2 + (2\pi\Delta f)^2}}{\sqrt{\left(\frac{1}{\Gamma}\right)^2 + (2\pi\Delta f)^2} \times \sqrt{\left(\frac{1}{\gamma}\right)^2 + (2\pi\Delta f)^2}}. \quad (37)$$

B. Average Coded bit error probability and Link budget Analysis

The average coded bit error probability is defined as the bit error rate (BER) performance after the convolutional decoder. The convolutional encoder with CC(3,1,7) is used for the data rate of 160Mbps.Its free distance, d_{free} is 15 and the sum of the hamming weight of all input sequences N_b is 7.Assuming each coded bit has an equal probability of being wrongly decoded, the average coded bit error probability is bounded by:

$$P_{bc} \le \frac{\partial T(W,I)}{\partial I}\Big|_{I=1, W=\sqrt{4P_{buc}(1-P_{buc})}}. \quad (38)$$

where $T(W,I)$ is the transfer function of the convolutional encoder[9], P_{buc} denotes uncoded bit error probability. (38) can be approximated to:

$$P_{bc} \approx N_b\left[4P_{buc}(1-P_{buc})\right]^{d_{free}/2} \approx N_b\left(1+\frac{E_b}{N_0}\right)^{-d_{free}/2}. \quad (39)$$

Where E_b / N_0 is signal to noise ratio in terms of average energy per bit and average white noise power. (39) can be used to trace the theoretical bit error rate for coded MB-OFDM UWB system. Since the S-V channel model does not take into account the issue of link budget, in real situation it needs to be considered. Link budget is a consideration of all the gains and losses in a transmission system. The link budget will take the form of the equation:

$$\begin{aligned} &\text{Re}ceived\ power(dBm) \\ &= Transmiited\ Power(dBm) \\ &\quad + Gains(dB) + Losses(dB) \end{aligned}. \quad (40)$$

The signal to noise ratio at the receiver side can be computed as

$$\begin{aligned} \frac{E_b}{N_0} &= P_{tx} - 20\log_{10}\left(\frac{4\pi f_g d}{c}\right) \\ &\quad - \left(k_B Temp + 10\log_{10}(R_b)\right) - Q \\ &\quad + 10\log_{10}\left(E\left\{|H(n)|^2\right\}\right) \quad in\ dB \end{aligned}. \quad (41)$$

Based on (11) we can derive result for $E\left\{|H(n)|^2\right\}$

$$
\begin{aligned}
&H(n) \\
&= \sum_{i=0}^{P-1} h(i)\exp(-j2\pi ni/N) \\
&= \sum_{i=-\infty}^{\infty} h(i)\exp(-j2\pi ni/N) \qquad . \qquad (42) \\
&= \sum_{i=-\infty}^{\infty}\sum_{c=0}^{C}\sum_{k=0}^{K} X\alpha_{ck}\, p(iT-T_c-\tau_{ck}) \\
&\quad \times \exp\left(-j2\pi\left(\frac{ni}{N}+f_{center}\left(T_c+\tau_{ck}\right)\right)\right)
\end{aligned}
$$

Assuming the length of the cyclic prefix is longer than the channel impulse response, that is $h(i)=0 \quad if \quad i \neq 0,1,...,P-1$. Since $E\{\alpha_{kc}\}=0 \Rightarrow E\{H(n)\}=0$. Let $p(t)$ is the pulse shape used for transmission and supposed to have a bandwidth of $1/2T$ and satisfies the Nyquist criterion. Its Fourier transform is given by

$$
P(f)=\begin{cases} T\exp(-j2\pi k_0 T), & -\dfrac{1}{2T} \le f \le \dfrac{1}{2T} \\ 0, & otherwise \end{cases} . \quad (43)
$$

For some integer k_0, using sampling theorem, we can write

$$
\begin{aligned}
&\sum_{i=-\infty}^{\infty} p(iT-T_c-\tau_{ck})e^{-j2\pi ni/N} \\
&= \begin{cases} \mathrm{e}^{-j\frac{2\pi n\left(k_0T+T_c+\tau_{ck}\right)}{NT}}, & 0 \le n \le \dfrac{N}{2}-1 \\ \mathrm{e}^{-j\frac{2\pi (n-N)\left(k_0T+T_c+\tau_{ck}\right)}{NT}}, & \dfrac{N}{2} \le n \le N-1 \end{cases} \qquad .(44)
\end{aligned}
$$

