Power and Energy Efficiency Analysis of Linearly Equalized Pulse Amplitude Modulated Cable Transmission

Christoph Lange [1,*] and Andreas Ahrens [2]

[1] School of Engineering−Energy and Information, Hochschule für Technik und Wirtschaft (HTW) Berlin, (University of Applied Sciences), Wilhelminenhofstraße 75A, 12459 Berlin, Germany

[2] Faculty of Engineering, Hochschule Wismar (University of Applied Sciences: Technology, Business and Design), Philipp-Müller-Straße 14, 23966 Wismar, Germany

Email: christoph.lange@htw-berlin.de (C.L.); andreas.ahrens@hs-wismar.de (A.A.)

*Corresponding author

***Abstract*—Contemporary telecommunication networks consume significant amounts of electricity. For this reason, energy-efficient systems have to be used for establishing such networks. Wired transmission and networking technologies form the basis of both, fixed and wireless networks. In wired access networks, among others, copper lines are exploited for high-performance data transfer using existing network infrastructure. The power demand and the energy efficiency of communication systems can be characterized by metrics such as transmit power or energy per bit, respectively. Often, transmission links are sought that show minimum power or energy demand at preset throughput and transmission quality. For the example of a linearly equalized pulse amplitude modulated transmission system operating via copper cable it is analyzed how constellation size and allocation of the equalization to transmitter and receiver can be utilized to minimize the necessary power or energy demand at constant bit rate and fixed error probability. First, the optimization of the constellation size leads to minima in the energy-related metrics depending on the combination of the given throughput and the band limitation imposed by the copper line. Second, the equalizer optimization shows that an equal segmentation of the equalizing characteristic is optimum, yielding minimum power or energy metrics for constant throughput and transmission quality. The results achieved reveal that an optimization of constellation size and equalizer segmentation yield clear energy savings with respect to conventional wired copper-cable communication systems operating in the baseband—supporting the energy efficient use of existing copper lines for highspeed data transmission in telecommunication networks.**

***Keywords*—digital baseband transmission, transmit power, energy efficiency, copper cable, constellation size, linear equalization, optimization**

I. Introduction

Modern telecommunication networks are significant electricity consumers [1, 2]. Often, they are large, frequently country-wide, communication infrastructures showing a substantial total electrical energy demand: Telecommunication operator networks typically consume approximately one percent of a national economy’s electricity [3], summing up to several hundreds of terawatt hours per year globally [2]. Therefore, their sustainability and their energy demand have been of notable interest over the last few decades before the background of public awareness regarding sustainability and scarce energy resources [4, 5], This article is an expanded version of a conference paper [1].

New networks are deployed and existing telecommunication networks are continuously extended for increasing their coverage and improving their performance capability, because communication services are indispensable and important in the modern society. The frequency and intensity of communication services’ usage is rising in both, the private and the business sector. In order to limit the networks’ aggregate energy demand, energy-efficient communication systems have to be used. The awareness concerning the sustainability of telecommunication networks stimulated related research and development activities in the past few decades. The research targeted at understanding [2, 6–8] and at limiting—or reducing—the electricity demand of telecommunication networks and transmission systems [9–26].

Twisted-pair copper cables, originally stemming from the era of analogue telephony, are largely used for broadband digital information transmission to small and medium businesses, apartment buildings and individual homes in wired access networks, in particular where fiber-based optical access is not yet on-hand widely at decent retail prices. Today, usually multicarrier techniques [27–31] are applied for high-speed information transfer in copper-based access networks. Wired access lines also form the basis for inhouse wireless networks by ensuring their connection to the telecommunication

Manuscript received April 1, 2026; revised April 28, 2026; accepted May 11, 2026; published September 18, 2026.

network. Fixed access networks usually are formed by a vast number of lines and so the operation of the copper-based access networks exhibit a considerable share in the aggregate telecommunication networks' energy consumption [6].

Therefore, it is of interest how copper-based cable transmission systems can be designed and operated energy efficiently. Degrees of freedom should be detected that allow for optimizing the transmission system during the design process in such a way that its energy or power demand can be minimized for given constraints such as bit-error probability and bit rate. Since the transmit power and the energy per bit are appropriate and meaningful indicators—or metrics—for the energy demand of transmission systems, in this article they are adopted as measures for the transmission's energy (or power) efficiency. When widening the view to entire telecommunication networks, this transmission energy used for the bit transfer constitutes a bottom limit to the total energy consumed in a network [1], since not only data traffic has to be carried from one network node to another, but also other network functions such as switching, routing, buffering, and network control have to be performed, they exhibit additional energy demands [32]. The analyses in this article focus on the task of data transmission; the effects of other network functions on networks' energy efficiency have been investigated in e.g. [16] and furthermore remain an interesting field of future research work.

The concrete problem tackled in the present article is as follows. A transmission link between a transmitter and a receiver is to be set up ensuring a certain throughput (bit rate) at a predefined quality (bit-error probability) via a prespecified transmission channel with minimum power and energy usage. Such a problem is typical when planning, deploying or extending networks energy efficiently. A generic model of a transmission link is shown in Fig. 1. In general, it consists of an information source, a transmitter, a transmission channel, a receiver, and an information sink [33, 34].

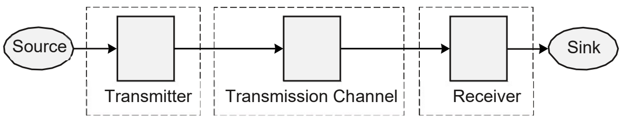

Fig. 1. Generic model of a transmission link between an information source and a related sink.

Since general insights on the introduced transmission and optimization problem are targeted by means of concise and comprehensible analytical calculations, a baseband system operating over a copper-line channel is investigated: The results are—with some related effort—transferrable to multicarrier modulation formats that are widely used in copper-based transmission, e.g., [28–31].

In this setup, the bit rate of a communication link is predefined as a required throughput and a preset quality, a bit-error probability, is needed for the transmission link to operate reliably. Typically, other channel and disturbance characteristics, such as bandwidth and noise properties, are also given.

In Pulse Amplitude Modulation (PAM) transmission systems the number of signaling levels, often referred to as constellation size of the modulation or transmission format, is a degree of freedom that can be optimized. In Refs. [35, 36] the energy-related metrics of a pulse amplitude modulated Additive White Gaussian Noise (AWGN) copper-cable baseband system with complete linear equalization in the receiver have been studied in regard to the constellation size. The results have shown that constellation size adaptations towards optimum values yield significant energy-per-bit savings.

Based on the results presented in Refs. [35, 36], this article extends the investigations by considering a generalized wired transmission model in which equalization is subdivided into transmitter- and receiver-side parts. As a foundation, firstly the transmit power and the energy per bit transmitted are studied for linearly equalized baseband cable communication systems with complete equalization in the receiver as functions of the number of signaling levels: Then the constellation size can (and should) be optimized in order to identify at which constellation size a minimal power or energy is required, respectively. It should be noted that, in contrast to the results in [35, 36], results for different, but network-typical, cable lengths are now presented. Beyond that, the equalization of the channel transfer function can be distributed to transmitter and receiver: The shares of equalization implemented in the transmitter and the receiver are another degree of freedom. The optimization of this partitioning results in a splitting of the equalization in such a way that the power or energy indicators, respectively, become minimal. In doing so, the power or energy demand can be reduced still more—beyond the case when only the constellation size is optimized. Again, results for different cable lengths typical in access networks are given.

The present article is structured in the following way. In Section II the energy-related metrics transmit power and energy per bit for a pulse amplitude modulated copper-cable baseband communication system with full linear equalization in the receiver are determined for preset bit-error probability and bit rate. Results are given as functions of the number of signaling levels, where new material is contained particularly in Section II as compared to Ref. [1]. In Section III the wired transmission model is generalized by splitting the equalization to the transmitters and receiver's side: Transmit power and energy per bit are derived for this enhanced transmission model and related results are presented as functions of a defined equalization portioning parameter, again for given bit-error probability and bit rate; new material (compared to Ref. [1]) appears especially in Section III. Also, achievable savings are shown when utilizing the mentioned degrees of freedom for system optimization. In Section IV important findings are summarized and some concluding remarks are provided. In Section V a brief outlook on further work in the context of the present article's topic is given.

II. Power and Energy Efficiency of Copper-Cable Transmission with Linear Receiver-Side Equalization

A. Model of the Wired Copper-based Transmission System

Fig. 2 shows the relevant transmission model. A digital data source generates and emits a sequence of bits for transmission over the wired channel formed by a copper cable. In the transmitter, a multilevel coder converts the binary data stream with a bit rate of f_{B} to a sequence of data symbols with s amplitude levels and a symbol rate of $f_{\mathrm{T}} = f_{\mathrm{B}}/\mathrm{ld}(s)$.[1] The transmit filter with transfer function $G_{\mathrm{s}}(f)$ thereof creates a pulse amplitude modulated time signal with s signaling levels, afterwards this signal is transmitted via the copper-line cable channel [1, 36].

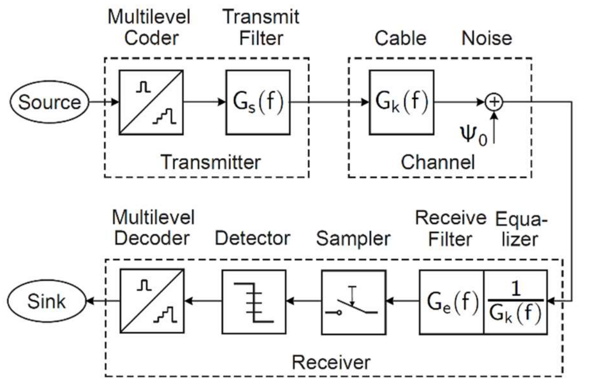

Fig. 2. Conventional copper-cable communication system with complete receiver-side equalization (figure reused from Ref. [1], with permission).

