Globally Optimal Coverage for Non-convex Environments with Mobile Sensor Networks

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***Abstract*— The paper investigates the problem of achieving globally optimal coverage for non-convex grid environments with a network of mobile sensor nodes. A distributed algorithm is proposed to solve the problem, which is locally implementable on individual sensor. That is, each mobile sensor only requires to update its current position to an adjacent location with certain probability relying on its coverage costs at current location and adjacent locations. It shows that under the proposed algorithm, a network of mobile sensors will eventually converge to a globally optimal configuration minimizing the coverage cost with probability one. Both rigorous analysis and simulations are provided to illustrate the global convergence result.**

***Index Terms*— Mobile sensor network, coverage, global optimality, non-convex environment.**

I. Introduction

Distributed coverage control as a fundamental problem in sensor networks attracts more and more attention in recent years. The goal is to optimize a configuration for a network of agents or sensor nodes in a given bounded and connected region to carry out sensing, surveillance, data collection, etc. [1]. The motivation behind is for better sensing or longer network lifetime with a good configuration in certain applications.

This locational optimization problem has been solved in a distributed way that results in (locally) optimal coverage [1]. Plenty of extensions are made recently from this inherently geometric strategy. Frasca et al. [2] propose a deployment and partitioning algorithm that requires only pairwise asynchronous gossip communication. The problem of dynamically covering a given region is discussed in [3]. Schwager et al. [4] propose a consensus based controller that greatly increases the convergence rate. The study of network configurations leading to optimal field interpolations is presented in [5]. For these schemes, the distribution of sensory information in the environment is required to be known a *priori* by all the sensors. This requirement is relaxed by Schwager et al. [6] by introducing a controller with a simple memoryless approximation from sensor measurements. Gao et al. [7] utilize the consensus algorithm dealing with the coverage problem over acyclic digraphs with fixed and controlled switching topology. Besides, other issues in wireless sensor networks closely related to the coverage problem are also studied, such as topology control and connectivity maintenance [8]–[10].

Due to practical requirements, there are more works appearing in recent years extending the coverage control problem from convex to nonconvex domains [11]–[13]. They are particularly interested in coverage in the presence of obstacles or holes. Caicedo et al. [11] show how a diffeomorphism can be used to transform a non-convex coverage problem to a convex one, for which the Lloyds algorithm can be applied. Caicedo-Nunez continues this work by constructing a diffeomorphism for convex regions with obstacles in their interior so that the solution to the transformed problem yields the solution of the original non-convex problem [14], [15] .

As most works result in only locally optimal coverage (particularly in a nonconvex environment), and usually they assume that no obstacle exists to block sensing and communication, the objective of this paper is to provide a globally optimal coverage result with a distributed control scheme for sensors in a non-convex environment which may contain obstacles and holes inside. The main goal of our work is to solve the traditional coverage problem with global optimality by discretizing the given environment into a grid map and applying a simulated-annealing-like algorithm for individual sensor. The environment is allowed to be nonconvex or even with obstacles inside. The algorithm is fully distributed and uses only local information from neighbors. That is, each mobile sensor updates its location to one of its adjacent spots with certain probability each step and the probability that governs the movement is determined by the local coverage cost observed by the sensor itself. It is shown in the paper that with the proposed scheme, the sensors will not be trapped in a locally optimal configuration but converge to a globally optimal configuration minimizing the overall coverage cost function. Simulations are provided to validate our results.

The remainder of this paper is organized as follows. We present our problem and design goal in Section II. We describe the proposed distributed algorithm and provide a main result in Section III. In Section IV, we show that the proposed distributed algorithm works well by simulations. Finally, we conclude the paper in Section V with a discussion of directions for future research.

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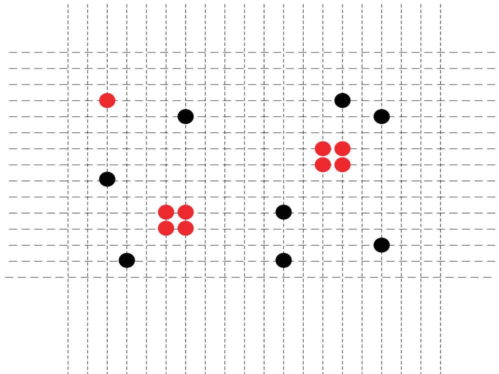

Figure 1: Sensors in a grid environment where each darker (black) dot represents a sensor and each lighter (red) dot indicates that the spot is occupied by obstacles or holes.