Let

$$
\bar{n}=\begin{cases} n, & 0 \le n \le \dfrac{N}{2}-1 \\ n-N, & \dfrac{N}{2} \le n \le N-1 \end{cases}
$$

Then (44) can be expressed as

$$
\begin{aligned}
&\sum_{i=-\infty}^{\infty} p(iT-T_c-\tau_{ck})\exp(-j2\pi ni/N) \\
&= \exp\left(-j\frac{2\pi\bar{n}\left(k_0T+T_c+\tau_{ck}\right)}{NT}\right)
\end{aligned} \quad . (45)
$$

$$
\begin{aligned}
&E\left\{\left|H(n)\right|^2\right\} \\
&= E\left\{\left|\sum_{i=-\infty}^{\infty}\sum_{c=0}^{C}\sum_{k=0}^{K} X\alpha_{ck}\, p(iT-T_c-\tau_{ck}) \times \mathrm{e}^{-j2\pi\left(\frac{ni}{N}+f_{center}\left(T_c+\tau_{ck}\right)\right)}\right|^2\right\} \\
&= E\left\{X^2\right\} E\left\{\sum_{c=0}^{C}\sum_{k=0}^{K}\left|\alpha_{ck}\,\mathrm{e}^{-j2\pi f_{center}\left(T_c+\tau_{ck}\right)}\right|^2 \times \left|\sum_{i=-\infty}^{\infty} p(iT-T_c-\tau_{ck})\,\mathrm{e}^{-j\frac{2\pi ni}{N}}\right|^2\right\} \\
&= E\left\{X^2\right\} E\left\{\sum_{c=0}^{C}\sum_{k=0}^{K}\left|\alpha_{ck}\,\mathrm{e}^{-j2\pi\left(f_{center}\left(T_c+\tau_{ck}\right)\right)}\right|^2 \times \left|\mathrm{e}^{-j\frac{2\pi\bar{n}\left(k_0T+T_c+\tau_{ck}\right)}{NT}}\right|^2\right\} \\
&= E\left\{X^2\right\}\times E\left\{\sum_{c=0}^{C}\sum_{k=0}^{K}\alpha_{ck}^2\right\} = E\left\{X^2\right\}\times 1 \\
&= \int_{-\infty}^{\infty}\left(10^{\frac{x}{10}}\right)^2 \times \frac{1}{\sqrt{2\pi\sigma_x^2}}\exp\left(-\frac{x^2}{2\sigma_x^2}\right)dx
\end{aligned} \quad (46)
$$

For large number of clusters C and rays K $x \approx normal(0,\sigma_x^2)$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2\pi\sigma_x^2}}\int_{-\infty}^{\infty}\mathrm{e}^{-\frac{1}{2\sigma_x^2}\left(x-\frac{\sigma_x^2\ln 10}{10}\right)^2+\frac{\sigma_x^2\ln^2 10}{200}}dx \\
&= \mathrm{e}^{\frac{\sigma_x^2\ln^2 10}{200}} = \mathrm{e}^{0.0265\sigma_x^2} = 1.2693
\end{aligned}
$$

For all channel model parameters.

Now (41) can be interpreted as : $10\log_{10}\left(E\left\{\left|H(n)\right|^2\right\}\right)$ which is fading gain captured by S-V channel model. It should be noted that k_B is Boltzman's constant in(J/k), $Temp$ is temperature in Kelvin, R_b is the bit rate in bps, c and d are the speed of light and distance of the transmission respectively, f_g is the geometric average of the upper frequency , $f_{\max}$ and the lower frequency, $f_{\min}$ in MHz of the transmission spectrum. Q represents the noise figure of the antenna which is 6.6dB and implementation loss 2.5dB[10]. It should be noted that the mathematical expressions $k_B Temp + 10\log_{10}\left(R_b\right)$ is average noise power per bit and $P_{tx} \le -41.25 + 10\log_{10}\left(f_{\max}-f_{\min}\right)$ is the average transmitted power in dBm