The channel is characterized by the copper cable's transfer function [37, 38]

$$G_{\mathrm{k}}(f) = \mathrm{e}^{-\ell\sqrt{\mathrm{j}\frac{f}{f_0}}}. \tag{1}$$

Here, ℓ represents the length of the cable (in km) and f_0 denotes a cable-specific constant, the characteristic cable frequency (in MHz × km^2): It depends primarily on the copper wire diameter, the cable construction, and on the material the wires are insulated with. The transfer function Eq. (1) usually is deduced by using the theory of transmission lines [38, 39] for the RC range[2]—describing a low-pass characteristic of the copper cable in the frequency range typically relevant for digital signal transmission. Noise disturbances are accounted for by AWGN with a power spectral density Ψ_0. Crosstalk impairments are not taken into account within the investigations throughout this article.

In Fig. 3 the transfer function's magnitude—the amplitude frequency response—of a twisted-pair copper line with a wire diameter of 0.6 mm is depicted for three different cable lengths: The low-pass properties of the channel copper line become evident. In addition, it is recognizable that a longer copper cable (e.g., ℓ = 5 km) exhibits a more pronounced low-pass effect than a shorter copper line (e.g., ℓ = 0.5 km).

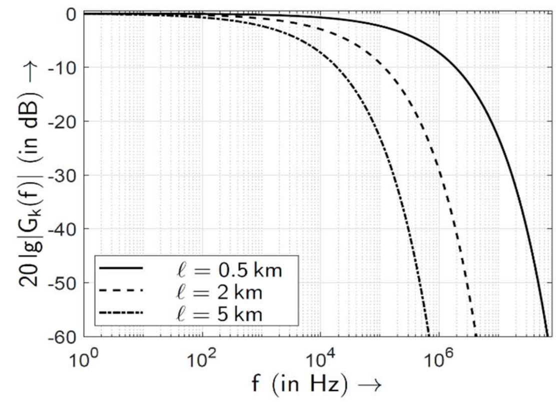

Fig. 3. Cable transfer function (amplitude frequency response) of a copper line with a wire diameter of 0.6 mm (f_0 = 0.178 MHz × km^2) for three cable lengths (figure reused from Ref. [1] and modified, with permission).

The cable transmission channel in Fig. 2 induces signal distortions causing Inter-Symbol Interference (ISI) [33, 34], which are a possible reason for bit errors. Therefore, equalizers are employed in order to compensate the distorting impact of the channel on the useful signal. Therefore, an equalizer has to invert the channel transfer function in the signal path in one way or another. The cable transmission model depicted in Fig. 2 comprises a conventional baseband cable transmission system where the cable's distorting impact is completely and linearly equalized by the inverse of the cable transfer function in the equalizer block $1/G_{\mathrm{k}}(f)$ at the receiver side. When considering the useful signal only, then the signal distortions caused by the channel $G_{\mathrm{k}}(f)$ are completely cancelled out or forced to zero. There are a lot of equalization methods and techniques established, often using digital signal processing methods, e.g., [33, 34, 40, 41]. For example, Decision Feedback Equalizers (DFE) [34] or Tomlinson Harashima Precoding (THP) techniques [41] can show superior equalization performance and lead to improved performance capabilities of transmission systems in particular at high signal-to-noise ratios when there is little or no bit-error propagation. Nevertheless, a zero-forcing linear equalizer like described above is presumed in order to obtain analytical results and insights on functional interdependencies by establishing a worst-case bound with regard to equalizer choice using a handleable and concise mathematical framework which is a main target of this article.

Usually, the receive filter transfer function $G_{\mathrm{e}}(f)$ is incorporated in a joint component together with the

[1] The notation $\mathrm{ld}(x)$ denotes the *dyadic* or binary logarithm: $\mathrm{ld}(x) = \log_2(x)$.

[2] The RC range is the frequency range where the resistance R and the capacitance C dominate the electrical behaviour of a transmission line [39]. This frequency range is especially important for digital signal transmission over copper wires.

doi:10.12720/jcm.21.5.617-630

equalizer in such types of receivers. At the equalizer-receive filter output a symbol rate sampling is performed and the detector decides on the received signal amplitude by thresholding. The multilevel decoder maps the detected symbols to bits and provides the information sink with a sequence of binary data.

Both, the transmit filter $G_\mathrm{s}(f)$ and the receive filter $G_\mathrm{e}(f)$ are square-root raised cosine filters [33] with identical roll-off factor r, respectively. The resulting received signal at the detector input is free from intersymbol interference since the cascade of transmit and receive filter fulfils the first Nyquist criterion [33] and, furthermore, the cable transfer function is completely equalized.

The signal-to-noise ratio:

$$\varrho = \frac{(\text{Half Vertical Eye Opening})^2}{\text{Noise Power}} = \frac{U_\mathrm{A}^2}{U_\mathrm{R}^2} \tag{2}$$

at the detector input is utilized for evaluating the transmission quality [1, 35, 36, 38]. The quantity U_s describes the half-level transmit amplitude at the transmitter output: Then the distance between directly neighboring transmit signal amplitude levels amounts to $2U_\mathrm{s}$. Since the first Nyquist criterion is satisfied over the cascade of transmit and receive filter the half vertical eye-opening U_A equals the half-level transmit amplitude U_s and ultimately $U_\mathrm{A} = U_\mathrm{s}$ is valid [1, 35, 36].

The noise with power spectral density Ψ_0 passes through the equalizer-and-receive filter transfer function $G_\mathrm{e}(f)/G_\mathrm{k}(f)$ (see Fig. 2); The noise power at the detector input then results in:

$$U_\mathrm{R}^2 = \Psi_0 \int_{-\infty}^{+\infty} \left|\frac{G_\mathrm{e}(f)}{G_\mathrm{k}(f)}\right|^2 \mathrm{d}f = 2\Psi_0 \int_0^{\infty} \left|\frac{G_\mathrm{e}(f)}{G_\mathrm{k}(f)}\right|^2 \mathrm{d}f. \tag{3}$$

Depending on the copper line's transmission properties, the cable transfer function Eq. (1) causes a noise enhancement that is typical for linear equalization—as the noise is amplified by the term $1/G_\mathrm{k}(f)$ in Eq. (3) [40].

When presuming gray coding [33], the bit-error probability as the targeted digital transmission quality criterion of the s-level baseband transmission with Eq. (2) is obtained as[3]:

$$P_\mathrm{b} = \frac{s-1}{s\,\mathrm{ld}(s)}\left[1 - \mathrm{erf}\left(\sqrt{\frac{\varrho}{2}}\right)\right]. \tag{4}$$

The bit-error probability P_b, derived in Eq. (4), is a monotone function of the signal-to-noise ratio ϱ, defined in Eq. (2), and furthermore depends on the constellation size s [36, 38].

B. Calculation of Transmit Power and Energy per Bit

As energy-related indicators of the wired baseband transmission system the transmit power P_s and the energy E_b per bit are calculated—taking the constraints preset bit rate f_B and targeted bit-error probability P_b into account [1, 35].

For a redundancy-free and uniformly distributed s-ary random baseband signal at the output of a square root raised cosine transmit filter the average transmit power[4] is given as [38]:

$$P_\mathrm{s} = \frac{U_\mathrm{s}^2}{3}(s^2 - 1). \tag{5}$$

The overall channel is supposed to be free from intersymbol interference by choice of transmit and receive filter combined with the complete compensation of the channel's distorting impact. Then the half vertical eye opening becomes $U_\mathrm{A} = U_\mathrm{s}$ and the signal-to-noise ratio results in (see Refs. [1, 36]):

$$\varrho = \frac{U_\mathrm{s}^2}{U_\mathrm{R}^2}. \tag{6}$$

The transmit power Eq. (5) can be reformulated by solving Eq. (6) for U_s^2 and subsequently inserting the outcome in Eq. (5). Then, the transmit power can be expressed by:

$$P_\mathrm{s} = \frac{U_\mathrm{R}^2 \times \varrho}{3}(s^2 - 1). \tag{7}$$

As a relationship of the transmit power P_s depending on the bit-error probability P_b and on the bit rate f_B is sought, now Eq. (4) is solved for the signal-to-noise ratio ϱ:

$$\varrho = 2\left[\mathrm{erf}^{-1}\left(1 - \frac{s\,\mathrm{ld}(s)}{s-1}P_\mathrm{b}\right)\right]^2. \tag{8}$$

Then, in a last step Eqs. (7) and (8) are combined. The transmit power P_s for a given bit-error probability P_b and a preset bit rate f_B finally yields [1, 36]

$$P_\mathrm{s} = \frac{(s^2-1)}{3} \times U_\mathrm{R}^2 \times 2\left[\mathrm{erf}^{-1}\left(1 - \frac{s\,\mathrm{ld}(s)}{s-1}P_\mathrm{b}\right)\right]^2. \tag{9}$$

The transmit power Eq. (9) is a function of the number of signaling levels s, which is a degree of freedom in the transmission system's design and thus subject to optimization when aiming at minimum transmit power P_s. Furthermore, the transmit power Eq. (9) depends on the required transmission quality, manifesting itself in the predefined bit-error probability P_b. The noise power U_R^2 is obtained by numerical integration of Eq. (3). Via the symbol rate f_T the noise power is a function of the given bit rate f_B and, in turn, it depends on the cable parameters (f_0 and ℓ, via the equalizer) as well as on the intensity of the noise disturbance (Ψ_0).

