Design of Degree Distribution with Maximum Distance Separable Codes for User Priority

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Abstract—In the super-dense future network communication, human and machine communications are mixed in a single network. They will trespass each other and could lead to an intersection. This paper develops Coded Random Access (CRA) using Maximum Distance Separable (MDS) codes as a multiple access scheme to serve millions of users, covering human and machines. This research aims to maximum number of user human and machines, where the priority access is given to the human. Two keywords of CRA, i.e., coding (solved by optimal degree distributions) and random. To analyze design degree distribution using Extrinsic Information Transfer (EXIT) Chart projection under binary erasure channels (BEC). The priority scheme for human communications is determined based on utility functions. The parameters used to see the performance of the EXIT chart are packet-loss-rate (PLR) and throughput. Based on the results, the degree design has succeeded in prioritizing humans, which is indicated by the small PLR value but large throughput, as well as the EXIT curve with its very small gap.

Keywords—CRA, EXIT Chart, MDS Codes, PLR, throughput

I. INTRODUCTION

Super-dense networks is a network supporting massive number of users that requires new technology to support very large number of users simultaneously. Internet of Things (IOT) transceivers often operate by generating short data packets in a sporadic, and sometimes unpredictable manner. In IoT paradigm, the network consisting of a massive number of nodes makes it difficult to devise an efficient resource allocation policy. This issue can be addressed by employing Random Access (RA) protocols. RA solutions used in modern cellular systems are 5G, SigFox, and LoRaWAN. The first RA protocols were introduced in 1970s, in particular to the family of ALOHA scheme [1]. This approach was known to be highly inefficient for IoT services, e.g. smart grid, industry 4.0, V2X communications, because the IoT devices deliver data through random access [2].

As a nature of massive number of users, random access technique is preferable for future multiple access scheme by its perfect scheduling. In RA, all users transmit their message randomly without over-head due to scheduling. Toni and Frossard [3] discuss an irregular slotted ALOHA (IRSA) based transmission, in which the process of conveying information to the BS uses the random IRSA concept to prioritize classes based on utility functions. A priority strategy was also performed [3], but priorities are focused on class, not humans or machines. A modification scheme using conventional LT codes for high priority data and low priority data was also proposed in this study [4].

Woo and Cheng mentioned that in [5], a random access with priority protocol using irregular repetition slotted ALOHA with priority (P-IRSA) has achieved a high throughput. P-IRSA is considered to access control traffic from user. P-IRSA is term of the packet leak rate, throughput and channel efficiency with compared with IRSA and AC-CDRSA protocols.

This research was conducted to get a better result from earlier studies [6, 7], where the previous studies only used one utility function and did not compare the fading effect on network quality. The difference in utility function will affect the design of degree distribution that will be made. This research proposes a multiple access Coded Random Access (CRA) technique which is considered suitable for super dense networks because it can provide access for a large number of users but with limited time slots. The random variable process in CRA uses the Successive Interference Cancellation (SIC) process. More work for random multiple access using SIC proposed in [8] and [9].

This study uses the Maximum Distance Separable (MDS) codes, a coding scheme to design the degree distribution. Other channel coding schemes such as irregular LDPC [10], polar codes [11], and convolutional codes [12] have been not used. The authors of [13] proposed Tornado codes based on the concatenation of polar codes, luby transform codes, and Accumulator with analyzed EXIT chart. MDS code was chosen because it has a high data rate required by future network transmission. MDS codes are important in coding theory because the theoretical significance and practical applications. Reed-Solomon codes are well-known family of MDS codes. MDS codes is a code has attracted a lot of attention in recent years because its application for quantum coding [14]. With MDS codes, each helper stores a segment of MDS coded...
data and then a user is able to recover the requesting data by collecting the segments from multiple helpers [15].