II. Problem formulation

Given a group of m sensors with limited communication and sensing capabilities in a connected region, the model of sensors is viewed in a grid lattice that substitutes the continuous space and each grid node is either free or is taken by a sensor or obstacles (see Fig. 1). When considering the motion on an upright square lattice, each sensor can move to only one of the four adjacent nodes (North, South, East, West) each step if the spot is free.

We state several notions and notations from locational optimization. More discussions are referred to [1], [16]. Due to practical requirements, we impose that the sensors are confined in a bounded and connected region $\mathscr{S}$. Suppose the sensor nodes are able to sense the boundary of the region with carry-on sensors. The upright square lattice in the bounded region $\mathscr{S}$ is then modeled as a graph $G=(V,E)$ with each node in V representing a grid node and an edge in E between two nodes meaning that the corresponding two grid nodes border each other. Such a graph is called a *grid graph*.

Consider an inertial coordinate system and let $p(v)$ denote the position of grid node v in the inertial coordinate system. Denote the locations of the m mobile sensors at time t by $z_1(t),\ldots,z_m(t)$. The aggregate position vector is

$$z=(z_1,\ldots,z_m)\in\mathbb{Z}^{2m}.$$

For a distribution z of a large number of mobile sensors in the grid, we define the Voronoi partition $\overline{V}(z):=\{V_1,V_2,\cdots,V_m\}$ of G as a set of disjoint subsets of V such that $V_1\bigcup V_2\bigcup\cdots\bigcup V_m=V$ and

$$V_i=\{v\in V|\parallel p(v)-z_i\parallel\leq\parallel p(v)-z_j\parallel,\forall j\neq i\}, \quad (1)$$

where $p(v)$ can be sensed by sensor i. We also call $V_i(t)$ sensor i's dominant region at t. Two sensors are called *neighbors* or *adjacent* if they are in the adjacent Voronoi partition regions. In addition, we define a new induced graph $\overline{G}:=(\overline{V},\overline{E})$, called *partition graph*, with an edge $\overline{e}\in\overline{E}$ between V_i and $V_j\in\overline{V}$ if and only if G has at least one edge $e\in E$ between $v'\in V_i$ and $v''\in V_j$. Examples of a partition graph without and with obstacles are shown in Fig. 2 and Fig. 3, respectively.

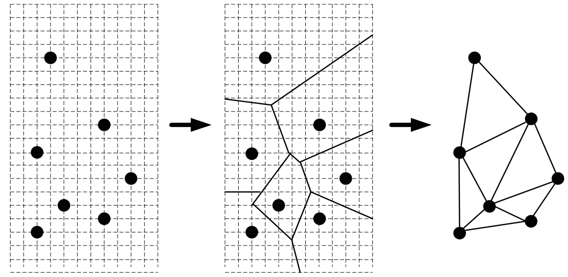

Figure 2: An example of Voronoi partition without obstacles (Left: a distribution of nodes; Middle: a Voronoi partition; Right: a partition graph).

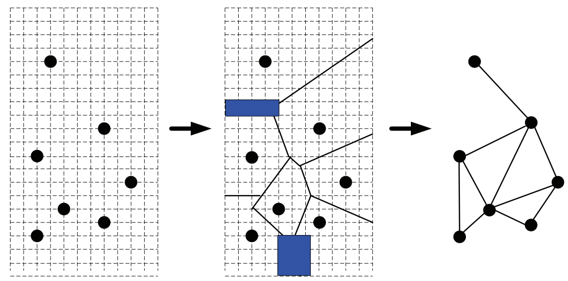

Figure 3: An example of Voronoi partition with obstacles (lighter (blue) squares in the figure).

Upon that, we formulate an individual coverage cost function for each sensor i as follows:

$$\mathscr{H}_i(z_i,\overline{V}(z))=\sum_{v\in V_i}f(\parallel p(v)-z_i\parallel)\phi(p(v)). \quad (2)$$

In the equation above, the *density function* $\phi:V\rightarrow\mathbb{R}_{>0}$ is a bounded measurable positive function that could be the cell phone users density at the communication range, the node burden in wireless sensor network, etc. in different applications. The *performance function* $f:\mathbb{R}_{\geq 0}\rightarrow\mathbb{R}_{\geq 0}$ is a monotone increasing function. The sensing performance at node v taken by the ith sensor at position z_i degrades with the distance $\parallel p(v)-z_i\parallel$. Accordingly, $f(\parallel p(v)-z_i\parallel)$ provides a quantitative assessment of how poor the sensing performance is. Then the overall coverage cost is the sum of all sensors' costs. That is,

$$\mathscr{J}(z,\overline{V}(z))=\sum_{i=1}^{m}\mathscr{H}_i(z_i,\overline{V}(z)). \quad (3)$$

The coverage problem is then to find a distributed control law for each sensor so that the m sensors come to a configuration minimizing the coverage cost function $\mathscr{J}(z,\overline{V}(z))$.