The energy per transmitted bit:

$$E_\mathrm{b} = \frac{P_\mathrm{s}}{f_\mathrm{B}} \tag{10}$$

[3] The notation $\mathrm{erf}(x)$ describes the Gaussian error function [33, 42, 43].
[4] In this article a signal- or system-theoretic power with dimension. (voltage)^2 and unit V^2 is utilized. At a real, constant resistance R this system-theoretic power can be converted into a physical power with the unit W by multiplying it by the factor $1/R$ [38].

is an accepted and meaningful energy-related technical metric in digital communications [15, 16]. With Eq. (9) the energy per bit Eq. (10) can be written as [1, 36]

$$P_\mathrm{s} = \frac{(s^2-1)}{3} \times \frac{U_\mathrm{R}^2}{f_\mathrm{B}} \times 2\left[\mathrm{erf}^{-1}\left(1 - \frac{s\,\mathrm{ld}(s)}{s-1}P_\mathrm{b}\right)\right]^2. \quad (11)$$

C. Numerical Results for the Energy-Related Metrics

Numerical results for the defined metrics (P_s and E_b) are shown based on a copper-line channel with a wire diameter of 0.6 mm (characteristic cable frequency f_0 = 0.178 MHz × km²). This exemplarily represents a typical copper line in fixed access networks. The numerical preconditions for the bit-error probability P_b and the noise power spectral density Ψ_0 are detailed in the respective figure captions. They are based on pertinent values for copper-line communication systems. For the roll-off factors of the transmit and receive filters (each being a square-root raised cosine filter) a typical value of r = 0.5 is assumed. The constellation size s is a degree of freedom. Therefore, the outcomes are displayed depending on s [1].

Fig. 4 shows the transmit power P_s depending on the constellation size s for different bit rates f_B transferred over a copper line of length ℓ = 2 km. Such a transmission medium is typical and on average representative for copper cables that connect individual homes or multi-dwelling buildings in a Fiber-To-The-Exchange (FTTEx) access network structure [44] to a traditional central office (a former telephone network exchange) as access node.[5]

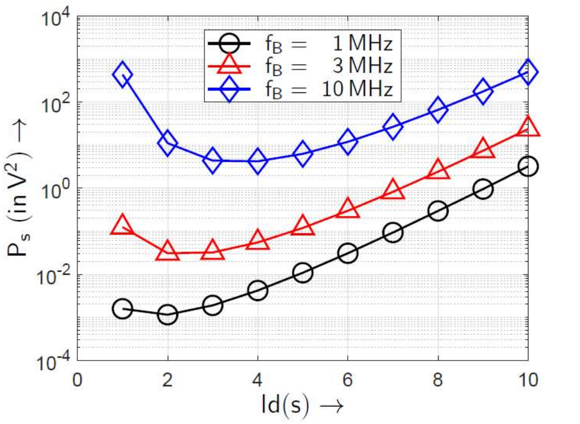

Fig. 4. Transmit power P_s as a function of the constellation size s for several bit rates f_B (ℓ = 2 km, $P_\mathrm{b} = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V²/Hz) (Figure reused from Ref. [1], with permission).

There is a minimum of the transmit power P_s at an optimum constellation size s_opt, depending on the bit rate f_B. With rising number of signaling levels s—seen from s_opt, the required transmit power for fixed transmission quality increases, because of higher-order modulation. When again starting from s_opt, towards smaller constellation sizes s the noise power increases because of extended signal and receive filter bandwidths. Therefore, in those cases higher transmit powers are necessary when aiming at a fixed bit-error probability at each given bit rate [1].

Fig. 5 shows the energy per transmitted bit E_b depending on the constellation size s for identical assumptions as in Fig. 4. The energies per bit E_b exhibit a similar behaviour regarding the number of signalling levels s as the transmit power P_s (in Fig. 4). The reason is that each of the transmit power functions is divided by a constant bit rate f_B (i.e., the transmit power is scaled according to Eq. (10)). However, for each curve the divisor f_B is a fixed but different one. Therefore, the constellation size s may be optimized regarding to minimum transmit power or energy per bit—leading to identical optimal constellation sizes s_opt.

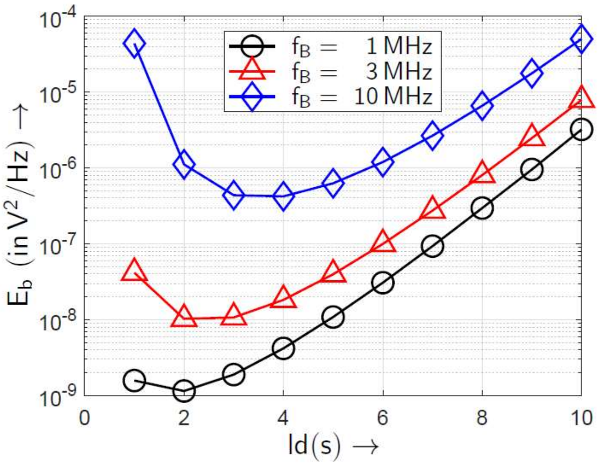

Fig. 5. Energy per bit E_b as a function of the constellation size s for several bit rates f_B (ℓ = 2 km, $P_\mathrm{b} = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V²/Hz) (figure reused from Ref. [1], with permission).

In Table I the optimum constellation sizes s_opt obtained from Figs. 4 and 5 respectively, are compiled.

TABLE I. OPTIMUM VALUES OF THE CONSTELLATION SIZES s_opt FOR SEVERAL FIXED BIT RATES f_B AT A CABLE LENGTH OF ℓ = 2 km

f_B (in MHz)	s_opt
1	$2^2 = 4$
3	$2^2 = 4$
10	$2^4 = 16$

In Fig. 6 the transmit power P_s is displayed as a function of the number s of signalling levels for some bit rates f_B for a shorter copper line of length ℓ = 0.5 km. This length is representative for copper cables that connect detached houses or apartment buildings to access multiplexers placed in street cabinets in a Fiber-to-the-Curb (FTTC) [45] access network (also termed Fiber to the Cabinet (FTTCab) [44]). In FTTC access networks the access multiplexers are usually linked backwards to further telecommunication network segments by optical fibers. Fig. 7 displays the related energy-per-bit curves, also as functions of s. Once more, the energies per bit E_b show a comparable behaviour regarding the constellation size s as seen for the transmit power P_s—both are,

[5] In the diagrams showing a dependency of powers and energies on the constellation size s the quantity $\mathrm{ld}(s)$ is used in order to allow for better perception. Technically this leads to a logarithmic horizontal scale and the quantity $\mathrm{ld}(s)$ represents the number of bits transferred per symbol.

according to Eq. (10), linked by the scaling factor bit rate f_B (constant per each curve).

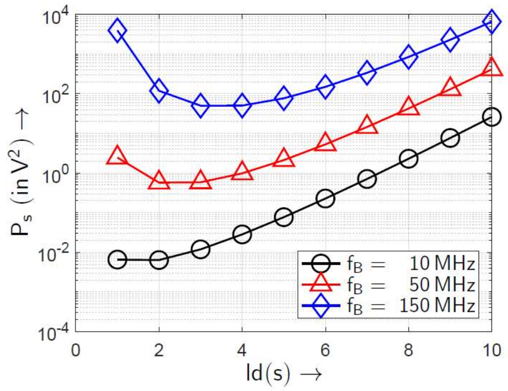

Fig. 6. Transmit power P_s as a function of the constellation size s for several bit rates f_B (ℓ = 0.5 km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V²/Hz) (figure reused from Ref. [1], with permission).

The characteristics of the transmit-power and the energy-per-bit curves for a shorter cable (with ℓ = 0.5 km) are very similar to those in case a longer cable (with ℓ= 2km) is utilized. A difference is evident with a view on the achievable bit rates. Significantly higher bit rates can be realized at similar communication quality on shorter cables. The reason for this behaviour is that the low-pass effect of the copper line is by far less pronounced at shorter cables than at longer cables. Thus, on the shorter cable a higher bandwidth can be used for information transmission in contrast to longer cables [1].

In Table II the optimum constellation sizes s_{opt} obtained from Figs. 6 and 7, respectively, are assembled.

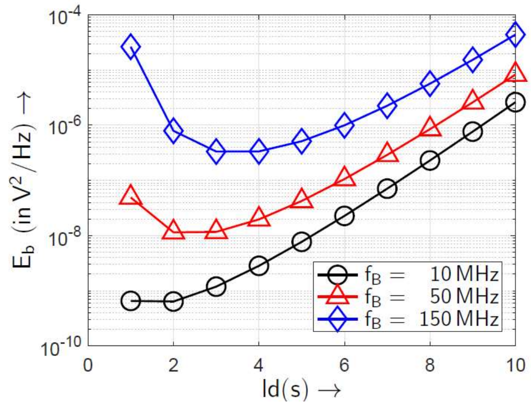

Fig. 7. Energy per bit E_b as a function of the constellation size s for several bit rates f_B (ℓ = 0.5 km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V²/Hz) (figure reused from Ref. [1], with permission).

TABLE II. OPTIMUM VALUES OF THE CONSTELLATION SIZES s_{opt} FOR SEVERAL FIXED BIT RATES f_B AT A CABLE LENGTH OF ℓ = 0.5 km

f_B (in MHz)	s_{opt}
10	$2^2 = 4$
50	$2^2 = 4$
150	$2^3 = 8$

D. Summarization of the Transmit-power and Energy-per-bit Results

Figs 8 and 9 show results achieved for the metrics $P_s(s)$ and $E_b(s)$ when utilizing the copper line of length ℓ = 2 km in the FTTEx network. In Figs.10 and 11 the transmit-power and the energy-per-bit results for a copper cable of length ℓ = 0.5 km are illustrated—in the FTTC network. Now, each of the figures contains a 3D diagram.

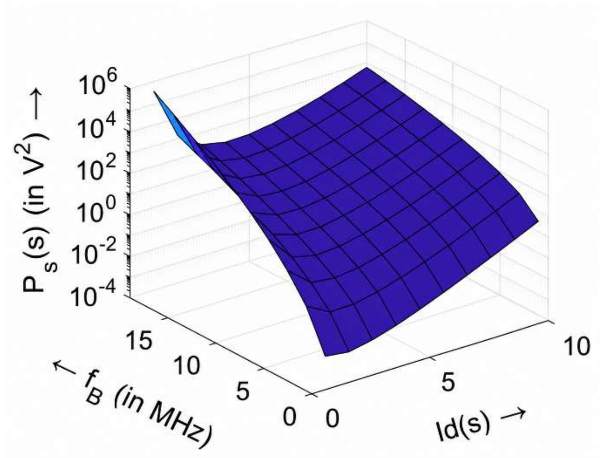

Fig. 8. Transmit power $P_s(s)$ as a function of bit rate f_B and constellation size s (f_0 = 0.178 MHz × km², ℓ = 2 km, $P_b = 1.5 \times 10^{-9}$, $\Psi_0 = 10^{-12}$ V²/Hz).