The expected outcome of this study is to determine the suitable degree for humans and machines using a coding scheme that fits to the channel and network capacity. The priority scheme was used to put the human communication on precedence over the machine. The final result should show a simple distribution degree design for humans and machines by the distribution of three utility functions. The theoretical foundations of information theory are based on Shannon, including Mutual Information which forms an introduction to the concept of EXIT Chart [16, 17]. EXIT chart was introduced by Stephen the Brink to analyze the iteration process in the decoder. The proposed codes are analyzed using EXIT Chart under BEC due to its simplicity. The measurement parameters for designing the degree distributions use Extrinsic Information Transform (EXIT) chart. The performance measurement is based on Packet Loss Rate (PLR) and Throughput parameters.

The contributions of this paper are summarized as follows: (i) degree distribution for human and machines with 3 schemes of utility functions (ii) EXIT projection analysis of the proposed CRA using MDS codes.

II. SYSTEM MODEL OF USER PRIORITY

The authors consider a system model, with structure of transmission as shown in Fig. 1. Fig. 1 shows bipartite-graph of CRA, where the users are divided into two groups, i.e. human and machine users. Both access several time-slot randomly. Human users are prioritized over the machines by giving access to all time-slot, and the latter is only provided with limited slot. This research focuses on three utility functions for the two user groups. The assumption is that humans need a high bit rate while machines use a low bit rate. This study used a simulation applying a Matlab software to do a performance analysis. The modulation used was BPSK and the channel model was based on Binary Erasure Channel (BEC), employing an equivalent Additive White Gaussian Noise (AWGN) fading channel to simplify analysis. The random variable process in CRA uses the Successive Interference Cancellation (SIC).

A. Degree Distribution of Nodes

The authors propose a level of user priority in this research represented by two utility functions. First, utility function for users, the authors used 3 schemes:

1) \( f_x(M_h, M_m) = (0.2 : 0.8) \), for network having numbers of human users of 20% and machines of 80%.

2) \( f_x(M_h, M_m) = (0.5 : 0.5) \), for network having equal numbers of human and machine users.

3) \( f_x(M_h, M_m) = (0.8 : 0.2) \), for network having numbers of human users of 80% and machines of 20%.

Second, utility function based on time-slots as \( f_s(N_h, N_m) = (1 : 0.9) \), meaning that networks having 100% access time-slot for human and 90% time-slot for machines. The target of prioritization is also to maximize number of users and minimize the number of time slots. The total number of user who can connect to the network is \( M = M_h + M_m \).

The design degree distribution is only the degree distribution for user node (UN), although, we cannot control or design for slot node (SN). Below is a polynomial to represent degree distribution for human (1) and machines (2) UN from node perspective as:

\[
\Lambda_s(x) = \sum_{k^i}^{n_h} x^i k^i \\
\Lambda_m(x) = \sum_{k^i}^{n_m} x^i k^i
\]

where, \( \Lambda_s(x) \) and \( \Lambda_m(x) \) is degree distribution for human and machines, respectively. Symbol \( k^i \) is the parameter indicating the network encoding scheme. Next steps, after getting node perspective for analysis of the network performance, we calculated edge-perspective for human and machines which is expressed as:

\[
\lambda_e(x) = \frac{1}{\lambda(1)} \sum_{i=1}^{n_h} \lambda_s^i p_e(x)
\]
\[ \lambda_m(x) = \frac{1}{\Lambda} \sum_{i=2}^{n_m} i \Lambda^m_i \gamma_i(x) \]  

(4)

where, \( \Lambda^h_i \) and \( \Lambda^m_i \) is probability of UN having degree \( i \) for human and machines, respectively. \( n_h^i \) and \( n_m^i \) are the maximum degree of user-node human and machines. The parameter, depending on Maximum Distance Sparable (MDS) codes, is \( \gamma_i(x) \). The calculation of \( \gamma_i(x) \) is expressed as:

\[ \gamma_i(x) = \sum_{j=0}^{k_i-1} \binom{n_h}{j} (1 - p_h)^j \left( p_h \right)^{n_h-j-1} \]  