The problem of reaching a locally optimal configuration in a bounded convex environment has been solved in [1]. However, in practice, it is very likely that the environment is nonconvex (even with obstacles inside) and a globally optimal configuration is expected to be reached via distributed schemes. In this paper, we assume each sensor can receive only local location information of its neighbors and leverage this information to adjust its own location. Collectively, the m sensors are expected to jump out of local optimum in the connected nonconvex environment and converge to a globally optimal configuration.

III. Local updating rule

In this section, we present a simulated-annealing-like algorithm to update the position of each sensor using only neighbor sensors' location information and provide rigorous analysis showing that a network of m sensors will converge to a globally optimal configuration minimizing the coverage cost function with probability one.

A. Algorithm

Let $N_i(t)$ be the set of adjacent nodes of $z_i(t)$ in the grid that are free at time t. Let $n_i(t)$ denote the cardinality of the set $N_i(t)$ at time t. Each sensor can move only to one of the four adjacent free nodes, so $n_i(t) \leq 4$. Denote $z^x_{-i}(t)$ the new aggregate position vector with the ith component $z_i(t)$ replaced by x at time t, i.e.,

$$z^x_{-i} = (z_1, \ldots, z_{i-1}, x, z_{i+1}, \ldots, z_m).$$

The m sensors are programmed to update their positions in turn. In other words, only at discrete time $t = lm + i$, $l = 0, 1, 2, \cdots$, sensor i is permitted to move. Next we present the algorithm for each sensor i to update its position with a random initial distribution. The notation $[\cdot]^+$ will be used in the algorithm to indicate that it is zero when the value in the square bracket is less than zero and it has the same value otherwise.

Step 1: Check its neighbors' locations in the sensing range and update the Voronoi partition.

Step 2: Compute the variation of the coverage cost
$\delta \mathscr{J}_i = \mathscr{J}(z^x_{-i}, \overline{V}(z^x_{-i})) - \mathscr{J}(z, \overline{V}(z))$
when sensor i moves to an adjacent location x.

Step 3: Update to location x with probability
$q_{z_i(t)\to x} = \frac{1}{n_i(t)} \exp \frac{-[\delta \mathscr{J}_i]^+}{\alpha(t)}$
where $\alpha(t)$ is a positive time-varying parameter.

From Step 3 of the algorithm, we can see that the probability for sensor i to remain in the current location is

$$1 - \sum_{x \in N_i(t)} q_{z_i(t)\to x}.$$

Note that $\delta \mathscr{J}_i$ in Step 2 can indeed be rewritten as

$$\begin{aligned} \delta \mathscr{J}_i &= \mathscr{J}(z^x_{-i}, \overline{V}(z^x_{-i})) - \mathscr{J}(z, \overline{V}(z)) \\ &= [\mathscr{H}_i(x, \overline{V}(z^x_{-i})) - \mathscr{H}_i(z_i, \overline{V}(z))] + \\ &\quad \textstyle\sum_j [\mathscr{H}_j(z_j, \overline{V}(z^x_{-i})) - \mathscr{H}_j(z_j, \overline{V}(z))], \end{aligned}$$

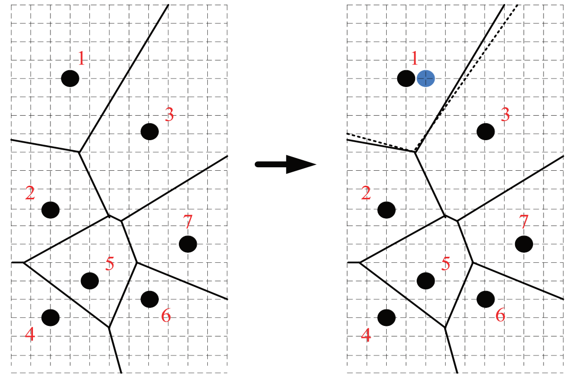

Figure 4: Sensor 1 moves to the lighter dot, and the Voronoi partition is updated as the dashed line. But the partition graph remains unchanged.