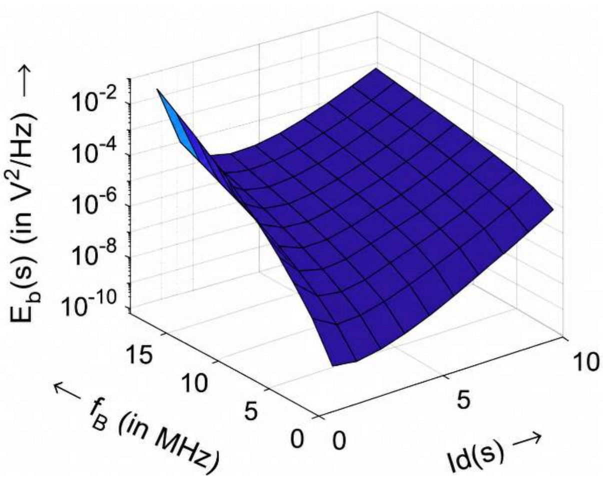

Fig. 9. Energy per bit $E_b(s)$ as a function of bit rate f_B and constellation size s (f_0 = 0.178 MHz × km², ℓ = 2 km, $P_b = 1.5 \times 10^{-9}$, $\Psi_0 = 10^{-12}$ V²/Hz).

The behaviour of the Eqs. (9)−(11) particularly concerning their dependency on the bit rate f_B and on the number of signalling levels s becomes easier visible in 3D: Both metrics, the transmit power $P_s(s)$ and the energy per bit $E_b(s)$, rise with increasing bit rate f_B (at fixed s), respectively. Regarding the constellation size s, both quantities ($P_s(s)$ and $E_b(s)$) have a minimum at an optimal $s = s_{opt}$ (at a preset bit rate f_B). Again, for the FTTC case (ℓ = 0.5 km) higher bit rates are possible than in the FTTEx case (ℓ = 2 km)—however, the principal behaviour is the same for different cable lengths.

In particular, the bit rates in the FTTC case are higher than those in the FTTEx case by roughly a factor of ten—due to the chosen cable lengths. The necessary transmit powers in Figs. 8–10 are comparable with regard to the order of magnitude: Therefore, the energies per bit in the FTTC case in Fig. 11 are lower than those in the FTTEx case in Fig. 9, approximately by a factor of ten, since the transmit powers are divided by higher bit rates in the latter case.

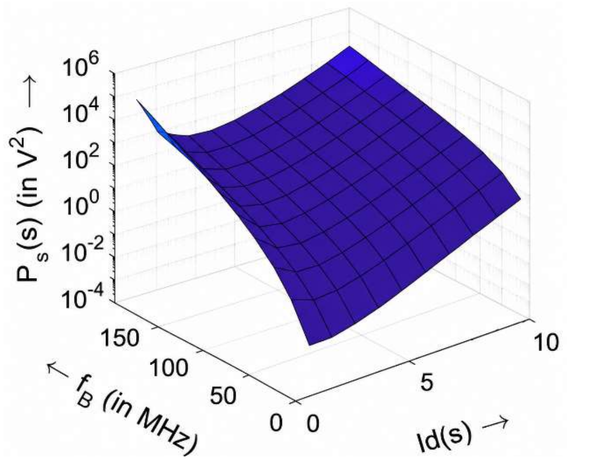

Fig. 10. Transmit power $P_s(s)$ as a function of bit rate f_B and constellation size s (f_0 = 0.178 MHz × km^2, ℓ = 0.5 km, P_b = 1.5 × 10^{-9}, $\Psi_0 = 10^{-12}$ V^2/Hz).

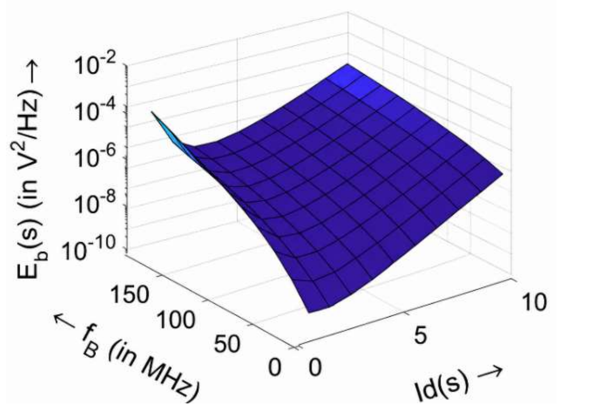

Fig. 11. Energy per bit $E_b(s)$ as a function of bit rate f_B and constellation size s (f_0 = 0.178 MHz × km^2, ℓ = 0.5 km, P_b = 1.5 × 10^{-9}, $\Psi_0 = 10^{-12}$ V^2/Hz).

III. Power and Energy Efficiency of Copper-Cable Transmission with Distributed Linear Equalization

A. Model of the Generalized Cable Transmission System

The linear equalization compensates the distortions caused by the copper-line channel and modelled by the cable transfer function Eq. (1) completely. Then the equalized useful signal at the receive filter output is free from intersymbol interference in the considered transmission system with square-root raised cosine transmit and receive filters. When sticking to linear systems the sequence of the signal processing is irrelevant to the output and thus to the equalization result. For that reason, the equalization or parts of it can be shifted in the signal path. Therefore, the equalizer transfer function $1/G_k(f)$ is partitioned into two portions: One part of the equalization is implemented in the transmitter and the remaining part is incorporated in the receiver. Then, the distribution of the shares of the equalizer's transfer function to transmitter and receiver is another degree of freedom. An optimization is possible for this partitioning.

In Fig. 12 the generalized model of the cable transmission system is displayed. The elements for multilevel coding and decoding, transmit and receive filtering (specifically, i.e., $G_s(f)$ and $G_e(f)$) as well as for sampling and threshold detection stay in the block diagram as shown in Fig. 12.

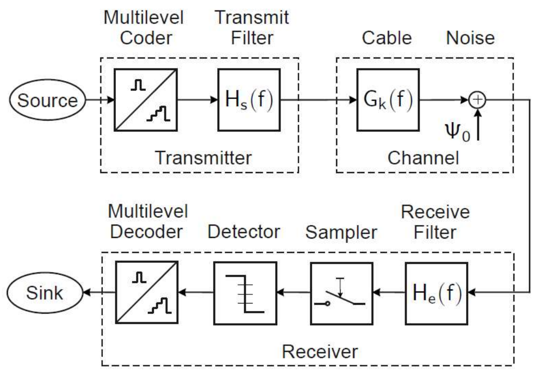

Fig. 12. Generalized copper-cable communication system with distributed transmitter- and receiver-side equalization (Figure reused from Ref. [1], with permission).

However, the equalization is changed. Now there are generalized transmit and receive filters with the transfer functions

$$H_s(f) = \frac{G_s(f)}{G_k(f)^{\frac{m}{m+1}}} \tag{12}$$

and

$$H_e(f) = \frac{G_e(f)}{G_k(f)^{\frac{1}{m+1}}} \ . \tag{13}$$

The impact of the copper cable's transfer function $G_k(f)$ on the useful transmit signal is now—as previously in Section II—entirely linearly equalized by its inverse $1/G_k(f)$. However, the equalization is now splitted to transmitter and receiver. Transmit filter $G_s(f)$ and receive filter $G_e(f)$ are each supplemented by components that perform parts of the equalization. A parameter m is introduced in Eqs. (12) and 13. It describes the degree of the equalization, i.e., what share of the equalization is implemented in the transmitter and what share in the receiver. The transmission system from transmits filter input to receive filter output remains clear of intersymbol interference for arbitrary m, since the first Nyquist criterion is fulfilled independently of m. This equalization method covers the following special cases [1]:

$m = 0$: Full receiver-side equalization.

$m \to \infty$: Full transmitter-side equalization.

$m = 1$: Equal equalization partitioning to transmitter and receiver.

In doing so, the setup with distributed equalization represents a generalization of the conventional cable transmission model with complete receiver-side equalization analyzed in Section II.