C. Outage Probabiltiy

The outage probability is defined as the probability that the combined SNR, ν, value gets below a specified threshold, ν_o:

$$P_{out} = P\left(\nu \le \nu_o\right) = \int_0^{\nu_o} p_\nu(x)dx. \qquad (47)$$

where $p_\nu(x)$ represents the PDF of ν. The combined SNR for the jointly encoded each data matrix $\mathbf{X}_p$ is defined as:

$$\nu = \frac{E_s \left\|\mathbf{X}_p \mathbf{H}_p\right\|^2}{E\left[\left\|\mathbf{Z}_p\right\|^2\right]} = \frac{\delta}{M}\upsilon. \qquad (48)$$

where δ is SNR defined earlier in (36) and $\upsilon = \sum_{n=0}^{M-1}\left|H_p(n)\right|^2$. The MGF of υ can be obtained as

$$\begin{aligned}\mathfrak{M}_\upsilon(s) &\approx \prod_{n=1}^{M}\frac{1}{1-s\varepsilon_n\left(\mathbf{R}_M\right)} \\ &= \sum_{n=1}^{M}\frac{B_n}{1-s\varepsilon_n\left(\mathbf{R}_M\right)}\end{aligned}. \qquad (49)$$

The probability density function (PDF) of υ can be computed by finding the inverse Laplace transform of (49)

$$\begin{aligned}p_\upsilon\left(x\right) &= \pounds^{-1}\left\{\mathfrak{M}_\upsilon\left(s\right)\right\} \\ &= \pounds^{-1}\left\{\sum_{n=1}^{M}\frac{B_n}{1-s\varepsilon_n(\mathbf{R}_M)}\right\} \\ &= \sum_{n=1}^{M}B_n\pounds^{-1}\left\{\frac{1}{1-s\varepsilon_n(\mathbf{R}_M)}\right\} \\ &= \sum_{n=1}^{M}\frac{B_n}{\varepsilon_n(\mathbf{R}_M)}.e^{-\frac{x}{\varepsilon_n(\mathbf{R}_M)}}. \quad x \ge 0\end{aligned}. \qquad (50)$$

Using (47) and (50), we can obtain the outage probability relation for a multiband OFDM system for any UWB channel model parameters as:

$$\begin{aligned}P_{out} &= P\left(\upsilon \le \frac{M\nu_o}{\delta}\right) \\ &= \int_0^{\frac{M\nu_o}{\delta}}\sum_{n=1}^{M}\frac{B_n}{\varepsilon_n(\mathbf{R}_M)}.e^{-\frac{x}{\varepsilon_n(\mathbf{R}_M)}} \\ &= \sum_{n=1}^{M}B_n\left(1-e^{-\nu_o M/\delta\varepsilon_n(\mathbf{R}_M)}\right)\end{aligned}. \qquad (51)$$

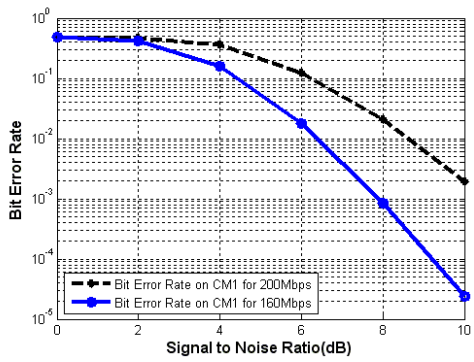
where, $B_n = \prod_{n'=1,n'\ne n}^{M}\frac{\varepsilon_n\left(\mathbf{R}_M\right)}{\varepsilon_n\left(\mathbf{R}_M\right)-\varepsilon_{n'}\left(\mathbf{R}_M\right)}$ that can be obtained from techniques of partial fractions. Example for CM1 and M=2, B_1 and B_2 can be computed as1.0075 and -0.0075 respectively and the outage probability can be expressed as

$$\begin{aligned}P_{out} = 1 &- 0.5(1+|R|^{-1})\cdot e^{\left(-\frac{2\nu_o}{\delta(1+|R|)}\right)} \\ &- 0.5(1-|R|^{-1})\cdot e^{\left(-\frac{2\nu_o}{\delta(1-|R|)}\right)}\end{aligned}. \qquad (52)$$