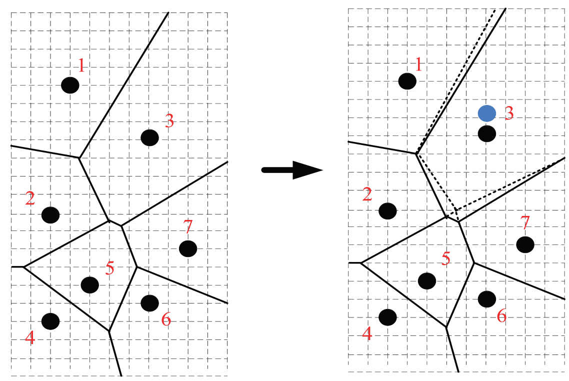

Figure 5: Sensor 3 moves to the lighter dot, and the Voronoi partition is updated as the dashed line. Sensors 3 and 5 were neighbors, but they are not after the repartition. So the partition graph changes.

where sensor j is a neighbor of sensor i in either $\overline{G}(z)$ or $\overline{G}(z^x_{-i})$. It means that to determine the updating probability, each sensor only requires the location information of neighbors.

We assume here each sensor i can sense neighbor sensor j's location in its sensing range. So once sensor i moves one step, $\delta \mathscr{J}_i$ can be calculated by sensor i based on its measurement from neighbors no matter the partition graph $\overline{G}$ changes or not (for instance, the partition graph does not change in Fig. 4, while it changes in Fig. 5 since sensor 3 has only three neighbors, namely $1, 2, 7$ after one step moving).

Remark 3.1: This local updating rule in the algorithm above indicates that if a sensor moves to an adjacent location for which the coverage cost becomes worse, then the probability of going to that location decreases in terms of an exponential function.

Remark 3.2: For implementation, it is required that each sensor is able to sense its neighbors' locations and the boundary of the region in its sensing range, such that it can calculate the Voronoi diagram.

B. Global optimality

Now we present our main result showing that a network of m sensors converges to a globally optimal configuration minimizing the coverage cost function with probability one when the proposed local updating rule is applied.

Theorem 1: Suppose that a network of m sensors is initially distributed in a given region $\mathscr{S}$ modeled by a grid graph G. If $\alpha(t) > 0$ is non-increasing and satisfies

$$\alpha(0) > \alpha(1) > \cdots > \lim_{t\to\infty} \alpha(t) = 0, \tag{4a}$$

$$\alpha(t) \geq \frac{rmD}{\ln(t+1)}, \tag{4b}$$

where r is the maximum path length of G, and $D = \max_i \max_z |\delta \mathscr{J}_i|$, then the m sensors converge to a globally optimal configuration minimizing the coverage cost function $\mathscr{J}$ with probability one.

The proof of Theorem 1 is similar to the one for the simulated annealing algorithm. We recall several notions and propositions from [17]–[19].

A square matrix P is *row stochastic* (stochastic for short) if it is nonnegative and every row sum equals 1. Given a stochastic matrix $P = (p_{ij}) \in \mathbb{R}^{n\times n}$, its coefficient of ergodicity $\lambda(P)$ [19] is given as

$$\lambda(P) = 1 - \min_{i_1,i_2,i_1\neq i_2} \sum_j \min(p_{i_1 j}, p_{i_2 j}).$$

Consider a time-inhomogeneous Markov chain with time-varying transition matrix $P(t)$. Then the s-step ($s \geq 1$) transition matrix is

$$P(t_0, t_0+s) = \prod_{k=0}^{s-1} P(t_0+k), \quad t_0 \geq 0.$$

Let $v(t)$ denote the state probability vector at t. So $v(t)$ can be written as

$$v(t) = v_0 P(t_0, t_0+t),$$

in which v_0 is the initial state probability vector at time t_0 and satisfies $|v_0| = 1$. Throughout this appendix, the notation $|\cdot|$ represents the sum of absolute values of all entries. Next, we list a few definitions and lemmas that will be used in the proof.

Definition 1: [19] A time-inhomogeneous Markov chain is weakly ergodic if for all t_0,

$$\lim_{t\to\infty} \sup_{v_0^1, v_0^2} |v^1(t) - v^2(t)| = 0,$$

where v_0^1 and v_0^2 are two different initial state probability vectors at time t_0.

Lemma 3.1: [17] A time-inhomogeneous Markov chain is weakly ergodic if and only if there is a strictly increasing sequence of positive integers $\{k_i\}, i = 0, 1, \cdots$ such that

$$\sum_{i=0}^{\infty} [1 - \lambda(P(k_i, k_{i+1}))] = \infty.$$

Note that weak ergodicity does not imply the existence of limits of vectors $v^1(t)$ and $v^2(t)$ but only a tendency towards equality of the rows of $P(t_0, t)$.