B. Calculation of Transmit Power and Energy per Bit

With the power spectral density of the transmit signal

$$\Psi_\mathrm{s}(f) = \frac{U_\mathrm{s}^2}{3f_\mathrm{T}}(s^2-1)|H_\mathrm{s}(f)|^2 \tag{14}$$

the transmit power in this generalized case is derived as:

$$P_\mathrm{s} = \int_{-\infty}^{+\infty} \Psi_\mathrm{s}(f)\,\mathrm{d}f = \frac{2U_\mathrm{s}^2}{3f_\mathrm{T}}(s^2-1)\int_0^{\infty}|H_\mathrm{s}(f)|^2\,\mathrm{d}f\,. \tag{15}$$

The noise power at the detector input becomes

$$U_\mathrm{R}^2 = \Psi_0 \int_{-\infty}^{+\infty}|H_\mathrm{e}(f)|^2\,\mathrm{d}f = 2\Psi_0\int_0^{\infty}|H_\mathrm{e}(f)|^2\,\mathrm{d}f\,. \tag{16}$$

By solving Eq.(2) for U_s^2 the relationship $U_\mathrm{s}^2 = \varrho \times U_\mathrm{R}^2$ is obtained. When inserting this and Eq. (16), Eq. (8) as well as $f_\mathrm{T} = f_\mathrm{B}/\mathrm{ld}(s)$ into Eq. (15), the expression for the transmit power finally becomes

$$P_\mathrm{s} = \frac{8}{3}\frac{(s^2-1)\Psi_0\times\mathrm{ld}(s)}{f_\mathrm{B}}\left[\mathrm{erf}^{-1}\left(1-\frac{s\,\mathrm{ld}(s)}{s-1}P_\mathrm{b}\right)\right]^2 \times \ldots \times \int_0^{\infty}|H_\mathrm{s}(f)|^2\,\mathrm{d}f \times \int_0^{\infty}|H_\mathrm{e}(f)|^2\,\mathrm{d}f\,. \tag{17}$$

From Eq. (17) the energy per bit $E_\mathrm{b} = P_\mathrm{s}/f_\mathrm{B}$ is obtained as:

$$E_\mathrm{b} = \frac{8}{3}\frac{(s^2-1)\Psi_0\times\mathrm{ld}(s)}{f_\mathrm{B}^2}\left[\mathrm{erf}^{-1}\left(1-\frac{s\,\mathrm{ld}(s)}{s-1}P_\mathrm{b}\right)\right]^2 \times \ldots \times \int_0^{\infty}|H_\mathrm{s}(f)|^2\,\mathrm{d}f \times \int_0^{\infty}|H_\mathrm{e}(f)|^2\,\mathrm{d}f\,. \tag{18}$$

In Eqs. (17) and (18) the bit rate f_B is included. Since the symbol rate f_T is connected to the bit rate f_B and the symbol rate f_T rules the bandwidths of the filters $H_\mathrm{s}(f)$ and $H_\mathrm{e}(f)$, the bit rate is also indirectly contained in Eqs. (17) and (18) by way of the integrals. In addition, the quality of transmission (P_b), the noise (Ψ_0) and the copper cable's properties (f_0, ℓ) influence the necessary transmit power or energy per bit, respectively [1].

When parts of the equalizing function are implemented in the transmitter the transmit signal is amplified frequency-dependently. Under unfavorable conditions high transmit signal amplitudes can occur. Then components of the transmitter, such as amplifiers, with higher dynamic range are needed than in case of linear equalization completely concentrated in the receiver [1]. Throughout this article, this requirement is assumed to be fulfilled.

C. Numerical Results for the Energy-related Metrics

1) Preamble

In this subsection, at first the equalization parameter m is numerically evaluated aiming at optimum equalization partitioning to transmitter and receiver with respect to minimum power P_s or energy E_b, respectively. After that, results for optimized m, i.e., with optimum equalization distribution among transmitter and receiver side, are given. They are compared to the results achieved in Section II for the case of entire linear equalization in the receiver. The numerical calculations and results are based on the same values for useful signal, disturbance and copper-cable properties given and used in Section II, changes and additions are explicitly stated.

2) Optimization of the equalization partitioning to transmitter and receiver

Fig. 13 displays the transmit power Eq. (17) as a function of the parameter m for a fixed constellation size s. The parameter m designates the degree of distribution of the equalizing function to transmitter and receiver after Eqs. (12) and (13). For $m = 0$ the equalization is entirely implemented in the receiver (as before in Section II) and for $m \to \infty$ it is performed completely in the transmitter.

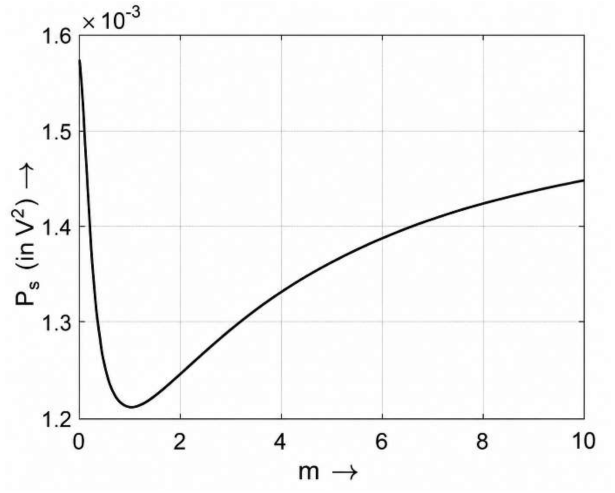

Fig. 13. Transmit power P_s as a function of the equalization parameter m for a bit rate $f_\mathrm{B} = 1$ MHz ($\ell = 2$ km, $s = 2$, $P_\mathrm{b} = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V^2/Hz) (Figure reused from Ref. [1], with permission).

From a pure transmission engineering point of view, both extreme cases, $m = 0$ and $m \to \infty$, result in the same transmit power—when potential practical implementation issues are neglected (e.g., regarding the dynamic range of the transmitter's elements). This same transmit power $P_\mathrm{s}(m = 0) = P_\mathrm{s}(m \to \infty)$ can be read off Fig. 4, too, for f_B = 1MHz and $\mathrm{ld}(s) = 1$. It can be observed, that P_s shows a minimum for $m = m_\mathrm{opt} = 1$. A uniform distribution of the linear equalization function to transmitter and receiver is optimum in terms of minimal power or energy demand. The optimal generalized transmit and receive filter transfer functions therefore become

$$H_\mathrm{s\,opt}(f) = \frac{G_\mathrm{s}(f)}{\sqrt{G_\mathrm{k}(f)}} \tag{19}$$

and

$$H_{\mathrm{e\,opt}}(f) = \frac{G_{\mathrm{e}}(f)}{\sqrt{G_{\mathrm{k}}(f)}}. \tag{20}$$

The transmit power P_s is shown in Fig. 14 depending on m, now for several constellation sizes s at a preset bit rate f_B. The minimum transmit power is identified at $m_{opt} = 1$ for all constellation sizes s. When adhering to fixed constellation sizes s and varying the bit rate f_B the result $m_{opt} = 1$ remains valid. Based on this insight, the conclusion is made that the optimum equalization parameter $m_{opt} = 1$ does neither depend on the bit rate f_B nor on the constellation size s. The optimal parameter $m_{opt} = 1$ is valid for all bit rates f_B and constellation sizes s—evidence is provided through Fig. 13 and Fig. 14.

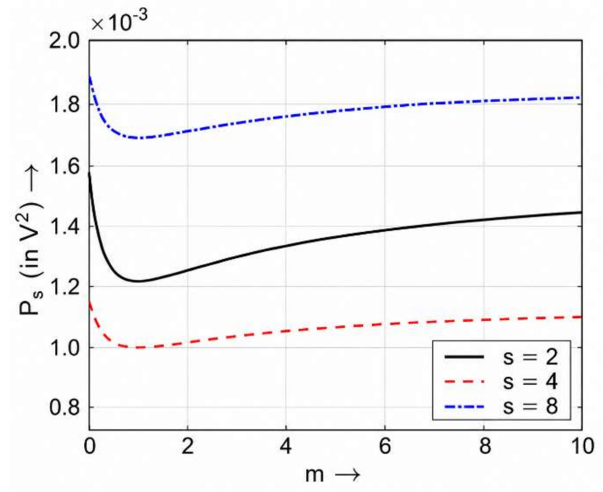

Fig. 14. Transmit power P_s as a function of the equalization parameter m for a bit rate $f_B = 1$ MHz for several constellation sizes s ($\ell = 2$ km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V^2/Hz) (Figure reused from Ref. [1], with permission).

The energies per bit $E_b = P_s/f_B$ follow a very similar curve, because they are scaled versions of the transmit power P_s. In particular, when optimizing with regard to a minimal energy per bit E_b the result is the same optimum parameter $m_{opt} = 1$ for the equalization distribution, too.

3) Results for optimized distribution of the equalization

The optimization of the equalization parameter m leads to $m_{opt} = 1$. This means that a uniform distribution of the linear cable equalization to transmitter and receiver side is the optimum choice when targeting the most performant transmission system. Therefore, in the context of this article it is of interest how the energy-related metrics' trajectories behave as functions of the constellation size s for both of the considered cases. Conventional transmission system with full receiver- side equalization (as in section II) with $m = 0$ and system with optimized linear equalization with $m = m_{opt} = 1$.

Fig. 15 shows the transmit power P_s and Fig. 16 displays the energy per bit E_b as functions of the constellation size s for two-bit rates each and for both equalization strategies, respectively.

In Table III the optimum constellation sizes s_{opt} obtained from Fig. 15 or Fig. 16, respectively, are summarized for the cases $m = 0$ and $m = 1$.

TABLE III. OPTIMUM VALUES OF THE CONSTELLATION SIZES s_{opt} FOR TWO FIXED BIT RATES f_B FOR $m = 0$ AND $m = 1$ AT A CABLE LENGTH OF $\ell = 2$ KM

f_B (in MHz)	m	s_{opt}
1	0	$2^2 = 4$
	1	$2^2 = 4$
10	0	$2^4 = 16$
	1	$2^3 = 8$

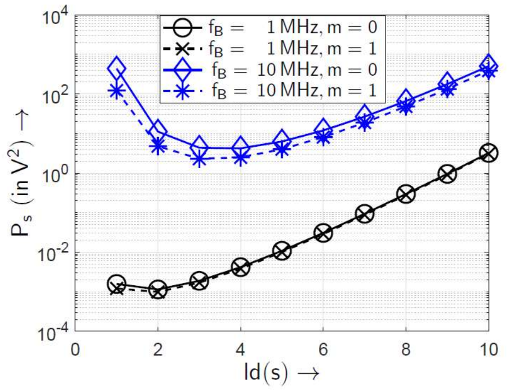

Fig. 15. Transmit power P_s as a function of the constellation size s for two-bit rates f_B for $m = 0$ and $m = 1$ ($\ell = 2$ km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V^2/Hz) (Figure reused from Ref. [1], with permission).