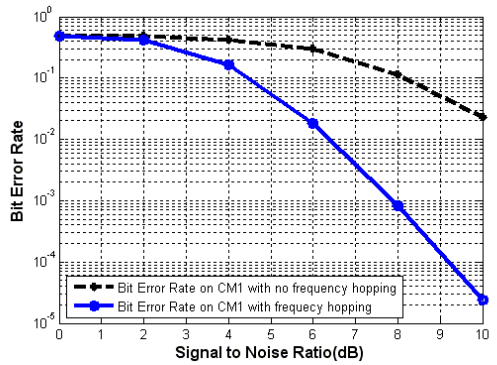
This is used to trace the theoretical outage probability

IV. Numerical and Simulation Results

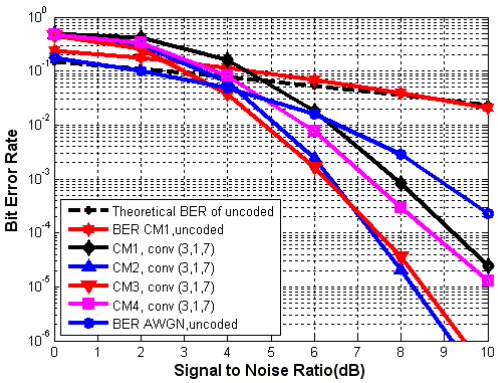
Fig.3a shows the bit error rate performance of 160Mbps is 1.5dB lower than that of 200Mbps for the same bit error rate of 10^{-2}. This is because in MB-OFDM system with the lowest data rate has the lowest Bit Error rate, since it uses the lowest coding rate. Fig.3b shows the merit obtained from frequency hopping. Frequency hopping from one band to another using time frequency code offers a system gain of about 3.5dB at bit error rate of 10^{-1} for the data rate of 160 Mbps. This gain is due to the frequency diversity achieved by use of a three times larger frequency band (diversity gain). From Fig.3c we can observe that the theoretical BER performance and simulated result for an uncoded system is the same for all UWB channles. However the simulation over the AWGN channel for an uncoded scenario shows better performance due to the absence of multipath fading. The performance over all channels for coded case is very satisfactory. As in [11] result for CM2 and CM3 above 8dB and CM4 and CM1 above 9dB with convolutional coding scheme shows acceptable performance.



(a)



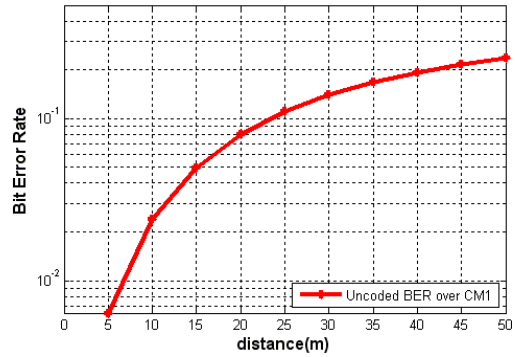
(b)



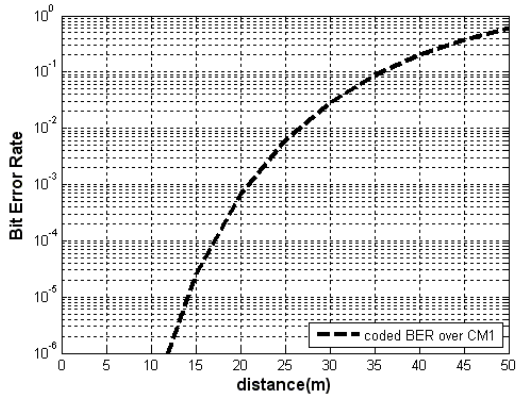
(c)

Figure 3 Simulation result for probability of error versus signal to noise ratio a) Simulation result for 200 and 160 Mbps speed mode b) effect of frequency hopping on performance c) performance comparison of all channel models

Fig. 4 shows the performance of the MB-OFDM UWB system in terms of signal to noise ratio, Bit error rate versus transmission distance when Average Tx power = -9dBm,f_{max}=4.8GHz, f_{min}=3.1GHz and R_b=160Mbps. The bit error rate performance for the uncoded system which is shown in Fig.4b is not satisfactory even for shorter distances. But the performance is improved when coding is introduced in the system as seen in Fig.4c. For instance the performance at 11.8 meters which corresponds to a bit error rate of 10^{-6} is satisfactory [11]. But for distance more than 11.8 meters,the performance deteriorates and this makes the MB-OFDM UWB system limited to short distances.