Definition 2: [17] A time-inhomogeneous Markov chain is strongly ergodic if there exists a nonnegative vector q satisfying $|q| = 1$ such that for all t_0

$$\lim_{t\to\infty} \sup_{v_0} |v(t) - q| = 0.$$

We now construct a Markov chain to describe the evolution of the configuration of a group of m sensors on the given region $\mathscr{S}$ modeled as a grid. Suppose the total number of grid nodes in $\mathscr{S}$ is n. Each state v represents a configuration of the m sensors in the grid. Thus, totally there are $|\mathscr{V}| = P_n^m = \frac{n!}{(n-m)!m!}$ states. We denote $\mathscr{V}$ the set of all the states.

In what follows, let states (configurations) $u, v \in \mathscr{V}$ correspond to position vectors z and z' of the m sensors. Denote $v \in N_{u_i}$ as the set of configurations related to configuration u that u can be updated to v by moving sensor i one step. The cardinality of the set N_{u_i} is denoted n_{u_i}. Define $\mathscr{V}^\star$ the set of globally optimal configurations, i.e., $\mathscr{V}^\star \triangleq \{u \in \mathscr{V} \,|\, \mathscr{J}(z, \overline{V}(z)) \leq \mathscr{J}(z', \overline{V}(z'))\ \forall v \in \mathscr{V}\}$. The transition matrix when sensor i moves at $t = lm + i$, $l = 1, 2\cdots$ can be written as

$$P_{uv}(i(t)) = \begin{cases} 0 & \text{if } v \notin N_{u_{i(t)}} \cup \{u\}, \\ \frac{1}{n_{u_{i(t)}}} \exp\frac{-[\delta \mathscr{J}_i]^+}{\alpha(t)} & \text{if } v \in N_{u_{i(t)}}, \\ 1 - \sum\limits_{w\neq u} P_{uw}(i(t)) & \text{if } v = u. \end{cases} \tag{5}$$

Proof of Theorem 1: We divide our proof into two parts: weak ergodicity and globally optimal convergence.

Firstly, we show that the time-inhomogeneous Markov chain is weakly ergodic. As we have assumed $n_{u_{i(t)}} \leq 4$ and $D = \max_i \max_z |\delta \mathscr{J}_i|$, then from (5) it is obtained that the transition probability from u to v at time t is greater than $\frac{1}{4}\exp\frac{-D}{\alpha(t)}$ for $u, v \in \mathscr{V}$ and $v \in N_{u_{i(t)}}$.

When considering only the ith sensor moving, the transition probability from configuration $u \in \mathscr{V}^\star$ to itself is monotonically increasing with increasing t, while the transition probabilities from configuration u to neighbors with higher costs are monotonically decreasing and converging to zero. Hence, there exists t' such that when $t \geq (t'-1)rm$, the transition probability from configuration u to itself is greater than $\frac{1}{4}\exp\frac{-D}{\alpha(t)}$ when only sensor i moves.

When $t \geq (t'-1)rm$, it is easily obtained by less than rm times updating that any two configurations u_1 and u_2 can reach the same configuration $u^\star \in \mathscr{V}^\star$ satisfying

$$\begin{aligned} P_{u_j u^\star}(t, t+rm) &\geq \prod_{h=t}^{t+rm} [(\frac{1}{4})\exp(\frac{-D}{\alpha(h)})] \\ &\geq (\frac{1}{4})^{rm} \exp(\frac{-rmD}{\alpha(t)}), \end{aligned} \tag{6}$$

where $j = 1, 2$.

So we have the coefficient of ergodicity of transition matrix $P(trm,(t+1)rm)$ when $t \geq t'$ as

$$\lambda[P(trm,(t+1)rm)] \leq 1-\min_{u_1\neq u_2}\sum_{u^\circ}\min(P_{u_1u},P_{u_2u}) \leq 1-(\frac{1}{4})^{rm}\exp(\frac{-rmD}{\alpha(trm)}). \quad (7)$$

As the term of time-varying $\alpha(t) \geq \frac{rmD}{\ln(t+1)}$ in (4b), we can obtain from (7) the following inequality

$$\sum_{i=0}^{\infty}[1-\lambda(P(k_i,k_{i+1}))] \geq \sum_{t=t'}^{\infty}(\frac{1}{4})^{rm}\exp(\frac{-rmD}{\alpha(trm)}) \geq (\frac{1}{4})^{rm}\sum_{t=t'}^{\infty}\frac{1}{trm+1}.$$

Obviously, the sequence $\sum_{t=t'}^{\infty}\frac{1}{trm+1}$ is divergent and the limit is ∞. So by applying lemma 4.1 we obtain the time-inhomogeneous Markov chain is weakly ergodic. It means for any two initial distribution, they tend to close each other.

Secondly, we prove that the Markov chain is also strongly ergodic and converges to globally optimal configurations.