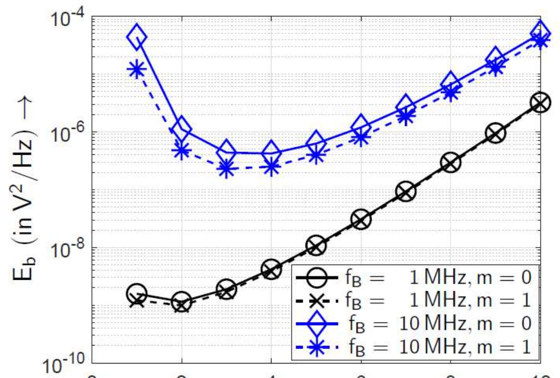

Fig. 16. Energy per bit E_b as a function of the constellation size s for two-bit rates f_B for $m = 0$ and $m = 1$ ($\ell = 2$ km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V^2/Hz) (Figure reused from Ref. [1], with permission).

From the results, significant improvements with respect to the power and energy indicators are observed especially at small constellation sizes.[6] For increasing constellation sizes the improvements from the optimization of the equalization partitioning become less important. The reason for this characteristic is that at smaller numbers of signaling levels there is a higher symbol rate and therefore a higher bandwidth of the useful signal and thus of the filters is observed. Then the equalization impacts the transmission more severely than at larger constellation

[6] Please note, that the vertical axes in Figs. 15–18 are scaled logarithmically.

sizes (resulting in smaller symbol rates and filter bandwidths at a given bit rate), because the spectral range the equalizer has to amplify is broader to achieve a useful signal free from ISI than at higher constellation sizes. Therefore, by optimizing the equalization parameter the system can strongly be improved in particular at small constellation sizes [1].

The energies per bit E_b and the transmit powers P_s again exhibit very similar characteristics as functions of the con-stellation size s, since the energy-per-bit curves are scaled versions of the transmit power trajectories.

In practical systems, optimizing the equalization partitioning can lead to significant energy efficiency improvements, since optimum constellation sizes tend to be found at lower constellation sizes s. Here, the optimization of the equalization parameter m has major impact.

Now results are presented for the shorter cable of length $\ell = 0.5$ km found in FTTC access networks. Figs. 17 and 18 show the transmit-power and the energy-per-bit curves as functions of the constellation size s for two-bit rates and both equalization methods, respectively. The results and insights resemble those discussed previously for the copper line of length $\ell = 2$ km—now at increased bit rates [1].

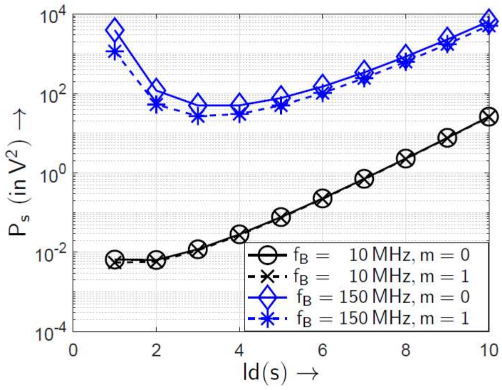

Fig. 17. Transmit power P_s as a function of the constellation size s for two-bit rates f_B for $m = 0$ and $m = 1$ ($\ell = 0.5$ km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V^2/Hz) (Figure reused from Ref. [1], with permission).

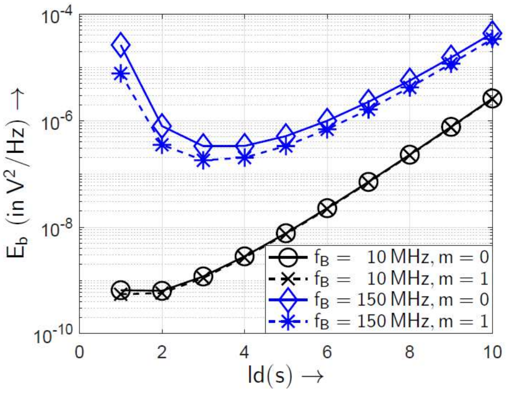

Fig. 18. Energy per bit E_b as a function of the constellation size s for two-bit rates f_B for $m = 0$ and $m = 1$ ($\ell = 0.5$ km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V^2/Hz) (Figure reused from Ref. [1], with permission).

In Table IV the optimum constellation sizes s_{opt} obtained from Fig. 17 or Fig. 18, respectively, are compiled for the cases $m = 0$ and $m = 1$.

TABLE IV. OPTIMUM VALUES OF THE CONSTELLATION SIZES s_{opt} FOR TWO FIXED BIT RATES f_B FOR $m = 0$ AND $m = 1$ AT A CABLE LENGTH OF $\ell = 0.5$ KM

f_B (in MHz)	m	s_{opt}
10	0	$2^2 = 4$
	1	$2^1 = 2$
150	0	$2^3 = 8$
	1	$2^3 = 8$

D. *Summarization of the Transmit-power and Energy-per-Bit Results*

Figs. 19 and 20 illustrate the results obtained for the energy-related metrics $P_s(s)$ and $E_b(s)$ when using a cable of length $\ell = 2$ km in the FTTEx network. In Figs. 21 and 22 related results for a cable of length $\ell = 0.5$ km—in the FTTC network—are shown, respectively. Each diagram contains the results for conventional receiver-side equalization ($m = 0$) and for optimized distribution of equalization ($m = m_{opt} = 1$) in a 3D plot as a function of the bit rate f_B and the constellation size s, respectively.

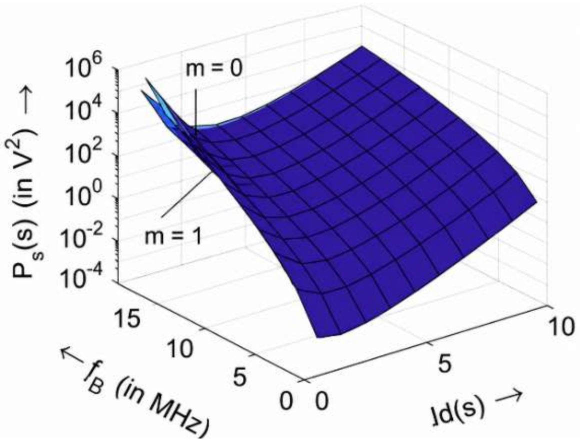

Fig. 19. Transmit power $P_s(s)$ as a function of the bit rate f_B and the constellation size s for $m = 0$ and $m = 1$ ($f_0 = 0.178$ MHz × km^2, $\ell = 2$ km, $P_b = 1.5 \times 10^{-9}$, $\Psi_0 = 10^{-12}$ V^2/Hz).

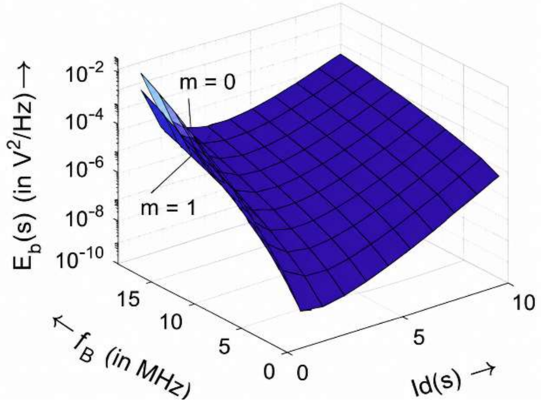

Fig. 20. Energy per bit $E_b(s)$ as a function of the bit rate f_B and the constellation size s for $m = 0$ and $m = 1$ ($f_0 = 0.178$ MHz × km^2, $\ell = 2$ km, $P_b = 1.5 \times 10^{-9}$, $\Psi_0 = 10^{-12}$ V^2/Hz).

The behaviour of the Eqs. (17) and (18) regarding their dependency on bit rate f_B and constellation size s becomes evident. Both, $P_s(s)$ and $E_b(s)$ increase with rising bit rate f_B (at fixed number of signalling levels s). Concerning the constellation size s both quantities exhibit a minimum at an optimum constellation size $s = s_{opt}$ (at a fixed bit rate f_B). Again, for the FTTC case (ℓ = 0.5 km) higher bit rates are possible than in the FTTEx case (ℓ = 2 km)—however, the principal behaviour is the same for different cable lengths. Furthermore, for optimized equalization distribution at m = 1 lower transmit powers or energies per bit, respectively, are required than in case of conventional equalization (m = 0). The advantage of the optimized equalization distribution becomes especially significant at increased bit rates f_B.

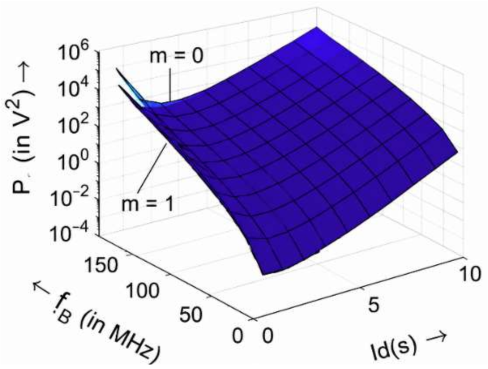

Fig. 21. Transmit power $P_s(s)$ as a function of the bit rate f_B and the constellation size s for m = 0 and m = 1 (f_0 = 0.178 MHz × km^2, ℓ = 0.5 km, $P_b = 1.5 \times 10^{-9}$, $\Psi_0 = 10^{-12}$ V^2/Hz).

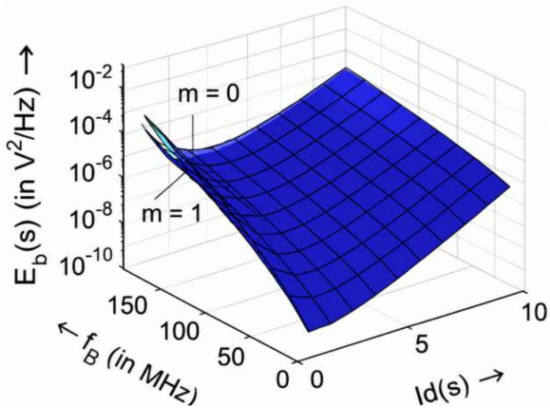

Fig. 22. Energy per bit $E_b(s)$ as a function of the bit rate f_B and the constellation size s for m = 0 and m = 1 (f_0 = 0.178 MHz × km^2, ℓ = 0.5 km, $P_b = 1.5 \times 10^{-9}$, $\Psi_0 = 10^{-12}$ V^2/Hz).