(5)

where, \( p_h \) is erasure probability for human. The design degree distribution for Slot-Node (SN) uses exponential algorithm, which approximate the probability of SN having degree \( d \) is:

\[ \omega(x) = \lim_{M \to \infty} \left( 1 - \frac{G_m}{R_m M^M} (1 - x_m) \right)^{M_m}, \]  

\[ \approx \exp \left\{ \frac{-G_m}{R_m} + \frac{G_h}{R_h} + \frac{G_m}{R_m} x_m + \frac{G_h}{R_h} x_h \right\} \]  

(6)

where \( G_m \) and \( G_h \) is offered traffic for human and machines. The average rate of human and machines is symbolized by \( R_h \) and \( R_m \).

**B. Maximum Distance Sparable (MDS) Codes**

MDS codes is the linear codes to meet the Singleton bound; its minimum distance is the code length minus the code dimension plus one, such that:

\[ k \leq n - \delta + 1, \]  

(7)

Code length is \( n \), minimum distance is symbolized by \( \delta \), and dimension code is \( k \). MDS codes have \( k - n \) redundant bit, with data rate of more than 1/2 (high data rate). This research assumes perfect MDS codes with only evaluated their erasure probability. The notations of MDS codes is \( (n, k) \), as seen in Fig. 2. Fig. 2 shows MDS codes describing that 2 message \( a_1, a_2 \) are encoded into 3 message \( b_1, b_2, b_3 \). The details MDS codes have been discussed in [18].

In this research, the calculation of average rate with MDS codes for human and machines is:

\[ R_h(x) = \frac{k_h}{n_h} = \frac{\sum_{i=0}^{n_h^i} \Lambda_h^i k^i}{\sum_{i=0}^{n_h^i} \Lambda_h^i n^i} \]  

(8)

\[ R_m(x) = \frac{k_m}{n_m} = \frac{\sum_{i=0}^{n_m^i} \Lambda_m^i k^i}{\sum_{i=0}^{n_m^i} \Lambda_m^i n^i} \]  

(9)

where, \( H_k^h \) and \( H_k^m \) is the maximum number of dimension and code length of user human. Similarly, \( H_k^m \) and \( H_m^m \) are for machine users.

**C. Extrinsic Information Transform (EXIT) Chart**

To analyze the decoder iteration process, Stephan Ten Brink introduces an EXIT chart. There are two processes in the EXIT chart, namely apriori mutual information \( (I_A) \) and extrinsic mutual information \( (I_E) \). In this research, we used EXIT chart projection analysis to determine the design degree distribution. Best performance means no intersection between UN and SN in EXIT curve, besides a smaller gap between UN and SN. Brannstom, Amat et al., [19] also applies the EXIT projection which is used in the analysis of convolutional codes in physical encoding and it is also applicable for networks. In this research, we used...
EXIT chart projection because there are two conditions affecting each other. The EXIT chart projection influenced two conditions, i.e., projected machine-SN with human-UN and projected human-SN with machines-UN as shown in Fig. 3.

The concept of EXIT chart used for analyzing three-stage concatenated system is found in [20, 21]. In [22, 23], the authors suggested projecting three dimensional EXIT functions onto a single two-dimensional EXIT chart.

We used channel model and the Networks is Binary Erasure Channel (BEC) channel model. Therefore, the networks is represented by erasure probability. Function of mutual information to present EXIT chart is expressed as:

\[ l_{A,UN}^h = 1 - p^h \]  \hspace{1cm} (10)

\[ l_{E,UN}^h = 1 - q^h \]  \hspace{1cm} (11)

Similar derivation is applied to machine groups with providing \( q^m \). The calculation of the erasure probability from user is expressed as:

\[ q^h = \sum_{i=2}^{n} i \Lambda_i^j(p) \]  \hspace{1cm} (12)