We define m quasi-stationary probability distributions $\{\pi(i(t)),\ i \in 1,2,\cdots,m\}$ with component

$$\pi_u(i(t)) = \frac{n_{u_{i(t)}}\exp[\frac{-\mathscr{J}(z,\overline{V}(z))}{\alpha(t)}]}{\sum_{v\in\mathscr{V}} n_{v_{i(t)}}\exp[\frac{-\mathscr{J}(z',\overline{V}(z'))}{\alpha(t)}]}. \quad (8)$$

Then $|\pi(i(t))| = 1$ holds. When $t = lm+i,\ l = 1,2,\cdots,$ we have that

$$\frac{\pi_v(i(t))}{\pi_u(i(t))} = \frac{n_{v_{i(t)}}}{n_{u_{i(t)}}}\exp[\frac{\mathscr{J}(z,\overline{V}(z))-\mathscr{J}(z',\overline{V}(z'))}{\alpha(t)}] = \frac{P_{uv}(i(t))}{P_{vu}(i(t))}$$

for $u \in N_{v_{i(t)}}$ or $u = v$ in $\mathscr{V}$. So we get

$$\pi_u(i(t))P_{uv}(i(t)) = \pi_v(i(t))P_{vu}(i(t)).$$

Thus,

$$\sum_{u\in\mathscr{V}}\pi_u(i(t))P_{uv}(i(t)) = \sum_{u\in\mathscr{V}}\pi_v(i(t))P_{vu}(i(t)) = \pi_v(i(t)),$$

which means

$$\pi(i(t))P(i(t)) = \pi(i(t)). \quad (9)$$

As we have mentioned that for any $u \in \mathscr{V}^\star$ and $v \in \mathscr{V}/\mathscr{V}^\star$,

$$\frac{\pi_u(i(t))}{\pi_v(i(t))} = \frac{n_{u_{i(t)}}}{n_{v_{i(t)}}}\exp[\frac{-(\mathscr{J}(z,\overline{V}(z))-\mathscr{J}(z',\overline{V}(z')))}{\alpha(t)}].$$

Since $\mathscr{J}(z,\overline{V}(z)) < \mathscr{J}(z',\overline{V}(z'))$ and $\alpha(t) \to 0$ when time goes to infinity, it follows that $\frac{\pi_u(i(t))}{\pi_v(i(t))} \leq \frac{\pi_u(i(t+1))}{\pi_v(i(t+1))}$ and

$$\lim_{t\to\infty}\frac{\pi_u(i(t))}{\pi_v(i(t))} = \infty.$$

Combining $\pi_v(i(t))$ and $\pi_u(i(t))$ belong to $[0,1]$, we have $\lim_{t\to\infty}\pi_v(i(t)) = 0$. On the other hand, since the sensors are distributed as uniformly as possible and away from the obstacle, we can always safely assume that $n_{u_{i(t)}} = n_{u_{j(t)}} = 4$ for $i \neq j$ and $u \in \mathscr{V}^\star$, which means $\lim_{t\to\infty}\pi_u(i(t)) = \lim_{t\to\infty}\pi_u(j(t))$.

In conclusion, $\lim_{t\to\infty}\pi_w(i(t)) = \lim_{t\to\infty}\pi_w(j(t))$ for any $w \in \mathscr{V}$ and $i \neq j$, i.e., $e^\star = \lim_{t\to\infty}\pi(t)$ is a globally optimal quasi-stationary probability distribution.

As we have shown that the time-inhomogeneous Markov chain here is weakly ergodic, according to Definition 4.1, when we choose $v^2(t) = \pi(t)$ for any two different initial state probability vectors v_0^1 and v_0^2, it is obtained that $\lim_{t\to\infty}v^1(t) = e^\star$. Thus, applying Definition 4.2, it concludes that $\lim_{t\to\infty}\sup_{v_0}|v(t)-e^\star| = 0$, i.e., the Markov chain is also strongly ergodic and the m sensors eventually converge to a globally optimal configuration minimizing the coverage cost function with probability one. ■

Remark 3.3: The monotonicity of $\alpha(t)$ decreasing to zero ensures that each sensor gets less and less opportunity to enter bad locations, while the condition (4b) guarantees the convergence to a globally optimal configuration rather than being trapped in a locally optimal one.

IV. Simulation results

In this section, we present several simulation results to validate the algorithm. For simplicity, we choose $f(\|p(v)-z_i\|) = \|p(v)-z_i\|$ and $\phi(p(v)) = 1$.