E. Transmit-power and Energy-per-Bit Savings

Figs. 15–18 as well as Figs. 19–22 show results for energy-related metrics plotted on a logarithmic scale. Improvements are observable; however, the orders of magnitude of the energy savings are difficult to read directly from these plots.

Therefore, a relative savings quantity is defined—to enhance the visibility of the savings. For that purpose, the conventional transmission system with two signalling levels (s = 2) and full linear receiver-side equalization (m = 0) is used as a reference [1].

A relative savings parameter $\varepsilon(s)$ is defined according to Refs. [1, 36]:

$$\varepsilon(s) = 1 - \frac{P_s(s)}{P_s(2)} = 1 - \frac{E_b(s)}{E_b(2)}. \qquad (21)$$

It reveals the savings clearly. The transmit power $P_s(2) = P_s(s = 2)$ and the energy per bit $E_b(2) = E_b(s = 2)$ are the transmit-power or energy-per-bit values obtained for the reference system (with s = 2 and m = 0). For the transmit power and energy-per-bit functions the notation $P_s(s)$ and $E_b(s)$ is used, respectively, to show and emphasize that they depend on the varying constellation size s as compared to the values $P_s(2)$ and $E_b(2)$ for a single and fixed constellation size (s = 2). Here, too, it is not relevant whether the savings are computed based on the transmit power or on the energy per bit according to Eq. (21)—since, again, transmit power and energy per bit are related by the fixed bit rate f_B [1, 36].

The quantity $\varepsilon(s)$ indicates the savings that can be achieved very explicitly. Figs. 23 and 24 show numerical results for the two cases of the cable lengths (ℓ = 2 km and ℓ = 0.5 km) considered earlier. The savings $\varepsilon(s)$ are displayed as functions of the number of signalling levels s. Only positive values of $\varepsilon(s)$ are included in Figs. 23 and 24 as merely true savings are plotted. Following Eq. (21), negative results for $\varepsilon(s)$ can occur, too, in particular for large constellation sizes: In such cases the power or energy consumed at these constellation sizes is larger than those the reference system requires (see e.g., Figs. 15–18), and such results are not depicted in Figs. 23 and 24.

For the longer cable with ℓ = 2 km it can be read off the diagram in Fig. 23 that at the lower bit rate of f_B = 1 MHz about 25% of savings can be realized by merely applying the optimum constellation size s_{opt} = 4. When sticking to the basic transmission system with s = 2 savings of 22% are registered if the optimum equalization partitioning with uniform distribution to transmitter and receiver (m_{opt} =1) is utilized.

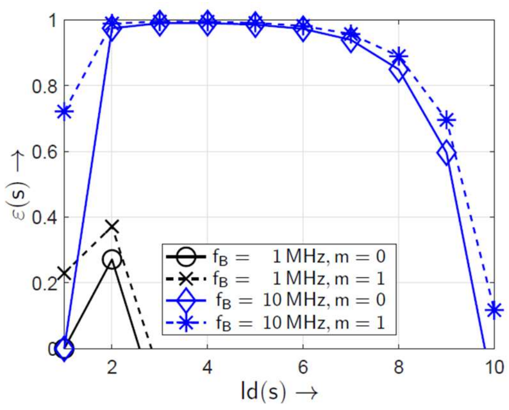

Fig. 23. Energy-related savings $\varepsilon(s)$ as function of the constellation size s for two-bit rates f_B for m = 0 and m = 1 (ℓ = 2 km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V^2/Hz) (figure reused from Ref. [1], with permission).

When combining both optimization options and thus the system operates at optimum constellation size (s_{opt} = 4) and optimal equalization partitioning (m_{opt} = 1) approximately 38 % of transmit power or energy per bit can be saved, respectively. Towards higher bit rates much more significant savings can be attained, e.g., > 90 % for a bit rate of f_B = 10 MHz under the given preconditions [1].

For the shorter cable with ℓ = 0.5 km the achievable savings can be recognized from the diagram in Fig. 24: The results show similar savings' orders of magnitude and tendencies. They therefore allow for similar insights as obtained and stated for the cable length of ℓ = 2 km, once more at increased bit rates [1].

The savings of optimized baseband transmission, when utilizing s_{opt} and m_{opt}, are moderate in a range of 15 % to 40 % at lower bit rates and they can reach large values of > 80 % at higher bit rates for the different cable lengths when taking the conventional baseband system with two-level transmission (s = 2) and full linear equalization in the receiver (m = 0) as Ref. [1].

The relative power and energy-related savings that can be achieved depend on the parameters taken as a basis, first and foremost the cable length ℓ (and characteristic cable frequency f_0) and the bit rate f_B. The interrelationship of useful signal's bandwidth and low-pass properties of the copper line is important for the improvements attainable by the optimization. The savings by optimizing constellation sizes and equalization partitioning in terms of transmit power or energy per bit, respectively, are notably significant when the transmission system operates at high bit rates related to the band limitation caused by the copper cable's low pass behavior [1].

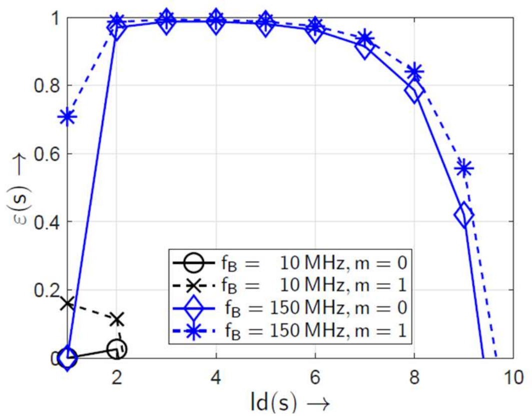

Fig. 24. Energy-related savings $\varepsilon(s)$ as function of the constellation size s for two-bit rates f_B for m = 0 and m = 1 (ℓ = 0.5 km, $P_b = 1.5 \times 10^{-9}$ and $\Psi_0 = 10^{-12}$ V^2/Hz) (figure reused from Ref. [1], with permission).

IV. Conclusion

Power and energy demand of transmission links are significant elements of telecommunication networks' energy efficiency. Therefore, transmit power and energy per bit of pulse amplitude modulated baseband communication systems for linearly equalized copper-cable transmission has been studied and degrees of freedom in the design and operation of such communication systems have been identified and investigated. Firstly, the dependencies of transmit power P_s and energy per bit E_b on the constellation size s were analyzed—representing a first degree of freedom in the design of communication systems. The bit rate f_B, the bit-error probability P_b, and the channel characteristics (cable parameters ℓ, f_0 and noise (Ψ_0)) have been assumed to be preconditions—as a practically relevant case when a transmission link is installed between two endpoints operating reliably at a required throughput via a given channel in an existing disturbance environment.

The results show, that the chosen power and energy metrics (P_s and E_b) depend on the constellation size s. There is an optimum constellation size s_{opt} where power or energy show a minimum, respectively. The optimum constellation size s_{opt} depends on the interrelationship between signal bandwidth and the band limitation induced by the cable, i.e., the interdependency between f_B, ℓ (and f_0).

Afterwards, the distribution of the equalization has been identified to be another degree of freedom in the engineering of cable transmission systems. The optimization led to the insight that a uniform partitioning of the equalization to transmitter and receiver results in a minimal transmit power P_s or energy per bit E_b, respectively.

When utilizing the optimized systems with respect to constellation size s and equalization distribution (m), significant power or energy savings can be achieved against conventional two-level systems with full linear equalization at the receiver side. If, in addition, crosstalk impairments are taken into account, it is expected that higher transmit powers or energies per bit are required at otherwise unchanged circumstances, since the disturbances in that case will be more severe than in the AWGN-only situation.

Finally, it can be concluded, that the optimization of transmission systems concerning available degrees of freedom in the systems' design and operation provides substantial opportunities for energy efficiency improvements of communication systems and networks.

V. Further Work

The optimization of constellation size and equalization in wired communication systems is an important means for energy-efficient transmission. It is an element for operating telecommunication networks energy-efficiently and therefore sustainably—as such networks typically consist of a multitude of various kinds of communication links and related systems. In addition, it enables load-adaptive link operation by adapting the transmission capabilities to temporally varying traffic volumes—which also helps to increase the telecommunication networks' energy efficiency. The evaluation of the energy savings potential for network segments, e.g. access networks, or whole telecommunication networks in the context of the optimization of constellation sizes and equalization partitioning to transmitter and receiver is an interesting question left to future research.

Further work on wired transmission systems' power and energy efficiency for copper cable information transfer should take into account the comparison of different equalization strategies as well as the assessment in terms of system's complexity. Here, stochastic simulations and practical validations will help to underpin the results obtained analytically in this article. Furthermore, from a methodical point of view a sensitivity analysis can be beneficial in order to classify and complement results of a numerical investigation. Also, an extension of the Pulse Amplitude Modulation (PAM)-related results to digital subscriber line systems using multicarrier Quadrature Amplitude Modulation (QAM) within the subchannels is of practical interest and hence a promising topic for further studies.

In real-world telecommunication networks a vast multitude of transmission links is established and operated in very different network environments: Then practical aspects like channels with non-Additive White Gaussian Noise (AWGN) disturbances and in particular crosstalk impairments, imperfect equalization, hardware constraints and requirements on the dynamic range of transmitter and receiver components can become important—unfolding yet other fields for interesting future work in the context discussed throughout the present article.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Christoph Lange performed conceptualization, methodology, formal analysis, investigation, validation, original draft writing and visualization; Andreas Ahrens contributed to methodology, formal analysis, investigation, validation, manuscript revision, visualization and supervision; both authors reviewed the manuscript and approved the final version.