Optimization process is to consider human group, in which to maximize the number of human UN. In this research, we describe expression our optimization process as maximize \( G = G_h + G_m \). Decoding process in this research used Successive Interference Cancelation (SIC) until stopping set is found [24]. There are several parameters needed to create a degree distribution design that prioritize human users:

1. Human rate is smaller than the machine’s \( (R_h < R_m) \).
2. Dimension MDS codes is less than equal to 3 \( (k \leq 3) \).
3. Human Throughput is higher than the machines \( (T_h > T_m) \).
4. Assume that \( N \leq 200 \) for practical analysis.
5. Evaluate the performance, i.e. EXIT chart projection, Packet Loss Rate (PLR) and Throughput.

III. RESULT AND DISCUSSION

This section presents the result of degree distribution for human and machines. The result analysis using EXIT chart projection is shown Tables I and II.

The results for human degree distributions with \( f_h = 0.2 : 0.8 \), means the number of human users is 20% and the machine users is 80%. The degree distribution obtained is \( \Lambda_h(x) = \{ (11, 2, 1) \} \), with the value of offered traffic of 0.15 and the human rate of 0.18. The offered traffic is the ratio between number of users and that of time-slot. The degree distribution for machines is \( \Lambda_m(x) = \{ ((4, 2), 0.1), ((6, 2), 0.9) \} \), with probability of 0.1 and 0.9. The offered traffic to machines obtained is 0.667 and the rate is 0.512. Total offered traffic \( (G) \) is the total amount offered traffic for human \( (G_h) \) and machines \( (G_m) \).

The results for the equal number of human and machines get the degree as displayed in Tables I and II.

The value of offered traffic obtained is 0.375 and 0.417 for humans and machines, respectively. In the third scheme, there are more humans than the machines \((80\% \) humans and \( 20\% \) machines). The total value offered of human traffic is 0.6 and that of machines is 0.167. Based on the results, it can be concluded that the utility function affects the value of offered traffic received.

To see the results of the degree distribution performance, in addition to the EXIT chart performance, PLR and Throughput values were also analyzed. This study also compares the results of PLR performance and throughput between fading and non-fading networks. The fading is interference on the transmission line caused by multipath reflections.

<table>
<thead>
<tr>
<th>TABLE I. DEGREE DISTRIBUTION MDS CODES FOR HUMAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_h )</td>
</tr>
<tr>
<td>0.2 : 0.8</td>
</tr>
<tr>
<td>0.5 : 0.5</td>
</tr>
<tr>
<td>0.8 : 0.2</td>
</tr>
</tbody>
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<thead>
<tr>
<th>TABLE II. DEGREE DISTRIBUTION MDS CODES FOR MACHINES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_m )</td>
</tr>
<tr>
<td>0.2 : 0.8</td>
</tr>
<tr>
<td>0.5 : 0.5</td>
</tr>
<tr>
<td>0.8 : 0.2</td>
</tr>
</tbody>
</table>

Figs. 3 and 4 show the results of the degree distribution based on the results of the EXIT chart projection for the number of human users 20% of the total network and the machine users of 80%. The EXIT chart projection results show a small gap. Fig. 3 shows the result of the EXIT chart projection where human users are affected by the slot node of the machine and Fig. 4 describes the result of the EXIT chart projection where machine users are affected by human slot nodes. Humans and machines use the same slot node in one network, the analysis process EXIT uses projection.
Figs. 4 and 5 show exit charts for human and machine users, assuming that the number of human and machine users is the same. The number of time slots used is 200. For the degree used in the slot node, the use of the utility function is 1:0.9. This means that humans are given 100\% time-slot access and machines are only allowed to access 90\% of the time slots. Fig. 5 shows the EXIT curve for the human degree marked with the symbol “•”, while for the machine degree marked with the black “*” symbol, as shown in Fig. 6.
Figs. 7 and 8 show the EXIT chat curve for the number of human users which is more than the machine. Based on the EXIT curve obtained, the gap obtained is quite wide when compared to $f_u = 0.2 : 0.8$ and $f_u = 0.5 : 0.5$. Based on utility function, we propose degree distributions to have good performance is $f_u = 0.5 : 0.5$, with degree for human $\Lambda_h(x) = \{(4,2), 0.1\}, \{(6,2), 0.9\}$ and degree for machines $\Lambda_m(x) = \{(5,2), 0.2\}, \{(5,2), 0.2\}$. The smaller gap between two EXIT curves are good (have small loss rate) of the proposed system.