We first give a simulation to illustrate that our algorithm can make the sensors get out of locally optimal configurations. The bounded regions $\mathscr{S}$ is a rectangle with the horizontal width longer than the vertical width. We initially set two sensors in the centers of the horizontal Voronoi partition, which is a locally optimal configuration for the cost function

$$\mathscr{J}(z,\overline{V}(z)) = \sum_{i=1}^{m}\sum_{v\in V_i}\| p(v)-z_i \|,$$

since the cost function will get increased whenever any sensor moves. However, the cost at current configuration is larger than the cost at the globally optimal configuration that the two sensors lie in the centers of the vertical Voronoi partition.

For this setup, $m = 2$ and $rD = 5$. We choose $\alpha(t) = \frac{10}{ln(t+1)}$. Applying the algorithm, then it is obtained that the two sensors converge to the centers of the vertical Voronoi partition that is a globally optimal configuration (see Fig. 6).

Moreover, we increase the sensor number to four in a square area as in Fig. 7. The original configuration in Fig. 7 is a locally optimal solution. After applying the algorithm, the four sensors converge to another one, which is the globally optimal configuration (Fig. 8).

To verify our algorithm in nonconvex environments, we consider five sensors in three letter-like regions: C, H and

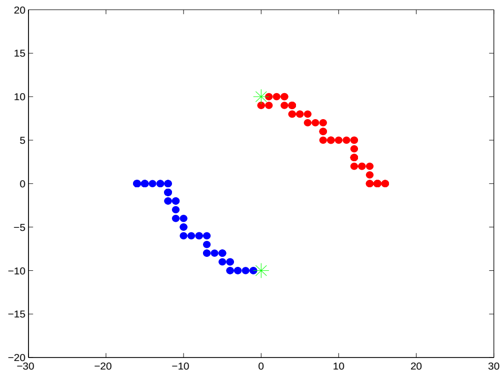

Figure 6: Two sensors jump out of locally optimal configuration, at the centers of the horizontal Voronoi partition denoted by $*$, and converge to the globally optimal one at the centers of the vertical Voronoi partition.

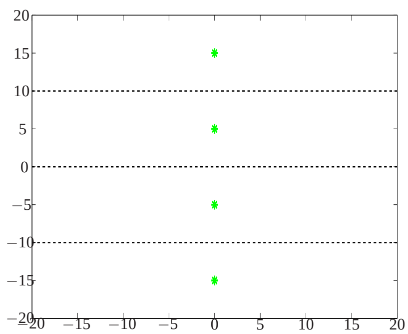

Figure 7: Four sensors are at the centers of the horizontal Voronoi partition denoted by $*$, constructing a locally optimal configuration.

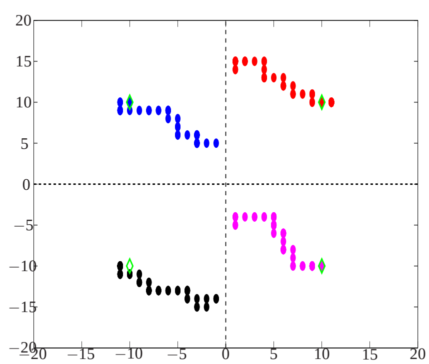

Figure 8: Four sensors jump out of locally optimal configuration, and converge to the globally optimal one. $\Diamond$ in the figure represents the final configuration they achieve.