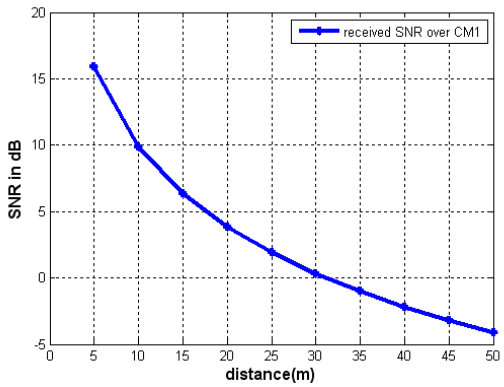
(b)



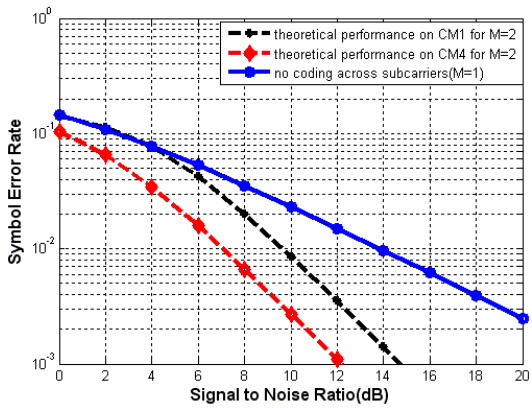
(c)

Figure 4 Received SNR versus transmission distance d. a) Received SNR b)Uncoded BER c)coded BER

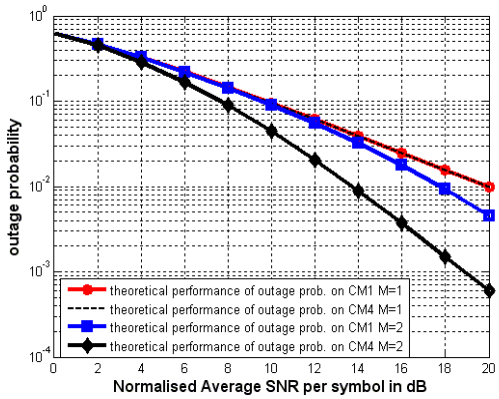
From Fig.5 we observe that for no coding (M=1) across subcarriers the theoretical performance of UWB system are similar for all UWB channel models and there is no performance gain from the multipath clustering property of UWB channels. Fig.5a shows that the performance of the MB-OFDM system with information jointly encoded across two subcarriers (M=2) is better than the system with no coding over subcarriers. The performance obtained under CM4 is superior to that of CM1 because CM4 has less correlation in the frequency response among different subcarriers. The theoretical outage probability performance shown in Fig.5b follows the same behavior as the average symbol error rate (PEP).



(a)



(a)



(b)

Figure 5 joint encoding across two subcarriers. a)signal to noise ratio versus symbol error rate b)Average SNR versus outage probability

V. Conclusion

In this paper,we have modeled, described the MB-OFDM UWB system and provided numerical derivations about its performance in accordance with the S-V channel model. Moreover the behavior of the system is examined in terms of transmission distance capability and its performance improvement when the data is jointly coded across subcarriers that are without using any channel coding schemes. The diversity gain of CM4 is better than CM1 when there is jointly encoding the information across subcarriers and this is due to the presence of less correlation in the frequency response among different subcarriers.

It should be noted that the diversity gain can be improved not only by increasing the number of jointly encoded subcarriers but also increasing the number of jointly encoded OFDM symbols, or the number of antennas. Nevertheless, increasing the number of jointly encoded subcarriers leads to the loss in coding gain as in [1].We have observed that MB-OFDM UWB system gains much from channel coding schemes and jointly encoding the data over subcarriers. However its performance can be enhanced further using MIMO techniques,Linear precoding and Dynamic Resource allocation. Hence more research and improvements on the existing UWB systems are required to increase the system range, transmission rate and robustness. For instance combining MIMO with Linear precoding[12] can make the MB-OFDM system more robust and can provide up to a data rate of 1 Gb/s with a reasonable additional system complexity [13].

Acknowledgment

This work is sponsored by the international exchange program of Harbin Engineering University for innovation oriented talents cultivation.

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