A. Packet Loss Rate (PLR)

Performance evaluations for prioritizing human over machines with MDS codes are packet loss rate (PLR) and throughput. PLR is ratio of packet loss for the total packet sent. PLR also describes the number of packet loss and total packet sent ($\hat{b}$) compared to number of packet in receive information ($b$). PLR is calculated as

$$\text{PLR} = \frac{\hat{b}}{b}$$  \hspace{1cm} (13)

Throughput is ratio number of correctly received packets to the total number of packet sent to the BS. The equation for PLR and throughput is expressed as:

$$T_h = G_h \times (1 - P_L^h) \hspace{1cm} (14)$$

$$T_m = G_m \times (1 - P_L^m) \hspace{1cm} (15)$$

where, $P_L^h$ and $P_L^m$ is the PLR value of human or machines. Throughput is also called a success rate of packet. Throughput analysis was needed to know the PLR graph. Similarly, the throughput analysis is also based on each degree distribution.

Figure 9. Packet Loss Rate with $f_u = 0.2 : 0.8$ and $f_u = 1 : 0.9$.

Figure 10. Packet Loss Rate with $f_u = 0.5 : 0.5$ and $f_u = 1 : 0.9$.

Figure 11. Packet Loss Rate with $f_u = 0.8 : 0.2$ and $f_u = 1 : 0.9$.

Figure 12. An example throughput with $f_u = 0.2 : 0.8$ and $f_u = 1 : 0.9$.

The results in Fig. 12 show the throughput results of the human and machine degrees with the utility function user of 0.2 and 0.8. The simulated throughput scheme was
divided into two, namely the condition when the degree has not been affected by fading and another in which it has been affected. The presence of fading causes a decrease in throughput value. The results obtained for PLR values and throughput can be seen in Table III. The best throughput value is when using the utility function 0.2 human and 0.8 machine with a human throughput value of 0.37 packet/slot and machine throughput of 0.3 packet/slot. This result shows that humans are prioritized because they have a higher throughput value than machines.

<table>
<thead>
<tr>
<th>$f_w$</th>
<th>PLR</th>
<th>No Fading</th>
<th>Fading</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 : 0.8</td>
<td>10^{-1}</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>0.5 : 0.5</td>
<td>10^{-1}</td>
<td>0.32</td>
<td>0.29</td>
</tr>
<tr>
<td>0.8 : 0.2</td>
<td>10^{-1}</td>
<td>0.31</td>
<td>0.3</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

In this study, we propose a CRA-based multiple access technique to prioritize human communication using the MDS codes coding scheme. MDS codes are designed using EXIT chart analysis to design sub optimal degree distributions. CRA with human communication priority was indicated by a smaller number of PLR and higher throughput. The different utility function gives different design of degree and performance value. The fading effect also affects a decrease in the throughput. The good performances of the proposed prioritization on human over machines with MDS codes is expected to provide significant contributions to the future wireless super-dense network.

CONFLICT OF INTEREST

The authors declare that this work bears no conflict of interest.

AUTHOR CONTRIBUTIONS

Solichah Larasati carried out all research processes, simulations, modeled the degree distribution, and validated the results. Khoirun Ni’amah conducted the program simulation, Reni Dyah Wahyuningrunt drafted the article, and Alfin Hikmaturakhman improved the language in the article. All authors had approved the final version.

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