N. The sensors are initially placed at random spots in the grid in the given regions as shown in Fig. 9 (a), (c) and

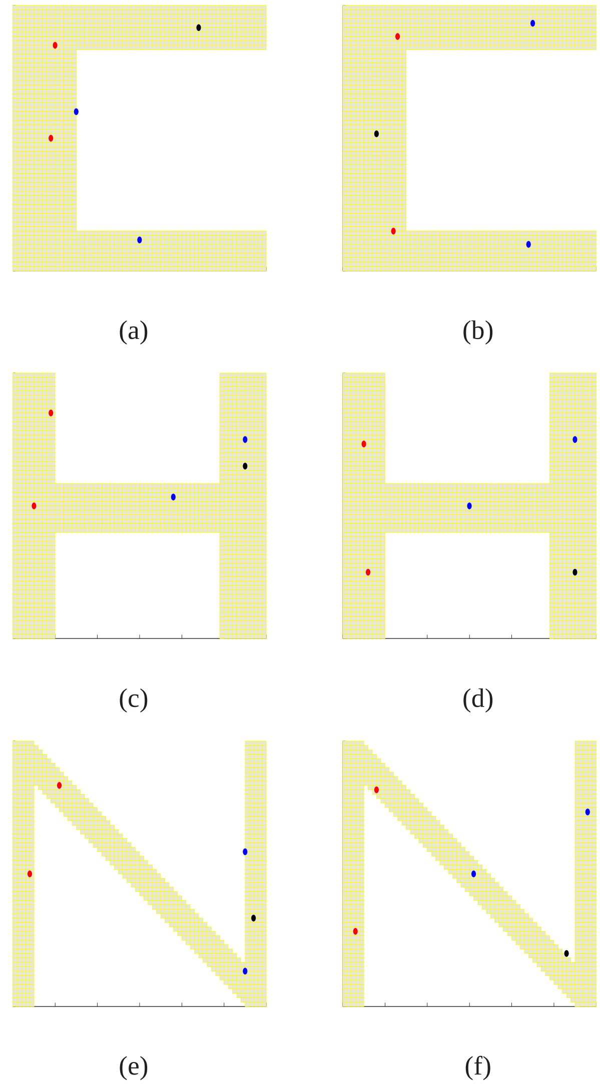

Figure 9: (a), (c), and (e): Initial configurations of 5 sensors in C, H, and N like regions. (b), (d) and (f): Final configurations.

(e). Here we again let

$$\mathscr{H}_i(z_i, \overline{V}(z)) = \sum_{v \in V_i} \| p(v) - z_i \| .$$

For this set up, $m = 5$ and $rD = 20$. We also choose $\alpha(t) = \frac{100}{ln(t+1)}$. The final configurations after applying the algorithm are depicted in Fig. 9 (b), (d) and (f), respectively. From the simulation results, we observe that the sensors asymptotically position themselves at optimal locations for coverage of the given region.

In addition, we extend the sensor number to 10 with the same setup and show how the sensors converge to optimal configuration with obstacles that may block sensors. The simulation results are shown in Fig. 10 and Fig. 11.

At last, a simulation with nonuniform density function ϕ is given. The density of the darker (pink) region on the left is twice of the right one. The 5 sensor nodes are initially distributed in Fig. 12 and converge to the globally optimal configuration recorded in Fig. 13, from which it

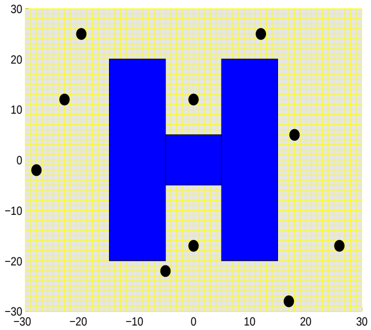

Figure 10: Initial configuration of 10 sensors in a square region with (blue H shaped area) inside.

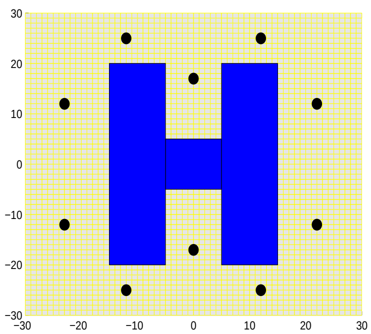

Figure 11: Final configuration of 10 sensors achieved after applying our algorithm.

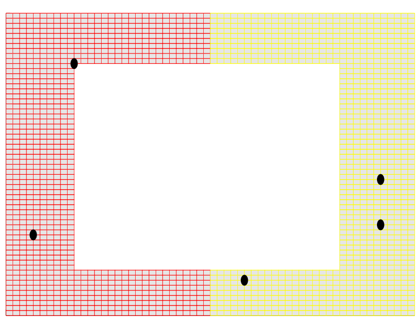

Figure 12: Initial configuration of 5 sensors in a nonuniform density region. The density of the darker (pink) region on the left is twice of the right one.

can be noticed that the sensor nodes are denser in the darker (pink) region.

V. Conclusion

This paper investigates a distributed coverage algorithm which results in global optimality for the traditional coverage problem. For a given (convex or nonconvex) bounded region, each mobile sensor updates its location to one of its adjacent positions with probability q, which

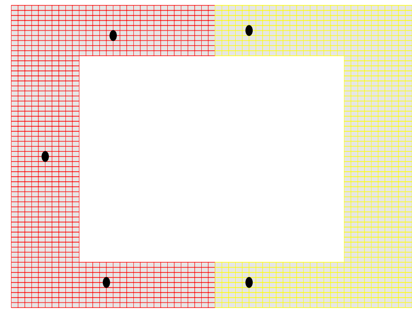

Figure 13: Final configuration of 5 sensors achieved after applying our algorithm, from which it can be seen that the sensor nodes are denser in the lighter (pink) region.

only depends on the locations of its neighbors and a time-varying parameter $\alpha(t)$. It is shown that by choosing appropriate $\alpha(t)$, the sensors are guaranteed to converge to a globally optimal configuration with probability one. As a future work, an important issue is to improve the convergence rate of the algorithm. Simultaneously updating the positions of sensors might be a direction to go.

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