REFERENCES

[1] C. Lange and A. Ahrens, "Improving power and energy efficiency of linearly equalized baseband cable transmission links," in *Proc. 11th Conference on Sensor Networks*, 2022, pp. 15–25.

[2] S. Lambert, W. V. Heddeghem, W. Vereecken, B. Lannoo, D. Colle, and M. Pickavet, "Worldwide electricity consumption of communication networks," *Optics Express*, vol. 20, no. 26, pp. B513–B524, 2012.

[3] C. Lange, D. Kosiankowski, A. Betker, H. Simon, N. Bayer, D. V. Hugo, H. Lehmann, and A. Gladisch, "Energy efficiency of load-adaptively operated telecommunication networks," *Journal of Lightwave Technology*, vol. 32, no. 4, pp. 571–590, 2014.

[4] J. Malmodin and D. Lundén, "The energy and carbon footprint of the Global ICT and E&M sectors 2010–2015," *Sustainability*, vol. 10, no. 9, 3027, 2018.

[5] D. Lundén, J. Malmodin, P. Bergmark, and N. Lövehagen, "Electricity consumption and operational carbon emissions of european telecom network operators," *Sustainability*, vol. 14, no. 5, 2637, February 2022.

[6] C. Lange, D. Kosiankowski, R. Weidmann, and A. Gladisch, "Energy consumption of telecommunication networks and related improvement options," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 17, no. 2, pp. 285–295, 2011.

[7] R. Bolla, R. Bruschi, F. Davoli, and F. Cucchietti, "Energy efficiency in the future internet: A survey of existing approaches and trends in energy-aware fixed network infrastructures," *IEEE Communications Surveys and Tutorials*, vol. 13, no. 2, pp. 223–244, 2011.

[8] J. M. H. Elmirghani, T. Klein, K. Hinton, L. Nonde, A. Q. Lawey, T. E. H. E. Gorashi, M. O. I. Musa, and X. Dong, "Green touch green meter core network energy-efficiency improvement measures and optimization," *Journal of Optical Communications and Networking*, vol. 10, no. 2, pp. A250–A269, 2018.

[9] M. Pickavet, W. Vereecken, S. Demeyer, P. Audenaert, B. Vermeulen, C. Develder, D. Colle, B. Dhoedt, and P. Demeester, "Worldwide energy needs for ICT: The rise of power-aware networking," in *Proc. 2nd International Symposium on Advanced Networks and Telecommunication Systems (ANTS)*, 2008, pp. 1–3.

[10] E. Agrell and M. Karlsson, "Power-efficient modulation formats in coherent transmission systems," *Journal on Lightwave Technology*, vol. 27, no. 22, pp. 5115–5126, 2009.

[11] D. C. Kilper, G. Atkinson, S. K. Korotky, S. Goyal, P. Vetter, D. Suvakovic, and O. Blume, "Power trends in communication networks," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 17, no. 2, pp. 275–284, 2011.

[12] R. Tan, Y. Shi, Y. Fan, W. Zhu, and T. Wu, "Energy saving technologies and best practices for 5G radio access network," *IEEE Access*, vol. 10, pp. 51747–51756, 2022.

[13] E. E. Tsiropoulou, G. K. Katsinis, and S. Papavassiliou, "Distributed uplink power control in multiservice wireless networks via a game theoretic approach with convex pricing," *IEEE Transactions on Parallel and Distributed Systems*, vol. 23, no. 1, pp. 61–68, 2012.

[14] D. Kilper, K. Guan, K. Hinton, and R. Ayre, "Energy challenges in current and future optical transmission networks," in *Proc. the IEEE*, 2012, vol. 100, no. 5, pp. 1168–1187.

[15] R. S. Tucker, "Green optical communications part I: Energy limitations in transport," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 17, no. 2, pp. 245–260, 2011.

[16] R. S. Tucker, "Green optical communications part II: Energy limitations in networks," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 17, no. 2, pp. 261–274, 2011.

[17] H. Khodakarami and W. Shieh, "On the energy efficiency of modulation formats for optical communications," *IEEE Photonics Technology Letters*, vol. 25, no. 3, pp. 275–278, February 2013.

[18] E. U. Biyikoglu, B. Prabhakr, and A. E. Gamal, "Energy-efficient packet transmission over a wireless link," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 487–499, 2002.

[19] S. N. Roy, "Energy logic: A road map to reducing energy consumption in telecommunications networks," in *Proc. International Telecommunications Energy Conference (INTELEC)*, San Diego, CA, September 2008, pp. 1–9.

[20] B. Puype, W. Vereecken, D. Colle, M. Pickavet, and P. Demeester, "Multilayer traffic engineering for energy efficiency," *Photonic Network Communications*, vol. 21, no. 2, pp. 127–140, April 2011.

[21] F. Idzikowski, S. Orlowski, C. Raack, H. Woesner, and A. Wolisz, "Dynamic routing at different layers in IP-over-WDM networks−Maximizing energy savings," *Optical Switching and Networking*, vol. 8, no. 3, pp. 181–200, 2011.

[22] H. S. Dhillon, H. C. Huang, H. Viswanathan, and R. A. Valenzuela, "Power-efficient system design for cellular-based machine-to-machine communications," *IEEE Transactions on Wireless Communications*, vol. 12, no. 11, pp. 5740–5753, 2013.

[23] M. Guenach, C. Nuzman, P. Tsiaflakis, and J. Maes, "Power optimization in vectored and non-vectored G.fast transmission," in *Proc. 2014 IEEE Global Communications Conference*, 2014, pp. 2229–2233.

[24] A. Deylamsalehi, D. A. P. Davis, P. Afsharlar, M. Bahrami, W. P. Chen, and V. M. Vokkarane, "Using machine learning to balance energy cost and emissions in optical networks," *Journal of Optical Communications and Networking*, vol. 10, no. 10, pp. D72–D83, 2018.

[25] N. Zhang, Z. Q. Yao, Y. Liu, S. P. Boyd, and Z.-Q. Luo, "Dynamic resource allocation for energy efficient transmission in digital subscriber lines," *IEEE* Transactions *on Signal Processing*, vol. 65, no. 16, pp. 4353–4366, 2017.

[26] A. Raj and P. Biyani, "Weighted-A^* based energy efficient resource allocation in g. fast," *IEEE Transactions on Communications*, vol. 67, no. 11, pp. 7744–7754, 2019.

[27] T. Starr, J. M. Cioffi, and P. J. Silverman, "Understanding digital subscriber line technology," *IEEE Communications Magazine*, 2000, vol. 38, no. 5, pp. 30–32

[28] J. A. C. Bingham, *ADSL, VDSL, and Multicarrier Modulation*, New York: Wiley, 2000.

[29] M. Timmers, M. Guenach, C. Nuzman, and J. Maes, "G.fast: Evolving the copper access Network," *IEEE Communications Magazine*, vol. 51, no. 8, pp. 74–79, 2013.

[30] J. M. Cioffi, K. J. Kerpez, C. S. Hwang, and I. Kanellakopoulos, "Terabit DSLs," *IEEE Communications Magazine*, vol. 56, no. 11, pp. 152–159, 2018.

[31] V. Oksman, R. Strobel, T. Starr, J. Maes, W. Coomans, M. Kuipers, E. B. Tovim, and D. Wei, "MGFAST: A new generation of copper broadband access," *IEEE Communications Magazine*, vol. 57, no. 8, pp. 14–21, 2019.

[32] D. C. Kilper, D. Neilson, D. Stiliadis, D. Suvakovic, and S. Goyal, "Fundamental limits on energy use in optical networks," in *Proc. 36th European Conference and Exhibition on Optical Communication*, 2010, pp. 1–6.

[33] J. G. Proakis and M. Salehi, *Digital Communications*, 5th ed. Boston; New York; San Francisco: McGraw-Hill, 2008.

[34] B. Sklar and F. Harris, *Digital Communications—Fundamentals and Applications*, 3rd ed. New York: Pearson, 2021.

[35] C. Lange and A. Ahrens, "Power and energy efficiency optimization of baseband transmission Links," in *Proc. Signal Processing Symposium (SPSympo)*, 2021, pp. 147–152.

[36] C. Lange and A. Ahrens, "Power and energy efficiency of multilevel baseband transmission systems: Analysis, optimization and improvements," *Journal of Communications*, vol. 17, no. 12, pp. 961–971, December 2022.

[37] D. Kreß and M. Krieghoff, "Elementary approximation and equalization for the transmission of PCM signals over coaxial cables," *Nachrichtentechnik Elektronik*, vol. 23, no. 6, pp. 225–227, 1973.

[38] C. Lange and A. Ahrens, *Digital Transmission Engineering*, Wiesbaden: Springer, 2025.

[39] C. W. Davidson, *Transmission Lines for Communications*, 2nd ed. Houndmills, Basingstoke, Hampshire, London: Macmillan Publishers, 1989.

[40] J. B. Anderson, *Digital Transmission Engineering*, 2nd ed. Piscataway, NJ: IEEE Press; John Wiley & Sons, 2005.

[41] R. F. H. Fischer and J. B. Huber, *Digital Communications: A Foundational Approach*, Cambridge: Cambridge University Press, 2024.

[42] J. Spanier and K. B. Oldham, *An Atlas of Functions*, Washington, New York, London: Hemisphere Publishing, 1987.

[43] G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists*, 5th ed. San Diego, San Francisco, New York: Harcourt/Academic Press, 2001.

[44] C. Lange, J. Preuschaft, M. Braune, and N. Gieschen, "Migration from current DSL-based architectures to future-proof pure optical access networks," in *Proc. 13th European Conference on Networks and Optical Communications (NOC)*, Krems (Austria), 2008, pp. 152–159.

[45] P. W. Shumate, "Fiber-to-the-home: 1977–2007," *Journal of Lightwave Technology*, vol. 26, no. 5, pp. 1093–1103, 2008.